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# Launch proposals for a space projectile

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## Abstract

Propulsion based on circular trajectory acceleration may, in the future, be feasible only for the transport of materials due to the stringent time limits on human tolerance to high accelerations. In addition to acceleration, the projectile is subjected to significant tensile and bending forces during circular motion before launch, which greatly restricts the types of materials that can be transported. However, once viscous drag due to the atmosphere becomes negligible, it may be possible to stabilize the orbit using a specially designed accelerator, potentially located at the apogee of the trajectory.

## 1 Introduction and the challenge

The Merriam-Webster dictionary defines *projectile* [1] as “a body projected by external force and continuing in motion by its own inertia”, thus it is as an object propelled initially by an external force and subsequently maintaining its motion through its own inertia, gravitational force, friction with its surrounding medium and other interactions. Unlike propelled objects, a projectile lacks its own means of propulsion and instead moves solely by the inertia imparted at the start of its trajectory.

Over the years, numerous solutions to the “Non-rocket spacelaunch” problem have been proposed [2, 3] and, in particular, [4]. This kind of problem attempts to propose alternative solutions to the limits imposed by the rocket equation. Since the rocket equation sets limits on the maximum velocity change for a rocket that must also carry its own fuel to operate, these solutions aim to both develop increasingly efficient propulsion systems and find

alternative equations that can lead to new propulsion techniques. Among these techniques, some involve the rocket not carrying any type of fuel and having the energy imparted during launch. In this article, we demonstrate the physics of such launches, where the rockets are effectively projectiles without internal propulsion.

For simplicity, we shall consider the non-relativistic case of projectile motion [5], especially given the focus on the practical realization of the apparatus. It's evident that velocities where relativistic effects play a significant role won't be reached. For this reason, we'll assume familiarity with the basics of Newtonian mechanics and their mathematical treatment. To be comprehensive, we'll endeavor not to neglect any effects arising from interactions with gravitational fields and the medium through which the motion occurs.

Currently, the most widely used propulsion strategy is thermal propulsion through chemical reactions [6], which leads to the release of large quantities of volatile and polluting substances into the atmosphere, similar to most combustion processes used in the propulsion of cars, trucks, and airplanes. Specifically, a rocket launched via thermal propulsion has its maximum mass at the moment of liftoff and gradually becomes lighter as mass is lost due to the combustion of fuel. This process of mass loss forms the basis of the physics underlying the rocket equation [7, 8].

This issue has already become significant today, prompting several attempts to mitigate both the environmental impact and the limitations imposed by the physics of the rocket equation. Notable among these are the SpinLaunch [9, 10], as well as [11, 12]. However, we will demonstrate that propulsion based on circular trajectory acceleration may, in the future, be feasible only for the transport of materials, while proving entirely unsuitable for human transport due to the strict time limits on human tolerance to high accelerations. In this paper, we aim to initiate a study of potential alternatives to the rocket equation by evaluating the validity of existing proposals and introducing new alternatives [13].

We shall denote the angular momentum vector of a particle relative to the origin  $\mathbf{L} = I\boldsymbol{\omega} = I\frac{\mathbf{r}\times\mathbf{v}}{|\mathbf{r}|^2}$  where [14]

- $I$  is the moment of inertia
- $\boldsymbol{\omega}$  is the orbital angular velocity
- $\mathbf{r}$  is the position vector

- $\mathbf{v}$  is the linear velocity and  $v = \|\mathbf{v}\|$ .

Let denote the torque vector by the symbol  $\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F} = \frac{d}{dt}\mathbf{L} = \dot{\mathbf{L}}$  where

- $\mathbf{F}$  is the force vector
- $\mathbf{L}$  is the angular momentum vector
- $\dot{\mathbf{L}}$  is the dot derivative (derivative with respect of time  $t$ )

We will also use the symbols indicated in Table 1 for the corresponding quantities.

## 2 Launching a projectile via a centrifuge

A centrifuge is a device that harnesses circular motion to accumulate kinetic energy in a body, which is subsequently disconnected from the centrifuge and continues moving due to inertia until the complete dissipation of energy, typically through friction. The rotational energy  $E_{rot}$  of a rotating body with moment of inertia  $I$  and orbital angular velocity  $\omega$  is  $E_{rot} = \frac{1}{2}I\omega^2$  (we want to point out the resemblance between the formula for kinetic energy  $E_{kin} = \frac{1}{2}mv^2$ ).

### 2.1 $\omega$ and angular acceleration in centrifuge with drag

Let  $P$  be the power delivered by a centrifuge required to set a body with inertia  $I$  into rotation. Power is defined as the rate of change of work. Hence power is work per unit time, given by  $P = \boldsymbol{\tau} \cdot \boldsymbol{\omega}$  where  $\boldsymbol{\tau} = \frac{d}{dt}(I\boldsymbol{\omega}) = I\dot{\boldsymbol{\omega}} + \dot{I}\boldsymbol{\omega} = I\boldsymbol{\alpha}$  and  $\boldsymbol{\alpha} = \dot{\boldsymbol{\omega}}$ . Assuming  $\dot{I} = 0$ .

