



HAL
open science

Digital Twin for Predicting Progressive Damage in Operating Pressure Vessels

Izat Khaled, Dmytro Vasiukov, Modesar Shakoor, Mohamed Bennebach, Salim Chaki

► To cite this version:

Izat Khaled, Dmytro Vasiukov, Modesar Shakoor, Mohamed Bennebach, Salim Chaki. Digital Twin for Predicting Progressive Damage in Operating Pressure Vessels. *Procedia Structural Integrity*, 2024, 57, pp.280 - 289. <10.1016/j.prostr.2024.03.030>. <hal-04706534>

HAL Id: hal-04706534

<https://hal.science/hal-04706534v1>

Submitted on 23 Sep 2024

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire HAL, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



Distributed under a Creative Commons CC BY-NC-ND 4.0 - Attribution - Non-commercial use - No Derivative Works - International License



Fatigue Design 2023

Digital Twin for Predicting Progressive Damage in Operating Pressure Vessels

Izat Khaled^{a*}, Dmytro Vasiukov^b, Modesar Shakoor^b, Mohamed Bennebach^a, Salim Chaki^b

^aCentre technique des industries mécaniques, 52 Av. Félix Louat, 60300 Senlis, France

^bCentre d'enseignement, recherche et innovation matériaux et procédés, IMT Nord Europe, 764 Bd. Lahure, 59500 Douai, France

Abstract

According to the definition of the French Alliance “Industry of the Future”, the digital twin is a virtual clone of a physical system or a process. It systematically implies the existence of a "digital model" coupled with the object it copies. Depending on the system concerned and the desired usage, it can be a geometric, multiphysical, functional, behavioral, and decision-making model. It must evolve over time like its real twin. It can be used to improve the control, security and optimization of production lines and factories, digital continuity at the product level, from its design to its end of life, monitoring and predictive maintenance.

The digital twin appears as a reliable way to monitor operation, to evaluate the resistance and safety of pressure vessels (PVs) in real service conditions. Via the connected transducers (IoT), we can integrate the life cycle information in an automatic data processing module, and finally to capitalize on all this data to optimize the design of new products.

To this end, the first objective of the thesis is to contribute to the development of a methodology for real-time assessment of the integrity of pressure vessels, by implementing a predictive maintenance strategy in place of the classical and costly, preventive or curative maintenance, based on IoT and digital twin technologies. The second technological goal of the thesis is to develop methods to optimize and make reliable the design and operation of new generations of pressure structures.

© 2024 The Authors. Published by Elsevier B.V.

This is an open access article under the CC BY-NC-ND license (<https://creativecommons.org/licenses/by-nc-nd/4.0>)

Peer-review under responsibility of the scientific committee of the Fatigue Design 2023 organizers

Keywords: digital twin; fatigue; pressure vessel; progressive damage

* Corresponding author. Tel.: +33-(0)667831043; fax: +0-000-000-0000 .

E-mail address: izat.khaled@cetim.fr

1. Introduction and state of the art

Pressure vessels are utilized across various industrial sectors, including petrochemical, power generation, chemical, and pharmaceutical industries. These pieces of equipment operate under harsh conditions, which can lead to a gradual degradation of their structural integrity, posing risks to both people and the environment.

The Ministerial Order of 20/11/2017 mandates regular in-service monitoring of these vessels. The frequency of this monitoring depends on the equipment's classification and aging. In-service monitoring consists of two main components: periodic inspections and periodic requalification. These compulsory strategies incur significant costs. Manufacturers face cumulative financial losses due to expenses associated with inspection agencies and the production downtime required for monitoring. Furthermore, visual inspections often fail to detect defects, and they do not provide comprehensive reports on the wear and tear that the equipment may have experienced up until the intervention date. This indicates that these interventions are carried out to adhere to safety protocols rather than out of necessity. Consequently, the industry has recognized the necessity for a system capable of predicting the remaining lifespan of pressure vessels and determining whether a technical intervention is warranted.

Given the hazardous nature of this equipment, the design, manufacture, and operation of pressure vessels (PVs) are rigorously regulated by laws and building codes. This strict oversight makes it challenging to replace traditional maintenance strategies with innovative ones.

The advancement in real-time monitoring of industrial equipment has become increasingly imperative to prevent unforeseen production downtimes and reduce maintenance costs. In this context, digital twins are emerging as an effective solution for monitoring industrial equipment in real-time and predicting their behavior. However, despite their potential, implementing digital twins to forecast the remaining lifespan and damage of PVs remains a challenge. This is primarily due to the need to consider multiple factors such as applied loads, material properties, equipment geometry, and environmental conditions, which renders the modeling process complex.

