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# Output-feedback controller synthesis with sign-indefinite quadratic forms

Santiago Pantano Calderón, Sophie Tarbouriech, Luca Zaccarian

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**Abstract:** We provide novel sufficient conditions for the synthesis of stabilizing dynamic output-feedback controllers for input-saturated plants. Based on Linear Matrix Inequalities (LMIs), together with appropriate transformations and sector conditions, the results exposed exploit the sign-indefinite quadratic forms involving the closed-loop state and the deadzone of the control input to define piecewise smooth Lyapunov functions. The proposed solutions leverage additional degrees of freedom with respect to the classical quadratic stabilization approach to construct global or regional exponential stability certificates of the origin for the closed-loop system.

## 1 Research goals

Consider the linear plant subject to input saturation

$$\mathcal{P} \sim \begin{cases} \dot{x}_p = A_p x_p + B_p \text{sat}(u) \\ y = C_p x_p \end{cases}, \quad (1)$$

with  $x_p \in \mathbb{R}^n$ ,  $y \in \mathbb{R}^m$  and, for all  $i = 1, \dots, m$ ,  $\text{sat}_i(u_i) := \max\{-\bar{u}_i, \min\{\bar{u}_i, u_i\}\}$ . The goal is to design the **dynamic output feedback controller with anti-windup compensation**

$$\mathcal{C} \sim \begin{cases} \dot{x}_c = A_c x_c + B_c y + E_c \text{dz}(u) \\ u = C_c x_c + D_c y \end{cases}, \quad (2)$$

where  $x_c \in \mathbb{R}^n$ ,  $u \in \mathbb{R}^m$  and  $\text{dz}(u) := u - \text{sat}(u)$ .

The controller-plant feedback system can be written as

$$\mathcal{H} \sim \begin{cases} \dot{x} = Ax + B \text{dz}(u) \\ u = Cx \end{cases}. \quad (3)$$

We extend the use of the sign indefinite quadratic form constructed in [3] to propose the Lyapunov function

$$V(x) := \begin{bmatrix} x \\ \text{dz}(u(x)) \end{bmatrix}^T \begin{bmatrix} P_{11} & P_{12} \\ P_{12}^T & P_{22} \end{bmatrix} \begin{bmatrix} x \\ \text{dz}(u(x)) \end{bmatrix}, \quad (4)$$

for the global synthesis [1], in which

- $P_{11} > 0$  is enforced
- but  $P_{22}$  may be sign-indefinite,

and, for the regional synthesis [2], the Lyapunov function

$$W(x) := \begin{cases} V(x) & \text{if } x \in \mathbb{R}^n : |H_1 x + H_2 \text{dz}(Cx)|_\infty \leq 1 \\ 1 & \text{otherwise} \end{cases} \quad (5)$$

with  $H_1$  and  $H_2$  being arbitrary design parameters.

