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# Observer-based Controller Design for LTI SISO Systems with Feedback Delay. A Partial Pole Placement Approach

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**Abstract:** In this paper, we address the observer based control problem for SISO (*Single-Input/Single-Output*) LTI (*Linear Time Invariant*) system subject to delay in the measurement data. The proposed design approach includes two steps: first, by using a partial pole placement approach based on the MID (*Multiplicity-Induced-Dominancy*) property, we construct a delay-based controller guaranteeing the stability of the closed-loop system. Second, we exploit the control gain to determine the observer-controller matrices. The effectiveness of the proposed methodology is illustrated through the problem of stabilizing a double integrator and of a wind-tunnel system with delayed feedback.

*Keywords:* Delay, observer, partial pole placement, spectral abscissa.

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## 1. INTRODUCTION

Transport, propagation, communication and heredity are the main ways in which delays appear in natural phenomena and engineering processes. For further examples and discussions on related applications, we refer to Stépán (1989); Gu et al. (2003); Michiels and Niculescu (2014), and the references therein.

In this work, a class of SISO system with delay in the input-output channel is studied. Some devices with ample computation resources exhibit faster calculation and uploading of model parameters so that the feedback time duration may vary. However, devices with limited computation resources may struggle to complete the timely updating and uploading of model parameters, and consequently generating feedback delays. To fill this aforementioned gap, we propose a delayed observer based controller to stabilize this class of systems.

Recent research has witnessed interesting advances in both stability analysis and the design of stabilizing control laws for delay systems. In frequency-domain, a recent pole placement paradigm, called *Partial Pole Placement (PPP)*, has been introduced in Boussaada et al. (2020); Mazanti et al. (2021); Boussaada et al. (2022) which guarantee the stability of the zero solution of the corresponding time-delay system. It derives from two properties called respectively *multiplicity-induced-dominancy* (MID) and *coexistent-real-roots-inducing-dominancy* (CR-RID), see for instance Amrane et al. (2018); Bedouhene et al. (2020); Schmoderer et al. (2023). It ensues from an intriguing observation that, in some cases, multiple

spectral values of the characteristic functions associated to linear *Delay-Differential Equations* (DDE) define the spectral abscissa<sup>1</sup> of the system, see, for instance, the discussions in Mazanti et al. (2021) (generic multiplicity in retarded case) Boussaada et al. (2022) (retarded and neutral cases, generic multiplicity) or, more recently, Boussaada et al. (2023) (over-order multiplicity).

In the present work, we aim to the design of observer-based controller using the structure of a functional observer but with a different purpose - estimating the stabilizing control law and not the 'classical' (system) state-estimation. In our opinion, one of the advantages of functional observer design for delayed systems is its application in designing observer-based controller which significantly decreases the computational burden. This controller design is inspired from Zasadzinski et al. (2007), where an uncertain linear system is considered. In fact, the design has been studied with a Lyapunov-based approach leading to LMI (*Linear Matrix Inequality*). Here, we extend this design to delayed LTI system with the use of a partial pole placement technique in the control design. To the best of the authors' knowledge, such a problem was not addressed yet in the literature. In fact, only recent works Rojas-Ricca et al. (2023) on predictor synthesis of delayed nonlinear system were proposed using the MID property to tune the gain ensuring the convergence of the prediction error dynamics.

The rest of the paper is organised as follows. Some prerequisites and preliminaries developed in some recent works, are given in section 2. The problem formulation is pre-

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<sup>1</sup> the real part of the rightmost (characteristic) root of the corresponding characteristic function

sented in Section 3 where the system, the observer-based controller model and the objectives are introduced. In Section 4, the decoupled control strategy is developed while the stabilization of closed-loop linear time-delay system, the existence conditions of the observer-based controller and the convergence of the error system, are analysed. Applications of this problem are presented in section 5. More precisely, the double integrator case study and the two finger robot which both involves time-delays in the measurements vectors are discussed. Finally, some concluding remarks end the paper.

