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Sliding Mode Control for Lateral Dynamics of Bicycle

Lama Ayad, Hocine Imine, Claudio Lantieri, and Francesca De Crescenzo

Abstract— Cyclists are vulnerable road users because they are at a higher probability of being involved in accidents due to different risk factors. One of the causes of the accidents is lane departure. Therefore, an active steering control system is developed and proposed to assist the cyclists to avoid the lane departure and preserve their safety. A PID control system and sliding mode control were proposed to control the lateral displacement. However, the most efficient control system to be used is a sliding mode control by applying super-twisting. Some simulation results are presented to indicate the difference between both types of control system and to show the reliable performance of the proposed controller.

I. INTRODUCTION

The majority of cities are seeking to preserve the safety of road users, so they are widely supporting the introduction of smart mobility and the development of intelligent transport systems to increase the traffic safety. In this context, different active safety systems were developed for vehicles and electrical bicycles for different purposes such as speed control, lateral stability, longitudinal stability, lane keeping, efficient breaking system, etc. [1] [2] [3] [4] [5] [6] [7] However, there is limited studies for developing assistance systems for a normal bicycle [8].

It is preferable to develop an assistance system for a normal bicycle because its use is so widespread, resulting in an increase in the number of accidents and the cyclists' fatalities. Cyclists represented a reasonable category of road traffic deaths in some countries of EU in 2020, 30.7% in Netherlands, and 17.1% in Belgium [9]. The safety of cyclists is subject to different risk factors resulting from different factors such as infrastructure features, facility features, the cyclists' behavior, the weather conditions, and the traffic conditions as identified in our previous research work. It is determined by different studies that the cyclists' behavior has a major impact on their safety. For example, 18% of the accidents was a result of carelessness and inattentiveness of the bicyclists in France [10], and 30 % of cyclist injuries are registered in the accidents because of rider inattentiveness in US [11]. One of the accidents types associated due to these behaviors is the lane departure [11]. It is important to add that lane departure may also be caused by other factors such as

the road conditions [12]. The lane departure was executed by some participants in previous real road experiments to avoid the skidding due to snow accumulation on bicycle lane as shown in videos captured by the virtual reality eye tracking glass (Pupils Lab GmbH) wore by the participants [12]. They left the dedicated trajectory path as not to fall from the bike because of snow. However, they deviated far from the safe and recommended track and put themselves in additional higher risk. They could be at greater risk of collision with other vehicles because they were riding alongside and so close to other cars and buses. Therefore, the objective of this paper is to develop a control system for lateral dynamics of a bicycle to prevent the lane departure. Controlling the lateral position of a bicycle to keep it within the desired trajectory is considered in the control system development to avoid the lane departure. This control system development and activation is the key to maintain the cyclists' safety, decrease the accidents, and obtain an integrated transportation system.

Lateral control system is proposed by acting on the steering angle of bicycle to keep it on the desired reference lane. It is proposed in this paper the development of a third-order sliding mode and the proportional integral derivative (PID) controller to ensure the lateral control. The simulation results show the effectiveness of both controllers as approved by previous research studies that the sliding mode control is one of the most promising control methods for trajectory tracking [6] and especially for non-linear dynamic models because it can achieve more robustness and stability. Also, PID controller is widely used in engineering practices because it is a simple and easy operated structure. It is especially applied for systems that have predictable and known behaviors [13].

This paper is organized as follows, in section two, the lateral Dynamics model of the bicycle is presented, in section three, the proposed steering sliding controller is described, in section four, the PID controller development is introduced, in section five, the simulation results are illustrated and finally in section six, conclusion, and future perspectives are described.

II. BICYCLE MODELLING

This section describes the lateral equations for a bicycle model, and they are represented in a state space form for lateral control design.

A. Bicycle Model description

The bicycle is considered as a complex system since it has many degrees of freedom and its geometry is intricate. The bicycle also presents yaw and significant roll that are both interconnected. Some important aspects to consider are the choice of bicycle bodies to include in the dynamic model, the elasticity behavior of the bicycle parts, the modeling of tire-road interaction, and the complexity of the rider model

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[14]. The suspension movement, road inclination, and aerodynamic influences are neglected. The lateral dynamics of bicycle model with the major variables are presented in Fig. 1 and table I [15]. The table I represents the parameters of the bicycle used in the mathematical model of the simulator based in university of Gustave Eiffel shown in figure 2 that will be connected to the developed control system model.