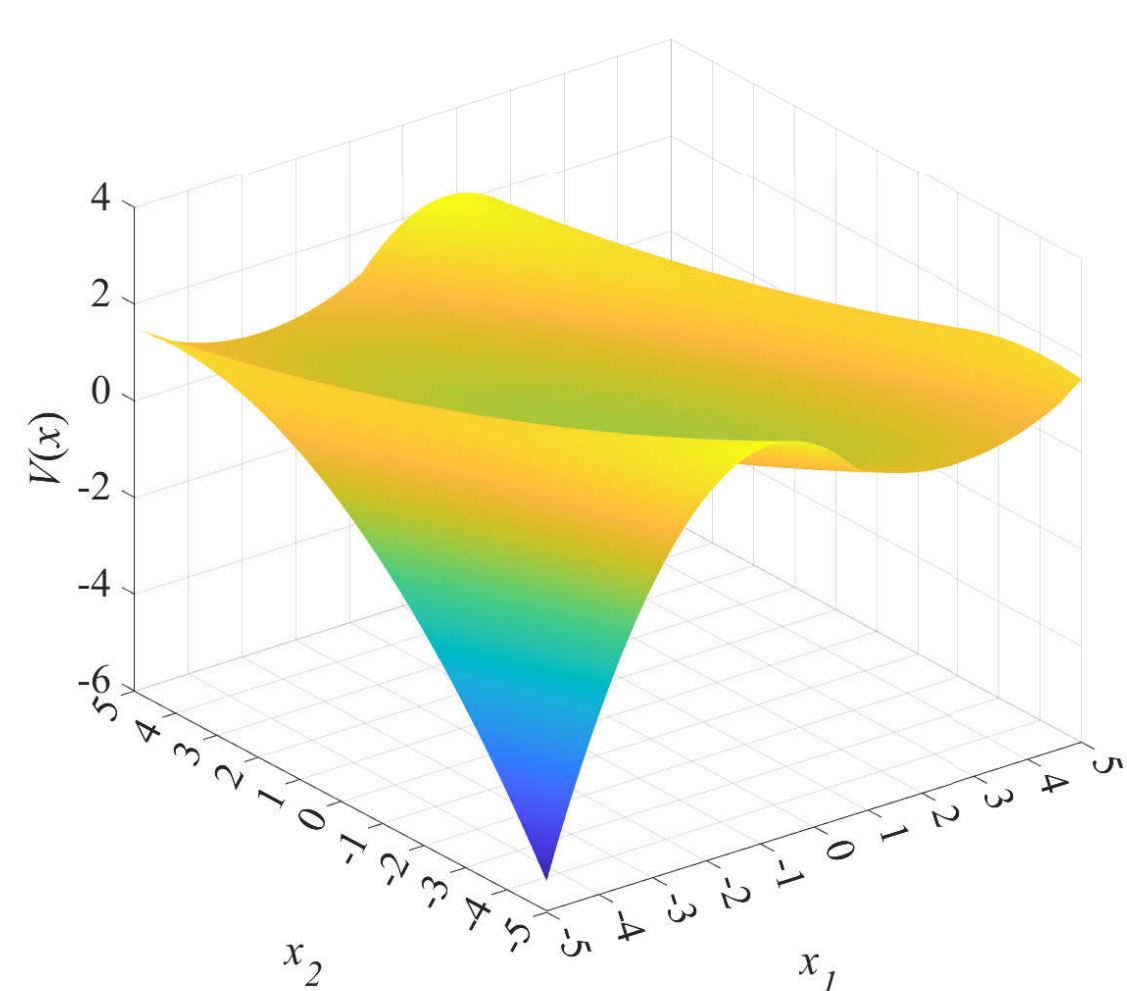


Figure 1: Example of the function  $V$  in the state space.

