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# From Doppler effect to black hole

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## Abstract

This conference paper contains the transcription and analysis of the lecture titled *From the Doppler Effect to Black Holes*, held on April 8, 2016 at AAM. The lecture presents a progression that culminates in the elaboration of the black hole concept, starting from a commonly experienced phenomenon: the Doppler effect. It moves from the classical Doppler effect to the relativistic Doppler effect, applicable to motions with relative velocities comparable to the speed of light. This result confirms the predictions of General Relativity, leading to the definition of the Schwarzschild radius of a mass.

## 1 Introduction

The objective of this lecture is to retrace the path of human thought, which has progressed from simple, everyday phenomena to the development of physical theories necessary to explain one of the universe's most mysterious phenomena: black holes. The lecture will not delve into the physics of black holes, but will instead outline the intellectual journey starting from simple phenomena like the Doppler effect. The Doppler effect is a phenomenon that most people have likely encountered, even without knowing its name. The main subject of study in the Doppler effect is waves [1].

Let us consider a 500 Hz wave (Hz is the unit of frequency, Hertz, named after the German physicist) and a 1000 Hz wave; we observe that the wave with the lower frequency produces a deeper sound, whereas the wave with the higher frequency results in a higher-pitched sound. If we analyze the shape of the sound wave, we can observe that it can be geometrically associated with

an ocean wave, with points of maximum intensity called peaks and points of minimum intensity called troughs. The period is the time interval between two peaks or two troughs, and the frequency is the inverse of the period, meaning how many periods occur within a 1-second interval.

## 2 Doppler effect

The Doppler effect involves:

- an observer
- a source that generates a constant sound or frequency (such as a car horn or an ambulance siren)

When the source gets closer to the observer, the frequency emitted by the source and measured by the observer increases. The observer can rationally conclude that the frequency is higher during the approach and lower during the retreat, implying that the space between two peaks or troughs is shorter for a wave approaching than for a wave receding. This occurs because, as the source approaches the observer, it compresses the sound it produces, shortening the time and increasing the frequency. In contrast, as the source moves away, the sound is stretched, increasing the duration and decreasing the frequency.

For this observation to be meaningful, it is essential to understand whether there is a relationship between the frequency emitted by the source when:

- it is stationary relative to the observer
- it is approaching the observer
- it is moving away from the observer

If such a relationship exists, then the phenomenon can be explained through physical and mathematical laws, making it possible to deduce the velocity of approach or retreat without directly measuring it but by observing other parameters and the changes they undergo. This is a crucial aspect of studies in astronomy and physics in general. If there is a relationship between quantities, it becomes possible to deduce the properties of an object without directly measuring them, but rather calculating them based on other quantities and how they vary [2].

In the case of the Doppler effect, we cannot directly determine the speed of the source without measuring it, but by measuring the frequency during approach or retreat and the frequency when the source is stationary relative to us. While the Doppler effect is most commonly experienced with sound, it can also be observed with light phenomena, though in this case, it is referred to as the relativistic Doppler effect [3, 4].

### 3 Relativistic Doppler effect

The relativistic Doppler effect is defined as such because the velocity of the source is comparable to the speed of light, and thus falls within the domain of the theory of relativity. In the case of a source emitting electromagnetic radiation, the phenomenon remains conceptually similar, but is described by different physical laws. As with the classical Doppler effect, an increase in frequency is observed during approach, and a decrease in frequency during retreat.

In the study of the motion of celestial bodies, it is possible to identify conditions under which relativistic Doppler phenomena manifest. By analyzing the light from stars, we can determine their velocity relative to us. For example, if we consider the light from certain classes of stars for which we know the exact emission frequency, we observe that objects like the M33 galaxy (the 33rd object in Messier’s catalog [5]) exhibit a shift of the entire electromagnetic spectrum towards blue (hence the term “blueshift” in English), indicating a higher frequency and thus a galaxy approaching us. In contrast, for the M108 galaxy, the frequencies in the electromagnetic spectrum are shifted more towards red, corresponding to a lower frequency and indicating a galaxy moving away from us (hence the term “redshift”).

