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► To cite this version:

Adrien Mélot, Emmanuel Rigaud, Joël Perret-Liaudet. Robust gear design with respect to the primary resonance induced by backlash nonlinearity. ENOC 2024 - 11th European Nonlinear Dynamics Conference, Jul 2024, Delft, Netherlands. pp.1-2. hal-04701338

HAL Id: hal-04701338

<https://hal.science/hal-04701338v1>

Submitted on 18 Sep 2024

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Robust gear design with respect to the primary resonance induced by backlash nonlinearity

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Summary. This study concerns the effect of uncertain tooth profile modifications on the primary resonance of a spur gear pair induced by the backlash nonlinearity. It should be noted that these corrections are micro geometric within tolerances. This primary resonance can exhibit dynamic responses with vibro-impacts between teeth. In this context, we have developed an original approach based on tracking the saddle-node bifurcations of the primary resonance. The equation of motion is solved using the harmonic balance method combined with an arc-length continuation method and a bordering technique. The control parameters are the meshing frequency associated with the rotation speed, and the tip relief associated with the profile correction. The evolution of the bifurcation points with respect to the parameters is calculated deterministically. To characterize the severity of the resonance, we introduce two criteria, namely the amplitude of the response at the resonance peak and the width of the resonance hysteresis. Uncertainty is then introduced by means of probability densities of the tip relief within the tolerance interval. Monte Carlo simulations are then used to conclude on the optimum profile correction, which differs significantly from that recommended in the purely deterministic case.

Introduction

Concerning the NVH requirement in the case of geared systems, the main goal is to minimize the housing vibration levels by acting on the internal source of excitation, *i.e.* the static transmission error (STE) under load [1]. Indeed, STE is the main source leading to the whining noise which is the dominant ones. This error is defined as the difference between the angular position of the output wheel and the position it should occupy if the gear system was perfect and infinitely rigid. Its physical origins are geometric faults such as profile deviations, localized deformations in the teeth, and global deformations of the entire transmission. Typically, these three sources lead to geometric errors of the order of $10 \mu\text{m}$ for gears with a module of 2 mm . As an excitation source, the STE can be modelled by two contributions, a displacement-type excitation and a parametric excitation associated with the periodic fluctuation of the mesh stiffness that couples the two meshing wheels. This generates the dynamic transmission error and dynamic forces on teeth propagating to the housing [2]. Due to necessary tooth backlash, the dynamic amplifications observed on resonances can reveal vibro-impacts responses. This is often the case of spur gears, where we can observe nonlinear primary softening resonance associated with contact losses [3]. In order to minimize the STE, micro geometric modifications of the tooth flanks are realized (tip relieves). Generally, the optimization is only based on the deterministic STE calculation [4,5], rather than on the dynamic response, known as the dynamic transmission error. In robust design, it is important to introduce the dispersion of these modifications (tolerances).

In this context, we propose to determine the robust optimum of the profile correction by considering not the static transmission error, but the dynamic transmission error observed at the primary resonance. The severity of the resonance is introduced via two ad hoc criteria, *i.e.* the amplitude of the response to the resonance peak and the width of the resonance hysteresis. The variability is then taken into account by assuming different probability densities for the value of the tip relief.

The dynamic modelling and the deterministic behaviour

The 2-degree-of-freedom modelling is shown in figure 1. It consists of two rigid discs (the pinion and the wheel) connected by the nonlinear meshing stiffness. The equation of motion can be written as $m\ddot{x} + c\dot{x} + F(x, e, t) = N$, and governs the dynamic transmission error $x = R_1\theta_1 + R_2\theta_2$.

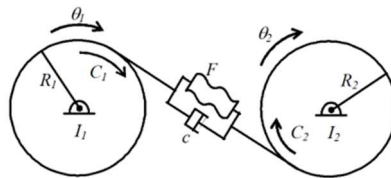


Figure 1: the dynamic modelling

The nonlinear force acting between teeth can be expressed as $F = k(t) [(x - j)H(x - j) + (x + j)H(-x - j)]$. $k(t)$ is the meshing stiffness which is periodic, H the Heaviside function, and $j(t) = b + e(t) + N/k(t)$ a periodic function taking into account the tooth backlash b and the excitation displacement $e(t)$ induced by the static transmission error. The equation of motion is solved by a harmonic balance method combined to an arc-length continuation method. Typical primary resonance can be shown in figure 2(a). We can observe the softening behaviour associated to the possible loss of contact between teeth. As the resonance is mainly governed by the amount of the tip relief, we have also developed a bifurcation tracking taking into account not only the rotational speed of the gear but also the tip relief [6]. Once again, we use an arc-length continuation method adapted to detect the saddle node bifurcation points. Result is shown in figure 2(b).