## 2. PREREQUISITES AND PRELIMINARY RESULTS

Consider the LTI dynamical system described by the DDE including a single delay:

$$y^{(n)}(t) + \sum_{k=0}^{n-1} a_k y^{(k)}(t) + \sum_{k=0}^q \alpha_k y^{(k)}(t - \tau) = 0 \quad (1)$$

under suitable initial conditions, where  $y(\cdot)$  represents the unknown real-valued function,  $\tau > 0$  denotes the delay, and the coefficients  $a_0, \dots, a_{n-1}, \alpha_0, \dots, \alpha_q$  stand as real coefficients. The DDE (1) is a specific instance of the subsequent matrix representation.

$$\dot{X}(t) + B_\tau \dot{X}(t - \tau) = AX(t) + A_\tau X(t - \tau) \quad (2)$$

where  $X(t) = (y(t) \ \dot{y}(t) \ \dots \ y^{(n-1)}(t))^T \in \mathbb{R}^n$  is the state vector and the matrices  $A$ ,  $A_\tau$  and  $B_\tau$  are known real matrices. Thus, the characteristic function corresponding to (1) is the quasipolynomial  $\Delta: \mathbb{C} \rightarrow \mathbb{C}$ , given by

$$\Delta(s, \tau) = P_0(s) + P_1(s)e^{-s\tau}, \quad (3)$$

where  $P_0$  and  $P_1$  are polynomials in  $s$ .

As mentioned in the Introduction, we will use a *partial pole placement* approach for the stability analysis of the closed-loop system. This approach consists in selecting the free parameters of the controller to determine the positions of a finite number of roots, ensuring that the dominant root (i.e. the rightmost characteristic root in the complex plane) is included among the selected ones. For further insights and/or illustrative examples, the reader is referred to Bedouhene et al. (2020); Boussaada et al. (2020); Mazanti et al. (2021); Ramirez et al. (2016). Unlike the finite pole placement (FPP) strategy employed in Manitius and Olbrot (1979), the controllers developed using these approaches do not aim to make the closed-loop system finite-dimensional and conserve the infinite-dimensional character of the system. Conceptually, they focus on controlling its rightmost spectral value by imposing an appropriate multiplicity or an appropriate number of coexisting real roots. Specifically, in this paper, we will use the MID property outlined in Balogh et al. (2021) where the case of real-rooted plants has been addressed.

So, we address the case when the plant is real-rooted.

*Assumption 1.* The polynomial  $P_0$  to be real-rooted.

Denote  $s_0$  a real root with multiplicity at least  $n$  of the characteristic function (3). It is established that if  $s_0$  represents the corresponding rightmost root, then the zero solution of system (2) is asymptotically stable, leading to a “stable” vector  $X(t)$ . The following propositions from Balogh et al. (2021) provide a detailed integral representation of the quasipolynomial.

*Proposition 2.* If the quasipolynomial (3) has a real root  $s_0$  with multiplicity at least  $n$  then it can be written as

$$\Delta(s) = (s - s_0)^n \left( n_n + \int_0^1 e^{-(s-s_0)\tau t} \frac{\tau R_{n-1}(s_0; \tau t)}{(n-1)!} dt \right), \quad (4)$$

where the family of polynomials  $R_k(s, \tau)$  is defined as

$$R_k(s; \tau) = \sum_{i=0}^k \binom{k}{i} P^{(i)}(s) \tau^{k-i}, \quad k \in \mathbb{N}^*. \quad (5)$$

The following Proposition from Balogh et al. (2021) establishes *sufficient conditions* to ensure the dominance of the multiple spectral value.

*Proposition 3.* Let  $s_0$  be a real root of the quasipolynomial (3) with multiplicity at least  $n+1$ . If  $R_k(s; \tau t) \leq 0, \forall t, 0 < t \leq 1$  then  $s_0$  is the dominant root of (3).

These results will be used in the sequel to prove the (exponential) stability of the closed-loop system. For more discussions we refer the reader to Balogh et al. (2021); Boussaada et al. (2023) and references therein.

