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Interval Methods applied to Signal Temporal Logic – Overview and Extension on Tubes

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Introduction

Designers and users of cyber-physical systems often require guarantees on the system's behavior, hence the need of a runtime verification process. This formal verification can be done using temporal logic, a formalism able to specify the system's requirements with temporal constraints. In 1977, Linear Temporal Logic (LTL) [9] is introduced, allowing to describe discrete time properties on a discrete signal. This is mainly used for formal verification of software or digital hardware, but reaches its limits when considering real-time constraints. When considering CPS, which are hybrid systems, man needs an extension of LTL able to handle real-time properties on real-value signal. This is the purpose of Signal Temporal Logic (STL) [8]. We consider an approach using interval analysis to ensure computation guarantees and take into account uncertainties in a robust manner.

STL is defined recursively by:

$$\phi := \mu \mid \neg\phi \mid \phi_1 \wedge \phi_2 \mid \phi_1 \mathcal{U}_{[a,b]} \phi_2 \mid \top \quad (1)$$

with ϕ an STL formula, and \mathcal{U} the *until* operator. The interval $[a, b]$ is defined with $a, b \in \mathbb{R}^+$. We have

$$(x, t) \models \phi_1 \mathcal{U}_{[a,b]} \phi_2 \iff \exists t' \in [a, b] (x, t + t') \models \phi_2 \text{ and } \forall t'' \in [t, t'] (x, t'') \models \phi_1. \quad (2)$$

μ is an atomic predicate: $\mu_x \equiv f(x_1(t), \dots, x_n(t)) > 0$.

Several operators can be derived from STL, such as the well known finally operator $\mathcal{F}_{[a,b]} \phi \equiv \top \mathcal{U}_{[a,b]} \phi$, noted \diamond , which is sometimes called *eventually*. The satisfaction of ϕ by a signal $\mathbf{x} = (x_1, \dots, x_n) \in \mathbb{R}^n$ is written $(\mathbf{x}, t) \models \mu$ and is true if and only if we have $f(x_1(t), \dots, x_n(t)) > 0$.

Monitoring using intervals

Related works

The qualitative satisfaction of an STL formula is a boolean value. This binary result can not contain an information of robustness of satisfaction. Quantitative satisfaction [5] introduces this notion of robustness to provide a measure of how much a trace satisfy or violate a given property. This approach can be used to handle some uncertainties, as in [4] where spatial and temporal uncertainties are considered, sometimes by the help of intervals. For instance, in [11], an offline monitoring algorithm with intervals for finite time STL formulas is proposed. In [3], an online monitoring is proposed using an interval of all the possible quantitative satisfaction of a partial signal with unbounded future.

In the literature, most of the researchers proposing STL monitors deal with single traces. However, in our approach, we want to go further by providing a robustness to numerical and model approximations. We then consider set of traces that can be represented by tubes (interval of trajectories).

Tubes

Interval STL is introduced in [1] to take uncertainties in predicates. Signals of intervals are considered, and quantitative satisfaction is returned as an interval. Then, depending on the inclusion or not of the zero value in this interval, the resulting satisfaction is a three-valued logical value: true, false, or undef. In [10], the authors cut the tubes in the time dimension and interpret STL formulas as successive discrete states in a new formalism called Reachset Temporal Logic. When there exists a trace in the set of trace that does not satisfy the constraint, then the proposed monitor returns false. The approach proposed in [6] is closer to [8]: it searches times when properties become true or false, and then propagates the information in a bottom up manner to deduce the result for the whole STL formula. In the latter, there is no completeness: the algorithm can return “unknown” when there is an ambiguity on the satisfaction or the violation of every trace.

Boolean interval extension

In our work, we propose to extend these previous results to boolean interval, such that the ambiguous “unknown” result is represented by the interval $[0, 1]$, and the ambiguity may be eliminated later if enough information are provided. Indeed, in the following example, both approaches from [10] and [6] lead to an unknown or false result, despite the signal does satisfy the formula.

Consider the tube $[x](t)$ represented in the Figure 1. We have a set point $\rho \in \mathbb{R}$ we want our system to reach in the time interval $[a, b] \subset \mathbb{R}^+$ (and not before). The corresponding STL formula Φ is:

$$\Phi \equiv \neg\phi \mathcal{U}_{[a,b]}\phi$$

with $\phi \equiv [x] \geq \rho$.

In this example, during the period $[a, b]$, the satisfaction of the predicate ϕ is ambiguous. However, the STL formula Φ is well satisfied by any trace taken in the tube $[x](t)$. In order to conclude the satisfaction rather than an unknown or false result, we propose to use

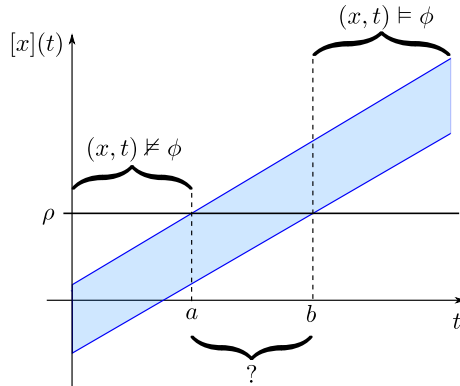


Figure 1: The tube is represented in blue. Before a and after b , there is no doubt on satisfaction or violation of the predicate $\phi \equiv x \geq \rho$. However, during the period $[a, b]$, some traces may satisfy ϕ while other not.

reachability analysis to obtain the result. The difficulty lies in the fact that the time t' from equation (2) may be different for every possible trace included in the tube, so there is an infinite number of trace and time to check.

The proposed solution is to use a branching algorithm on the time interval $[a, b]$, such that we can conclude the satisfaction of the formula when the set $[\mathbf{x}](0)$ is completely covered by the union of the inner backward reachsets starting from the set X_ϕ of states satisfying ϕ and from times $[t'] \subset [a, b]$ (see e.g. [7]).

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¹CIEDS: French Interdisciplinary Center for Defense and Security.

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