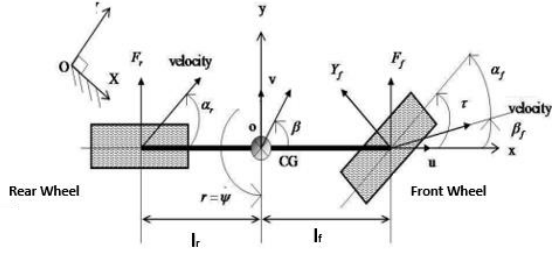


Figure 1. Lateral Bicycle Dynamics

Table I. List of Major Bicycle Variables

Symbol	Bicycle Parameters		
	Meaning	Amount	Unit
m	Total Mass (Bicycle and Rider)	85	kg
m_F	Front Wheel Mass	23	kg
m_R	Rear Wheel Mass	62	kg
l_F	Distance from COG to Front Axle	0.293	m
l_R	Distance from COG to Rear Axle	0.808	m
h	COG Height	1.09	m
C_f	Tire Lateral Stiffness Coefficient	490	N/m
I_z	Moment of Inertia around z	0.165	Kg.m ²
I_x	Moment of Inertia around x	0.387	Kg.m ²
K_r	Bicycle Roll Stiffness	1106000	N/m
B_r	Bicycle Roll Damping	1135	N.s/m
δ_F	Front Tire Steering Angle	-	Rad
φ	Roll Angle Along X-Axis (longitudinal)	-	Rad
ψ	Yaw Angle Along Z-Axis (Vertical)	-	Rad
β	Side Slip Angle	-	Rad
v	Bicycle Longitudinal Speed	-	m/s

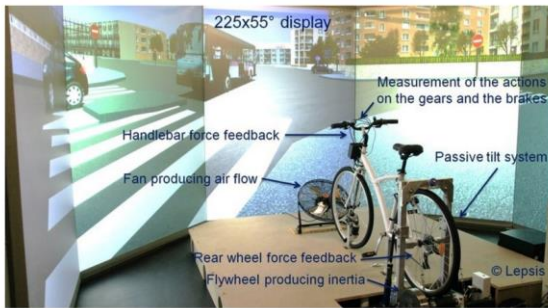


Figure 2: PICSL-L Bicycle Simulator

B. Lateral Bicycle Equations

The dynamic model is based on the extension the kinematic model by taking additional factors such as the effects of forces. Two components of the motion shall be maintained: the longitudinal direction in the direction of yaw angle, and the lateral direction perpendicular to the yaw. The common set of equations representing the lateral dynamics of a bicycle can be described as [16]:

$$\begin{aligned} a_y &= \ddot{y} + \omega^2 * R \\ &= v\dot{\beta} + v\dot{\psi} \end{aligned} \quad (1)$$

The lateral acceleration is expressed in terms of the rate of change of side slip angle and the yaw rate change. Then, the lateral dynamics can be described as taking into consideration the two forces affecting the dynamics being the lateral forces on the front and rear tires since they are the most important components in bicycle dynamics:

$$mv(\dot{\beta} + \dot{\psi}) = F_{yf} + F_{yr} \quad (2)$$

$$I_z\ddot{\psi} = l_f F_{yf} - l_r F_{yr} \quad (3)$$

The lateral forces acting on the system can be linearized and calculated as follows by considering the tire slip angles are small:

$$F_{yf} = C_f \alpha_f = C_f \left(\delta - \beta - \frac{l_f \dot{\psi}}{V} \right) \quad (4)$$

$$F_{yr} = C_r \alpha_r = C_r \left(-\beta - \frac{l_r \dot{\psi}}{V} \right) \quad (5)$$

By substituting the tire force equations in the equations 1, 2 and 3, the lateral, and yaw dynamics can be calculated by the following equations:

$$\begin{cases} \ddot{y} = -\frac{C_r + C_f}{mV} \dot{y} - \left(\frac{C_f l_f - C_r l_r}{mV} + V \right) \dot{\psi} + \frac{C_f}{m} \delta \\ \dot{\beta} = \frac{-C_r + C_f}{mV} \beta + \left(\frac{C_f l_f - C_r l_r}{mV^2} - 1 \right) \dot{\psi} + \frac{C_f}{mV} \delta \\ \ddot{\psi} = \frac{C_r l_r - C_f l_f}{I_z} \beta - \left(\frac{C_r l_r^2 + C_f l_f^2}{I_z V} \right) \dot{\psi} + \frac{C_f l_f}{I_z} \delta \end{cases} \quad (6)$$

C. States Vector Representation

The motion equations of the bicycle model are written in state-space form [17]:

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx \end{cases} \quad (7)$$

where $x = [y_l, \varphi, \psi, \beta, \dot{y}_l, \dot{\varphi}, \dot{\psi}]^T$ is the state vector composed of respectively lateral displacement, roll angle, yaw angle, side slip angle, lateral speed, roll rate and yaw rate, $y = [y_l, \varphi, \psi]^T$ is the outputs vector. $u = \delta$ is the system input.