Let the drag equation  $F_D = \frac{1}{2}\rho v^2 C_D A$ .  $C_D = C_D(R_e)$  is a function of the shape of the object and another dimensionless parameter  $R_e = \frac{\rho v D}{\mu}$ .

Consider the system illustrated in figure 1, where a body consists of two rods of lengths  $r_m$  and  $r_M$  respectively, connecting two masses  $m$  and  $M$  to the central pivot undergoing rotation.

For a circular motion of radius  $r$ ,  $\omega = \frac{v}{r}$ . In circular motion, the drag force is not constant at every position vector. Intuitively,  $F_D$  depends on the linear velocity  $v$ , which in turn depends on  $r$  because, as an object moves further from the center (increasing radius), it has to cover a greater distance in the same amount of time to complete one revolution. Therefore, to maintain

Symbol	Quantity	Unit of measure
$m, M$	mass	$[kg]$
$v$	speed	$[\frac{m}{s}]$
$\rho$	density	$[\frac{kg}{m^3}]$
$C_D$	dimensionless coefficient called drag coefficient	$[-]$
$A$	specific area of the object	$[m^2]$
$D$	specific linear dimension of the moving object	$[m]$
$\mu$	dynamic viscosity of the medium	$[\frac{kg}{ms}]$
$\alpha$	angular acceleration vector	$[\frac{rad}{s^2}]$
$Re$	Reynolds number	$[-]$
$\delta$	elastic deflection	$[m]$
$F_D, \tau_D, P_D$	drag force, torque and power respectively	$[N], [Nm], [W]$
$E$	modulus of elasticity	$[\frac{N}{m^2}]$
$I_A$	area moment of inertia	$[m^4]$
$\zeta$	standard gravitational parameter	$[m^3/s^2]$
$h$	specific relative angular momentum	$[\frac{m^2}{s}]$
$\epsilon$	specific orbital energy	$[J]$
$a$	semi-major axis	$[m]$
$b$	semi-minor axis	$[m]$
$c$	linear eccentricity	$[m]$
$e$	eccentricity	$[-]$
$R_{\oplus}$	radius of the Earth	$[m]$
$\eta$	reduced mass	$[kg]$

Table 1: Symbols used in this paper. Same quantities in different contexts use the same symbol with an added superscript, subscript or both.

