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Exact algorithms for the bilevel scheduling of uniform parallel machines in the context of coupling maintenance and scheduling decisions

Quentin Schau ^{*} Federico Della Croce [†] Han Hoogeveen [‡]
Olivier Ploton [§] Vincent T'kindt [¶]

1 Introduction

This study focuses on a scenario with one leader and one follower composed of parallel uniform machines where scheduling decisions occur periodically in the context of bilevel optimization. To the best of our knowledge, the literature on bilevel scheduling is relatively limited, we refer here to [2, 5, 6, 7, 8].

Assume we are given a set \mathcal{J} of N jobs, each job j being characterized by its processing time p_j , weight w_j and due date d_j . On one hand, we have a leader that manages the system health and global scheduling. On the other hand, we have a follower that schedule jobs on a set of given parallel uniform machines. The leader decides of the jobs the follower schedules by minimizing the number of weighted tardy jobs. The follower aims to minimize the total completion time. Following the classic bilevel optimization framework, we consider the optimistic case, i.e. the follower returns the schedule that leads to the minimum number of weighted tardy jobs among schedules optimal for the total completion time.

We assume that the leader periodically makes scheduling decisions every T units of time. Prior to each period, the leader receives data from the shop floor. Using this data, the leader makes decisions, potentially altering the speed of certain machines by setting them to V_{max} or V_0 , or removing them from the follower. These decisions are relevant within the context of Industry 4.0. Next, the leader selects a subset $\mathcal{I} \subset \mathcal{J}$ of n jobs and assigns them to the follower. In the follower's problem, there is a lexicographical objective function. Correspondingly, following the extended three-field classification in [1], we denote this bilevel problem as $Q|V_i \in \{V_0, V_{max}\}, OPT - n | \sum_j C_j^F, \sum_j w_j U_j^L$, while

^{*}quentin.schau@univ-tours.fr. Laboratoire d'Informatique Fondamentale et Appliquée, Université de Tours, 64 rue Jean Portalis, 37200 Tours, France.

[†]federico.dellacroce@polito.it. DIGEP, Politecnico di Torino, Corso Duca degli Abruzzi 24, 10129 Torino, Italy.

[‡]j.a.hoogeveen@uu.nl. Department of Information and Computing Sciences, Utrecht University, Princetonplein 5, 3584 CC Utrecht, The Netherlands.

[§]olivier.ploton@univ-tours.fr. Laboratoire d'Informatique Fondamentale et Appliquée, Université de Tours, 64 rue Jean Portalis, 37200 Tours, France.

[¶]tkindt@univ-tours.fr. Laboratoire d'Informatique Fondamentale et Appliquée, Université de Tours, 64 rue Jean Portalis, 37200 Tours, France.

the related follower problem is denoted as $Q|V_i \in \{V_0, V_{max}\}|Lex \left(\sum_j C_j^F, \sum_j w_j U_j^L \right)$. Notice that, when $n = N$ the two problems coincide.

2 Complexity and solution approaches

In terms of complexity, we can show the \mathcal{NP} -hardness in the strong sense of problem $Q|V_i \in \{V_0, V_{max}\}|Lex \left(\sum_j C_j^F, \sum_j w_j U_j^L \right)$ (and correspondingly of the bilevel problem). Indeed, the special case with identical parallel machines and unit weights, that is problem $P|Lex \left(\sum_j C_j^F, \sum_j U_j^L \right)$, can be shown to be \mathcal{NP} -hard in the strong sense by reduction from the numerical 3-dimensional matching (*NUM-3DM*).

When the number of machines m is not part of the input, we can already show that problem $P2|Lex \left(\sum_j C_j^F, \sum_j U_j^L \right)$ is \mathcal{NP} -hard by reduction from the well known even-odd partition problem. To complete the complexity analysis, we can show that all problems from $P2|Lex \left(\sum_j C_j^F, \sum_j U_j^L \right)$ to $Qm|V_i \in \{V_0, V_{max}\}|Lex \left(\sum_j C_j^F, \sum_j w_j U_j^L \right)$ with m constant are \mathcal{NP} -hard in the ordinary sense.

Before solving the bilevel problem, we focus on the follower problem. The lexicographical objective function, especially the first criteria, leads to a block structure for the scheduling. Assume we have a set \mathcal{J} of n jobs given by the leader, then let N_{max} (respectively N_0) be the number of machines with high-speed V_{max} (respectively, with low speed V_0). We can show that an optimal schedule of the problem $Q|V_i \in \{V_{max}, V_0\}| \sum_j C_j$ is given by the repetition of patterns. In Figure 2, an illustration depicts a pattern consisting of α blocks on high-speed machines and one block on low-speed machines. Jobs highlighted with ■ are part of the first pattern, while those marked with ■ belong to a other pattern. Jobs within the blue-dashed group signify their membership to a block and can subsequently undergo permutation without changing the optimal value of the first criteria, i.e. $\sum_j C_j$. Let us define \mathcal{B}_{max} (respectively \mathcal{B}_0) as the set of all blocks that can be assigned to high-speed machines (respectively low-speed machines).

Hence, we know for all jobs j on which set \mathcal{B}_{max} or \mathcal{B}_0 it is scheduled. Using this characterization, by extending to the considered problem the approach on parallel machines scheduling proposed in [4], a dynamic programming recursion can be formulated running in pseudo-polynomial time. In addition, a MIP can be formulated for the follower problem. We are currently working on a column generation approach, by extending the works proposed in [3]. At the time of the conference we will present these three exact solution approaches and how they can be generalized to solve the bilevel problem.

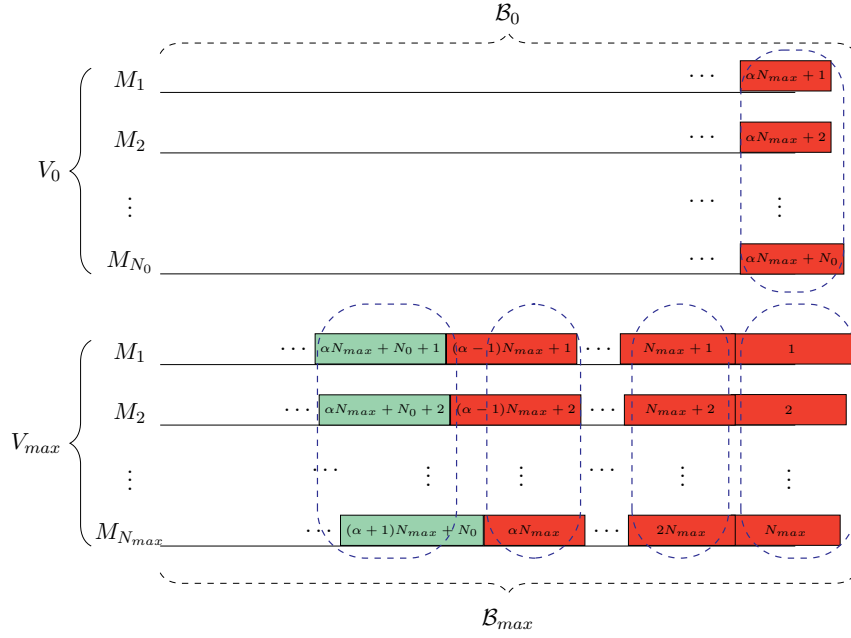


Figure 1: Scheduling pattern with block structure for the optimal solution of problem $Q|V_i \in \{V_{max}, V_0\}|\sum_j C_j$

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