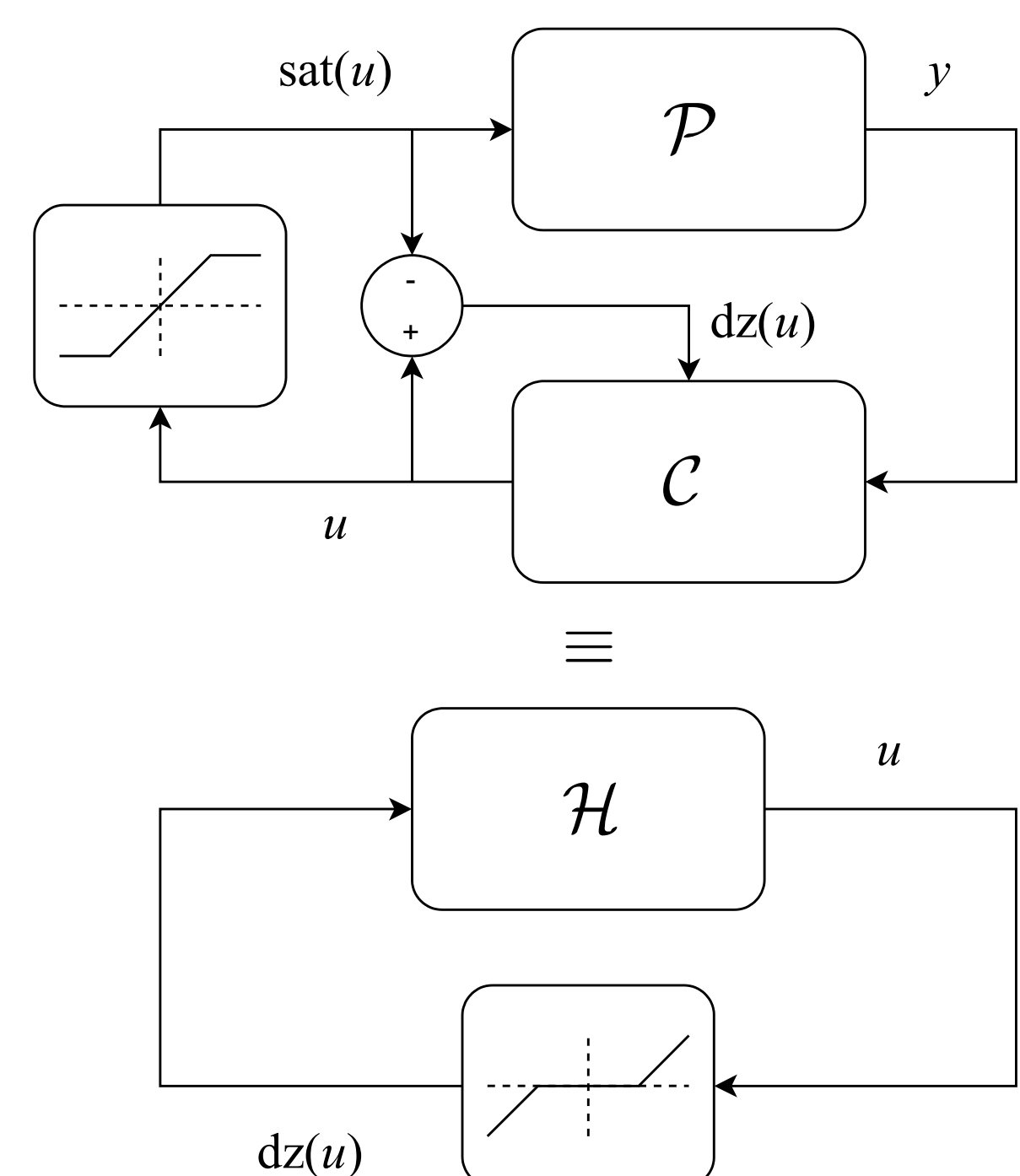


Figure 2: Saturated closed-loop system.

## 2 Main results

**Theorem:** Defining  $\bar{U} := \text{diag}(\bar{u})$ , if there exist matrices  $\mathbf{X} \in \mathbb{S}_{>0}^n$ ,  $\mathbf{Y} \in \mathbb{S}_{>0}^n$ ,  $\mathbf{Z}_p \in \mathbb{R}^{n \times m}$ ,  $\mathbf{Z}_c \in \mathbb{R}^{n \times m}$ ,  $\hat{\mathbf{P}}_{22} \in \mathbb{S}^m$ ,  $\mathbf{S} \in \mathbb{D}_{>0}^m$ ,  $\mathbf{G}_p \in \mathbb{R}^{n \times n}$ ,  $\mathbf{G}_c \in \mathbb{R}^{n \times n}$ ,  $\hat{\mathbf{H}}_2 \in \mathbb{R}^{n \times m}$ ,  $\hat{\mathbf{A}}_c \in \mathbb{R}^{n \times n}$ ,  $\hat{\mathbf{B}}_c \in \mathbb{R}^{n \times m}$ ,  $\hat{\mathbf{C}}_c \in \mathbb{R}^{m \times n}$ ,  $\hat{\mathbf{D}}_c \in \mathbb{R}^{m \times m}$  and  $\hat{\mathbf{E}}_c \in \mathbb{R}^{n \times m}$  satisfying

$$\Psi_1 = \text{He} \begin{bmatrix} \frac{1}{2}\mathbf{Y} & 0 & \mathbf{Z}_p & 0 \\ \mathbf{I}_n & \frac{1}{2}\mathbf{X} & \mathbf{Z}_c & 0 \\ \bar{\mathbf{U}}\mathbf{G}_p - \hat{\mathbf{C}}_c & \bar{\mathbf{U}}\mathbf{G}_c - \hat{\mathbf{D}}_c\hat{\mathbf{C}}_p & \frac{1}{2}\hat{\mathbf{P}}_{22} + \bar{\mathbf{U}}\hat{\mathbf{H}}_2 + \mathbf{S} & 0 \\ \mathbf{G}_p & \mathbf{G}_c & \hat{\mathbf{H}}_2 & \frac{1}{2}\mathbf{I}_m \end{bmatrix} > 0, \quad (6)$$