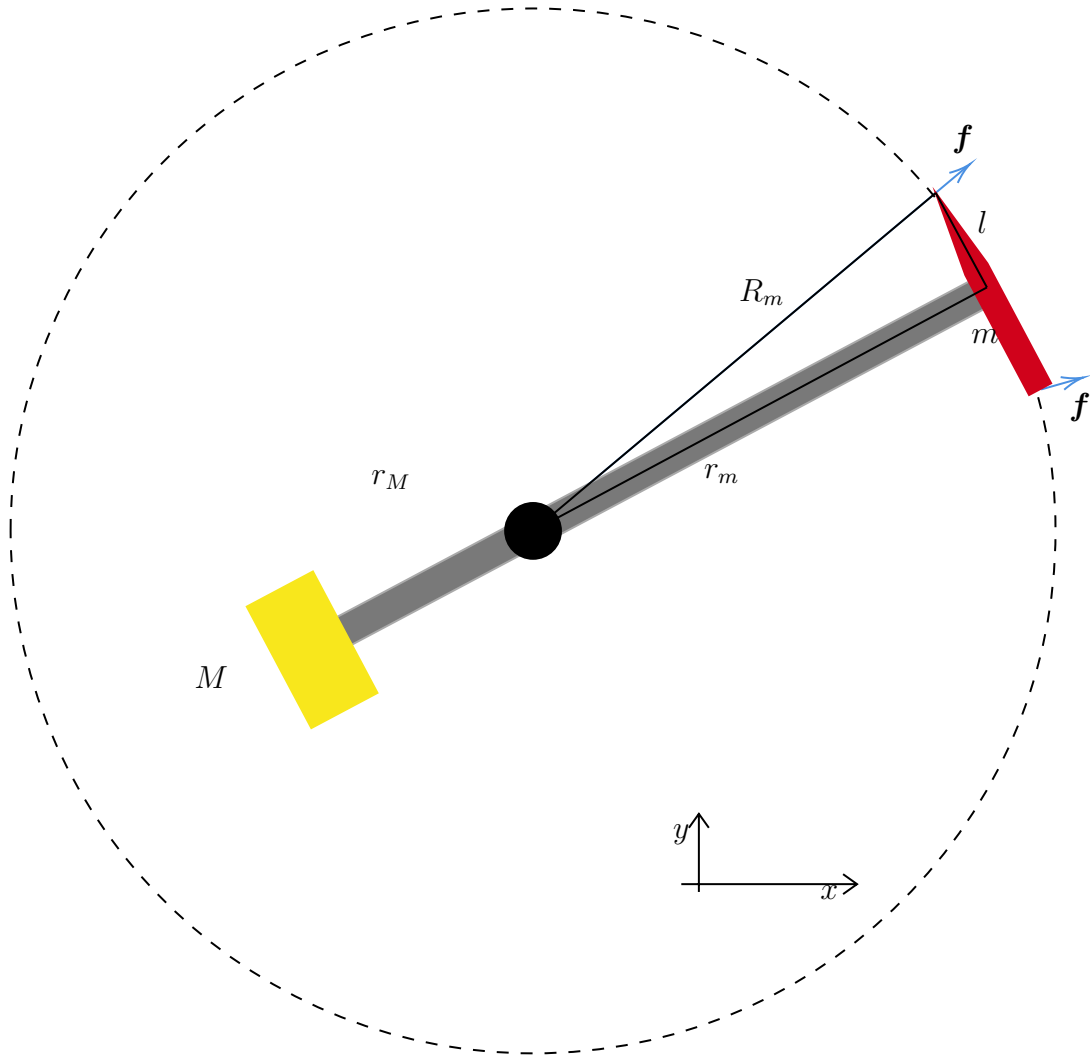


Figure 1: The basic setup of the centrifuge involves rotating the projectile of mass  $m$  and length  $l$  at distance  $r_m$ . A counterweight of mass  $M$  is placed in opposition to the projectile at distance  $r_M$  to balance the centrifuge.

the same angular velocity, the object must increase its linear velocity as the radius increases. Hence, we can either assume that all the drag force is applied at outermost end of the rotating body  $\|\mathbf{r}\|$  and consider  $F_D = \frac{1}{2}\rho\omega^2r^2C_D A$ , or integrate  $F_D$  for each element and derive a more general formula. We notice that for our purposes, the shape of the two rotating

bodies  $m$  and  $M$  (as long as they are chosen to minimize drag) is not crucial for the study of motion. Therefore, we shall consider the simplest case where  $F_D = \frac{1}{2}\rho\omega^2r^2C_D A$ ,  $\tau_D = rF_D = \frac{1}{2}\rho\omega^2r^3C_D A$  and  $P_D = \omega\tau_D = \frac{1}{2}\rho\omega^3r^3C_D A$ .

$P_D$  opposes  $P$  delivered by the centrifuge. Then, we have  $I\alpha\omega = P - P_D$  or  $P = I\dot{\omega}\omega + \frac{1}{2}\rho\omega^3r^3C_D A$  which is a nonlinear differential equation in  $\omega(t)$  where we assume that the motion start from rest  $\dot{\omega}(0) = 0$  and at the starting position  $\omega(0) = 0$ . For a spherical object,  $D$  is its diameter and  $A = \pi\frac{D^2}{4}$ . Then,  $F_D = 6\pi\omega r D\mu$ ,  $\tau_D = 6\pi\omega r^2 D\mu$  and  $P_D = 6\pi\omega^2 r^2 D\mu$ . And the previous differential equation reduces to  $P = \omega(I\dot{\omega} + 6\pi\omega r^2 D\mu)$ , from which we

obtain  $\omega(t) = \frac{1}{r}\sqrt{\frac{P\left(1 - e^{-\frac{12\pi D\mu r^2 t}{I}}\right)}{6\pi D\mu}}$  and  $\alpha(t) = \frac{rP}{I}e^{-\frac{12\pi D\mu r^2 t}{I}}\sqrt{\frac{6\pi D\mu}{P\left(1 - e^{-\frac{12\pi D\mu r^2 t}{I}}\right)}}$  =  $\frac{P}{I\omega(t)}e^{-\frac{12\pi D\mu r^2 t}{I}}$ . The terminal angular velocity is  $\omega_\infty = \lim_{t \rightarrow \infty} \omega(t) = \frac{1}{r}\sqrt{\frac{P}{6\pi D\mu}}$ , while the terminal angular acceleration is  $\alpha_\infty = 0$ .

The moment of inertia is given by the sum of the individual moments of inertia:  $I = I_M + I_m + I_{r_M} + I_{r_m}$  where  $I_M = Mr_M^2$ ,  $I_m = mr_m^2$ ,  $I_{r_M} = \frac{1}{3}m_{r_M}r_M^2 = \frac{1}{3}\rho_{r_M}^L r_M^3$  where  $\rho_{r_M}^L$  is the linear density of the rod.  $I_{r_m} = \frac{1}{3}m_{r_m}r_m^2 = \frac{1}{3}\rho_{r_m}^L r_m^3$ .  $\rho_{r_M}^L = \rho_{r_m}^L = \rho_r^L$  and  $M$  serves to counterbalance the motion of  $m$ . Then,  $Mr_M = mr_m$  and the total moment of inertia is  $I = mr_m(r_M + r_m) + \frac{1}{3}\rho_r^L(r_m^3 + r_M^3)$ . This is valid for an infinitely thin but rigid rotating rod.

