



**HAL**  
open science

# Structural optimization for controlling isolated response curves

Adrien Mélot, Enora Denimal Goy, Ludovic Renson

► **To cite this version:**

Adrien Mélot, Enora Denimal Goy, Ludovic Renson. Structural optimization for controlling isolated response curves. ENOC 2024 - 11th European Nonlinear Dynamics Conference, Jul 2024, Delft, Netherlands. pp.1-3. hal-04701318

**HAL Id: hal-04701318**

**<https://hal.science/hal-04701318v1>**

Submitted on 18 Sep 2024

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



Distributed under a Creative Commons Attribution 4.0 International License

## Structural optimization for controlling isolated response curves

Adrien Mélot\*, Enora Denimal Goy† and Ludovic Renson ‡

\*Univ. Gustave Eiffel, Inria, COSYS-SII, I4S team, FR

†Inria, CMAP, Ecole Polytechnique, FR

‡Dynamics Group, Department of Mechanical Engineering, Imperial College London, UK

*Summary.* We introduce a numerical analysis framework for controlling isolated response curves (isolas), i.e., curves that form closed loops and are not connected to the main branch of solutions. The methodology relies on bifurcation tracking analyses to monitor the evolution of node-collocation bifurcations in a codimension-2 space. Singularity theory is employed to differentiate points of isolas formation and merger from codimension-2 bifurcations. An optimization problem is defined to advance or delay the formation or merger of isolas.

### Introduction

Isolated response curves, or isolas, are solution branches of nonlinear dynamical systems that form closed curves in parameter space. Although they are not connected to the main solution branch, i.e. the one continued from quasi-linear solutions at low energies, they can merge with the main branch when a parameter, such as the forcing amplitude, is varied. Numerous studies confirm the presence of isolas in the response of nonlinear systems, with the earliest dating back to the 1950s. Isolass pose a significant risk as they can induce multi-stable regions where high-amplitude responses coexist with low-amplitude responses. They can also lead to substantial increases in amplitude and frequency shifts when the merger of an isola with the main branch occurs at a nonlinear resonance. Recent works have shown that isolas can be exploited, for instance, isolas induced by symmetry breaking have been utilized for the detection of infinitesimal masses [1]. From a numerical perspective, the most popular method to investigate the presence of isolas is bifurcation tracking, which involves monitoring the evolution of the position of a bifurcation point in a codimension-2 space. Since isolas are closed curves, they have at least two fold bifurcations. It is thus possible to track a fold bifurcation as a function of the excitation frequency and an additional parameter, here referred to as the tracking parameter in the following, and detect an isola when the bifurcation curve has multiple solutions for certain values of the tracking parameter [2]. However, although the basic concepts of numerical tools are relatively well-established, the detection and classification of turning points in bifurcation tracking curves to identify points of isolas formation and merger still requires a posteriori interpretation that is incompatible with an optimization procedure. For the first time, we propose here a mathematical framework to control the location of isolated response curves in a codimension-2 parameter space through structural optimization.

### Computational strategy

#### Optimization problem

Multiple objectives can be considered concerning the location of isolas in parameter space. Here, we consider the position in terms of the value of an operational parameter  $\mu$ , whose variation can lead to the formation or merger of an isola, a common example of such a parameter being the amplitude of external forcing applied to the system. The simplest objective is to delay the occurrence of an isola formation point to increase the interval of parameter  $\mu$  without multistability. Without loss of generality, we assume that the formation and merger of an isola occur for increasing values of the parameter  $\mu$ . The optimization problem can be written in the general form:

$$\begin{aligned} \underset{\boldsymbol{\theta}}{\text{maximize}} \quad & \min(\mu_{\text{birth}}^{(k)}) \quad \forall k \in \llbracket 1, N \rrbracket \\ \text{s.t.} \quad & b_i^l \leq \theta_i \leq b_i^u \quad \forall i \in \llbracket 1, p \rrbracket, \end{aligned} \quad (1)$$

where the min operator accounts for the potential presence of multiple isolas,  $\boldsymbol{\theta} \in \mathbb{R}^p$  is the vector of optimization parameters, i.e., design parameters. Here, we only consider simple constraints intended to bound the values of optimization parameters with vectors  $\mathbf{b}^l$  and  $\mathbf{b}^u$  to ensure that the values are realistic, for example, with a strictly positive mass.

#### Computation and detection of points of isola formation and merger

The next step of the methodology consists in computing a fold bifurcation curve associated to an isola. This can be readily carried out with bifurcation tracking. Without loss of generality, we employ the harmonic balance method (HBM) to recast the equations of motion into the following zero problem:

$$\mathcal{R}(\mathbf{q}, \Omega, \mu) = 0 \quad (2)$$

where  $\mathbf{q}$  is the vector of the system's state,  $\Omega$  the forcing frequency and  $\mu$  an auxiliary parameter. In order to compute fold bifurcation curves, we free parameters  $\Omega$  and  $\mu$  and employ an arclength continuation algorithm on a minimally extended system consisting of the dynamic force equilibrium equation  $\mathcal{R}$  and two additional scalar equations to account for the free

parameters:

$$\mathbf{R}(\mathbf{q}, \Omega, \mu) = \begin{pmatrix} \mathcal{R}(\mathbf{q}, \Omega, \mu) \\ p(\mathbf{q}, \Omega, \mu) \\ g(\mathbf{q}, \Omega, \mu) \end{pmatrix} = \mathbf{0}. \quad (3)$$

Equation  $p(\mathbf{Q}) = 0$  is the arclength equation and  $g(\mathbf{Q}) = 0$  characterizes fold bifurcation points [2, 3]. Solving Eq. (3) in a parameter interval yields a fold bifurcation curve which can exhibit a number of turning points with respect to the tracking parameter  $\mu$ . These turning points are easily detected by monitoring the sign of  $\mu$ -component of the tangent vector to the curve at each iteration. The detected points can be codimension-2 cusp bifurcation, points of isola formation or points of isola merger. In order to classify those, we resort to using singularity theory to derive test functions whose zeros indicate a specific point.

