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Fostering cooperation between public transport and on-demand services through tradable credits: a modeling framework based on the trip-based macroscopic fundamental diagram

--Manuscript Draft--

Full Title:	Fostering cooperation between public transport and on-demand services through tradable credits: a modeling framework based on the trip-based macroscopic fundamental diagram
Abstract:	<p>Ride-hailing (RH) business boomed in the last decade. RH activities may overlap with the public transportation (PT) offer and thus decrease the overall transportation network efficiency. Especially, RH vehicles operating in the city center compete with PT ridership where the PT network is dense with a high level of service. RH services can, however, extend the PT offer in the suburbs where the PT services are sparse and less attractive.</p> <p>This paper proposes implementing a Tradable Credit Scheme (TCS) to nudge RH drivers to operate in the suburbs instead of the city center. Credits are distributed to the RH drivers, and operating in the areas with satisfying PT coverage requires credits. RH drivers choose where to operate to maximize their expected revenues, thus balancing RH revenues and credit costs. The TCS encourages them to operate outside the city center and complete the PT offer instead of competing against it. The TCS also promotes a shift in the travelers' behaviors and fosters combined trips: riding the PT in the city center and then shifting to RH for the trip portion in the suburbs.</p> <p>This paper presents a modeling and evaluation framework to adjust the TCS settings. The trip-based Macroscopic Fundamental Diagram (MFD) represents the movement of the RH vehicles to pick up and drive the customers to their destinations. The equilibrium of the RH vehicles assignment in the city regions and the credit price is computed based on heuristics.</p> <p>An example illustrates the application of the TCS in a day-to-day process.</p>
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1 **FOSTERING COOPERATION BETWEEN PUBLIC TRANSPORT AND ON-DEMAND**
2 **SERVICES THROUGH TRADABLE CREDITS: A MODELING FRAMEWORK BASED**
3 **ON THE TRIP-BASED MACROSCOPIC FUNDAMENTAL DIAGRAM**

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1 ABSTRACT

2 Ride-hailing (RH) business boomed in the last decade. RH activities may overlap with the public
3 transportation (PT) offer and thus decrease the overall transportation network efficiency. Espe-
4 cially, RH vehicles operating in the city center compete with PT ridership where the PT network
5 is dense with a high level of service. RH services can, however, extend the PT offer in the suburbs
6 where the PT services are sparse and less attractive. This paper proposes implementing a Tradable
7 Credit Scheme (TCS) to nudge RH drivers to operate in the suburbs instead of the city center.
8 Credits are distributed to the RH drivers, and operating in the areas with satisfying PT coverage
9 requires credits. RH drivers choose where to operate to maximize their expected revenues, thus
10 balancing RH revenues and credit costs. The TCS encourages them to operate outside the city cen-
11 ter and complete the PT offer instead of competing against it. The TCS also promotes a shift in the
12 travelers' behaviors and fosters combined trips: riding the PT in the city center and then shifting to
13 RH for the trip portion in the suburbs. This paper presents a modeling and evaluation framework
14 to adjust the TCS settings. The trip-based Macroscopic Fundamental Diagram (MFD) represents
15 the movement of the RH vehicles to pick up and drive the customers to their destinations. The
16 equilibrium of the RH vehicles assignment in the city regions and the credit price is computed
17 based on heuristics. An example illustrates the application of the TCS in a day-to-day process.

18

19

20 *Keywords:* trip-based MFD; tradable credit scheme; on-demand mobility

1 INTRODUCTION

2 Ride-Hailing (RH) companies like Uber, Didi, and Lyft introduced new options for mobility in
3 many cities. Due to lesser regulations than traditional taxi companies, the fees are usually lower,
4 and RH services have expanded significantly (1). However, these on-demand mobility services
5 may negatively affect the overall multimodal transportation network. RH companies contributed
6 to the congestion increase in San Francisco between 2010 and 2016, according to (2). The study
7 of (3) concludes that RH companies mainly compete with Public Transportation (PT) alternatives,
8 even if RH companies also provide trips not covered by the transit network.

9 RH services' objective is, as a private player, to increase their profits and thus dispatch
10 their resources in high-demand areas which may already have good transit coverage. Those behav-
11 iors are usually not aligned with the collective optimum settings. We propose a Tradable Credit
12 Scheme (TCS) for RH drivers. Contrarily to traditional taxi license schemes, which regulate and
13 redistribute the number of operating taxis in a given area in the long run, we envision a short-term
14 and flexible framework. For now, TCS has only been proposed for demand management, see the
15 contributions of (4–6) for examples, or the report on TCS of (7) for an overview of TCS for traf-
16 fic management. Here, we apply the concept to the supply side. The objective is to encourage
17 RH drivers to shift from the city center to the suburbs, where they can propose first-/last-mile al-
18 ternatives and complement the actual PT services. The TCS restricts the number of RH vehicles
19 driving in the city center and between the first rings. Thus, operating in the city center becomes
20 less attractive due to the costs of acquiring the required credits. Travelers in the city center would
21 find fewer RH drivers available to drive them and use PT. Increasing RH operations in the suburbs
22 only should foster cooperation between PT and RH. Customers traveling from the suburbs use RH
23 services to the border of central regions and then ride PT.

24 For evaluating the TCS on RH services, the operation, and competition with other modes, it
25 is essential to keep track of the transportation system dynamics, as congestion significantly impacts
26 travel times and, thus, the service quality. We must also consider the service's full spatial extent
27 and reproduce the vehicles' trajectories. The trip-based MFD framework (8, 9) is an excellent
28 candidate to simulate the RH trips under those conditions without the computational burden of
29 detailed simulations. Several recent contributions regarding RH services are founded on the MFD
30 concept. (10) study the equilibrium between RH offer, passenger demand, and service pricing. The
31 model predictive controller represents the traffic dynamics with an MFD framework. (11) nudge
32 passengers to share their rides and park unmatched vehicles to reduce the impact of RH vehicles
33 on congestion.

34 We implement a TCS to shift RH drivers from the city center to the suburbs. The trip-based
35 MFD framework is used to track the position of the RH drivers and compute their trajectories:
36 driving to pick up the passengers and then drop them at their destination. The RH drivers receive
37 an initial credit allocation. Operating in different city regions requires credits. The credit charge is
38 lower the further we are from the city center. The goal of the regulator is to promote the use of PT
39 in the city center and a combination of RH and PT for trips between the center and the suburbs.
40 In the following, we present the methodology in Sect. 2, i.e., how we represent the RH operations
41 and travelers' trips with the MFD framework. The constraints of the TCS introduction are then
42 developed in Sect. 3. The computation of the transportation network at equilibrium is described.
43 A day-to-day process is proposed to investigate the transitions between different TCS as the local
44 authority adjusts the TCS from time to time. Sect. 4 develops a case study to illustrate the proposed
45 TCS. The numerical results are then discussed in Sect. 5. Sect. 6 concludes this paper.

1 METHODOLOGY

2 The road network is divided into N_R different regions. Each region has a different travel demand
 3 distribution and PT coverage. The regions are indexed by increasing order from the center to the
 4 outskirts. We note \mathcal{C} the set of travelers. Each traveler uses a travel mode $m \in \mathcal{M}$. The alternatives
 5 are RH service, riding the PT, or combining both. The transportation system is also populated with
 6 background traffic: regular users driving their own cars, influencing the congestion level and the
 7 network speed. We note \mathcal{D}_0 the set of potential drivers and $\mathcal{D} \subset \mathcal{D}_0$ the set of active RH drivers
 8 operating in the network. As we consider driver elasticity, some potential drivers may not operate
 9 because the earnings are too low. We assume each driver i has a reservation price P_i^{res} . When the
 10 average RH revenue exceeds the driver's reservation price, it will join the RH service. Otherwise,
 11 it leaves the service. The equilibrium is not straightforward: the more RH drivers, the more the
 12 total RH revenue is, but at the same time, the higher the denominator of the average revenue (the
 13 number of RH drivers) is.