The matrices $A \in \mathcal{R}^{7 \times 7}$, $B \in \mathcal{R}^7$ and $C \in \mathcal{R}^{3 \times 7}$ are given by:

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & a_{44} & 0 & 0 & a_{47} \\ 0 & 0 & 0 & 0 & a_{55} & 0 & a_{57} \\ 0 & a_{62} & 0 & 0 & 0 & a_{66} & a_{67} \\ 0 & 0 & 0 & a_{74} & 0 & 0 & a_{77} \end{bmatrix} B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ b_4 \\ b_5 \\ b_6 \\ b_7 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

$$\text{where } a_{44} = -\frac{(c_f+c_r)}{mv}; a_{47} = \frac{(-c_f l_f + c_r l_r)}{mv^2} - 1;$$

$$a_{55} = -\frac{(c_f+c_r)}{mv}; a_{57} = \frac{(-c_f l_f + c_r l_r) - mv^2}{mv};$$

$$a_{62} = -\frac{(K_r - m_2 g h)}{I}; a_{66} = -\frac{B_r}{I}; a_{67} = \frac{h m_2 v}{I};$$

$$a_{74} = \frac{(-c_f l_f + c_r l_r)}{I_z}; a_{77} = -\frac{(c_f l_f^2 + c_r l_r^2)}{I_z v};$$

$$b_4 = \frac{c_f}{mv}; b_5 = \frac{c_f}{m}; b_6 = -\frac{c_f}{mh}; b_7 = \frac{l_f c_f}{I_z};$$

$$\text{and } I = I_x + h^2 m_2$$

III. SLIDING MODE CONTROL

Lane keeping assistance is based on controlling the lateral position. Therefore, a sliding mode control is developed which its aim is to ensure that the variable $y_e = y_l + \psi$ converges toward 0 in a finite time. The variable is the sum of lateral displacement and yaw angle. This can be reached by producing the appropriate control input to drive the model to the desired state and remain on the sliding surface. The control input is the steering angle and the lateral displacement and the yaw angle are the outputs. The super twisting algorithm is used to develop the control mode and to reduce the chattering.

The sliding mode controller is developed using MATLAB Simulink based on the following equations.

A. Sliding Mode Control Design

The following functions are defined:

$$\begin{cases} f(x) = A_2 x_2 + D_2 \beta \\ g(x) = B_2 u \end{cases} \quad (8)$$

The matrices A_2 , B_2 , and D_2 are obtained when the principal dynamic model described in the section II is rewritten in a reduced state form where the outputs vector is $[y_l, \varphi, \psi]^T$. Those matrices are the submatrices of A, and B and defined in the following forms where $A_2 \in \mathcal{R}^{3 \times 3}$, $B_2 \in \mathcal{R}^3$, and $D_2 \in \mathcal{R}^3$:

$$A_2 = \begin{bmatrix} a_{55} & 0 & a_{57} \\ 0 & -B_r/I & h m_2 v/I \\ 0 & 0 & a_{77} \end{bmatrix} B_2 = \begin{bmatrix} c_f/m \\ 0 \\ l_f c_f/I_z \end{bmatrix} D_2 = \begin{bmatrix} 0 \\ 0 \\ a_{74} \end{bmatrix}$$

A sliding mode surface is also defined as following to ensure the convergence attaining:

$$S = \dot{y}_e + \lambda y_e \quad (9)$$

Where \dot{y}_e is the error derivative, and λ is a positive constant.

The derivative of sliding surface can be written as the following equation [18]:

$$\dot{S} = \frac{\partial S}{\partial t}(t, x) + \frac{\partial S}{\partial x}(t, x) \cdot (f(x) + g(x, u)) \quad (10)$$

The control input of the system is composed of two terms. The first one is defined by means of its discontinuous time derivative, while the other is a continuous function of sliding variable. The control algorithm is defined by the following control law:

$$\begin{cases} u = u_{eq} - k_1 |S|^{1/2} \text{sign}(S) + u_1 \\ \dot{u}_1 = -k_2 \text{sign}(S) \end{cases} \quad (11)$$

where $k_1 > 0$ and $k_2 > 0$ are the control gains.