Over the past few years, the utilization of digital twins has experienced rapid growth. In the realm of fatigue, Vanderhorn (2022) developed a numerical twin for diagnosing fatigue damage in ships and providing design support for future generations. Jiang et al. (2021) proposed an approach for accurately predicting the service life of steel bridges under various sources of uncertainty.

The state-of-the-art in pressure vessel (PV) modeling has seen significant advancement in recent years, thanks to the emergence of numerical technologies such as numerical twins. Numerous studies have been conducted on numerical twins for pressure equipment, aiming to enhance the monitoring and maintenance of such crucial assets. Notably, Burov and Burova (2020) undertook the development of a digital twin to predict the mechanical behavior of composite pressure vessels (COPVs) under real-world conditions, with the goal of optimizing their design and performance. Another notable endeavor, as presented in the work of Jaribion et al. (2020), involved the creation of a digital twin to enhance the risk management and safety of high-pressure hydrogen tanks.

Concerning fatigue approaches, as extensively outlined in Weber's article (1999), three primary methodologies are commonly employed for post-processing fatigue analysis of materials. The first approach is predicated on an equivalent stress, determined in accordance with the stress state, whether uniaxial or multiaxial, induced by the loading, Crossland (1956), Vu (2009), Del Cero Coelho (2014). The second method, known as the ONERA-type approach or incremental calculation, relies on numerical techniques to project fatigue life, Chaboche (1974), Mesmacque (2005). It considers local stress variations within the structure, as well as the effects of multiaxial loading. This approach is more intricate, demanding the application of advanced numerical scaling methods. Finally, the third approach, referred to as the continuous damage approach, was pioneered by Rabotnov (1968) and Kachanov (1958). It places emphasis on predicting the gradual accumulation of damage in materials subjected to cyclic loading. This approach factors in the cumulative impacts of stresses and strains on fatigue life. It necessitates precise characterization of the material and modeling of the damage mechanisms, Lemaitre et al. (1999), Flaceliere et al. (2007), Vu (2010).

Each of these approaches possesses its own set of advantages and limitations. The choice of which approach to employ hinges on factors such as material properties, structural geometry, applied loads, and the specific objectives of the fatigue analysis. In our case, dealing with a substantial structure composed of 600,000 elements, the conventional use of incremental calculations for fatigue post-processing is rendered impractical due to the considerable computational time it would require. Such an approach would prove too costly and unwieldy given our circumstances. This, unfortunately, means that we are unable to fully realize the potential of a digital twin in its ideal,

highly responsive form. Consequently, considering our computational time constraints and the imperative to adhere to pressure equipment construction codes, we have made the strategic decision to employ S/N curve approaches for our fatigue post-processing. These methods allow us to estimate the lifespan of the structure in relation to the applied stress level, utilizing reference curves established by the prevailing standards and codes.

In the realm of fatigue calculation methodologies widely employed in the pressure equipment industry, the first and foremost is based on CODAP (2020), a code that offers specific fatigue criteria and holds extensive usage within the field. This code furnishes comprehensive guidance for the design and validation of pressure equipment, accounting for cyclic loading and incorporating appropriate safety margins. The application of this approach entails the utilization of analytical techniques to estimate the fatigue life of components.

CODAP offers detailed and streamlined methodologies for ascertaining the design stress, contingent upon the applied loads and their nature, be they proportional, non-proportional, or non-proportional with variable principal directions. Subsequently, various correction factors come into play, taking into consideration parameters like stress level, surface condition, average stress, and other pertinent factors. These correction factors serve to incorporate the distinct impacts of each parameter on the structural damage. This methodology, founded on correction factors, enables the consideration of diverse loading conditions and the specific attributes of each structure, thereby ensuring a more precise evaluation of equipment integrity and safety. Moreover, this code provides specific S/N curves tailored for the distinct critical areas of the equipment, encompassing both welded and non-welded sections.

The second methodology, employed through nCode (2013) software, leverages advanced modeling and simulation techniques to predict fatigue life. This approach enables a more precise analysis of the stresses and strains endured by components, considering multi-axis effects and factors affecting material durability. It relies on sophisticated numerical models and algorithms to provide a more accurate estimation of fatigue life.

