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Uncertainty Quantification for Conformal Patch Antenna Installed on Infinite Cylinder.

E.M. Djelloul, A. Chabory, C. Morlaas, and R. Douvenot

ENAC, Université de Toulouse, Toulouse, France, e-mail: el-mehdi.djelloul@enac.fr

Abstract—Small antennas installed on large structures can be critical elements which need to be fully characterized radiation wise. Such an issue naturally arises for airborne antennas for example. This paper presents a crucial step in the modeling of such a radiation in a complex environment. Considering a patch antenna installed on an infinite cylinder, its radiation is analyzed while taking into account variability in the antenna parameters, as well as in the cylinder geometry. By the Polynomial Chaos Expansion of the analytical field, the resulting stochastic radiation pattern can be derived to quantify uncertainty and thus bound the possible realizations of the antenna radiation with respect to variable parameters.

I. INTRODUCTION

In an attempt to get a realistic representation of the radiation pattern of a small antenna installed on a large structure, the work presented here tackles the modeling of such an antenna while taking into account the uncertainties appearing in both the antenna model and the geometry on which it is installed. This very paper focuses on the analytical treatment of the radiation of a patch antenna installed on an infinite cylinder. This situation can illustrate issues such as airborne antennas, for which the size of the aircraft and the operating frequency of the antenna can make prohibitive the use of anechoic chambers or even the use of commercial electromagnetic simulators. Section II of the paper presents the geometry of the problem and gives the radiation of the patch antenna in the axial configuration from the expression derived from the electric field. This radiation is used in section III to define the variable parameters, while presenting the Polynomial Chaos Expansion process to get the stochastic radiation pattern of the antenna. This method has proven to provide accurate metamodels for the antenna itself [1]. Section IV illustrates the methodology in the case of an antenna radiating at 1 GHz, typical for an aviation system such as the distance measurement equipment.

II. RADIATION OF A PATCH ANTENNA ON A CYLINDER

If a small antenna is installed on a very large cylindrical structure, it is legitimate to consider the cylinder as infinite. Here is derived the analytical radiation of a conformal patch installed on such a cylinder by modeling the patch as two radiating magnetic line current distributions.

A. Equivalent radiation of the conformal patch antenna

The infinite z -oriented cylinder has radius $a > 0$ and a conformal patch antenna is installed on it, with width w , length l , height h and relative permittivity of the substrate ε_r .

As illustrated in figure 1, by the surface equivalence principle, the radiation of the conformal patch is the same as the

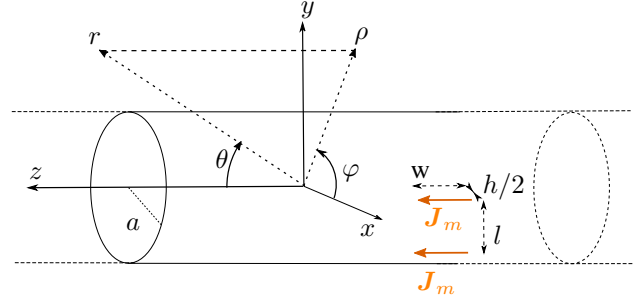


Fig. 1. Patch antenna on cylinder with magnetic currents equivalence.

radiation produced by two line magnetic currents \mathbf{J}_m , concentrated at height $h/2$ and separated by $l = \lambda_0/2\sqrt{\varepsilon_r}$, where λ_0 is the wave length associated with the resonant frequency f_0 . The current density is z -oriented, $\mathbf{J}_m(\varphi, z) = J_m^z(\varphi, z)\hat{\mathbf{z}}$. Moreover, for $|z| \leq w/2$,

$$J_m^z(\varphi, z) = \frac{J_0 h}{2} \{ \delta[\varphi + \varphi_l(a, h)] + \delta[\varphi - \varphi_l(a, h)] \} \quad (1)$$

where J_0 is the current magnitude, δ the Dirac distribution and $\varphi_l(a, h) = \frac{\lambda_0}{2(2a + h)\sqrt{\varepsilon_r}}$. Equation (1) is further used to determine the radiation of the currents antenna on the cylinder.

B. Analytical expression of the radiated field

The radiation of magnetic currents on a cylinder provides a φ -oriented electric far-field, $\mathbf{E}(\theta, \varphi) = E_\varphi \hat{\boldsymbol{\varphi}}$. Exploiting the Fourier transform of J_m^z and further computation leads to [2]

$$E_\varphi(\theta, \varphi) = -jJ_0 h \frac{e^{-jk_0 r}}{\pi^2 r} \frac{\sin\left(\frac{k_0 w \cos \theta}{2}\right)}{\cos \theta} \times \sum_{p \in \mathbb{N}} \frac{\nu j^p \cos(p\varphi)}{H_p^{(2)'}(ak_0 \sin \theta)} \cos(p\varphi_l(a, h)), \quad (2)$$

where $\nu = 1$ if $p = 0$ and 2 otherwise, and $H_p^{(2)'}$ is the derivative of the Hankel function of second kind. To illustrate the influence of the variability of parameters on the radiation of the antenna, the far-field is considered as a random variable, as explained in the next section.

III. RANDOMIZATION OF THE ELECTRIC FIELD

After the analytical expression of the radiated field has been derived, it is possible to consider it as a random variable, resulting from the variations of its parameters. This section presents how an orthonormal stochastic expansion basis can be exhibited for a square-integrable function, and then shows how the variable parameters are chosen.

