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# Nonlinear system identification with control-based continuation of bifurcation curves

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*Summary.* We propose a methodology to carry out nonlinear system identification based on bifurcations. The proposed approach relies on fold bifurcation curves identified experimentally through control-based continuation and an optimization framework to minimize the distance between the experimental and numerical curves computed with bifurcation tracking analyses. The approach is demonstrated on a nonlinear model of base-excited energy harvester with magnetic force nonlinearity.

## Introduction

The design of modern mechanical systems necessitates the development of numerical models for anticipatory assessment of the systems' responses and identification of potential deficiencies in the proposed designs. Hence, the development of accurate numerical models capable of qualitatively and quantitatively predicting the behaviour of physical systems is of utmost importance. Due to potentially strong nonlinear effects, traditional system identification features based on linear modes often prove suboptimal. Local bifurcations are critical phenomena occurring in nonlinear system which give rise to both qualitative and quantitative changes in the system's response and act as organizing centres for the local dynamics. Consequently, bifurcation curves define the limits of stability of nonlinear systems and regions of multi-stability. Consequently, they provide both quantitative and qualitative information on systems operating in strongly nonlinear regimes. Control-based continuation (CBC) is a means of applying numerical continuation directly to a physical experiment for bifurcation analysis without the use of a mathematical model or a post-processing stage as is often the case for more traditional experimental approaches. CBC enables the detection and tracking of bifurcations directly by exploiting control feedback to stabilize the observed responses [1, 2, 3]. The experimental curves exploited hereafter are taken from [2].

## Numerical procedure

The next step of the methodology consists in computing curves of the experimentally-tracked bifurcation points. Without loss of generality, we consider mechanical systems whose periodic responses are solutions of the following nonlinear algebraic system:

$$\mathcal{R}(\mathbf{q}, \Omega, \mu) = 0 \quad (1)$$

where  $\mathbf{q}$  is the vector of discretised states (here using Fourier series),  $\Omega$  the forcing frequency and  $\mu$  an auxiliary parameter. In order to compute fold bifurcation curves, we free parameters  $\Omega$  and  $\mu$  and employ an arclength continuation algorithm on a minimally extended system consisting of the dynamic force equilibrium equation  $\mathcal{R}$  and two additional scalar equations to account for the free parameters:

$$\mathbf{R}(\mathbf{q}, \Omega, \mu) = \mathbf{R}(\mathbf{Q}) = \begin{pmatrix} \mathcal{R}(\mathbf{q}, \Omega, \mu) \\ p(\mathbf{q}, \Omega, \mu) \\ g(\mathbf{q}, \Omega, \mu) \end{pmatrix} = \mathbf{0}. \quad (2)$$

Equation  $p(\mathbf{Q}) = 0$  is the arclength equation and  $g(\mathbf{Q}) = 0$  characterizes fold bifurcations points [4, 5]. Solving Eq. (2) in a parameter interval yields a fold bifurcation curve, i.e. a one-dimensional submanifold of a high dimensional space, which defines areas in multi-stability in the parameter space. In order to carry out system identification and model updating, we formulate an optimization problem to minimize the distance between experimental bifurcation curves and numerical ones as illustrated in Fig. 1. More specifically, the optimization problem is formulated as:

$$\begin{aligned} \underset{\boldsymbol{\theta}}{\text{minimize}} \quad & \mathcal{F} = \frac{1}{N} \sum_{j=1}^N \|\mathbf{diag}^{-1}(\mathbf{Q}_j^{exp}) (\mathbf{Q}_j^{num} - \mathbf{Q}_j^{exp})\| \\ \text{subject to} \quad & b_i^l \leq \theta_i \leq b_i^u \quad \forall i \in \llbracket 1, p \rrbracket, \end{aligned} \quad (3)$$

where  $\boldsymbol{\theta}$  is the vector of optimization parameters,  $\mathbf{Q}_j^{exp}$  and  $\mathbf{Q}_j^{num}$  denote the experimental and numerical fold bifurcation curves, respectively, and  $N$  the number of points used to sample each curve.