## 3. PROBLEM STATEMENT

We consider a SISO LTI system with a delay in the output signal given by the following equations:

$$\dot{x}(t) = Ax(t) + Bu(t) \quad (6a)$$

$$y(t) = Cx(t - \tau) \quad (6b)$$

$$x(0) = \phi(t) \quad t \in [0, \tau] \quad (6c)$$

where  $x \in \mathbb{R}^n$ ,  $u \in \mathbb{R}^m$  ( $m = 1$ ), and  $y \in \mathbb{R}^p$  ( $p = 1$ ) are the state vector, the control input and the measurements vector, respectively. Here,  $\tau \in \mathbb{R}_+$  is a constant delay. The function  $\phi(t)$  is a continuous one defining the corresponding initial condition. The matrix  $A \in \mathbb{R}^{n \times n}$  and the vectors  $B \in \mathbb{R}^n$  and  $C^T \in \mathbb{R}^n$  are time-invariant.

As outlined in the Introduction, our aim is to design an observer-based controller with the following structure

$$\dot{\eta}(t) = N\eta(t) + My(t) + Fu(t - \tau) \quad (7a)$$

$$u(t) = \eta(t) + Ey(t) \quad (7b)$$

where  $\eta \in \mathbb{R}^m$ . The matrices  $N$ ,  $M$ ,  $F$  and  $E$  are unknown, and will be determined later.

The approach used in this paper involves a two-step controller design: first, constructing a state feedback gain  $L$  verifying  $u(t) = Lx(t - \tau)$  for subsystem (6). Once this gain is found (i.e. assumed to be known for the sequel), the second-step consists in proposing an observer-based controller (7) allowing to construct this gain  $L$ .

More precisely, we introduce the following notion:

*Definition 4.* System (6) is said to be stabilizable based on the functional observer if there exist gain matrix  $L$ , a functional observer (7a) and a control law (7b) such that

- (i)  $\lim_{t \rightarrow \infty} u(t) - Lx(t - \tau) = 0$
- (ii) the designed controller must be an observer-based one for the nominal system (i.e. a separation principle like condition is satisfied : the eigenvalues of  $A$  and  $N$  are those of the state matrix of the nominal closed-loop system),
- (iii) the closed-loop system (6)-(7) is stable.

## 4. CONTROL STRATEGY

The control strategy for system (6) can be divided into two parts: the state-feedback gain design and the design of a functional filter which estimates the control law without estimating the system states.

### 4.1 Synthesis of the state feedback gain

The linear state feedback controller can be determined by a nominal linear model. Thus, we replace  $u$  by  $Lx(t-\tau)$  in system (6a)-(6b) and get the following closed-loop system

$$\dot{x}(t) = Ax(t) + BLx(t-\tau) \quad (8a)$$

$$y(t) = Cx(t-\tau) \quad (8b)$$

The characteristic function  $\Delta : \mathbb{C} \times \mathbb{R}_+ \rightarrow \mathbb{C}$  corresponding to (8) reads as follows

$$\Delta(s, \tau) = \det(sI_n - A - BLE^{-\tau s}) \quad (9)$$

which yields to the form of function (3).

*Remark 5.* In previous work Sassi et al. (2022, 2023), we assumed that the matrices product  $BL$  is a one-rank matrix. This assumption guarantee that the quasipolynomial (3) involves a single delay, so one can use results in section 2. However, we consider, in this work, a class of SISO system. Thus, with the following rank property

$$\text{rank}(AB) \leq \min(\text{rank}(A), \text{rank}(B))$$

one can see easily that the rank of the matrix  $BL$  is equal to 1 as the rank of the matrices  $B$  and  $L$  is equal to 1. So, we can use results of section 2 without considering a one-rank matrix assumption. Furthermore, the previous results are for the design of observers. So, here we extend our design to the design of observer-based controller.