$$\Psi_2 = \text{He} \begin{bmatrix} A_p\mathbf{Y} + B_p\hat{\mathbf{C}}_c & A_p + B_p\hat{\mathbf{D}}_c\hat{\mathbf{C}}_p & -B_p\mathbf{S} & 0 & 0 & \mathbf{Z}_p \\ \hat{\mathbf{A}}_c & \mathbf{X}A_p + \hat{\mathbf{B}}_c\hat{\mathbf{C}}_p & \hat{\mathbf{E}}_c & 0 & 0 & \mathbf{Z}_c \\ \hat{\mathbf{C}}_c - \bar{\mathbf{U}}\mathbf{G}_p & \hat{\mathbf{D}}_c\hat{\mathbf{C}}_p - \bar{\mathbf{U}}\mathbf{G}_c & -\mathbf{S} - \bar{\mathbf{U}}\hat{\mathbf{H}}_2 & \mathbf{Z}_p^T - \hat{\mathbf{C}}_c & \mathbf{Z}_c^T - \hat{\mathbf{D}}_c\hat{\mathbf{C}}_p & \hat{\mathbf{P}}_{22} \\ A_p\mathbf{Y} + B_p\hat{\mathbf{C}}_c & A_p + B_p\hat{\mathbf{D}}_c\hat{\mathbf{C}}_p & -B_p\mathbf{S} & -\mathbf{Y} & -\mathbf{I}_n & 0 \\ \hat{\mathbf{A}}_c & \mathbf{X}A_p + \hat{\mathbf{B}}_c\hat{\mathbf{C}}_p & \hat{\mathbf{E}}_c & -\mathbf{I}_n & -\mathbf{X} & 0 \\ 0 & 0 & \mathbf{S} & \hat{\mathbf{C}}_c & \hat{\mathbf{D}}_c\hat{\mathbf{C}}_p & -\mathbf{S} \end{bmatrix} < 0, \quad (7)$$

then the origin of the closed loop system is locally exponentially stable from all  $x \in \mathbb{R}^n : W(x) < 1$  with the controller matrices  $A_c$ ,  $B_c$ ,  $C_c$ ,  $D_c$  and  $E_c$  as selected in [1], [2]. Additionally, if (6) and (7) are satisfied with for all  $x \in \mathbb{R}^{2n}$ , then the origin of the closed loop system is globally exponentially stable.