Unlike the CODAP methodology, which relies on safety coefficients, this approach takes a more refined stance, incorporating precise theories to account for various factors. These include considerations for mean stress effects (such as Goodman, Gerber, etc.), plasticity (as per Neuber (1961)), and surface condition. This refined approach yields results that are more closely aligned with the actual operating conditions of the components, accommodating a broad spectrum of design constraints akin to those outlined in CODAP, including maximum principal stresses or those calculated through the critical plane. By leveraging these stresses, the software can pinpoint critical areas susceptible to damage and forecast residual life based on fatigue accumulation, Bennebach et al. (2021).

Following an initial comparison of the methodologies, we pinpointed the one that aligns best with our specific case study. This assessment underscored the value of adopting the nCode approach, primarily due to its capacity for real-time monitoring and its superior accuracy when juxtaposed with the methodologies advocated by the construction code.

2. Digital twin proposal

While digital twins offer significant advantages, they are not without their limitations. These include potential inaccuracies in damage prediction and suboptimal sensor utilization, often due to cost and resource constraints. Furthermore, the fatigue criteria and approaches outlined in construction codes for sizing pressure equipment, such as those set forth by organizations like the American Society of Mechanical Engineers (ASME), International Institute of Welding IIW (2008), and CODAP, may not directly align with the requirements of in-service equipment monitoring, introducing potential challenges.

We propose a new hybrid digital twin for real-time residual life and damage monitoring of PVs. This model combines data-driven and physical approaches, drawing on fatigue criteria and curves available in CODAP. The objective is to develop a scalable model capable of predicting equipment behavior using real-time measured data. As shown in Figure 1, this approach aims to reduce maintenance costs, optimize maintenance planning, and ensure the safety of pressure equipment. Ultimately, it should contribute significantly to improving the safety and reliability of industrial equipment while reducing maintenance expenses.

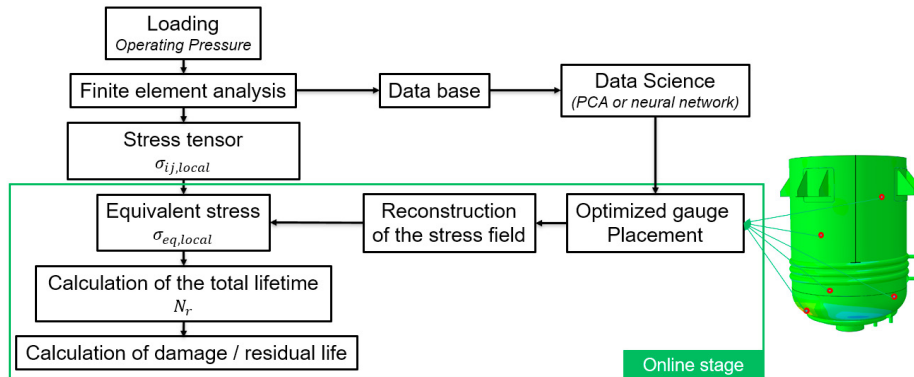


Figure 1 : Algorithm for the calculation of residual life and damage of pressure vessel in real time

We introduce an innovative method employing a hybrid digital twin that integrates finite element techniques for stress and damage calculation with data science for stress field reconstruction and sensor placement optimization. This advanced digital twin is engineered to provide real-time predictions of damage rate and residual life based on minimal data from strain gauges. To enhance monitoring efficiency, we incorporate principal component analysis (PCA) in tandem with sensor placement optimization. This combination maximizes the coverage of the strain field while utilizing a minimal number of sensors, consequently reducing the costs and time associated with sensor installation and maintenance.

Consequently, this approach holds the potential to revolutionize the monitoring of PVs, enhancing both efficiency and cost-effectiveness. This, in turn, can elevate their reliability and extend their operational lifespan.

3. Numerical modeling of the pressure vessel

The primary goal of this phase is to construct a highly accurate numerical model of the PV, serving as a representative case study. This model will enable us to pinpoint critical zones in terms of stress levels and potential occurrences of defects. Furthermore, it offers the capacity to calibrate the model using experimental test data, ensuring that the simulation results align closely with the actual measurements obtained through experimentation. By identifying these critical areas prior to instrumentation, the modeling process will also facilitate the optimization of sensor placement, allowing for the focused monitoring of the most highly stressed regions within the equipment.

3.1. Presentation of the case study

A physical demonstrator has been designed and manufactured in a reduced scale, representative of a chemical reactor of polymerization in service. The demonstrator shown in Figure 2 is 1.7 m high and 1.1 m in diameter. It is made of P265GH steel whose mechanical properties are detailed in EN10028 (2009).