14 The regulator aims to enforce a TCS to reduce competition between RH vehicles and PT in
 15 the city districts where the transit coverage is satisfying, usually the city center and inside the first
 16 rings. Its strategy is to foster multimodal trips where RH drivers permit travelers from the outskirts
 17 to ride an RH vehicle to a transit hub at the border of the city center and then use the PT. Fig. 1
 18 presents a schematic representation of the different travel alternatives for a traveler going from the
 19 suburbs (region 3) to the city center (region 1).

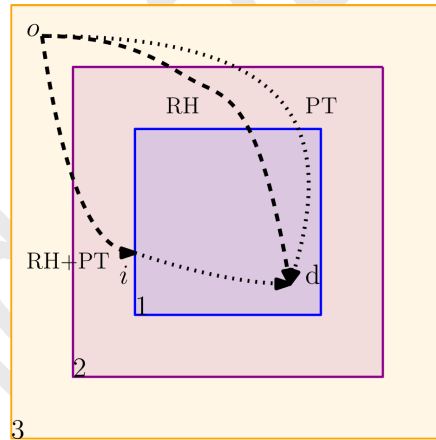


FIGURE 1: A trip between an origin o in region 3 and a destination d in region 1 has three alternatives: RH, PT, or RH till the border i and then PT.

20 We set a framework based on the trip-based MFD to study the effect of the TCS. It con-
 21 sider the congestion dynamics and the heterogeneity of the trips: each traveler has its own origin,
 22 destination, and departure time. Its trip length is retrieved from the actual multimodal network
 23 topology. We track the position of the RH vehicles. The drivers start at an initial position that
 24 corresponds to their homes. They move only to pick up or drive a customer to its destination. The
 25 rest of the time, they park on the street and wait to pick up another customer. Those idle drivers do
 26 not contribute to the congestion.

At their departure time t , the travelers request a trip through a Mobility-as-a-Service (MaaS) platform. The platform then chooses their travel alternatives (RH and/or PT) in order to minimize the total travel costs. The user travel costs from origin o to destination d are defined by the travel

times and the service prices:

$$C_{o,d,PT}^t = \alpha_j T_{PT,o,d} + f_{PT}; \quad (1)$$

$$C_{o,d,RH}^t = \alpha_j \left(L_{pu,o,d}/V_{r_o}(t) + \sum_{r=r_o}^{r_d} L_{o,d}^r/V_r(t) \right) + f_{RH}L_{o,d}; \quad (2)$$

$$C_{o,d,RH-PT}^t = \alpha_j \left(L_{pu,o,i}/V_{r_o}(t) + \sum_{r=r_o}^{r_i} L_{o,i}^r/V_r(t) + T_{PT,i,d}^* \right) + f_{RH}L_{o,i} + f_{PT}; \quad (3)$$

$$C_{o,d,PT-RH}^t = \alpha_j \left(T_{PT,o,i}^* + L_{pu,i,d}/V_{r_i}(t) + \sum_{r=r_i}^{r_d} L_{i,d}^r/V_r(t) \right) + f_{RH}L_{i,d} + f_{PT}. \quad (4)$$

- 1 α_j is the Value of Time (VoT) of the traveler j . The PT travel cost consists of the travel time
 2 $T_{PT,o,d}$ plus the price of a unitary ticket f_{PT} . We assume the ticket price is independent of the trip.
 3 The RH travel cost consists of the pick-up time, the travel time, and the RH charge. The pick-up
 4 time is the pick-up distance $L_{pu,o,d}$ over the current average speed in the origin region $V_{r_o}(t)$. The
 5 estimated travel time is decomposed over the different regions. In each region r , the travel time
 6 is the trip length in this region $L_{o,d}^r$ over the regional speed $V_r(t)$. The RH charge is the distance-
 7 based fee f_{RH} multiplied by the trip length. For the RH-PT alternative (RH then PT), the travel
 8 cost is the sum of the RH travel cost until the border i of the destination region and then the PT
 9 travel cost from this border to the destination. The same applies to PT-RH (PT then RH) in reverse:
 10 the traveler rides the PT and then takes an RH vehicle. We note $T_{PT,o,d}^*$ the PT travel time when
 11 combined with RH. It is usually smaller than $T_{PT,o,d}$ because the RH driver will pick up/drop the
 12 passenger close to the PT stop, reducing the access time.

Travelers starting at t from o to d send a request to the matching platform. The MaaS platform waits a few minutes to create a batch of departing travelers. It then assigns the travelers to a mode and an available RH driver. The assignment takes place to reduce the total travel cost of the current batch. The platform first removes the travelers for which the PT option is the cheapest alternative, even without accounting for the pick-up distance. Then if there are more travelers than drivers, the travelers with the highest pick-up distance are assigned to PT to ensure at least as many available RH drivers as travelers. These first steps reduce the size of the assignment problem. The assignment process corresponds to the following Integer Linear Problem (ILP):

$$\min \sum_{i,j,m \in \mathcal{D} \times \mathcal{C} \times \mathcal{M}} y_{i,j}^m C_{i,j}^m + \sum_{j \in \mathcal{C}} \left(1 - \sum_{i,m \in \mathcal{D} \times \mathcal{M}} y_{i,j}^m \right) C_i^{PT} \quad (5)$$

$$\sum_{j,m \in \mathcal{C} \times \mathcal{M}} y_{i,j}^m \leq 1, \forall i \in \mathcal{D} \quad (6)$$

$$\sum_{i,m \in \mathcal{D} \times \mathcal{M}} y_{i,j}^m \leq 1, \forall j \in \mathcal{C} \quad (7)$$

$$y_{i,j}^m = 0 \text{ if } lic_i > r_j^m, \forall i, j, m \in \mathcal{D} \times \mathcal{C} \times \mathcal{M} \quad (8)$$

- 13 m is the mode: RH or RH+PT (RH then PT or PT then RH); $C_{i,j}^m$ is the travel cost of j matched
 14 with i for mode m ; C_i^{PT} the cost of the transit-only alternative; $y_{i,j}^m = 1$ if and only if the driver i is
 15 matched with passenger j for the alternative m (pure RH or combined trips), and zero otherwise.
 16 The first constraint Eq. (6) states that each driver is matched to at most one customer. The second

1 Eq. (7) ensures each customer is matched to at most one driver. The third Eq. (8) ensures that
 2 the driver's license lic_i allows it to serve the trip. r_j^m is the required license to serve customer j
 3 following the alternative m .

4 REGULATING FLEET SIZE IN EACH REGION WITH TCS

5 The regulator introduces and enforces a TCS to regulate the RH operations in the city. Each
 6 active driver gets κ credits for free from the regulator per day. The drivers need to spend τ_r
 7 credits to acquire a license to operate (i.e., pick-up or drop-off passengers) in the regions with an
 8 index higher or equal to r for a day. Since the regions are defined for TCS purposes, we assume
 9 $\tau_r < \tau_{r-1}$, $\forall r \in [1, N_R - 1]$. We let the option of operating in the region N_R free of credit charge,
 10 i.e., $\tau_{N_R} = 0$. The TCS can be enforced by GPS tracking or CCTV-based license plate recognition.
 11 Drivers connect to an online platform to buy the licenses (with credits) and exchange their credits
 12 with the dedicated bank. The bank regulates the credit price p according to the offer and demand.

13 We note x_r the number of drivers with a license for region r , i.e., who can operate in regions
 14 $r' \geq r$. For an RH trip from an origin in region 3 to a destination in region 1, as in Fig. 1, the driver
 15 needs a license to operate in region 1 since it allows the driver to operate in regions 1, 2, and 3.
 16 However, for a combined trip (RH then PT) from 3 to 1, only a license for region 2 is needed as
 17 the last leg of the trip uses PT.