The term u_{eq} has the role of a feedforward approaching the system to the sliding surface and it is obtained by resolving $\dot{S} = 0$ as follows:

$$u_{eq} = \frac{-(\lambda + a_{55})\dot{y}_l - (\lambda + a_{77} + a_{57})\dot{\psi} - a_{74}\beta}{\Gamma} \quad (12)$$

$$\Gamma = \left(\frac{C_f}{m} + \frac{l_f C_f}{I_z} \right) \quad (13)$$

B. Convergence Analysis

The convergence of the controller shall be analyzed to ensure that the designed control model is stable and robust. This can be analyzed by verifying the 4 following assumptions if they have been reached, as developed by [19] [20]

- The control input u is such that $|u| \leq U_{max}$, with U_{max} is a real positive constant and any solution of the system (4) is defined for all t .
- There exists $u_1 \in [0, U_{max}]$ such that for any continuous function $u(t)$ with $|u(t)| > u_1$, there exists time t_l such that, $S(t)u(t) > 0$ for all $t > t_l$. Then, the control input $u = -U_{max} \text{sign}(S(t_0))$, where t_0 is the initial value, permits to attend the sliding surface $S = 0$ in finite time.
- There are positive constants $S_0, G_1, G_2, u_0 < I$, such that if $|S(t, x)| < S_0$, then:

$$0 < G_1 \leq \frac{\partial \dot{S}}{\partial u}(t, x, u) \leq G_2 \quad \forall x, u$$

- There exists a constant H such that if $|S(t, x)| < S_0$, then

$$\left| \frac{\partial \dot{S}}{\partial t}(t, x, u) + \frac{\partial \dot{S}}{\partial x}(t, x, u) \cdot (f(x) + g(x, u)) \right| \leq H \quad (14)$$

The corresponding sufficient conditions for the finite time convergence to the sliding manifold are:

$$\begin{cases} K_2 > \frac{H}{G_1} \\ K_1 > \frac{2}{G_1} \sqrt{\frac{G_2 H (K_2 + H)}{G_1 (K_2 - H)}} \end{cases} \quad (15)$$

All the above assumptions are satisfied. Therefore, it can be concluded that the control system proposed is behaved appropriately. The system also converges to the sliding surface, and the sliding mode is reached in finite time t_0 .

IV. PID CONTROLLER

The PID controller is considered as one of the simplest types of the control mode in control systems. The PID controller is designed to adjust the error between the desired setpoint and the measured variable to reach zero by generating the control signal by acting on the error signal to drive the system into the desired setpoint [21] [22].

The control signal is obtained through three parameters: Proportional (P), Integral (I), and Derivative (D). Therefore, the PID controller is defined as follows:

$$u(t) = k_p + k_d \frac{d}{dt} e(t) + k_i \int e(t) dt \quad (16)$$

where k_p is the proportional gain, k_d is the derivative gain, and k_i is the integral gain. These parameters can be weighted, or tuned to reach the desirable signal.

The proportional gain k_p indicates the sensitivity of the controller's response to changes in the error between the desired setpoint and the measured process variable. Increasing the proportional gain can make the controller respond more quickly to changes in the error, but too high of a value can cause the system to become unstable and oscillate. The integral gain k_i is tuned to achieve the desired performance control and to reduce steady state error. This gain is not separately tuned but combined with PD control. The derivative gain k_d acts on the rate of change of the error signal to produce a control output that responds quickly to changes in the error signal [13].

A PID control system is developed using MATLAB Simulink by connecting it to the bicycle model defined in section II.

V. SIMULATION RESULTS

The sliding mode control and PID control systems have the same principal as shown in figure 3. The system controls the lateral displacement by acting on the steering to reach the

desired setpoint. The speed values used in the control system are extracted from sensors installed on an instrumented bicycle used by one of the participants in the previous real road experiments conducted in Sweden. It was measured with the corresponding sensor: speed sensor 2 from Garmin.

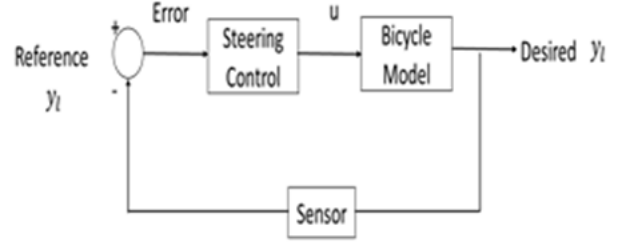


Figure 3. Bicycle Control System

This control system has been developed to the sliding mode and PID using MATLAB Simulink, using all the required specifications. The deviation from the specified trajectory to be followed by the cyclist in the lateral direction should not exceed 0.01 m.