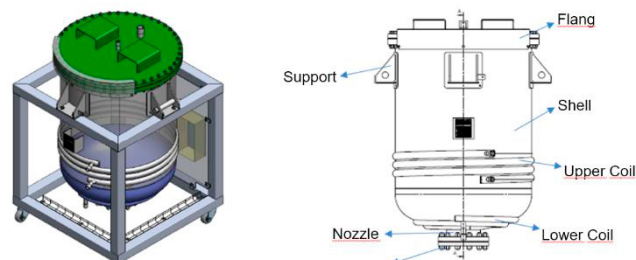


Figure 2 : Reduced model of the PV (case study)

A finite element modeling of the equipment subjected to inner pressure loadings was done in commercial software Abaqus (2009), which allowed us to identify the critical areas of the PV, and to validate the numerical model using experimental results to obtain reliable life and damage predictions. This step is crucial for sensor placement and to ensure quality data acquisition during subsequent experimentation, and in preparation for the fatigue study of PV.

The equipment's exact geometry comprises numerous intricate welds, which pose challenges for precise modeling. In this initial modeling phase, we opt for a simplification by omitting these welds to facilitate the process. While this simplification may introduce a minor degree of inaccuracy in determining stress levels and critical areas, it still yields a satisfactory approximation for our specific objectives. As a result, we find the simplified equipment geometry to be adequate for our study.

Additionally, considering the relatively modest thickness of the steel sheets composing the equipment, coupled with the minimal temperature variation anticipated under operational conditions, we do not anticipate notable mechanical or thermal stress gradients across the thickness of the equipment. For this reason, we opt to model the PV using shell elements (S4R type as per Abaqus terminology). This choice is made to reduce calculation time and enhance result accuracy.

First, the numerical model will be subjected to the following loading cases to calibrate the numerical results with the experimental ones.

- Case a: pressure loading in the shell only: 0.53 MPa
- Case b: pressure loading in the coils only: 0.38 MPa
- Case c: perform a combined pressure loading (case a + case b)

However, it will also be important to understand the effect of thermal cycling on the results obtained. For that, it will be planned in the continuation of work, to numerically carry out thermal cycles to quantify the influence of the latter on our results.

The influence of the three loading cases (Cases a, b, and c), which represent the putting into service stages of pressure equipment, was compared. The maximum principal stresses field results are shown in Figure 3.

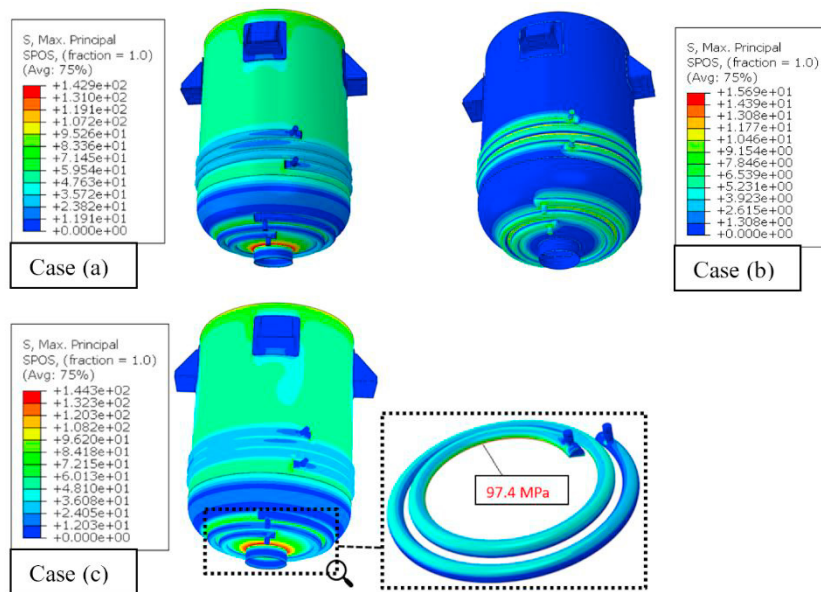


Figure 3 : Stress field distribution on the PV, for three loading cases

From Figure 3 it is obvious that loading Case (a) generates the dominant stress field on the structure. The effect of the major discontinuity gives us, the first critical zone with the highest stress level of 144.2 MPa, at the level of the

nozzle. The second critical zone is detected at the level of the lower coil, with a stress level of 97.4 MPa. It should be noted the detected critical areas are welded areas in real geometry. The finite element model will be the main source for further numerical data generation for classical fatigue damage modeling, and for intelligent sensor placement. The results obtained in this study allow us to better understand the influence of different loading configurations on the stress in the pressure equipment. Moreover, the detection of critical zones thanks to our numerical model will allow us to define the locations where the strain gauges will have to be installed during the next experimental phase.