The framework distinguishes two timescales, as presented in Fig. 2. The drivers' activity

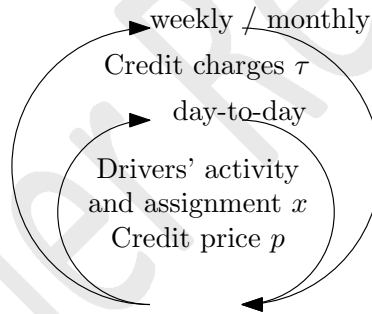


FIGURE 2: The two timescales of TCS: drivers' activity and assignment, and credit charge changes by the regulator.

18 and assignment, along with the credit price, are updated each day. The regulator adjusts the credit
 19 charge τ on a longer-term (weekly, monthly, or even yearly). In the following, we investigate two
 20 aspects. The first is the traffic state computation to estimate and predict the effect of TCS on the
 21 drivers' assignment and, thus, on the mode shares. The second is the evolution of the drivers'
 22 choice as a day-to-day process to represent the transition linked to the introduction of the TCS.
 23

24 Calculating the equilibrium

25 We first focus on calculating the equilibrium. The drivers' assignment x , i.e., the choice of operat-
 26 ing regions, balances two markets: the RH operation market, where travelers require RH services,
 27 and the credit market, where drivers buy and sell credits. Fig. 3 summarizes the different interac-
 28 tions. Travelers' mode choice impacts RH revenue for drivers, which, with the credit price, will
 29 change drivers' assignments. The average pick-up distance decreases as the number of drivers able
 30 to serve the trip increases. The pick-up distance affects the RH travel costs, thus modifying mode
 31 choices. The main mode shift is expected in the city center, where the TCS significantly increases

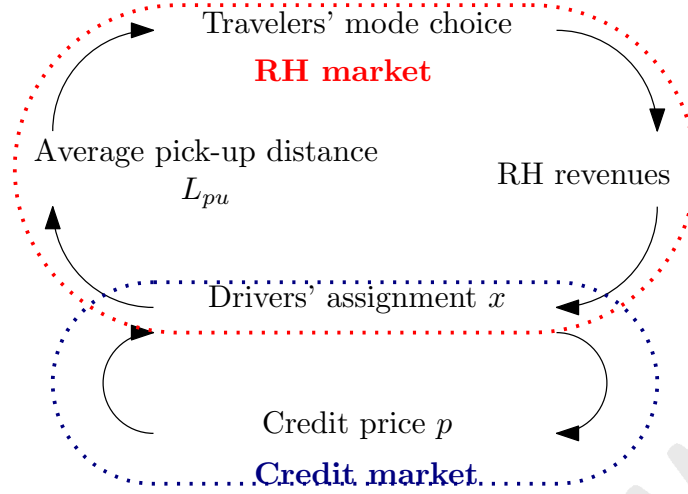


FIGURE 3: Inter-dependencies between drivers, travelers, and credit market.

1 RH operation costs: the travelers will find fewer RH drivers able to serve their trips, so longer
 2 waiting times. The RH option will then lose its attractiveness to the profit of PT. In the suburbs, on
 3 the opposite, we expect more RH drivers to operate, but without the required license to drive in the
 4 center. Thus, the attractiveness of combined trips will increase thanks to the lower RH waiting time.

5 We jointly formulate the equilibrium of the number of active drivers, their assignment x ,
 6 and credit price p . The RH revenue R_r is the volume of fees travelers pay for using RH (alone or
 7 combined with PT) for a trip requiring access to region r but not $r - 1$.

The average RH gain G_r^{avg} for operating with license r is the sum of the average revenue for each region the license permit access to minus the price of the license:

$$G_r^{\text{avg}} = \sum_{r' \geq r} \frac{R_{r'}}{\sum_{r'' \leq r'} x_{r''}} - p\tau_r. \quad (9)$$

Note that we do not account for the value of the initial allocation as we only use the difference between average gains. We also define the average RH revenue for all regions combined R^{avg} :

$$R^{\text{avg}} = \frac{\sum_{r \in [1, N_R]} R_r}{|\mathcal{D}|}. \quad (10)$$

The equilibrium is reached when the chosen licenses correspond to the maximum gain value

$$G_{\max}^{\text{avg}} = \max_r (G_r^{\text{avg}}) \text{ over the licenses. The equilibrium is formulated as } (G_{\max}^{\text{avg}} - G_r^{\text{avg}})x_r = 0, \forall r \in [1, N_R - 1]; \quad (11)$$

$$x_r \geq 0, \forall r \in [1, N_R]; \quad (12)$$

$$\sum_{r=1}^{N_R} x_r = |\mathcal{D}|; \quad (13)$$

$$P_i^{\text{res}} \leq R^{\text{avg}} \iff i \in \mathcal{D} \forall i \in \mathcal{D}_0; \quad (14)$$

$$\sum_{r=1}^{N_R} x_r (\tau_r - \kappa) \leq 0; \quad (15)$$

$$p \sum_{r=1}^{N_R} x_r (\tau_r - \kappa) = 0; \quad (16)$$

$$p \geq 0. \quad (17)$$

1 Eq. (11) means that any licenses chosen by at least one driver must yield the maximum gain.
 2 Eq. (12) states that the number of license holders is non-negative. Eq. (13) is the conservation of
 3 the number of drivers. Eq. (14) states that a driver is active if and only if its reservation price is
 4 below the average revenue. Eq. (15) is the credit cap (CC): the drivers cannot spend more credits
 5 than the distributed amount. Eq. (16) is the market clearing condition (MCC): all credits are used,
 6 or their price is zero. Eq. (17) means the credit price is non-negative. The last three constraints are
 7 specific to the TCS.

8 The equilibrium presented in Eq. (11-17) is theoretical and cannot be reached for most of
 9 the scenarios because the numbers of license holders are integer values. The computed equilib-
 10 rium will be compared to the results of the day-to-day process. Furthermore, the computation of
 11 different parts of the equilibrium is implicit, such as the RH matching Eq. (5), or nonlinear, such
 12 as the average gains and revenues Eq. (9) and Eq. (10). For these reasons, we formulate a cost
 13 function corresponding to a state close to the equilibrium. This cost function is then minimized
 14 with heuristic methods.

We reformulate the equilibrium into the minimization of an objective function J . The first part J_1 means the average gains for which there are license holders are close to the maximum gain.

$$J_1 = \frac{1}{|\mathcal{D}_0|N_R} \sum_{r=1}^{N_R} (G_{\max}^{\text{avg}} - G_r^{\text{avg}})x_r \quad (18)$$

The second part J_2 replaces constraint Eq. (14).

$$J_2 = \sum_{i \in \mathcal{D}_0} \delta(i, R^{\text{avg}}) \quad (19)$$

with $\delta(i, R^{\text{avg}})$ equals one when the driver i is active even though the average revenue is below its reservation price or when i is inactive, and the average revenue is higher than its reservation price, i.e.,

$$\delta(i, R^{\text{avg}}) = 1 \iff (i \in \mathcal{D} \wedge R^{\text{avg}} < P_i^{\text{res}}) \vee (i \notin \mathcal{D} \wedge R^{\text{avg}} > P_i^{\text{res}}). \quad (20)$$

It is zero otherwise. The objective function is the sum of both costs.

$$J = J_1 + J_2 \quad (21)$$

15 Note that J is always non-negative.

We use the driver conservation Eq. (13) and the MCC Eq. (16) to reduce the size of the minimization problem. We assume the price is non-zero. Otherwise, the TCS is non-effective, and

the state of the system is the same as without TCS, where all drivers can operate in all regions, i.e., $x_1 = |\mathcal{D}|$ and $x_r = 0, \forall r \in [2, N_R]$. Then the equality holds for the CC Eq. (15). We combine it with driver conservation to remove two variables. We choose to replace x_{N_R-1} and x_{N_R} with

$$x_{N_R-1} = \frac{|\mathcal{D}|(\kappa - \tau_{N_R}) - \sum_{k=1}^{N_R-2} (\tau_k - \tau_{N_R})x_k}{\tau_{N_R-1} - \tau_{N_R}};$$

$$x_{N_R} = |\mathcal{D}| - \sum_{r=1}^{N_R-1} x_r.$$
(22)

1 The equilibrium computation finds N_R variables: the credit price, the number of active
 2 drivers, and the assignment in the first $N_R - 2$ regions. The set of active drivers \mathcal{D} is retrieved from
 3 the number of active drivers by selecting the ones with the lowest reservation prices.