## 2.2 Tendency of a projectile to deflect before being launched

A projectile subjected to a high angular velocity on a circular orbit experiences a centrifugal force outward from the system. The projectile is an extended body with a length  $l$  (see figure 1), so its ends are farther from the center of rotation than the anchoring point with the rotating rod. In particular  $R_m = \sqrt{r_m^2 + \frac{l^2}{4}} > r_m$ . Therefore, the centrifugal force exerted on the upper and lower ends is greater than the centrifugal force at its anchoring point. The difference  $\mathbf{f}$  between this two forces causes stress in the structure of the projectile, resulting in its deflection.

To simplify the calculations (and provide an acceptable upper limit) of the deflection, we shall assume that  $m$  is evenly distributed between its lower end, at the anchoring point and at the upper end (respectively  $\frac{m}{3}$ ,  $\frac{m}{3}$  and  $\frac{m}{3}$ ). The

angular velocity of  $m$  is the same for each element of mass  $\frac{m}{3}$ , but the upper and lower ends move on a trajectory with a radius  $R_m > r_m$ . Let  $\omega_{R_m} = \frac{v_{R_m}}{R_m}$  and  $\omega_{r_m} = \frac{v_{r_m}}{r_m}$  respectively the angular velocity of the lower (or upper) end and the anchoring point.  $\omega_{R_m} = \omega_{r_m}$ . Then,  $v_{R_m} = \frac{R_m}{r_m} v_{r_m}$ . Let  $\mathbf{f}$  be the difference between the centrifugal force exerted by the mass at the lower (or upper) end and the centrifugal force exerted by the mass at the anchoring point. Therefore,  $f = \|\mathbf{f}\| = \frac{m}{3} (a_{R_m} - a_{r_m}) = \frac{m}{3} \left( \frac{v_{R_m}^2}{R_m} - \frac{v_{r_m}^2}{r_m} \right)$ . But  $v_{R_m} = \frac{R_m}{r_m} v_{r_m}$  from the equality of angular velocities. Then,  $f = \frac{m}{3} a_{r_m} \left( \frac{R_m}{r_m} - 1 \right) = \frac{l}{3} \rho_m^L a_{r_m} \left( \sqrt{1 + \frac{l^2}{4r_m^2}} - 1 \right)$  where  $\rho_m^L$  is the linear density of  $m$ ,  $l$  its length and  $a_{r_m}$  the centrifugal acceleration at the anchoring point.

Basically, the deflection of the projectile subjected by  $f = \|\mathbf{f}\|$  can be approximated to the elastic deflection  $\delta = \frac{fl^3}{24EI_A}$  of an end-loaded cantilever beams with length  $\frac{l}{2}$ , area moment of inertia  $I_A$  and modulus of elasticity  $E$ . The longitudinal section of the projectile can be considered a filled rectangle (if we consider the projectile a cylinder). The area moment of inertia (or second moment of inertia) along the  $x$ -axis of a filled rectangular area with base width  $2r_m$  and height  $l$  is  $I_A = \frac{2r_m l^3}{12}$ . Then,  $\delta = \frac{l}{12} \rho_m^L a_{r_m} \left( \sqrt{1 + \frac{l^2}{4r_m^2}} - 1 \right) \frac{1}{Er_m} = \frac{l}{12E} \rho_m^L \omega^2 \left( \sqrt{1 + \frac{l^2}{4r_m^2}} - 1 \right)$ .

### 3 Motion of a projectile in a gravitational field

A projectile is, by definition, a body devoid of internal propulsion: its motion is solely due to the imparted motion at the moment of launch. Let's briefly analyze the thought experiment attributed to Isaac Newton, from which it takes its name, "Newton's cannonball". It involves a cannon placed at a great height above the Earth's surface (sufficiently high to avoid colliding with other natural or artificial obstacles). In Newton's experiment, air friction is neglected, so the projectile is fired from the cannon and, depending on its initial velocity, traces different orbits. If the initial velocity is low (less than the orbital velocity at the height where the cannon is placed), then the projectile will fall to Earth after a short path. If the initial velocity is equal to the orbital velocity, the projectile will return to the starting point after tracing a circular orbit around the Earth. If the velocity is greater than the

orbital velocity but less than the escape velocity, the projectile will follow an elliptical orbit according to Kepler's laws. If the velocity is equal to or greater than the escape velocity, the projectile will leave Earth's orbit.