4. Intelligent placement of sensors

Optimizing sensor placement stands as a pivotal aspect of equipment monitoring and maintenance. Sensors are instrumental in gathering crucial data and enabling real-time monitoring of vital parameters. A well-organized sensor placement strategy ensures comprehensive coverage of the stress field, allowing for early detection of anomalies and potential failures. Moreover, the judicious placement of sensors leads to a reduction in costs related to their installation and upkeep. This results in a more cost-effective utilization of sensors and a more efficient harnessing of the collected data. Thus, sensor placement optimization emerges as a critical factor in ensuring both reliable and economically viable equipment monitoring.

The objective is to expound upon an intelligent sensor placement algorithm grounded in advanced data analysis techniques, specifically Principal Component Analysis, as outlined by Jackson (1991) and Jolliffe (2002). The primary aim is to fine-tune the positioning of strain gauges to ensure precise reconstruction of the strain field across the entire equipment. Leveraging the principles of Principal Component Analysis (PCA), the algorithm is adept at identifying strategic sensor locations, thereby achieving optimal coverage of the strain field with the minimal number of sensors.

4.1. Operation of the algorithm

One of the commonly used methods for stress loading history reconstruction is the modal superposition method He and Fu (2001), Craig and Kurdila (2006), Chabod (2022). The modal superposition method is mainly used in the field of vibration dynamics, where it allows the reconstruction of forces and stresses from the measured modal responses.

More advanced approaches based on machine learning techniques have been developed to improve the stress monitoring, Quiroga et al. (2017), and stress field reconstruction. These approaches exploit the complex relationships between local strain measurements and the corresponding stresses, using mathematical models and learning algorithms to estimate the unmeasured stresses. The advantage of these more advanced approaches is their ability to consider non-linearities and complex interactions between different parts of the structure.

Two algorithms, namely Principal Component Analysis (PCA) and discrete empirical interpolation method (DEIM), Chaturantabut and Sorensen (2010), are then used. PCA identifies the principal components that contain the most information, while the DEIM method optimizes sensor placement. This approach aims to maximize the use of the most meaningful data while reducing computational complexity. The algorithm will be trained using stresses data from different failure scenarios, represented by a degraded Young's modulus, on specific “defect areas” of the structure (see Figure 4).

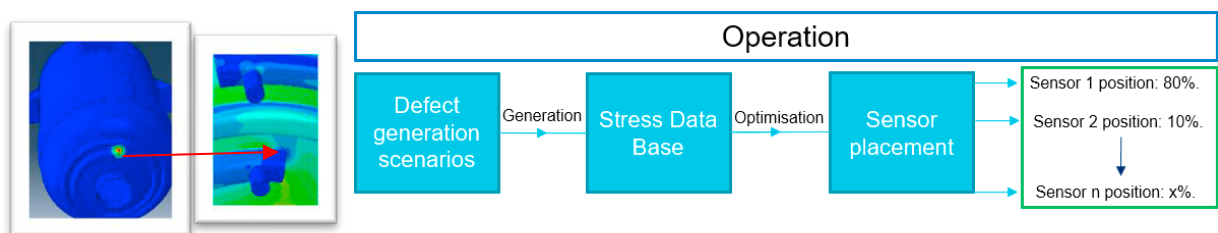


Figure 4: Sensor placement algorithm

4.2. Results

The results of the algorithm show the optimal location of the sensors obtained. It generates the desired stress reconstruction area, represented here by the lower part of the equipment, the elements on which the gauges must be positioned. Also, it provides a graph that expresses the percentage of error that we will have on the reconstruction of stresses according to the gauges used.

Thus, through this method, the operator will have the choice of the number of gauges to be used according to the percentage of admissible error and the influence of this error on the reliability of the fatigue life of the structure. The figure 5 shows the placement of the first five gauges through the stress field will be reconstructed with 3.2% error on the lower part of the equipment. The error is calculated by comparing the stress field reconstructed through the chosen number of transducers, and the original complete stress field.