4 **Simulating the day-to-day markets evolution and the transition to equilibrium**

5 To assess the equilibrium prediction quality, the convergence speed, and the smoothness of the
 6 transition, we represent the day-to-day transition between traffic states under different TCS con-
 7 straints. The credit market size is tiny compared to the one presented in the previous TCS-related
 8 works, as the number of RH drivers dwarfs the number of commuters. We thus assume the drivers
 9 do not trade directly with each other, as they may have trouble finding a seller or buyer. Instead,
 10 they exchange with a credit bank that buys and sells credits at a regulated price. However, the credit
 11 cap might be violated in the transition between two TCS as every request to buy or sell credits at
 12 the regulated price is accepted. The price depends on the difference between the credit offer and
 13 demand as the bank aims to sell as many credits as it buys to reach a neutral budget. The different
 steps of the day-to-day process are presented in Fig. 4. Each day begins with an update of the TCS

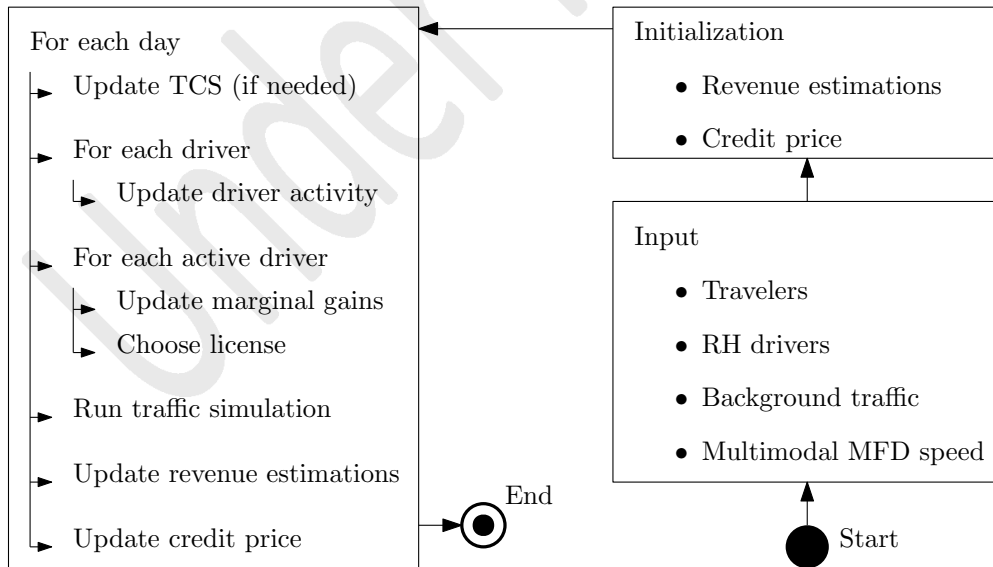


FIGURE 4: Simulation of the day-to-day RH operations.

14
 15 if the current TCS differs from the previous day. Each driver chooses if it joins the RH service
 16 for the day or not. The active drivers then choose their licenses. The revenue estimations and the
 17 credit price are updated to provide a basis for the next day's decisions. Each step is detailed in the
 18 following paragraphs.

1 *Updating active car drivers*

2 Every day, the number of active car drivers is updated using an estimation of the average RH
 3 revenue in the whole city (all regions combined) \tilde{R}^{avg} based on past days' observations. The
 4 drivers decide if they want to join the RH service for the day. Note that the license choice occurs
 5 later. The RH drivers for whom the estimated RH revenue is higher than their reservation price
 6 will join the RH market and thus get their free allocation of credits. The other drivers do not take
 7 part in the RH services. They wait the next day to reevaluate the estimation of the RH revenue.
 8 Note that the credit price does not impact the average RH revenue because the credit consumption
 9 is balanced in the long term. Thus, the total money flow from the RH drivers to the regulator is
 10 null at equilibrium.

11 *Car drivers choice of license*

The active drivers then choose the license they will acquire with credits, buy the required credits
 or sell the extra ones. They consider the estimation of marginal gains for changing their license to
 a more expensive or cheaper one. We first define the marginal gain MG_r of adding access to region
 r for a driver who already can access the region $r+1$. The marginal gain is the average revenue of
 region r minus the price of the additional credit charge:

$$\begin{aligned}
 MG_r &= \tilde{G}_r^{\text{avg}} - \tilde{G}_{r+1}^{\text{avg}} \\
 &= \frac{\tilde{R}_r(\text{day})}{\sum_{r' \leq r} x_{r'}} - p(\text{day})(\tau_r - \tau_{r+1}), \forall r \in [1, N_R - 1].
 \end{aligned}
 \tag{23}$$

12 A positive marginal gain for region r means switching from license $r+1$ to r will increase the
 13 driver's profit. It means the additional revenue a driver can earn by operating in this additional is
 14 higher than the additional money needed to buy the required license. Conversely, negative marginal
 15 gain means accessing the new market is not worth the extra credit cost.

16 After each driver choice, the marginal gains MG_r are updated as the distribution of license
 17 holders x changes, and thus the denominator of the first term in Eq. (23) varies. For the case of a
 18 city split into $N_R = 3$ regions, the current RH driver chooses its next license following the diagram
 in Fig. 5. The license choice depends on the signs of MG_1 , MG_2 , and the sum $MG_1 + MG_2$. For

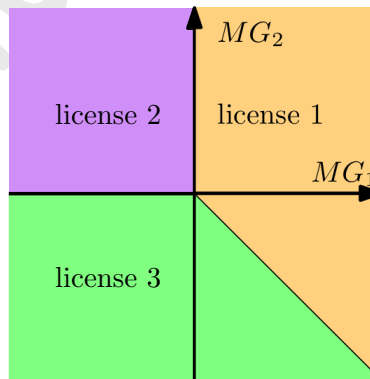


FIGURE 5: Driver assignment depending on the evaluation of the marginal gains.

19

20 example, MG_1 negative and MG_2 positive means a RH driver currently owning a license 3 has an
 21 interest in switching to license 2 (extra RH revenue higher than credit cost) but not to license 1
 22 (extra credit cost higher than additional RH revenue).

1 *Updating revenue estimation*

2 After the traffic simulation, the estimated RH revenue for each region \tilde{R}_r is updated using the
3 observed revenue for the current *day*:

$$\tilde{R}_r(\text{day} + 1) = \tilde{R}_r(\text{day}) - \frac{1}{\sqrt{\text{day} - T_\tau}} (\tilde{R}_r(\text{day}) - R_r). \quad (24)$$

4 The update factor decays with time and is reset at each time T_τ the regulator modifies the credit
5 charges. T_τ satisfies $T_\tau \leq \text{day} < T_{\tau+1}$, $T_{\tau+1}$ being the date of the next TCS change.