For a precise analysis of this type of phenomenon, we rely on light spectra, where specific frequencies are used as reference points to determine whether a shift has occurred. Referring to the earlier example, if we use a particular frequency from the Sun (which does not exhibit any appreciable shift from Earth and can be considered stationary relative to us), we can observe that the same frequency in the light spectrum of the M108 galaxy is shifted more towards the red compared to the light spectrum of the M33 galaxy, which is shifted more towards the violet.

Through this observation, it is possible to deduce the velocity at which a celestial object is receding or approaching by measuring the frequency and its

shift relative to its value for a source stationary with respect to the observer, which, in this case, is the Sun.

## 4 General theory of relativity

In 1915, Albert Einstein published the theory of relativity [6], culminating in a system of equations that describe the motion of bodies within a field shaped by the force of gravity. These equations allow us to describe the movement of a body immersed in a gravitational field. We apply this theory to describe the motion of galaxies after making two assumptions about the nature of the universe [7]:

- Homogeneous: it has the same density at every point, meaning every point in the universe has the same density as any other point.
- Isotropic: any given volume of the universe is identical to any other volume taken from a different location in the universe.

The properties of homogeneity and isotropy refer to two distinct concepts: if we consider a certain volume of a given element, it will have the same density as another volume of the same element, making them homogeneous with respect to each other. Isotropy, on the other hand, means that the universe exhibits the same characteristics in all directions and is uniform regardless of which volume we consider [7].

The model of the universe derived from the general relativity equations, under the assumptions of homogeneity and isotropy, describes a dynamic universe in which bodies recede from each other at speeds that increase with their distance. In other words, there is a relationship between the velocity of a body's recession from the observer and its distance. Therefore, if the velocity is known, the distance can be calculated, and thanks to the relativistic Doppler effect, the velocity can easily be deduced from the shift in spectral lines.

Now, empirical confirmation of this result is necessary. If the theory is not experimentally verified, it cannot hold scientific value but would remain purely speculative.

## 4.1 Hubble law

Around 1929, American scientist Edwin Hubble, working at one of the largest observatories of the time, acquired a series of photographs of galaxies, determining their velocities (using the spectral line shift method, known as the relativistic Doppler effect discussed earlier) and measuring their distances. The result was astonishing. It demonstrated a direct proportionality between the velocity of recession and the distance of the galaxy. This proportionality constant is known as Hubble's constant [8].

This result provides two crucial insights:

- The universe is continuously expanding.
- It is consistent with the predictions of general relativity.
- It aligns with the calculations performed by Friedmann and other scientists, applying the results from general relativity within the framework of an isotropic and homogeneous universe.

## 4.2 Schwarzschild radius

An experimental validation of the correctness of general relativity indicates that other results obtained through the same theory may potentially be correct [4]. One of the mathematical foundations of general relativity is the concept of a field and metric. From the study of these two concepts, it is possible to define specific quantities such as the Schwarzschild radius, the Kerr-Newman radius, and the Reissner-Nordström radius. The concept particularly relevant for understanding black holes is the Schwarzschild radius.

What is it? Intuitively, the Schwarzschild radius is a length value associated with any body that has mass: given a mass, one can calculate a length value that provides a particular property describing the body. This radius allows us to relate a body's mass to a characteristic measure. For example, consider Earth. Its Schwarzschild radius is 0.887 cm. Thus, if we were to compress Earth (or a body with the same mass as Earth) to the size of a sphere with a diameter of approximately 1.7cm, Earth would have reached its Schwarzschild radius. Similarly, for the Sun, the Schwarzschild radius is 2.95km (a sphere with a diameter of about 6km), and even for a rock weighing just 5kg, its Schwarzschild radius is only  $7.43 \cdot 10^{-27}m$ . The greater the mass, the larger the Schwarzschild radius.

To fully understand the significance of the Schwarzschild radius, we can refer to another common experience: the concept of escape velocity. In the simplest application of this concept, consider the case of throwing a stone into the air. Our experience shows that the stone returns to Earth because we impart a relatively small velocity to it. However, this would not be the case if we gave the stone a velocity equal to or greater than the escape velocity of Earth—the velocity required to overcome Earth’s gravitational field.