Now, let us define the following polynomial  $\mathbb{P}$  called in Boussaada et al. (2023) as the *elimination-produced polynomial*, defined by

$$\mathbb{P}(s, \tau) = \sum_{k=0}^n \binom{n}{k} P_0^{(k)}(s) \tau^{n-k}. \quad (10)$$

Using Propositions 2 and 3 in Section 2, the stability of the closed-loop control design can be guaranteed by the following theorem.

*Theorem 6.* Consider that Assumption 1 holds and the quasipolynomial (9) associated to the closed-loop system (8) admits a root at some complex number  $s_0$  with multiplicity  $n+1$ . Then,  $s_0$  is necessarily algebraic and the corresponding characteristic polynomial is  $\mathbb{P}$  defined in (10). Moreover, if  $s_0$  is the abscissa of  $\mathbb{P}$  and  $\mathbb{P}$  is is real-rooted and Hurwitz, then the closed-loop system given by (8) exhibits exponential stability with  $s_0$  as a decay rate.

### Sketch of the proof

The vanishing of the characteristic equation  $\Delta(s) = 0$  can be expressed as  $e^{-s\tau} = -\frac{P_0(s)}{P_1(s)}$ . Subsequently, an algebraic system of  $n$  equations  $\Delta'(s) = 0, \dots, \Delta^{(n)}(s) = 0$  is considered. Initially, due to the linearity of the system  $\Delta'(s) = 0, \dots, \Delta^{(n-1)}(s) = 0$  with respect to the coefficients of  $P_1$ , one proceeds by systematically eliminating these coefficients order by order. Subsequently, the obtained values of these coefficients are substituted into the remaining equation  $\Delta^{(n)}(s) = 0$ , yielding to the

elimination-produced polynomial  $\mathbb{P}$ , in the variable  $s$ , the delay  $\tau$ , and the coefficients of  $P_0$ .

Assuming that condition 1 is satisfied, the proof of the theorem relies on proposition 3, as the quasipolynomial corresponding to the closed-loop system (8) is described by equation (9).  $\square$

### 4.2 Synthesis of the observer based controller

Using item (i) of definition 4, an observation error signal can be defined as

$$e(t) = Lx(t-\tau) - u(t) = \Psi x(t-\tau) - \eta(t) \quad (11)$$

where  $\psi = L - EC$ . Note that the matrix  $L$  is known and deduced from the section before, so that the closed-loop system is stable. The error admits the following dynamics

$$\dot{e}(t) = Ne(t) + (\Psi A - N\Psi - MC)x(t-\tau) + (\Psi B - F)u(t-\tau) \quad (12)$$

The unbiasedness of the nominal part corresponding to item (i) of definition 4 is achieved if and only if the closed-loop is stable and the following conditions hold:

$$F - \Psi B = 0 \quad (13a)$$

$$\Psi A - N\Psi - MC = 0 \quad (13b)$$

Replacing  $\Psi$  by its expressions, equations (13) can be written with the following form.

$$F = LB - ECB \quad (14a)$$

$$LA = ECA + NL + KC \quad (14b)$$

with  $K = M - NE$ . One can see that we have just to solve equation (14b) to determine the existence condition of the observer, i.e. the expression of matrices  $N$ ,  $K$  and  $E$ , and then deduce the matrix  $F$  from equation (14a) and the matrix  $M$  from the expression of  $K$ .

Thus, we rewrite equation (14b) with the following form

$$\underbrace{[LA]}_{\mathcal{A}} = \underbrace{[N \ K \ E]}_{\overline{\mathcal{X}}} \underbrace{\begin{bmatrix} L \\ C \\ CA \end{bmatrix}}_{\mathcal{B}} \quad (15)$$

where the unknown matrices  $N$ ,  $K$  and  $E$  are gathered in one vector, denoted in the sequel  $\overline{\mathcal{X}}$ . Then equation (15) can be written with a compact form  $\mathcal{A} = \overline{\mathcal{X}}\mathcal{B}$ . This type of equation admits a solution if and only if the following rank condition is satisfied Darouach (2000).

$$\text{rank}(\mathcal{B}) = \text{rank} \begin{bmatrix} \mathcal{A} \\ \mathcal{B} \end{bmatrix} \quad (16)$$