The estimated average revenue is also updated following the current day RH revenue:

$$\tilde{R}^{\text{avg}}(\text{day} + 1) = \tilde{R}^{\text{avg}}(\text{day}) - \frac{1}{\sqrt{\text{day} - T_\tau}} \left(\tilde{R}^{\text{avg}}(\text{day}) - \frac{\sum_r R_r}{|\mathcal{D}|} \right). \quad (25)$$

6 *Updating credit price*

The credit price is updated according to the credit consumption to reach a budget-neutral state.

$$p(\text{day} + 1) = \max \left(0, p(\text{day}) + \frac{\Delta_p}{\sqrt{\text{day} - T_\tau + 1}} \sum_{i \in \mathcal{D}} (\tau_{lic_i} - \kappa) \right), \quad (26)$$

7 The factor associated with the credit balance also decays with the number of days since the last
8 change of TCS, to smoothen the price and ensure convergence. The regulator sets the sensitivity
9 of the price change to the credit balance Δ_p . lic_i is the license chosen by the driver i .

10 **CASE STUDY**

11 We illustrate the proposed methodology with an example. The case study is designed to quantify
12 the effect of different TCS on different stakeholders at different aggregated levels, such as mode
13 shares, drivers' assignments, and total travel costs.

14 **Transportation network**

The fictive city is a square with a side length of 12 km. It is split into $N_R = 3$ regions, as presented in Fig. 1. The distance between two points is expressed with the Manhattan distance (the sum of the absolute difference between each coordinate), assuming the road network is a grid. PT mean speeds and access times depend on the departure and arrival regions, as presented in Table 1. Transit is faster and more frequent in the city center. The access times also account for the walking time from origin to station and from station to destination determined by the trip's highest region. The access time is divided by two for combined trips (RH and PT), as the RH vehicle picks up or leaves the passenger close to the transit station. The PT fare f_{PT} is set to 1 EUR per trip.

Origin/Destination	1	2	3
1	7 (5)	6 (10)	6 (15)
2	6 (10)	6 (10)	5 (15)
3	6 (15)	5 (15)	5 (15)

TABLE 1: PT mean speeds (m/s) and headtimes (between brackets, min) for the different region OD pairs.

The PT fare system is simplified as it does not depend on the distance traveled or the number of visited regions. It usually holds for small to medium cities but not for large metropolis. The city is

represented as a unique MFD region for computing the trips:

$$V(n) = V_{\max} \left(1 - \frac{n}{n_{\max}} \right). \quad (27)$$

1 The maximum speed V_{\max} and the maximum accumulation n_{\max} are set to 10 m/s and 5 000 vehi-
2 cles, respectively.

3 Demand

4 We generate 1 000 travelers as individual travelers. Each agent has a VoT drawn from a uniform
5 distribution between 20 and 100 EUR/h. Their departure times are drawn from a uniform dis-
6 tribution between 0 and 1 hour. The trips are generated by randomly picking up the origin and
7 destination regions. The probability associated with each pair of regions is presented in Table 2.
More trips depart or arrive in regions close to the center. The origin and destination points are

Origin/Destination	1	2	3
1	5/34	4/34	4/34
2	4/34	4/34	3/34
3	4/34	3/34	3/34

TABLE 2: Demand distribution for the different region OD pairs.

8
9 selected by randomly drawing a point for each region.

10 Background traffic is generated with 3 000 cars following the same distribution of trip
11 lengths. The departure time distributions of the background traffic and MaaS customers are shown
12 in Fig. 6.

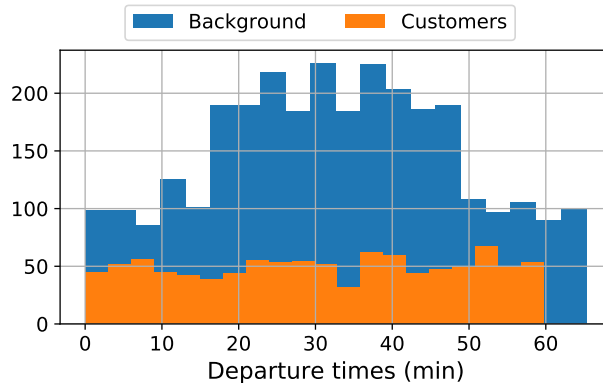


FIGURE 6: Departure time distribution of the background traffic and the MaaS customers.

13 These background vehicles only affect the traffic conditions, as those users do not change
14 mode (thus do not request RH rides) and do not take part in the TCS. We simulate with only the
15 background traffic first and add the car accumulation due to this background traffic when running
16 the primary simulation with MaaS operations. We neglect the impact of RH drivers on background
17 trips, as the number of RH vehicles is significantly lower than the number of background vehicles.

1 RH offer

2 We generated a pool of $|\mathcal{D}_0| = 150$ potential drivers, with reservation prices uniformly generated
 3 between 10 and 50 EUR. Note that the RH operations have a secondary impact on the traffic
 4 condition. The background traffic significantly affects the congestion level. We set the RH fee at
 5 2 EUR/km. The matching period is 2 min. The RH requests are buffered during each matching
 6 period. Then a matching process assigns available RH drivers to the passengers. If a passenger is
 7 not matched, it will be sent to the next batch. After three failures, the passenger will ride PT.

8 TCS design

9 The regulator provides free $\kappa = 10$ credits to each *active* driver. It successively introduces TCS to
 10 restrict the access of regions 1 and 2. For the first ten days, no TCS is applied, then on day 10, the
 11 credit charges are set to [15,10,0], [20, 10, 0] on day 50, and finally [40, 20, 0] on day 90. For
 12 example, a driver needs to spend 20 credits daily to operate in region 1 between day 50 and day
 13 89. It will require 40 credits afterward.

14 NUMERICAL RESULTS

15 We first estimate the effect of TCS on the RH service at equilibrium. Thanks to the problem
 16 size reduction (22), there are only three unknowns: the number of active drivers $|\mathcal{D}|$, the number
 17 of license 1 holders x_1 and the credit price p . We use the tool *differential evolution* from the
 18 Python toolbox *SciPy* (12) to minimize the cost function J . The tool is based on genetic algorithm
 19 techniques, more precisely on the algorithm developed by (13). We only have to compute the
 20 number of active car drivers for the no TCS case. The cost function is J_2 , i.e., only the error
 21 associated with active drivers.

22 We then look into the transition from the status quo to different TCS. The regulator grad-
 23 ually introduces the TCS for RH drivers in the following day-to-day scenario. The introduction
 24 of the TCS affects the average RH revenue and thus the number of active RH drivers, as seen in
 Fig. 7. As expected, the RH revenue (Fig. 7(a)) decreases with the TCS as the RH drivers are more

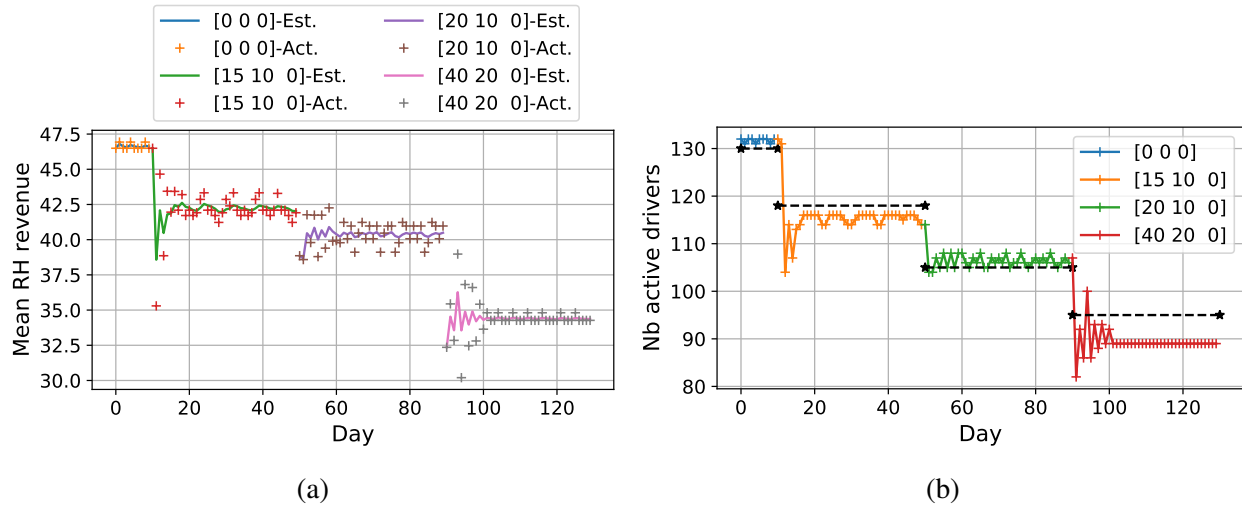


FIGURE 7: Evolution of (a) the average actual (Act.) RH revenue and its estimation (Est.) and (b) number of active RH drivers. The black dashed lines are the equilibrium values for the corresponding TCS computed with the heuristic method.