Consider the planetary system in figure 2, where the red circle represents the projectile with linear velocity  $\mathbf{v}$  orbiting around the planet (gray circle). According to Kepler's laws, the planet is approximately located at one of the two foci of the ellipse traced by the projectile itself. Specific orbital energy  $\epsilon$  is a constant quantity for a system consisting of two orbiting bodies.  $\epsilon = \left(\frac{v^2}{2} - \frac{\zeta}{d}\right) \eta$  where  $\zeta = G(m + M)$  is the sum of the standard gravitational parameters of  $m$  and  $M$  and  $\eta = \frac{mM}{m+M}$  is the reduced mass. As usual,  $a$  is the semi-major axis,  $b$  is the semi-minor axis,  $c = \sqrt{a^2 - b^2} = ea$  is the linear eccentricity where  $e = \frac{c}{a} = \sqrt{1 - \frac{b^2}{a^2}}$  is the eccentricity. It is demonstrated that  $\epsilon = -\frac{\eta \zeta^2}{2h^2} (1 - e^2) = -\frac{\zeta \eta}{2a}$  where  $h$  is the specific relative angular momentum. Then, given  $d$  and  $v$ . It is straightforward to prove that

$$\begin{aligned} a &= \frac{\zeta d}{2\zeta - v^2 d} \\ e &= \frac{v^2 d}{\zeta} - 1 \\ b &= \frac{vd}{2\zeta - v^2 d} \sqrt{d(2\zeta - v^2 d)} \end{aligned} \tag{1}$$

Example. Consider the case of a circular orbit ( $e = 0$ ) of radius  $d$ . Then,  $v^2 d = \zeta$ . Substituting into formula for the semi-major axis yields  $a = d$ , and substituting into the formula for the semi-minor axis yields  $b = d$ . As expected, the semi-major and semi-minor axes are equal to the radius of the circular orbit.

Based on these formulas, we have completely characterized the ellipse traced by the projectile in its orbit, knowing only the initial values of  $m$ ,  $d$  and  $\mathbf{v}$ . When the projectile starts from the surface of the Earth,  $d = R_{\oplus}$ . Regardless of the initial velocity (and obviously neglecting initial friction with the atmosphere), the orbit will be unstable in the sense that after one orbit around the Earth, the projectile will fall back to Earth because the ellipse it traces necessarily intersects the Earth's surface (at a position generally different from the starting point) as in figure 3.

The polar equation from a focus of the ellipse is  $d(\beta) = \frac{a(1-e^2)}{1+e \cos(\beta)}$  where  $\beta \leq 0 < 2\pi$ . Using  $x = d(\beta) \cos(\beta)$  and  $y = d(\beta) \sin(\beta)$  and differentia-

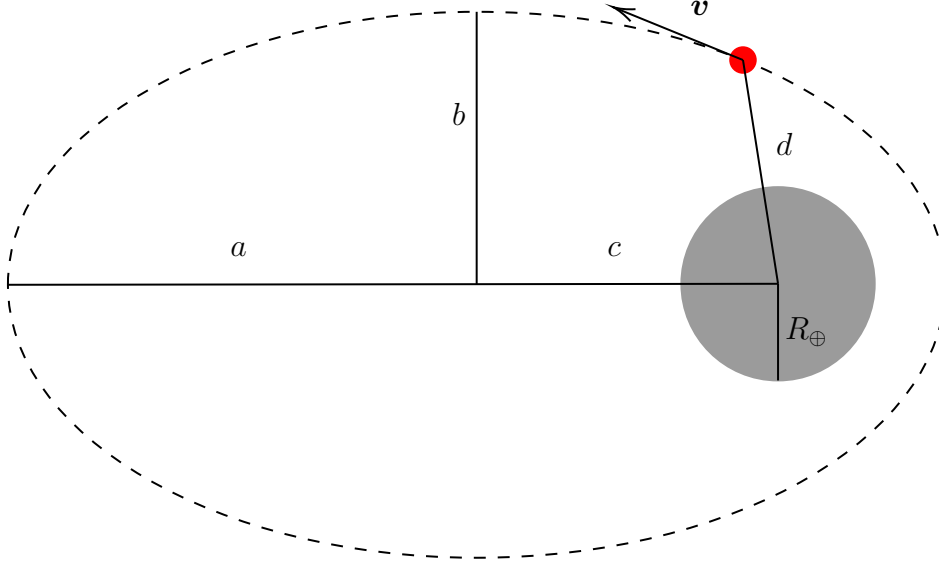


Figure 2: The stable elliptical orbit for  $d > R_{\oplus}$ .

tion with respect to  $\beta$  we get  $\frac{dx}{d\beta} = -d(\beta) \sin(\beta)$  and  $\frac{dy}{d\beta} = d(\beta) \cos(\beta) + d'(\beta) \sin(\beta)$  and  $\frac{dy}{dx} = \frac{\left(\frac{dy}{d\beta}\right)}{\left(\frac{dx}{d\beta}\right)} = \frac{d(\beta) \cos(\beta) + d'(\beta) \sin(\beta)}{d'(\beta) \cos(\beta) - d(\beta) \sin(\beta)}$ . To compute the tangent to ellipse at point we use the ellipse polar equation and we get  $\frac{dy}{dx} = \frac{e \cos(\gamma) + \cos(\beta)}{e \sin(\gamma) - \sin(\beta)}$ . At angle  $\beta = \theta$  the tangent is equal to  $\tan(\xi)$ . Then, the rotation of the ellipse is the angle  $\gamma$  such that  $\frac{e \cos(\gamma) + \cos(\theta)}{e \sin(\gamma) - \sin(\theta)} = \tan(\xi)$ . The problem can be further generalized by considering both elevation  $\varphi$  and azimuth of the projectile.