If we were to give the stone a velocity of  $40,320\frac{km}{h}$ , the same speed achieved by spacecraft during launch, the stone would not return to Earth but would escape Earth’s gravitational field and start wandering through the universe. If the radius of Earth were to decrease, the stone would experience a stronger gravitational attraction, and thus the escape velocity would increase. Therefore, we can state that as the radius decreases, the escape velocity increases. This process cannot continue indefinitely. A fundamental postulate of the theory of relativity is that we cannot achieve infinite velocity, but only speeds less than the speed of light, which is  $299,792,458\frac{m}{s}$ .

Thus, if we were to attempt to escape from a hypothetical Earth with an infinitesimal radius, we would have to accept that the maximum speed we can travel is the speed of light, and not any greater.

So far, we have established that if the radius of Earth decreases, a greater escape velocity is required, and that we can reduce Earth’s radius until reaching a maximum escape velocity, which is the speed of light. But what if Earth’s escape velocity were the speed of light? What would be the radius of Earth in that case?

If Earth’s escape velocity were equal to the speed of light, then the radius of Earth would be equal to its Schwarzschild radius. This is the meaning of the Schwarzschild radius: it is the radius associated with a mass such that its escape velocity equals the speed of light.

### 4.3 Recap

Now that this important result has been reached, it is appropriate to review the path followed:

1. Through the Doppler effect, it is possible to evaluate the velocity at which a body is moving by studying the frequency received during approach, the frequency received during recession, and the frequency when the body is at rest.

2. There is a relativistic counterpart to the Doppler effect, which allows for the study of a galaxy by calculating its velocity relative to us based on the shift in the electromagnetic spectrum.
3. Thanks to the theory of general relativity and the observations made by Edwin Hubble, we know there is a direct relationship between the distance of a galaxy and its velocity relative to the observer.
4. By applying the concepts of field and metric, it is possible to quantify a parameter known as the Schwarzschild radius, for which a mass with a radius equal to this value has an escape velocity equal to the speed of light.

If it is true that the speed of light is the maximum velocity in the universe and that a body with a Schwarzschild radius has an escape velocity equal to the speed of light, it logically follows that a body with a Schwarzschild radius cannot be observed. This is because any electromagnetic radiation emitted by it would be unable to escape. Therefore, a body with a Schwarzschild radius is effectively comparable to a black hole.

## 5 Black hole

From the results obtained, we can establish that a black hole is an object that has reached the dimensions of its Schwarzschild radius. Such a black hole is specifically referred to as a Schwarzschild black hole. Since below this dimension, the escape velocity is equal to or greater than the speed of light, no electromagnetic radiation can escape from within this region. For this reason, a black hole is distinguished by two regions separated by an event horizon, within which lies the singularity, from which no information can be obtained. This event horizon has a dimension equal to the Schwarzschild radius from the center of the black hole [4].

Outside the event horizon, we can experience extremely strong gravitational attraction and gravitational field distortions, leading to observable phenomena such as light deflection. The greater the mass of a black hole, the larger its Schwarzschild radius. For instance, we previously calculated the Schwarzschild radius of the Sun; if we were to compress it to a sphere with that radius, it would become unobservable because its escape velocity would exceed the speed of light.



## 5.1 Conclusions

If the logical steps followed thus far are correct and if black holes indeed exist, then it is possible to observe them somewhere in the universe. Although we cannot receive information directly from the center of a black hole because it emits no radiation, we can observe the effects these objects have on their surrounding space. One such effect is gravitational lensing: phenomena where light coming from behind a black hole or a massive object is distorted due to the presence of the massive object. This phenomenon creates a lensing effect caused by gravitational phenomena, hence the term “gravitational lensing”. Based on the analysis of images of gravitational lenses, it is possible to calculate the curvature generated by massive objects on the background image, and thus infer the object’s size and mass without any information about the singularity inside the event horizon.

The current goal of theoretical physics is to develop a comprehensive description of phenomena occurring inside the event horizon, in scenarios where intense gravity manifests in infinitesimal spaces. This involves creating a theory of gravity applied to quantum scenarios, known as quantum gravity. This is a potential theory capable of unifying quantum mechanics with general relativity, and macroscopic phenomena with microscopic phenomena: a possible theory of everything. Our current understanding of black holes is based on observations and theoretical analyses from outside the event horizon, and there is, as of now, no knowledge of what happens inside this horizon.

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