## 3 Numerical examples

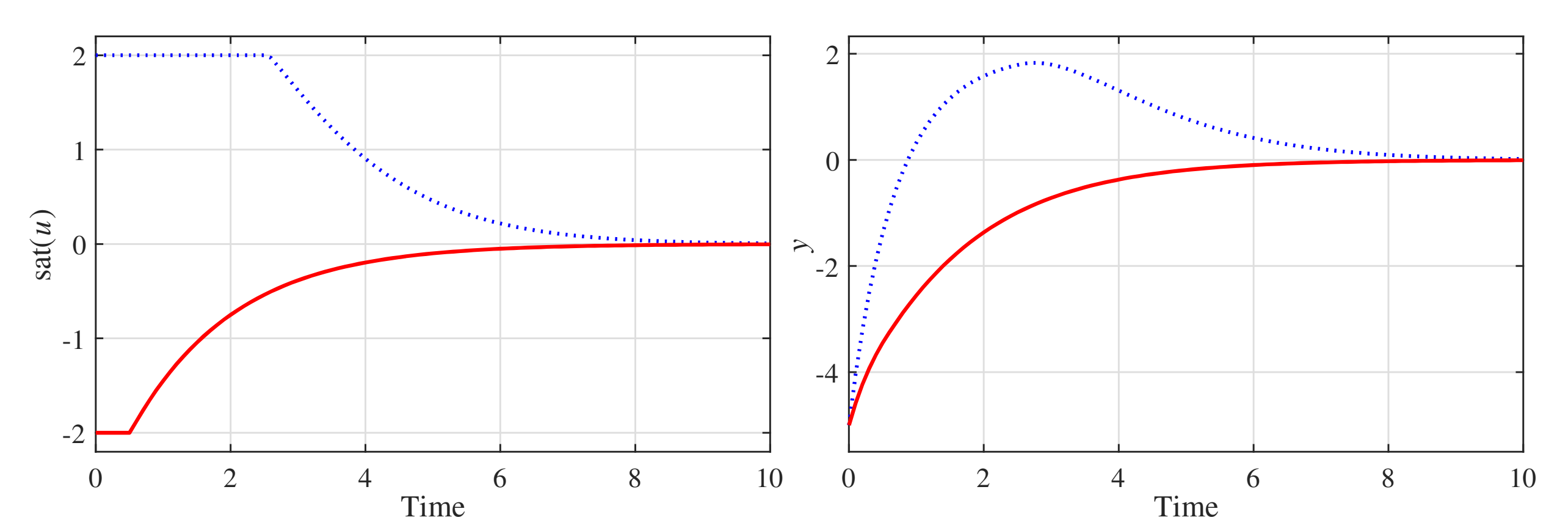


Figure 3: Global results example.

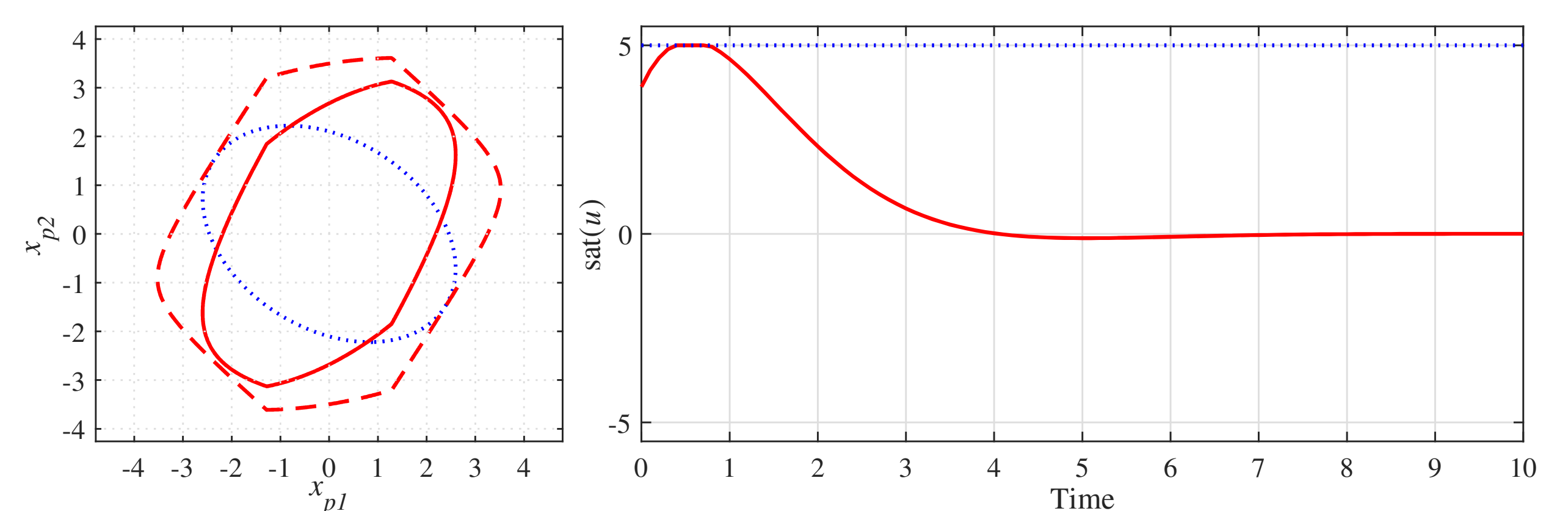


Figure 4: Regional results example.

## References

- [1] S. Pantano Calderón, S. Tarbouriech, and L. Zaccarian. Global exponential saturated output feedback design with sign-indefinite quadratic forms. In *European Control Conference (ECC)*, 2024.
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- [3] I. Queinnec, S. Tarbouriech, G. Valmorbida, and L. Zaccarian. Design of saturating state feedback with sign-indefinite quadratic forms. *IEEE Transactions on Automatic Control*, 67(7):3507–3520, 2022.