Let us note  $r = m + 2p$  (with  $m = 1$ , dimension of the system input), The general solution of equation (15) is given by

$$\overline{\mathcal{X}} = \mathcal{A}\mathcal{B}^\dagger + Z(I_r - \mathcal{B}\mathcal{B}^\dagger) \quad (17)$$

where  $Z$  is an arbitrary matrix with appropriate dimension, which will be chosen in order to satisfy the convergence of the estimation errors,  $\overline{\mathcal{X}}$  is defined hereafter in equation (19) and  $\mathcal{B}^\dagger$  is any generalised inverse of  $\mathcal{B}$ , which fulfils the following equation Rao and Mitra (1971)

$$\mathcal{B} = \mathcal{B}\mathcal{B}^\dagger\mathcal{B} \quad (18)$$

Then, the observers matrices can be expressed through a single gain matrix  $Z$  as

$$\underbrace{[N \ K \ E]}_{\overline{\mathcal{X}}} = \underbrace{[N_1 \ K_1 \ E1]}_{\mathcal{A}\mathcal{B}^\dagger} + Z \underbrace{[N_2 \ K_2 \ E2]}_{I_r - \mathcal{B}\mathcal{B}^\dagger} \quad (19)$$

The matrix  $Z$  will be determined from the convergence analysis of the estimation error hereafter and will allow us computing the different parameter matrices of the functional observer (7).

Since the existence conditions of the functional observer are satisfied with the resolution of (13b), the dynamics of the estimation error is described by:

$$\dot{e}(t) = Ne(t) \quad (20)$$

As the control problem is decoupled, and the convergence of the closed-loop system is derived by similarity to the one presented in the previous paragraphs, one can see that since the matrix  $N$  is Hurwitz, (7) is an *observer-based controller* for the system (6). Thus, one can assign the decay rate, denoted  $s_0$  of the observation error system by choosing an appropriate matrix such that

$$Z = (s_0 - N_1)N_2^{-1} \quad (21)$$

Based on the remarks above, we state the following result:

*Theorem 7.* With the notations above, assume that the rank condition (16) holds. Then the system (7) is an observer-based unbiased controller for system (6) where  $L$  is given by the resolution of the corresponding quasipolynomial (9) in function of the delay parameter  $\tau$ , if:

- (i)  $N$  is Hurwitz and
- (ii) for an assigned decay rate  $s_0$ , the matrix  $Z$  is given by equation (21).

## 5. ILLUSTRATIVE EXAMPLES

### 5.1 Double integrator system

We consider a system of a double integrator with sensor delay as in Espitia et al. (2022)

$$\dot{x}_1(t) = x_2(t) \quad (22a)$$

$$\dot{x}_2(t) = u(t) \quad (22b)$$

$$y(t) = x_1(t - \tau) \quad (22c)$$

The system is unstable. However, it is observable and controllable. In closed-loop, assuming that  $u(t) = Lx(t - \tau)$ , system (22a) can be written as:

$$\dot{X}(t) = AX(t) + BLX(t - \tau),$$

where  $X(t) = [x_1(t) \ x_2(t)]^T$ . The matrices  $A$  and  $B$  can be easily deduced and  $L = [\ell_1 \ \ell_2]$  is the control gain to be determined in function of the delay  $\tau$ . The quasipolynomial corresponding to the closed-loop system is explicitly given by

$$\Delta(s, \tau) = s^2 - e^{-\tau s}(\ell_2 s + \ell_1).$$

It is easy to observe that  $\Delta$  admits a triple root at some complex number  $s$  if and only if  $s = s_{\pm}$  where

$$\begin{cases} s_{\pm} = \frac{-2 \pm \sqrt{2}}{\tau}, \\ \ell_1 = -\frac{2(-7 \pm 5\sqrt{2})e^{s_{\pm}\tau}}{\tau^2} \\ \ell_2 = -\frac{2(-1 \pm \sqrt{2})e^{s_{\pm}\tau}}{\tau}, \end{cases} \quad (23)$$

leading to:

$$\Delta(s, \tau) = s^2 - e^{-\tau s} \left( \frac{2s(\sqrt{2} - 1)e^{-2+\sqrt{2}}}{\tau} + \frac{2(-7 + 5\sqrt{2})e^{-2+\sqrt{2}}}{\tau^2} \right) \quad (24)$$