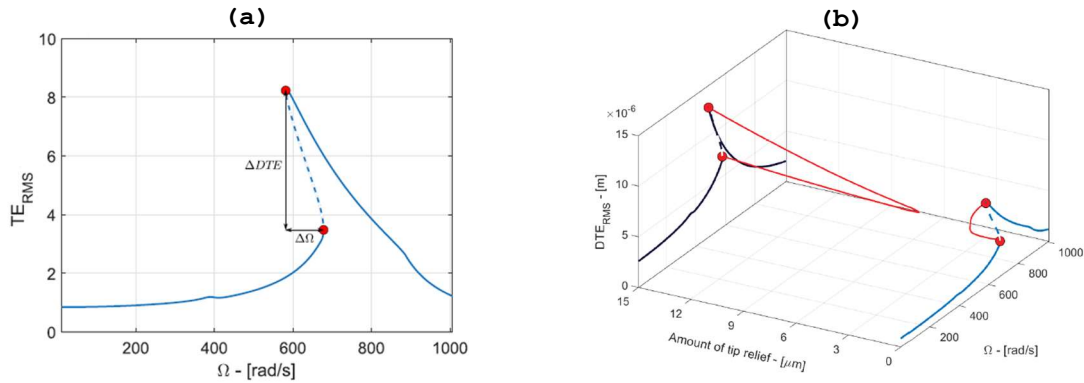


Figure 2: (a) a typical primary resonance; (b) the bifurcation tracking (red line).

Uncertainty and robust design

Uncertain amount of tip relief η on both gears is introduced by considering different probability density functions $f_\eta(x)$, which are the uniform, the normal, the Weibull and the beta distributions over a tolerance range of $5 \mu\text{m}$. Indeed, information regarding distribution laws of tooth profile modification is scarce. In order to quantify the severity of the resonance, we introduce two parameters, namely the amplitude of the response to the resonance peak ΔDTE and the width of the resonance hysteresis $\Delta\Omega$, as depicted in figure 2(a). Then, to consider robust design, we introduced two ad hoc criteria $C_{freq} = \int_{\mu_{min}}^{\mu_{max}} \Delta\Omega(x) f_\eta(x) dx$, and $C_{amp} = \int_{\mu_{min}}^{\mu_{max}} \Delta DTE(x) f_\eta(x) dx$, to be minimized. $\Delta\Omega(x)$ and $\Delta DTE(x)$ are obtained from the above deterministic bifurcation tracking. Results are compared with the ones calculated by considering classic optimization of the peak-to-peak amplitude of the static transmission error.

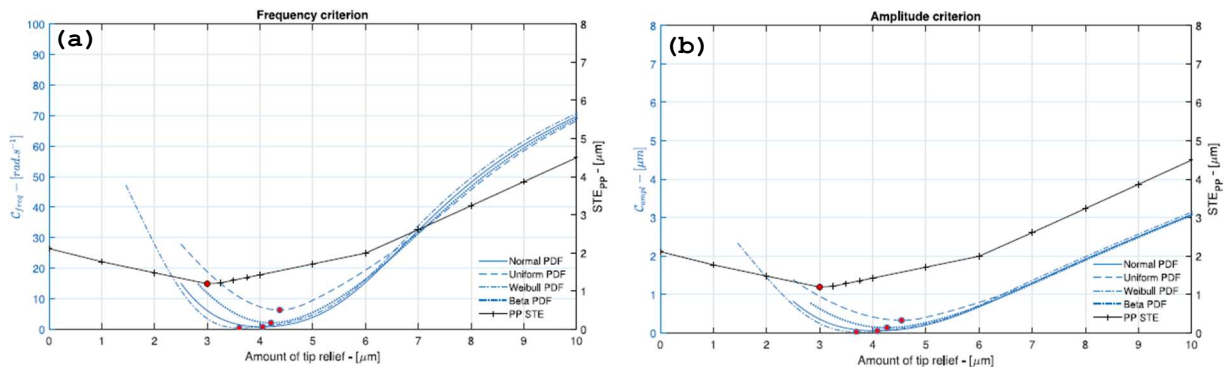


Figure 3: (a) C_{freq} criterium versus tip relief η ; (b) C_{amp} criterium versus tip relief η . For the two, in red, the optimum obtained from a classic peak-to-peak amplitude of the static transmission error (found equal to $3 \mu\text{m}$).

As we can see, figure 3, the optimum amount of tip relief, regards to the severity of the primary resonance is obtained around $4 \mu\text{m}$, depending on the considered distribution of η . So, our proposed new criteria for optimizing the tip relief is around 25% higher than a conventional static optimization ($3 \mu\text{m}$).

Conclusion

A computational procedure based on deterministic bifurcation tracking was developed to design tooth profile modifications. By the knowledge of the bifurcation tracking, and considering uncertainty via probability density function of the tip relief, the proposed approach allows us to define an optimal robust design. In particular, it was found that considering uncertainties and the primary resonance severity leads to a different optimal value compared to the one obtained with static computations. More generally, the principle of the proposed method can be extended to other practical problems (other types of bifurcation, large-scale geared system models, other mechanical systems than gears, etc.).

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