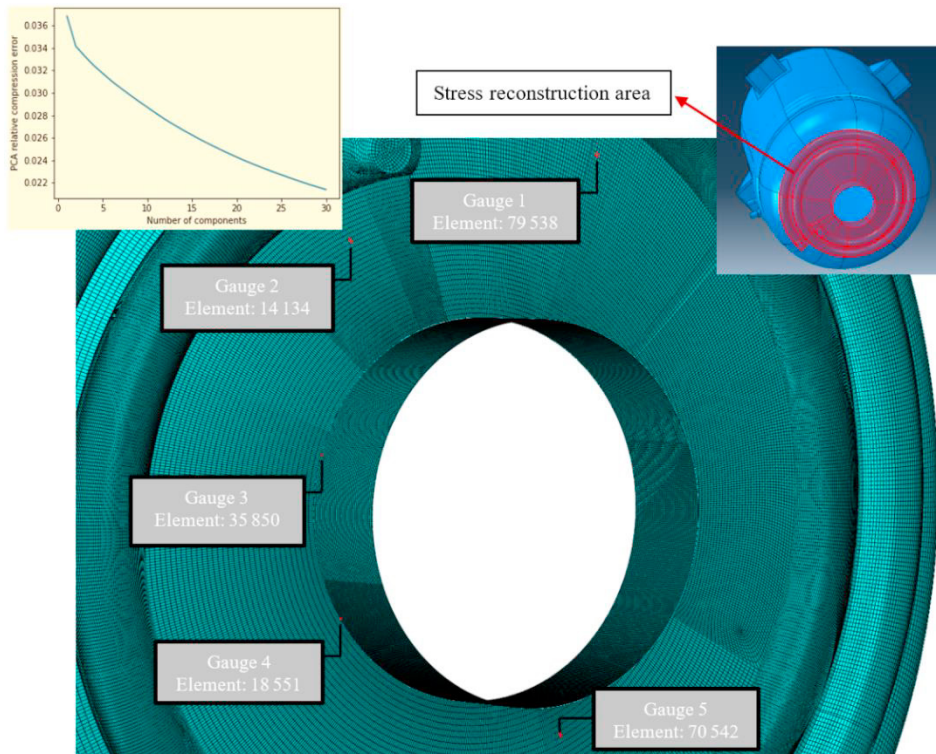


Figure 5 : Placement of the first five strain gauges on the lower part of the PV according to the algorithm

5. Experimental campaign

To substantiate the outcomes of our numerical modeling, we conducted a series of rigorous experimental tests. This section of the study provides an exhaustive account of the tests undertaken and the corresponding results obtained. The primary aim of this experimental phase is twofold: first, to pinpoint the optimal gauge placement zones; and second, to affirm the dependability of the numerical model developed for analyzing the fatigue of the structure.

To inspect critical areas, the multi-encoded ultrasonic inspection method NF EN ISO 9712 (2022), NF EN ISO 13588 (2019), and NF EN ISO 19285 (2017) is involved. This method is based on the use of incidence angles of the

ultrasonic waves emitted from a 5L16 type probe, to cover different inspection areas. It allows the detection of defects such as cracks, porosities, and inclusions in the welded areas.

These selected areas were chosen based on two key criteria: firstly, the outcomes of the finite element analysis which pinpointed regions subjected to elevated stresses, and secondly, insights provided by experts who identified potentially critical areas based on their experience or through visual inspections revealing manufacturing defects. The aim of this approach is twofold: first, to accurately identify any manufacturing defects, and second, to strategically position gauges within these areas of interest. This positioning allows for the measurement of strain levels during testing, as well as an assessment of their impact on the fatigue behavior of the equipment.

Five critical welds were selected for monitoring: the longitudinal weld of the shell, the circumferential weld that joins the shell and the half-sphere, the two corner welds that join the two coils to the vessel, and the weld that joins the half-sphere to the nozzle (see Figure 6).