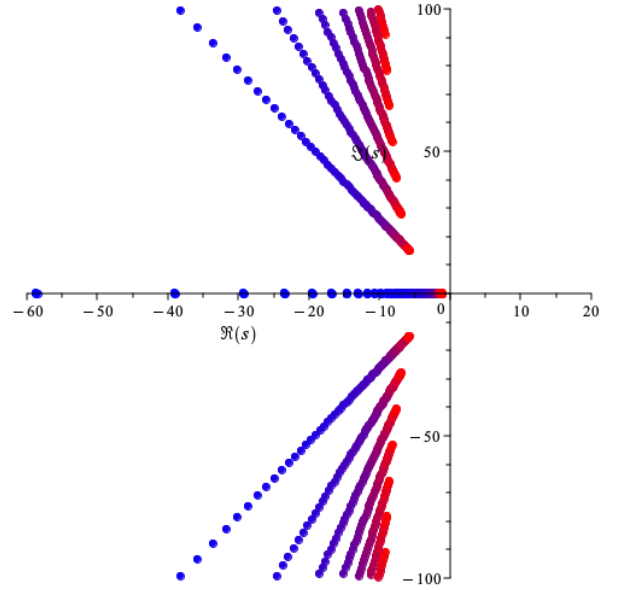


Fig. 1. Migration of the roots of (24) with respect to the delay variation  $\tau \in [\frac{1}{200}, \frac{1}{2}]$  (from blue to red).

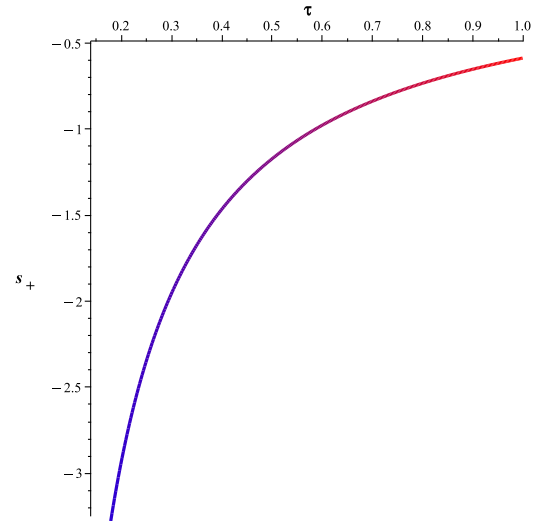


Fig. 2. Variation of the spectral abscissa of (24) with respect to the delay (parameter) variation.

Using first Theorem 6 one can compute the gain  $L$  as in equations (23). Consider two cases given by  $\tau_1 = 0.4$  and  $\tau_2 = 2$ . Then, the matrix  $L = [-0.4945 \ -1.1529]$  for  $\tau = \tau_1$  and  $L = [-0.0198 \ -0.2306]$  for  $\tau = \tau_2$ . Next, using Theorem 7, we compute the observer gain  $N$  to stabilize the error system (20) and the characteristic function is given by

$$\Delta(s, \tau) = s - N = s - N_1 - ZN_2$$

So that, one can assign the decay rate  $s_0 = -2$  of the observation error by choosing an appropriate  $Z = \frac{(s_0 - N_1)}{N_2} = [-2.2215 \ -1.0986 \ -2.5612]$  for  $\tau = \tau_1$ .

Then, the observer-based controller matrices are given by  $N = -2$ ,  $M = 4.6116$ ,  $F = -1.1529$  and  $E = -2.8003$ . For  $\tau = \tau_2$  and with the same decay rate  $s_0 = -2$ , we obtain the matrix  $Z = [-2.0043 -0.0396 -0.4622]$ , which gives the following observer-based controller  $N = -2$ ,  $M = 0.9223$ ,  $F = -0.2306$  and  $E = -0.4809$ .