1 constrained in the type of trip they can serve. When the TCS changes, the estimation of the average
 2 revenue undergoes some oscillations. The RH market needs some days to adapt to the new TCS. It
 3 converges after two to three days. As a consequence, the number of active RH drivers (Fig. 7(b))
 4 decreases as the RH revenue does not exceed their reservation price anymore. The equilibrium
 5 computation reasonably estimates the number of RH drivers for the no TCS and the first two TCS
 6 with an error of less than 4%. The equilibrium computation overestimates the number of active
 7 drivers by 7% for the last TCS. We evaluate the value of the cost function with the average value
 8 over the last 10 days. The value is 24.9. It is far higher than the costs associated with the result
 9 of the equilibrium computation 0.2. The explanation is the equilibrium computation of minimiz-
 10 ing J (difference with the maximum gain weighted by the number of license holders, Eq. (18)) is
 11 not equivalent to the day-to-day process, which involves assigning drivers to the most profitable
 12 licenses.

13 The cap of credits is not a hard constraint in the transient simulation as the bank can punctu-
 14 ally sell more credits than the number it buys from RH drivers. The evolution of the credit balance
 (consumption minus allocation) and credit price is presented in Fig. 8. The credit balance and,

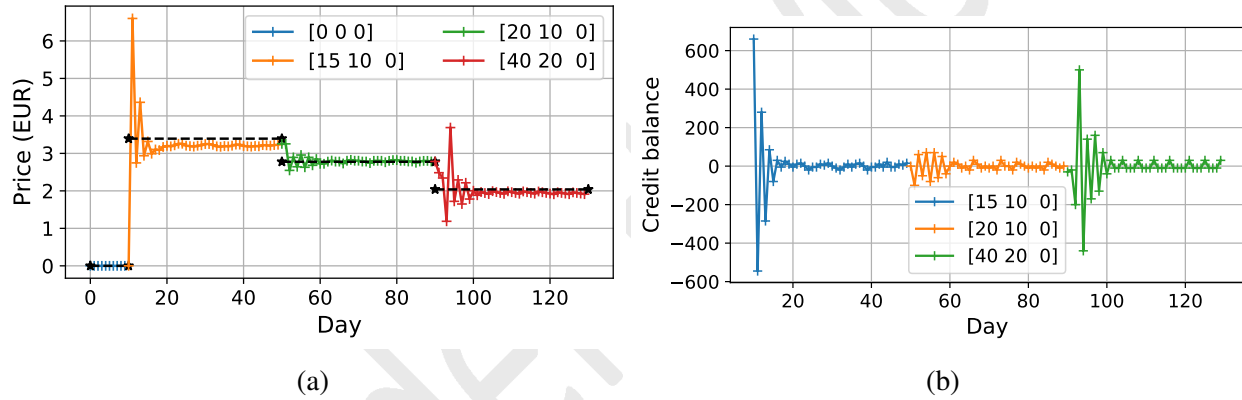


FIGURE 8: Evolution of (a) the credit price and (b) credit balance. The black dashed lines are the equilibrium values for the corresponding TCS.

15 consequently, the credit price are destabilized after each change of TCS, and it takes a few days to
 16 reach the new equilibrium with a balance of credit consumption. After introducing the first TCS,
 17 the overshoot represents about 100% of the equilibrium value. Based solely on the previous day's
 18 observation (no TCS), the RH drivers chose their assignment as per no TCS, resulting in a great
 19 credit imbalance and, thus, a proportional and strong price correction. The equilibrium computa-
 20 tion (black dashed lines) gives a close estimate of the credit price, with an error of less than 7%.
 21 The credit price decreases as the TCS becomes more constraining. Note that, as the drivers require
 22 more credits to operate in the inner regions, the money required to operate in the city center usually
 23 increases even if the credit price decreases.

24 The evolution of the equivalent license prices, i.e., the out-of-pocket money spent or earned
 25 by buying or selling the required credits $(\tau_r - \kappa)p$, is presented in Fig. 9. The number of RH
 26 drivers buying the license for region 1 is also shown. The more constraining the TCS is, the more
 27 expensive the access to region 1 for RH drivers is. During the first two TCS, where $\tau_2 = \kappa$, the
 28 free allocation covers the need to access region 2. Its access is thus free. As no credits are required
 29 for region 3 ($\tau_0 = 0$), only drivers accessing region 3 earn money in the credit market as they
 30

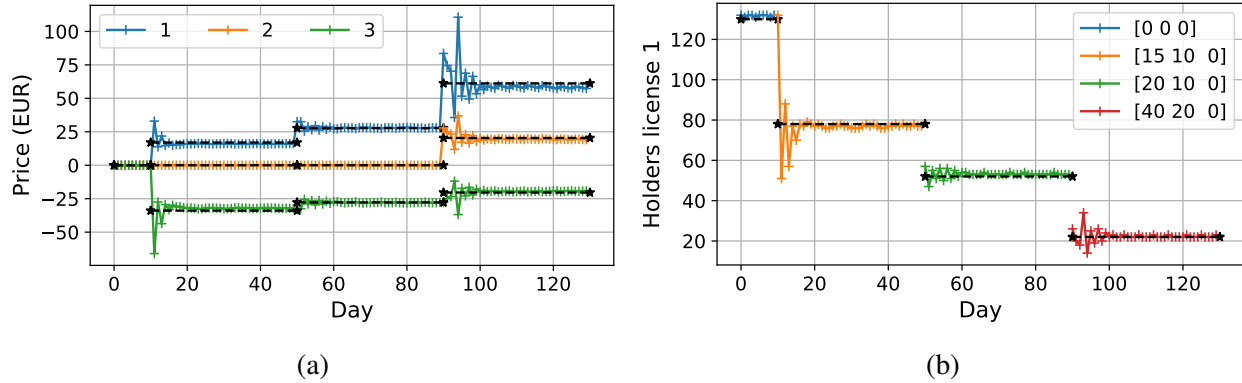


FIGURE 9: Evolution of (a) the equivalent license prices and (b) the number of RH drivers active in the city center. The black dashed lines are the equilibrium values for the corresponding TCS.

1 sell their allocation. The equilibrium computation allows for a good prediction of the RH drivers’
 2 assignment. The error is at the maximum of 4%. The number of drivers operating in the city center
 3 decreases as the TCS becomes more constraining. Without TCS, 87% of the workforce operates
 4 in region 1 against 14% with the last TCS.