## Results

The methodology is exemplified on the following model of a 2-degree-of-freedom base-excited oscillator:

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{pmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{pmatrix} + \begin{bmatrix} c_1 & 0 \\ 0 & c_2 \end{bmatrix} \begin{pmatrix} \dot{q}_1 \\ \dot{q}_2 \end{pmatrix} + \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 + k_3 \end{bmatrix} \begin{pmatrix} q_1 \\ q_2 \end{pmatrix} + \begin{pmatrix} f_{nl}(q_1) \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (4)$$

with  $f_{nl}(q_1)$  a third order polynomial force which includes the base excitation. The reference values of the model parameters can be found in [4].

Figure 1a shows that the system initially exhibits two isolas merging with the main solution branch ( $\blacklozenge$ ) for forcing amplitudes equal to  $d = 2.3$  mm and  $d = 3.0$  mm. Figure 1b shows that the optimization considering all of the model parameters as optimization variables allows one to efficiently control the isola by delaying the point of isola formation ( $\blacklozenge$ ) to higher values of the forcing amplitude ( $d = 2.3$  mm compared to  $d = 4.8$  mm) and leads to the disappearance of one isola.

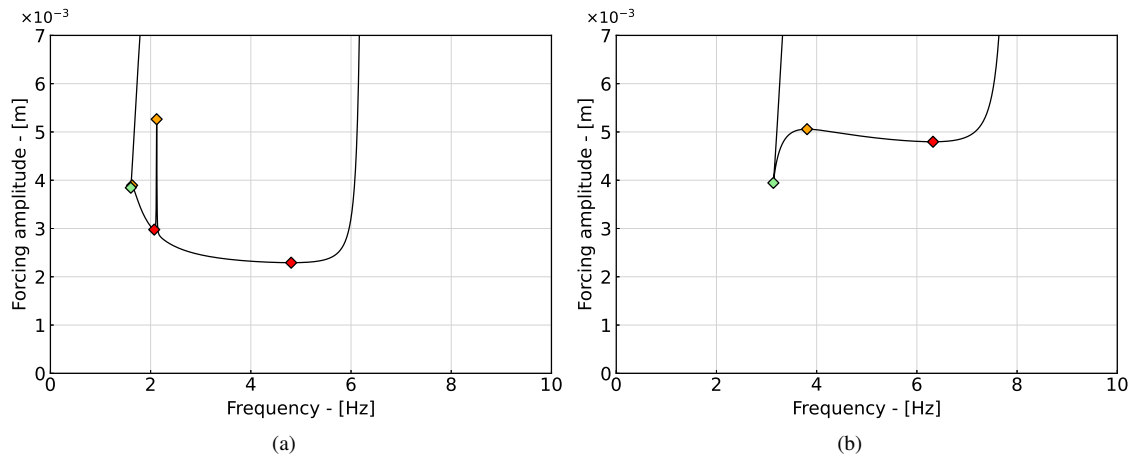


Figure 1: Projections in the forcing amplitude-frequency plane of the fold curve of the reference (a) and optimized (b) systems. Cusp bifurcations and points of isola formation and merger are denoted by green ( $\blacklozenge$ ), red ( $\blacklozenge$ ) and orange ( $\blacklozenge$ ) diamond markers, respectively.

## Conclusions

Isolated response curves are a significant nonlinear phenomenon that can have profound implications for the performance and durability of mechanical systems. We have proposed a general methodology for controlling their position in a codimension-2 space. This methodology utilizes the harmonic balance method and minimally augmented systems to track the evolution of fold bifurcation points. Indicators have also been defined using singularity theory to detect and classify cusp bifurcations as well as points of isolas formation and merger.

## References

- [1] Grenat, C., Baguet, S., Lamarque, C. H., and Dufour, R. (2019). A multi-parametric recursive continuation method for nonlinear dynamical systems. *Mechanical Systems and Signal Processing* **127**:276-289.
- [2] Detroux T., Renson L., Masset L., Kerschen G. (2015) The harmonic balance method for bifurcation analysis of large-scale nonlinear mechanical systems. *Comput. Methods Appl. Mech. Eng.* **296**:1838
- [3] Mélot A., Rigaud E., Perret-Liaudet J. (2022) Bifurcation tracking of geared systems with parameter-dependent internal excitation. *Nonlinear Dynamics* **107**:413431
- [4] Detroux, T., Noël, J. P., Virgin, L. N., and Kerschen, G. (2018). Experimental study of isolas in nonlinear systems featuring modal interactions. *Plos one* **13**(3):e0194452.