The initial conditions considered for the simulation are  $x(0) = [-12 \ 10]$  and for the controller  $\eta(0) = 0$ . Figures 3 and 4 show the convergence of the closed-loop states system for  $\tau = 0.4\text{s}$  and  $\tau = 2\text{s}$  respectively.

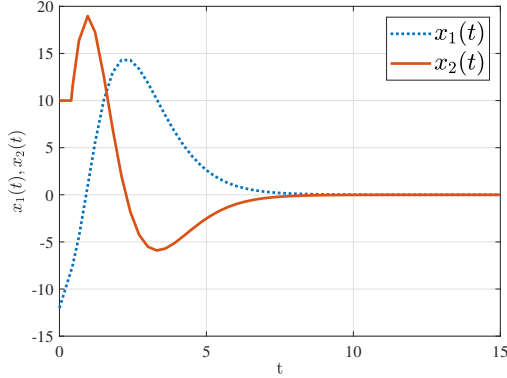


Fig. 3. Closed-loop states  $x(t)$  for  $\tau = 0.4\text{s}$ .

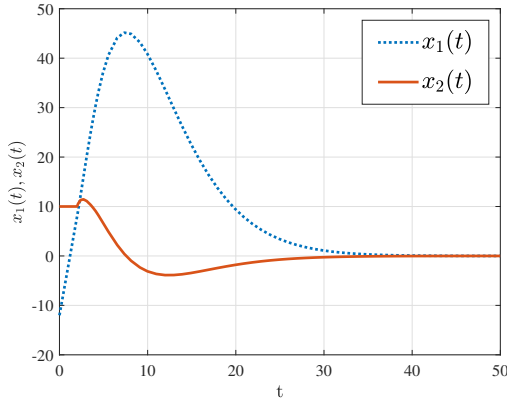


Fig. 4. Closed-loop states  $x(t)$  for  $\tau = 2\text{s}$ .

## 5.2 Wind Tunnel System

As a second example, we consider the problem of control of a transonic flow in a wind tunnel described in Armstrong and Tripp (1981); Maniatis (1984):

$$\begin{cases} \kappa m'(t) + m(t) = k\vartheta(t), \\ \vartheta''(t) + 2\zeta\omega\vartheta'(t) + \omega^2\vartheta(t) = \omega^2 u(t), \end{cases} \quad (25)$$

in which  $m$ ,  $\vartheta$ , and  $u$  represent, respectively, perturbations of the Mach number of the flow, the guide vane angle, and the input of the guide vane actuator, with respect to steady-state values. The parameters  $\kappa$  and  $k$  depend on the steady-state operating point and are assumed to be constant as long as  $m$ ,  $\vartheta$ , and  $u$  remain small, and satisfying  $\kappa > 0$  and  $k < 0$ . The parameters  $\zeta \in (0, 1)$  and  $\omega > 0$  come from the design of the guide vane angle actuator and are thus independent from the operating point. Equation (25) may be written under a state-space model representation as follows

$$\dot{X}(t) = \begin{bmatrix} -2\zeta\omega & -\omega^2 & 0 \\ 1 & 0 & 0 \\ 0 & \frac{k}{\kappa} & \frac{-1}{\kappa} \end{bmatrix} X(t) + \begin{bmatrix} \omega^2 \\ 0 \\ 0 \end{bmatrix} u(t), \quad (26)$$

where  $X(t) = [\vartheta(t) \ \vartheta'(t) \ m(t)]^T$ .

In closed-loop, assuming that  $u(t) = Lx(t - \tau)$ , with  $L = [\ell_1 \ \ell_2 \ \ell_3]$ , the system can be written as in (8a) and the available measurement is assumed to be single and delayed such that  $y(t) = \vartheta(t - \tau)$ .