Fig. 10 shows the evolution of mode choice. The TCS decreases the number of active

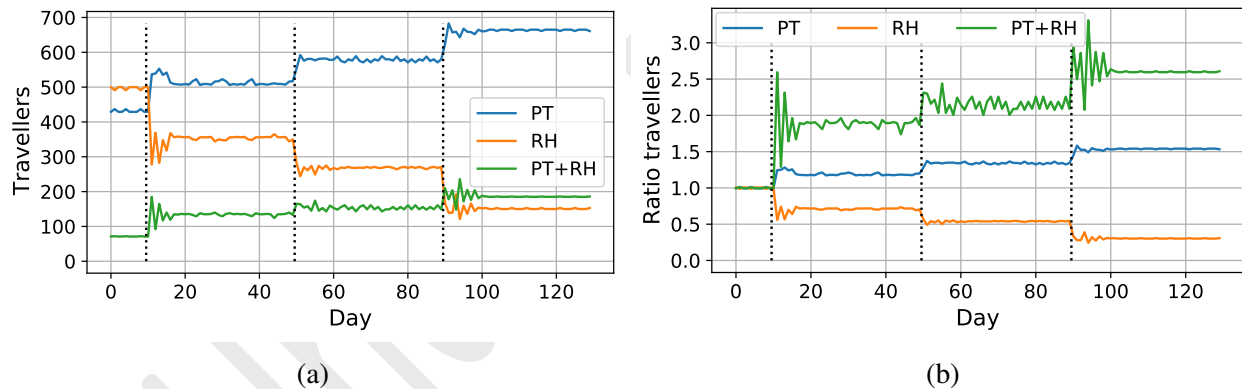


FIGURE 10: Mode choice evolution: (a) absolute and (b) relative. The reference is the no TCS case. The black vertical dotted lines mark the TCS changes.

5
 6 drivers and the number of drivers operating in the city center. Customers tend to shift to PT or
 7 combined trips (PT+RH) because they face higher RH travel costs, or RH drivers are unavailable.
 8 In absolute (Fig. 10(a)), the majority of the customers report on PT. However, the relative change
 9 (Fig. 10(b)) is higher for combined trips (PT+RH), with an increase of more than 2.5 for the last
 10 TCS.

11 Table 3 details the mode changes between the no TCS case and the last TCS ([40,20,0])
 12 for each pair of regions of origin and destination. The TCS makes the shares of RH-only rides
 13 decrease for all trips, except for trips inside the outermost region 3, where RH share increases by
 14 17 points. PT share increases, except for trips between 2 and 3 and inside 3, where it decreases
 15 by 8 to 19 points. The share of combined trips increases between regions 1 and 3 and 2 and 3.
 16 It reaches 0 for trips between 1 and 2. However, it was already small without TCS. The induced

O/D	1			2			3		
	PT	RH	PT+RH	PT	RH	PT+RH	PT	RH	PT+RH
	no TCS								
1	48	52	-	43	54	3	33	51	16
2	42	38	16	45	55	-	39	55	6
3	47	41	12	41	53	6	48	52	-
	TCS [40,20,0]								
1	95	5	-	98	2	0	43	19	37
2	95	5	0	98	2	-	28	22	49
3	48	11	41	33	10	57	31	69	-

TABLE 3: Mode shares (%) without TCS and with the last TCS ([40,20,0]) for the different pairs of regions. **Red** means the mode share decreases with the TCS and **blue** means it increases.

1 shortage of RH drivers allowed to operate in region 1 nudge travelers to use PT in the city center.
2 Instead, the RH drivers are available in the periphery to drain passengers from the outside regions
3 to PT stations at the center border and pick up passengers at those stations to drive them to their
4 destination in the suburbs.

5 The transition of TCS leads to large overshoots for the credit price, number of active drivers,
6 and driver assignment. We assume the regulator does not tolerate large oscillations, as the lack of
7 smoothness renders the TCS unpopular with RH drivers and travelers having to change their travel
8 habits drastically on consecutive days. We propose to use the knowledge of the credit price from
9 the equilibrium computation to provide a warm start. At the beginning of a new TCS period,
10 the credit price is arbitrarily fixed to the equilibrium price calculated for the new policy setting.
11 It becomes flexible again after three days of operation with the new policy. The evolution of the
12 number of active drivers, credit price, and number of drivers operating in the city center is presented
13 in Fig. 11. Thanks to the warm start from the equilibrium computation, the system avoids large
14 overshoots when transitioning from one TCS to another. Without a warm start, the amplitudes of
15 the oscillations for the number of active drivers (Fig. 7(b)) are up to 14%, more than 100% for
16 the credit price (Fig. 8(a)), and 34% for the number of license 1 holders (Fig. 9(b)). With the
17 warm start, the magnitude of the oscillations magnitudes are respectively 10% (Fig. 11(a)), 22%
18 (Fig. 11(b)), and 30% (Fig. 11(c)).

19 The communication of the equilibrium price value simultaneously with the introduction of
20 the TCS allows the RH drivers to adapt to the new TCS faster. The equilibrium values are similar
21 with and without warm starts. It is important for acceptability that the stakeholders can predict the
22 effect of the TCS before its introduction. They can thus plan their response to the new TCS and
23 not only react to the new schemes.

24 The global effect on the transportation system can be assessed by the total travel time and
25 the total driven distance in Fig. 12. The first TCS leads to an increase of the travel time of 22%,
26 travel cost of 20%, and a decrease of RH distance of 13%. The second TCS increases the travel
27 time by 40%, travel cost by 35%, and decreases the RH distance by 21%. The third increases
28 the travel time by 61%, the travel cost by 55%, and decreases the RH distance by 36%. To put
29 the TCS into perspective, removing the RH service completely and having all travelers riding the
30 PT increases the total travel time by 87% and the travel cost by 75%. The RH-traveled distance

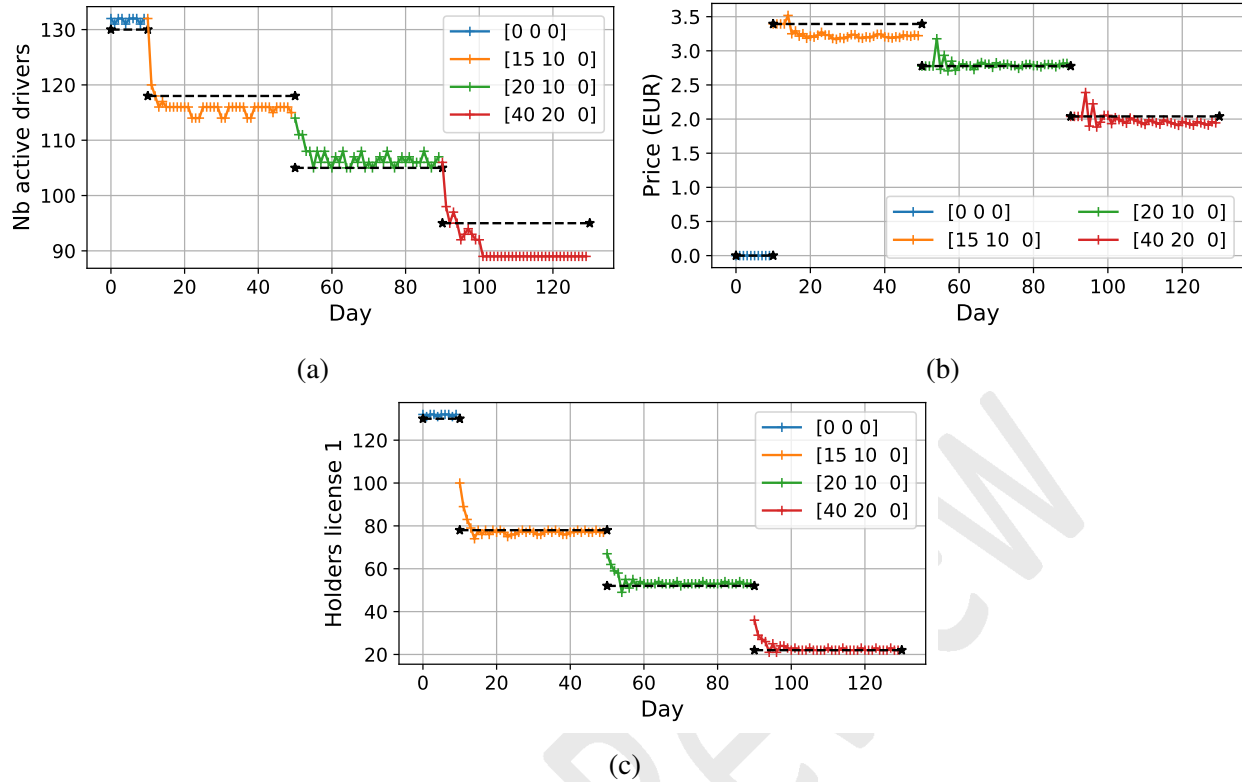


FIGURE 11: (a) Number of active drivers, (b) credit price, and (c) number of license 1 holders when providing a warm start for each TCS change.