By intersecting the orbit with the Earth (supposed to be perfectly circular) therefore an ellipse with a circumference it is possible to calculate the latitude of the impact point.

### 3.1 Drag force during launch

Regardless of the launch mode, the projectile will be subject to two non-negligible forces opposing its motion away from Earth: the gravitational force and the frictional force. From Newton's second law,  $m\ddot{x} = -mg - 6\pi\mu D\dot{x}$  where  $x = x(t)$ ,  $x(0) = 0$  and  $\dot{x}(0) = v_0$ . The analytic solution is  $x = \frac{m\left(-e^{-\frac{6\pi D\mu t}{m}}(6\pi D\mu v_0 + gm) - 6\pi Dg\mu t + 6\pi D\mu v_0 + gm\right)}{36\pi^2 D^2 \mu^2}$ . We report  $\dot{x} =$

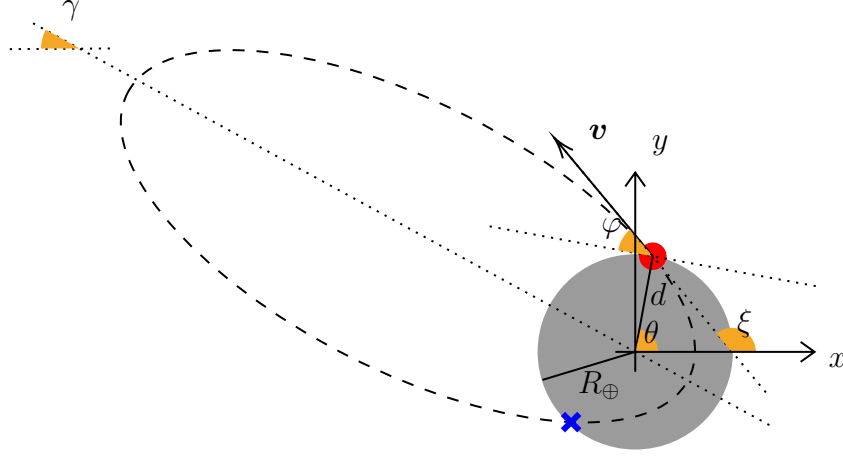


Figure 3: Unstable elliptical orbit for  $d = R_{\oplus}$ . The figure shows the  $x$ - and  $y$ - axes of the reference frame. The latitude angle  $\theta$  of the launch point, the elevation  $\varphi$  above the horizon of the projectile at the launch point, the rotation angle of the ellipse  $\gamma$  and the convenience angle  $\xi = \theta + \frac{\pi}{2} - \varphi$ .  $\mathbf{v}$  is tangent to the ellipse at the launch point. The blue cross represents the point where the elliptical orbit intercepts the earth (point of impact of the projectile).

$\frac{e^{-\frac{6\pi D\mu t}{m}} \left( 6\pi D\mu v_0 - gm \left( e^{\frac{6\pi D\mu t}{m}} - 1 \right) \right)}{6\pi D\mu}$  and  $\ddot{x} = -\frac{e^{-\frac{6\pi D\mu t}{m}} (6\pi D\mu v_0 + gm)}{m}$ . Let  $t_A$  be the ascent time, then  $\dot{x}(t_A) = 0$  which is the time required for the projectile to exhaust the energy imparted at the start. It is found that the  $t_A = \frac{m \left( \log \left( \frac{6\pi D\mu v_0}{gm} + 1 \right) \right)}{6\pi D\mu}$ , and substituting this into  $x(t_A)$  yields the maximum height  $x_A = \frac{m \left( 6\pi D\mu v_0 - gm \log \left( \frac{6\pi D\mu v_0}{gm} + 1 \right) \right)}{36\pi^2 D^2 \mu^2}$ . The Taylor series of  $x_A$  at  $\mu = 0$  (low viscosity) is  $x_A = \frac{v_0^2}{2g} - \frac{2\pi D\mu v_0^3}{g^2 m} + \frac{9\pi^2 D^2 \mu^2 v_0^4}{g^3 m^2} + O(\mu^3)$ . Specific orbital energy  $D$  will evidently be the remaining energy with respect to the specific orbital energy  $\epsilon > \epsilon_D$  at the time of launch. Given that the work is given by the integral of the force,  $\epsilon_D = \epsilon - m \int_0^X \ddot{x}(t(x)) dx$  where  $X$  is the height of the projectile for which the effects of drag are negligible ( $\mu \approx 0$ ).