A. Sliding Mode Simulation Results

For the sliding mode control, the desired lateral displacement, and the steering angle are shown respectively in the figure 4 and 5 respectively. Based on the tuning and selecting the appropriate gains, the control system is well-behaved and achieves the desired performance. It can be concluded that the control model design is effective.

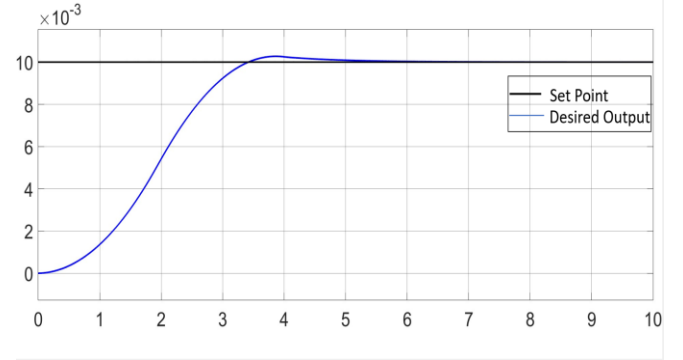


Figure 4. Lateral Displacement Output of Sliding Mode Control

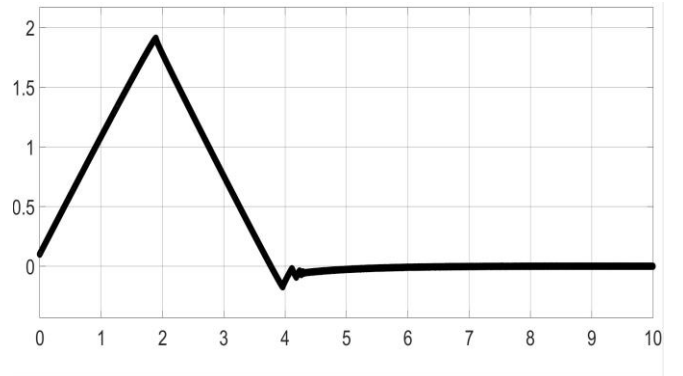


Figure 5. Steering Angle Response of Sliding Mode Control

The figure 4 indicates that the desired lateral displacement converges to the setpoint. So, the error converges in finite time to zero in a small period of time of 6 seconds.

The steering angle graph presents the variation of steering angle over time. This graph shows how the sliding mode control is adjusting the steering angle to keep the bicycle on the desired path. The figure 5 shows that the steering angle does not continue to oscillate over time.

The sliding mode control law, the sliding surface, and the gains are well designed and well-tuned because the system responses efficiently and tracks the desired trajectory.

B. PID Simulation Results

For the PID controller, the desired lateral displacement is shown in figure 6.

The figure 6 presents the variation of lateral displacement with time. After extensive trial and error of the gains, the desired lateral displacement could be reached the set point with zero error.

However, since the lateral bicycle equations used in our model are non-linear and may represent disturbances due to its complexity. For this reason, the sliding mode control system will be considered for our further works since it is more robust than PID for non-linear system.

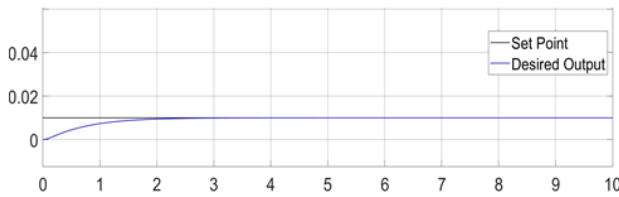


Figure 6. Lateral Displacement Output of PID Controller

VI. CONCLUSION

A steering control system is developed for a bicycle to prevent the lane departure and preserve the cyclist safety to decrease the number of accidents involved due to this type of risk. The purpose of control system is to be behaved once the cyclist didn't react properly in the risk situation, or due to his inattentiveness.

The system is to control the lateral displacement and yax angle by acting on the steering angle. The control system is developed using sliding mode control and PID control. The results obtained from the simulations shows that both controllers give favorable results and the steering is adjusted properly to reach the desired path. It can be concluded that these designs are effective.

The sliding mode control system is considered in our future research works. However, the performance can be improved by developing a sliding mode observer and combining it with the sliding mode controller to estimate the dynamic states of the bicycle. Also, the lateral stiffness, the spring stiffness and damping coefficient of the bicycle used are obtained based on literature review, so the purpose is to identify them as well in the future work to obtain an integrated sliding control model of good quality.

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