1 is zero as no RH vehicle operates. Total travel time (Fig. 12(a)) increases since some passengers
 2 using RH need to take PT with the introduction of the TCS. The effect of RH on traffic conditions
 3 is secondary to the background traffic. Thus, the decrease in RH activities does not significantly
 4 improve the driving conditions. The total travel cost (Fig. 12(b)) includes the travel times weighted
 5 by the VoT and the fees paid for using RH and PT. The total driven distance (Fig. 12(c)) decreases
 6 since more commuters use PT only or in combination with RH. As a benchmark, the total travel
 7 time is represented when travelers can only ride transit.

8 The impacts of TCS on total travel time and driven distance are presented as a Pareto
 9 front in Fig 13 to highlight the compromises between the two objectives. The first seven days are
 10 removed to keep the states close to equilibrium. The total driven distance is a proxy for the negative
 11 externalities of RH. The TCS reduces the total driven distance and increases the total travel time.
 12 TCS is a tool to regulate RH continuously between the unregulated RH operations and the ban
 13 of RH services. It proposes different compromises regarding sustainability and minimum level of
 14 service for the regulator to match its objectives.

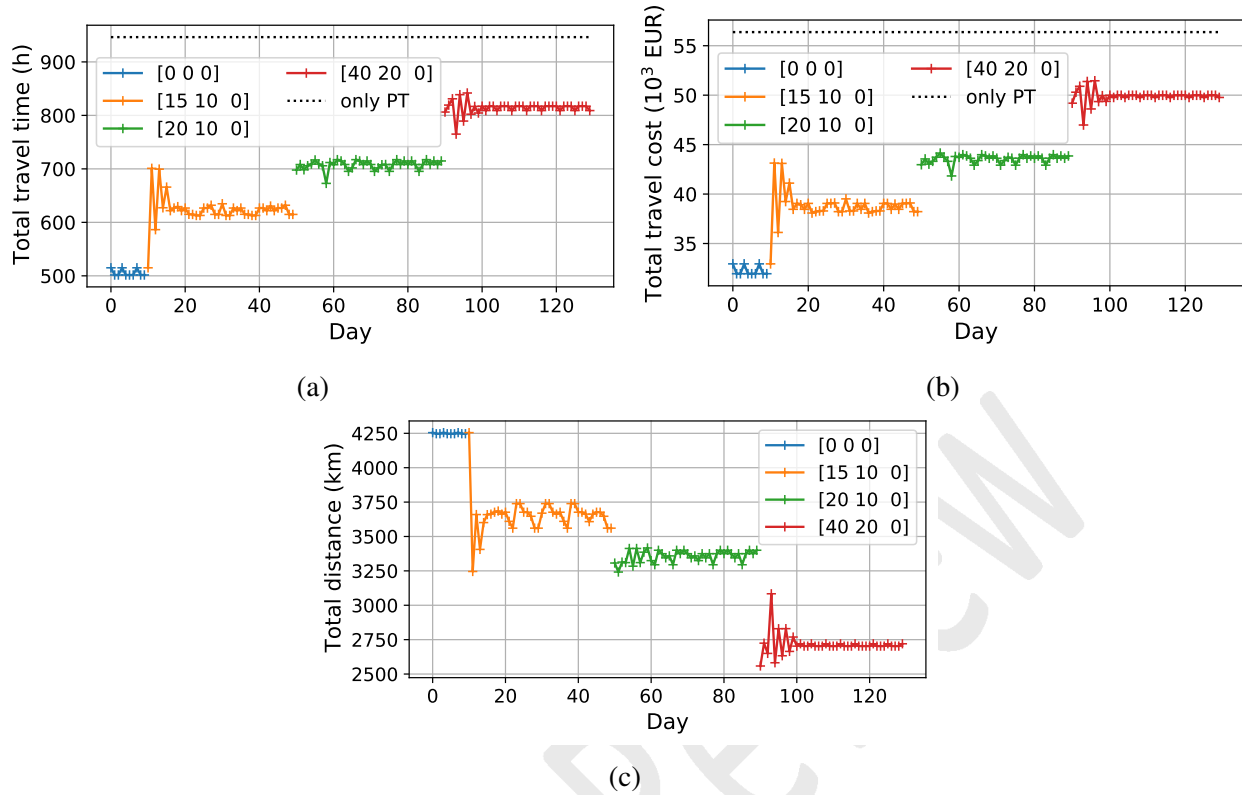


FIGURE 12: (a) Total travel time, (b) travel cost, and (c) driven distance (including pick-up distance). Without warm starts of the credit price.

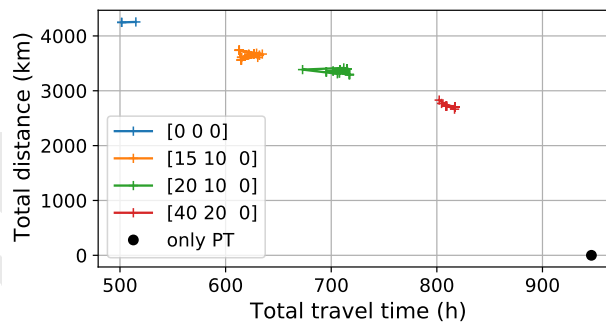


FIGURE 13: Total travel time and PT revenue.

1 CONCLUSIONS

2 We propose a policy based on TCS to regulate the RH services and foster a combination of RH
 3 and PT. The regulator provides a free allocation of credits to each active RH driver. Operating
 4 in different regions requires credits. The more central the region is, the more credits are needed,
 5 as the regulator wants RH drivers to operate in regions with low PT service levels. The novelty
 6 lies in regulating RH operations within a TCS framework. Indeed, TCS has been proposed in the
 7 literature to regulate the travel demand but not the transportation offer. We developed an MFD-
 8 based framework to compute the travel times and track the position of the RH drivers. Travelers

1 are matched with RH drivers or assigned to PT via a MaaS platform. We formulate the equilibrium
2 under a TCS regarding the assignment of the drivers in the different regions, the offer elasticity,
3 and the credit price. The equilibrium is computed with a heuristic method. The transition between
4 the no TCS case and the TCS is represented by a day-to-day process where the credit price and
5 driver choices (activity and operating regions) are updated daily depending on the credit balance
6 and the RH revenues. A numerical case study illustrates the effect of the TCS. The TCS forces RH
7 drivers to operate less in the city center and more in the suburbs. Consequently, travelers prefer
8 riding PT or combining PT and RH to reach their destination. The total travel time increases, and
9 the total traveled distance decreases. It offers a range of compromises for the regulator to protect
10 the PT from the unfair RH competition instead of the binary choice of allowing/forbidding RH
11 operations.

12 This study does not account for demand elasticity. Depending on the level of service of the
13 proposed MaaS and the effect of the TCS on the operation of the RH drivers, some travelers may
14 switch from private cars to PT+RH or, on the opposite, switch from RH to private cars. Future
15 work should investigate TCS to reduce private car trips by replacing them with a combination of
16 PT and RH and, in the long term, reduce car ownership.

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