For simplicity, let's assume a vertical launch  $\epsilon_D = \epsilon - \frac{g^2 m^3}{72\pi^2 D^2 \mu^2} w(p_1, p_2)$  where  $w(p_1, p_2) = (W(p_1) - W(p_2))(W(p_2) + W(p_1) + 2)$  as a function of the Lambert  $W$  function [15].  $p_1 = -\frac{e^{-\frac{6\pi D\mu v_0}{gm}} - 1}{gm} (6\pi D\mu v_0 + gm)$  and  $p_2 =$

$-\frac{(6\pi D\mu v_0 + gm)e^{\frac{6\pi D\mu(6\pi D\mu X - mv_0)}{gm^2}}}{gm}$ . The Taylor series of  $\epsilon_D$  at  $\mu = 0$  (low viscosity) is  $\epsilon_D = \epsilon - gmX + \frac{2\pi D\mu v_0^3}{g} - \frac{2\pi D\mu v_0^2 \sqrt{v_0^2 - 2gX}}{g} + 4\pi D\mu X \sqrt{v_0^2 - 2gX} + O(\mu^2)$ .

A more reasonable equation should also take into account other factors, particularly the decrease in the gravitational field as the projectile moves away from Earth  $g = g(x)$  and the reduction in air viscosity with increasing altitude  $\mu = \mu(x)$ . Therefore,  $m\ddot{x} = -mg \frac{R_\oplus^2}{(R_\oplus + x)^2} - 6\pi D\mu(x)\dot{x}$  but is no longer analytically solvable. However, under certain assumptions, we can assume that  $g$  does not vary during ascent (at least for  $x < \frac{R_\oplus}{10}$ ), while  $\mu(x)$  decreases exponentially in accordance with the barometric formula [16, 17]:  $m\ddot{x} = -mg - 6\pi\mu_1 e^{-\mu_2 x} D\dot{x}$  for some characteristics constants  $\mu_1$  and  $\mu_2$ .

### 3.2 Stabilize the projectile in orbit

We have already proved that the orbit of a projectile launched from the surface of the Earth is unstable (it will necessarily fall back to Earth after completing an orbit around it). This holds true unless the projectile is accelerated while in orbit. This will increase its kinetic energy, and thus the trajectory will no longer intersect the surface of the Earth, stabilizing the orbit. Figure 3 shows the intersection of the unstable orbit with the Earth's surface but in figure 4, we propose a solution based on an accelerator that stabilizes the projectile by accelerating it to orbital velocity.

At the moment of launch on the surface of the Earth, the formula  $\epsilon = \left(\frac{v^2}{2} - \frac{\zeta}{R_\oplus}\right)\eta$  holds (friction has not yet dissipated any energy) but during the ascent phase, the projectile loses velocity (energy). Then,  $\epsilon_D = \left(\frac{v_D^2}{2} - \frac{\zeta}{X}\right)\eta$  where  $\epsilon_D < \epsilon$ ,  $X > R_\oplus$  and  $v_D < v$ . The orbit traced taking into account the energy lost during the ascent phase is significantly different from the orbit traced in the absence of friction. Therefore, it is necessary to consider the parameters  $\epsilon_D$ ,  $v_D$  and  $X$  and describe the orbit again using equations 1 with  $d = X$ ,  $v = v_D$  and  $\epsilon = \epsilon_D$ .

Replacing values into equation 1 gives  $a_D$ ,  $b_D$ ,  $c_D$  and  $e_D$ . Let  $d_D(\beta) = \frac{a_D(1-e_D^2)}{1+e_D \cos(\beta)}$  where  $\beta \leq 0 < 2\pi$  be the polar equation from a focus of the dashed ellipse in figure 4. The orbital velocity for the elliptical orbit as a function of  $\beta$  is  $v_D^{\text{elliptical}}(\beta) = \sqrt{2 \frac{d(\beta)\epsilon_D + \zeta\eta}{d_D(\beta)\eta}}$ , while the orbital velocity for a circular orbit is  $v_D^{\text{circular}}(\beta) = \sqrt{\frac{GM}{r(\beta)}}$ . Clearly,  $v_D^{\text{circular}}(\beta) > v_D^{\text{elliptical}}(\beta)$ , but the