## Results

The studied system consists of a thin steel plate fixed to an aluminum armature, with two sets of neodymium magnets attached at one end (cf. Fig. 2a). This setup forms a single-degree-of-freedom (SDOF) oscillator. The system, subjected to base excitation, experiences magnetic interactions between moving magnets and an iron laminated stator with a coil. These interactions introduce a nonlinear restoring force, which can be adjusted by controlling the air gap between magnets and the iron stator.

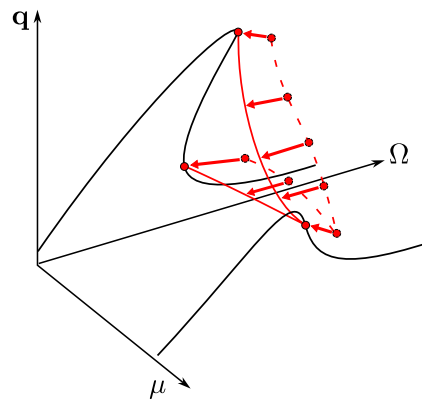


Figure 1: Schematic representation of the system identification procedure. The experimental and numerical bifurcation curves are drawn as solid and dashed red lines, respectively.

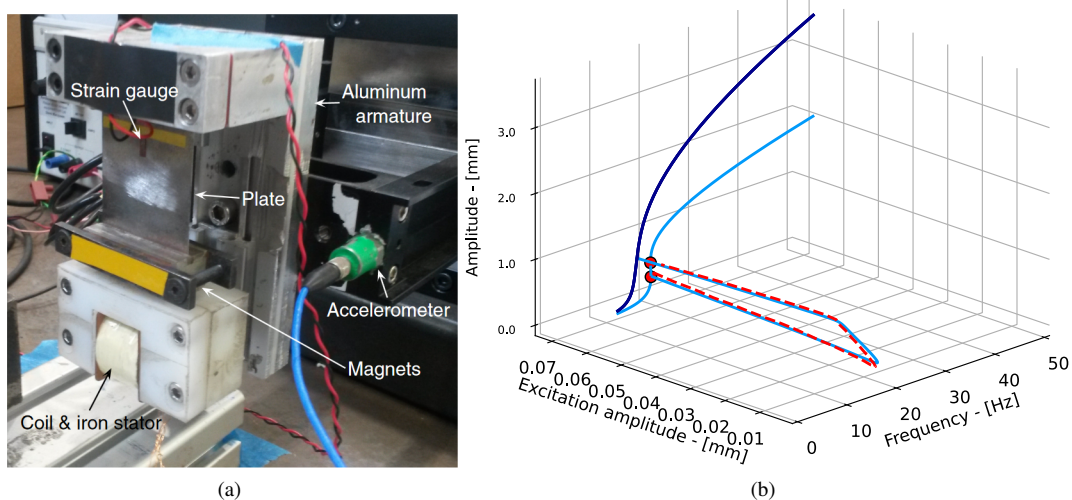


Figure 2: Experimental set-up with the nonlinear oscillator - adapted from [2] - (a) and example of result on the softening part of the forced response curve (b). The dashed red line corresponds to the experimental data while the dark and light blue lines are the initial and identified forced response and bifurcation curves, respectively.

Figure 2b shows the results obtained by considering an optimization on both structural and nonlinear parameters of the model. One can see that the methodology allows one to identify parameters leading to a good quantitative and qualitative agreements with experimental data.

## Conclusions

In this paper, a nonlinear system identification method based on experimental fold bifurcation curves was presented. The proposed approach exploits the information contained in bifurcation curves to identify the system's parameters for several values of the forcing amplitude in strongly nonlinear regimes. The approach is based on both control-based continuation, to obtain experimental curves, and numerical bifurcation tracking to evaluate and minimize an objective functional formulated as the distance between experimental and numerical fold bifurcation curves. Leveraging bifurcation curves offer as many features as there are bifurcations occurring in the considered parameter interval, thus improving the reliability of the identification process compared to standard techniques. Future work may look into extending the methodology to other types of bifurcations (Neimark-Sacker, period-doubling,...).

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