A. Introduction to the polynomial chaos expansion

If g is a square-integrable function of a random vector \mathbf{X} of size M , then it can be expanded on an orthonormal polynomial basis, called the Polynomial Chaos (PC) basis, i.e. there exists a basis $\{\psi_{i_k}\}_{i_k \in \mathbb{N}}$ of multivariate polynomials, such that [5]

$$g(\mathbf{X}) = \sum_{i_k \in \mathbb{N}} p_{i_k} \psi_{i_k}(\mathbf{X}). \quad (3)$$

The elements of $\{\psi_{i_k}\}_{i_k \in \mathbb{N}}$ are determined according to the distributions of the random variables X_{i_k} via the Wiener-Askey scheme. Moreover, p_{i_k} is obtained as the projection of g on the vector ψ_{i_k} via the classical L^2 -inner product. Equation (3) characterizes $f(\mathbf{X})$ via deterministic coefficients p_{i_k} , transferring all the stochastic in the known basis polynomials.

B. Polynomial chaos expansion of the radiated field

To perform uncertainty quantification, it is required to define the parameters of the field which will be considered as random variables. Afterwards, the field can be expanded by PC, as described in III-A. For the patch to resonate at f_0 , it is necessary to leave λ_0 as deterministic, as well as ε_r . As a result, the cylinder radius a , the patch width w and height h can be considered as variable. As w is chosen accordingly with λ_0 via $w = \lambda_0 / \sqrt{2(\varepsilon_r + 1)} = \kappa \lambda_0$ [3]. Consequently, in the following, κ is considered as variable instead of w . Table I provides the connection between the deterministic parameters and the uniform distributions of their stochastic counterparts.

TABLE I
RANDOM VARIABLES DEFINITION

Parameter	Random variable	Distribution
a	A	$\mathcal{U}([2; 4])$
κ	K	$\mathcal{U}([0.9w; 1.1w])$
h	H	$\mathcal{U}([2 \cdot 10^{-3}; 5 \cdot 10^{-3}])$

As a result, the stochastic counterpart of equation (2) writes

$$\tilde{E}_\varphi(\theta, \varphi) = -jJ_0 \mathbf{H} \frac{e^{-jk_0 r}}{\pi^2 r} \frac{\sin(\pi \mathbf{K} \cos \theta)}{\cos \theta} \times \sum_{p \in \mathbb{N}} \frac{\nu j^p \cos(p\varphi)}{H_p^{(2)'}(Ak_0 \sin \theta)} \cos\left(\frac{p\lambda_0}{2(2A+H)\sqrt{\varepsilon_r}}\right) \quad (4)$$

Once the series in equation (4) is truncated to encompass the minimum sphere of the problem, \tilde{E}_φ is expanded on a PC basis. The chosen distributions being uniform, the Legendre polynomials is the adequate basis [5].

IV. RESULTS AND ANALYSIS

The methodology described in sections II and III is applied on a patch antenna with frequency $f_0 = 1$ GHz and $\varepsilon_r = 2.15$. Its optimal width is $w = 12$ cm, its height is $h = 3.5$ mm and the cylinder radius is $a = 3$ m in the deterministic case. To compute the electric field from equation (2), the series is truncated to order $k_0(a+h) + 10 = 109$ terms [4]. Another series appears in equation (4) and is truncated to order $\binom{M+\partial}{\partial} = \binom{3+4}{4} = 35$ terms [5]. Here, ∂ is the fixed total degree of the PC expansion, set at 4.

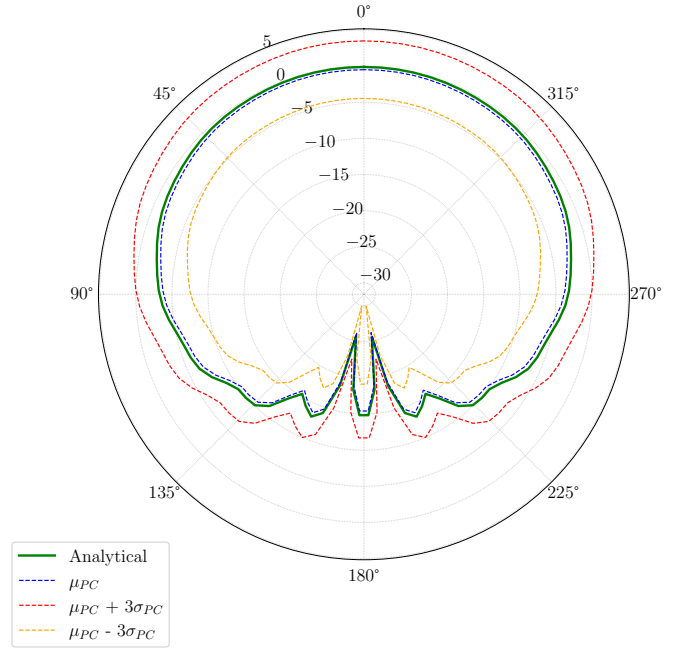


Fig. 2. Normalized Stochastic Electric Field (dB) {Cylinder+Patch} ($\theta = 0^\circ$).

Figure 2 displays the normalized analytical radiated electric field pattern derived from equation (2). It also displays μ_{PC} , the mean of the reconstructed field with PC, over 30 realizations. Patterns $\mu_{PC} \pm 3\sigma_{PC}$ show how the mean stochastic field behaves with respect to the 3- σ limit. This means that for the chosen distributions, the radiation patterns of the conformal patch antenna installed on an infinite cylinder have a probability of over 99.73% to be confined within the boundaries provided by the $\mu_{PC} \pm 3\sigma_{PC}$ patterns.

V. CONCLUSION

This paper has presented a methodology to characterize the stochastic radiation pattern of a patch antenna on an infinite cylinder. The stochastic part lies in the antenna parameters and in the cylinder geometry. The boundaries of the resulting radiation patterns confine almost all possible realizations of the random field. The PC expansion has proven to be computationally efficient, even for large electromagnetic scenes.

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