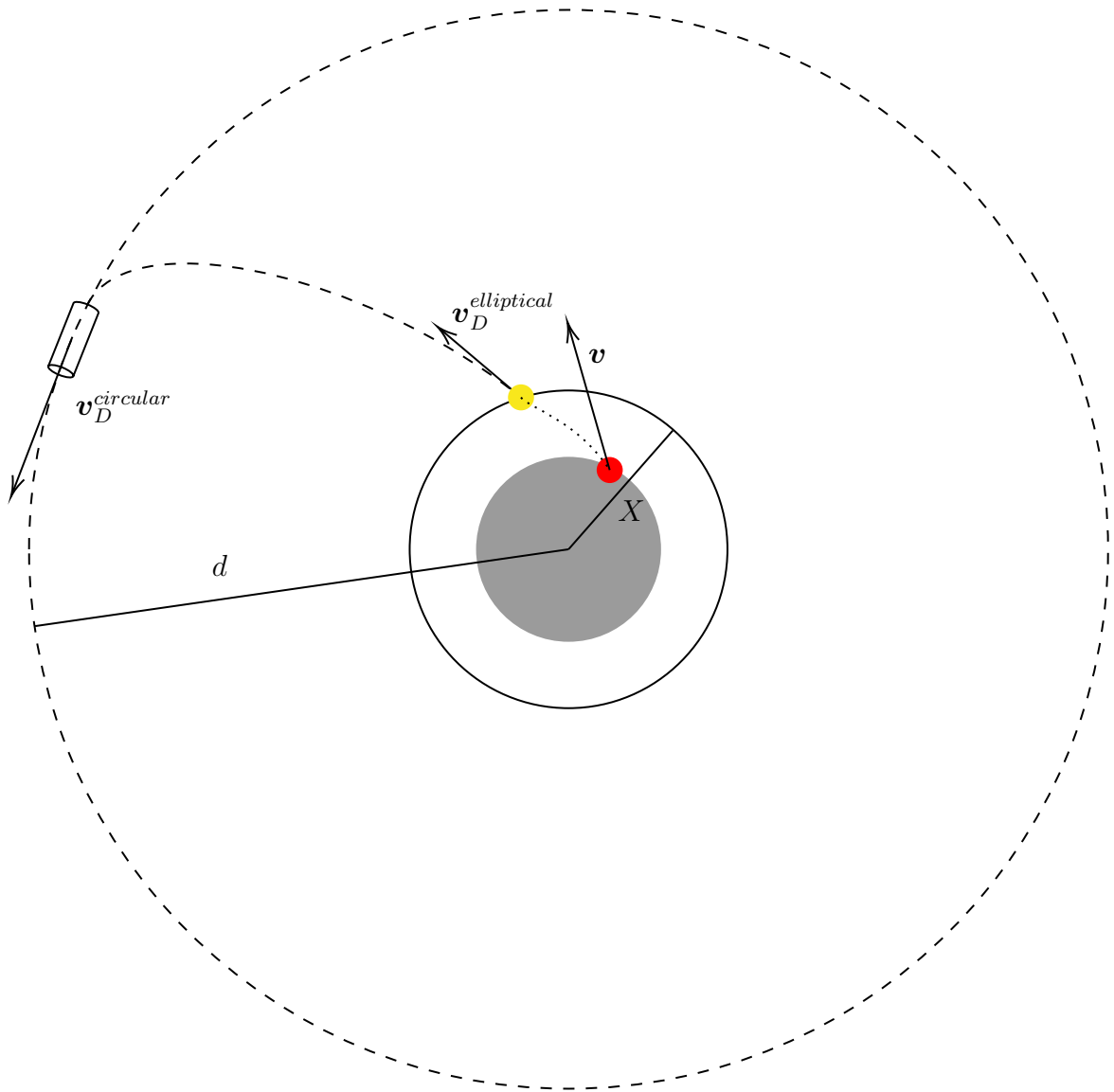


Figure 4: The projectile on the Earth's surface (red circle) and its motion subjected to atmospheric friction (dotted line) until it reaches the altitude  $X$  where friction can be considered negligible ( $\mu(X) \approx 0$ ). Upon surpassing the altitude  $X$  (yellow circle), the projectile traces an unstable elliptical orbit if it weren't for the presence of the accelerator that brings it to orbital velocity  $v_0$ .

ratio between circular and elliptical velocities is minimum for  $\beta = \pi$ . Hence,

$$\min \frac{v_D^{circular}}{v_D^{elliptical}} = \frac{(2\zeta - Xv_D^2)\sqrt{GM\eta}}{\sqrt{(4\zeta - 2Xv_D^2)(X^2v_D^2\epsilon + \zeta\eta(2\zeta - Xv_D^2))}}$$

## 4 Conclusions and future work

In this paper, we have analyzed alternative possibilities for launching satellites into orbit. Specifically, we have focused, and will continue to focus in the future, on launch techniques that do not require the consumption of large amounts of fuel, as is currently the case. The aim is to eventually enable space travel, or at least achieve stable orbits, without relying on fuel, instead harnessing gravitational fields or other forms of propulsion distinct from thermal propulsion.

From the studies conducted thus far on circular motion as a propulsion method, several challenges have clearly emerged, particularly due to air resistance during the initial phase of launch when velocity is highest. This issue could potentially be mitigated by launching the rocket from the lower atmosphere, which would partially reduce the impact of air resistance. However, this approach still necessitates the use of a support rocket propelled by thermal propulsion, thus remaining subject to the physics of the rocket equation. Nonetheless, once the initial phase is overcome, we have demonstrated that orbit stabilization is achievable through the use of an accelerator, which, without significant complications, can impart the necessary velocity to the projectile.

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