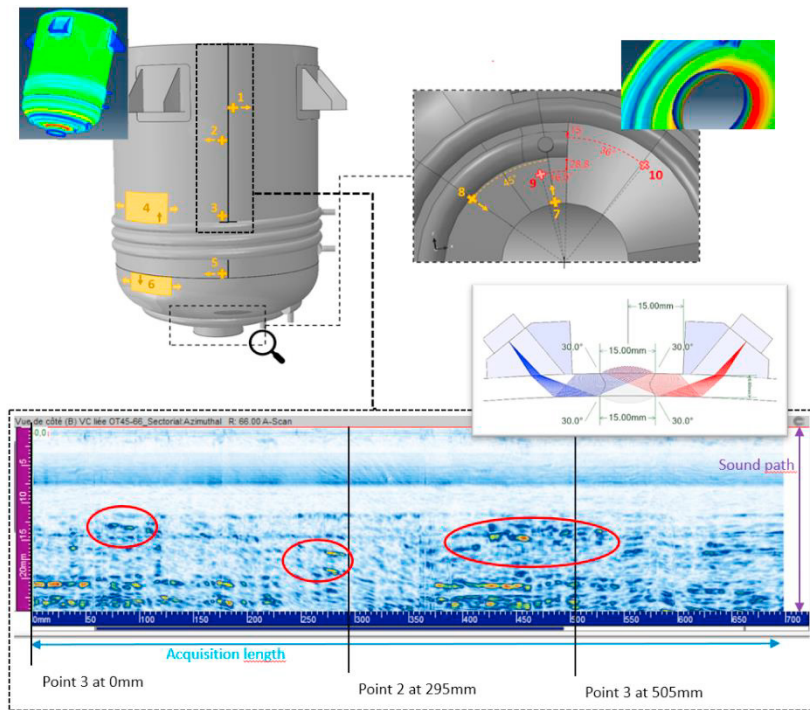


Figure 6 : Identification of inspection areas and results of longitudinal weld inspection

Ultrasound inspections revealed several defects, indicated by the red circles in Figure 6. These defects were identified as reference points for instrumentation. A total of eight strategic locations were pinpointed, each in proximity to a defect uncovered during the examination of the five welds. Additionally, two other locations were selected beyond these welds, serving the purpose of model calibration. Specifically, points 3 and 5, corresponding to junctions between two welds, experience multiaxial stresses that can lead to stress concentration and an increased risk of cracking and failure. To capture this multiaxial state, these two zones will be monitored using rosettes.

6. Conclusion and perspectives

The initial phase of developing a numerical twin for the PV has yielded a comprehensive understanding of the critical behavior of the equipment and identified its most stressed areas. Additionally, we have honed in on the methodologies and damage calculation approaches that are most pertinent to our specific case study. Leveraging the intelligent sensor placement algorithm, we've achieved an initial approximation of optimal gauge locations, allowing for precise reconstruction of the stress field with a minimal number of gauges. Moving forward, our next steps

involve enhancing the numerical model of the PV. This will encompass considerations for thermal loading, welds, and adaptation of the mesh for fatigue post-processing in the welded areas. Subsequently, we will embark on the calibration and validation of the numerical results, comparing them against the experimental data. In the ensuing phase, we will juxtapose the results obtained from the two methodologies and various damage calculation approaches, selecting the one best suited for our objectives.

The subsequent segment of our work will zero in on the optimized placement of gauges and the reconstruction of the stress field on the equipment. Prior to its application on the equipment itself, we will validate this algorithm on representative specimens from the critical zones. Lastly, we will delve into simulating crack propagation in the initiation zones utilizing the Z-cracks software.

Acknowledgements

We would like to express our deep gratitude to CETIM and ANRT, the funders who made this research possible. We also thank the IMT Nord Europe for sharing their knowledge, ideas, and experiences to improve the quality of this research.

We would like to thank all the people who contributed to the realization of this article, and all the organizations and individuals who devoted their time to provide us with their expertise.