The quasipolynomial corresponding to the closed-loop system is explicitly given by

$$\Delta(s) = s^3 + \underbrace{\frac{23s^2}{6} + \frac{80s}{3} + \frac{25}{2}}_{P_0(s)} e^{-s\tau} \times \left( 25\ell_1 s^2 + \left( \frac{25\ell_1}{2} + 25\ell_2 \right) s - \frac{3025\ell_3}{361} + \frac{25\ell_2}{2} \right) \quad (27)$$

This latter quasipolynomial admits a quadruple root (as shown in figure 5)  $s = -4.3891$ , for a delay  $\tau = 1/3\text{s}$ , if and only if

$$\begin{cases} \ell_1 = 0.0111 \\ \ell_2 = 0.2067 \\ \ell_3 = 1.3512 \end{cases} \quad (28)$$

Since the gain  $L$  is known, we compute the observer-based

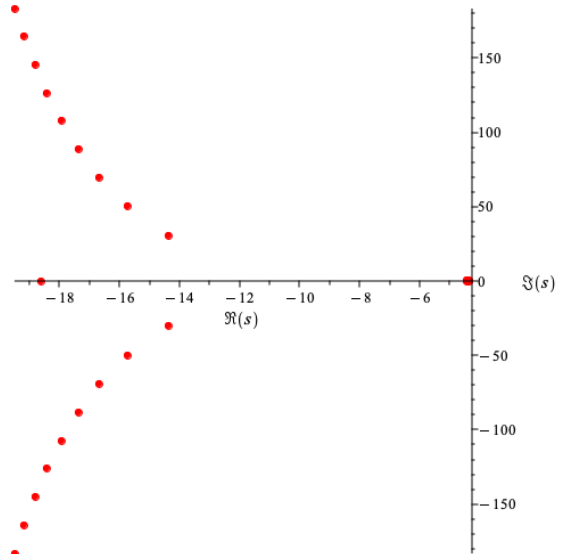


Fig. 5. The spectrum distribution of the quasipolynomial (27)

controller matrices using equation (17), and by assigning a decay rate  $s_0 = -1$ , one can compute easily the matrix  $Z$  using (21). So, the controller matrices are as follows.  $N = -1$ ,  $M = -1.037$ ,  $F = 0.2775$  and  $E = 0.46316$ .

We consider the initial conditions as  $X(0) = [0.25 \ 0 \ 0.15]^T$  and  $\eta(0) = 0$ . Figure 6 shows the convergence of the states vector to zero and consequently the stabilization of the considered system.

## 6. CONCLUSION

A decoupled controller design for SISO delayed systems has been investigated using a functional filter approach

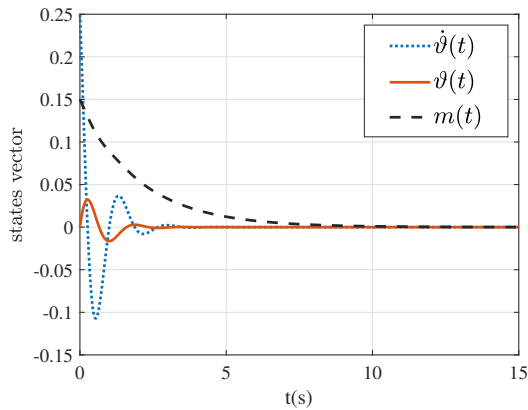


Fig. 6. Closed-loop states  $x(t)$  for  $\tau = 1/3s$ .

to estimate the control law. The optimal gain matrices of the functional observer-based controller are obtained in two steps. First, we determine the controller gains which stabilize the closed-loop system. Then, using these gains for some delay value, the nominal unbiasedness condition of the observer error dynamics are given and the functional observer matrices are easily obtained with a partial pole placement approach. The practicality and efficiency of the proposed stabilization approach is illustrated through two examples. *Multiple-Inputs Multiple-Outputs* (MIMO) systems will be considered in future works with another type of control strategy to overcome the one rank condition on the delayed matrix given in previous work and not considered here due to the considered class of system, i.e. a SISO one (see remark 5).

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