References

- CODAP, 2020, Le code de construction des appareils à pression non soumis à l'action de la flamme, division 2.
- Vanderhorn, E., Wang, Z., Mahadevan, S., 2022, Towards a digital twin approach for vessel-specific fatigue damage monitoring and prognosis. *Reliability Engineering & System Safety*, Volume 219.
- Jiang, F., Ding, Y., Song, Y., Geng, F., Wang, Z., 2021, Digital twin-driven framework for fatigue life prediction of steel bridges using a probabilistic multiscale model: Application to segmental orthotropic steel deck specimen. *Engineering Structures*, Volume 241.
- Burov, A., Burova, O., 2020, Journal of Physics: Development of digital twin for composite pressure vessel. *Conference Series*, University of Bristol, Vol. 1441.
- Jaribion, A., Khajavi, S., Ohman, M., Knapen, A., Holmstrom, J., 2020, A digital twin for safety and risk management: A prototype for a hydrogen high-pressure. *Designing for digital transformation*, pp 369-375.
- Weber, B., 1999, *Fatigue multiaxiale des structures industrielles sous chargement quelconque*. EN 10028-2, 2009, *Steel flat non-alloy alloy elevated temperature properties*.
- IIW, 2008, *recommandations pour la conception en fatigue des assemblages et des composants soudés*. IIW.
- NF EN ISO 9712, 2022, *Essais non destructifs, Qualification et certification du personnel END*
- NF EN ISO 13588, 2019, *Essais non destructifs des assemblages soudés, Contrôle par ultrasons, Utilisation de la technique multi-éléments automatisés*.
- NF EN ISO 19285, 2017, *seuil d'évaluation à -14dB d'un trou générateur (TG) Ø3, Essais non destructifs des assemblages soudés, Technique ultrasons multi-éléments (PAUT), Niveaux d'acceptation*.
- Crossland, B., 1956, Effect of large hydrostatic pressures on the torsional fatigue strength of an alloy steel. *Proc. Int. Conf. on Fatigue of Metals*.
- Vu, Q., 2009, *Fatigue polycyclique multiaxiale de l'acier C35 : caractérisation et modélisation des mécanismes d'endommagement*. ISAE-ENSMA Ecole Nationale Supérieure de Mécanique et d'Aérotechnique.
- Del Cero Coelho, F., 2014, *Maitrise de la tenue en fatigue des cordons de soudure*. Autre. ISAE-ENSMA Ecole Nationale Supérieure de Mécanique et d'Aéronautique – Poitiers. NNT : 2014ESMA0016. Tel-01127338
- Chaboche, J.L., 1974, Une loi différentielle d'endommagement de fatigue avec cumulation non linéaire. *Revue Française de Mécanique*.
- Mesmacque, G., Garcia, S., Amrouche, A., Rubio-Gonzalez, C., 2005, Sequencial low in multiaxial fatigue, a new damage indicator. *International Journal of Fatigue*.
- Rabotnov, Y.N., 1968, creep rupture of proc. XII Int. cong. Appl. Mech, Stan for Springer.
- Kachanov, L.M., 1958, Time of rupture process under creep condition, *Izv. Akad. Nauk. SSR, old tech*, Vol 8. pp 26-31
- Lemaitre, J., Sermage, J.P., Desmorat, R., 1999, A two scale damage concept applied to fatigue. *International Journal of Fracture*, pp 67-81.
- Flaceliere, L., Morel, F., Dragon, A., 2007, Coupling between meso-plasticity and damage in high-cycle fatigue. *International Journal of Damage Mechanics*, pp 473-509.
- Vu, Q., 2010, *Fatigue polycyclique multiaxiale de l'acier C35 : caractérisation et modélisation des mécanismes d'endommagement*. Sciences de l'ingénieur. ISAE-ENSMA Ecole Nationale Supérieure de Mécanique et d'Aérotechnique – Poitiers.
- Neuber, H., 1961, Theory of stress concentration for shear-strained prismatical bodies with arbitrary non-linear stress-strain law. *ASME Journal of Applied Mechanics*, Vol. 28, pp. 544-550.
- Bennebach, M., Duval, R., Amuzuga, P., Huther, I., Marzin, M., Rohart, P., Chaib, Z., 2021, *guide de dimensionnement en fatigue des composants et des structures*. CETIM, collection fatigue.
- Jackson, J.E., 1991, *A user's guide to principal components*. John Wiley & Sons, New York.

- Jolliffe, I.T., 2002, Principal component analysis for special types of data. In: Principal component analysis. Springer Series in Statistics. Springer, New York, NY.
- He, J., Fu, Z.F., 2001, Modal analysis. Butterworth-Heinemann, pp. 25-28, p. 33.
- Craig, R., Kurdila, A., 2006, Fundamentals of structural dynamics, Second Edition, pp 325-327.
- Chabod, A., 2022, Digital twin for fatigue analysis. *Procedia Structural Integrity*, Volume 38.
- Quiroga, J., Mujica, L. & Villamizar, R., Ruiz, M., Camacho, J., 2017, PCA based stress monitoring of cylindrical specimens using PZTs and guided waves. DO.10.3390/s17122788, *Sensors*.
- Chaturantabut, S., & Sorensen, D. C. (2010). Nonlinear Model Reduction via Discrete Empirical Interpolation. *SIAM Journal on Scientific Computing*, 32(5), 2737–2764. <https://doi.org/10.1137/090766498>
- nCode, 2013, DesignLife theory guide. V.9.
- Smith, M., 2009, Abaqus/standard user's manual. V.6.9.