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The transient P.E.T comfort index : A Description and analysis of the physiological model

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RÉSUMÉ. Ce travail présente la version transitoire du modèle physiologique utilisée dans l'indicateur de confort Physiological Equivalent Temperature (PET). Les hypothèses de modélisation sont détaillées dans un premier temps, puis on compare les grandeurs physiologiques simulées à un cas issu de la littérature expérimentale. Enfin l'influence du schéma numérique de résolution des équations constitutives est discutée. Les résultats montrent que le modèle de la PET ne reproduit pas aussi bien les variations de température de peau et du corps humain que celui de Gagge (SET). De plus, le schéma d'intégration temporelle explicite conduit à une prédiction instable ou imprécise des températures, et il convient donc d'utiliser un schéma implicite pour faire face à la non-linéarité des coefficients du modèle.

MOTS-CLÉS. confort, PET, physiologie

ABSTRACT. The present work deals with the physiological model used in the transient version of the Physiological Equivalent Temperature (PET) comfort index. The modelling assumptions are first presented, then the simulated physiological quantities are compared with a case from the experimental literature. Finally, the influence of the numerical integration schemes is explored. The results show that the PET model does not reproduce skin and body temperature variations as well as the Gagge (SET) model. In addition, the commonly used explicit scheme leads to unstable or imprecise prediction of the skin and core temperatures and hence an implicit scheme should be used for time integration, thus handling the non-linearity of the model's coefficients.

KEYWORDS. thermal comfort, PET, physiology

1. INTRODUCTION

Among a variety of existing comfort indexes, the "two-node" model initially developed by (Gagge et al., 1971), (Gagge et al., 1986) is used for the prediction of comfort in indoor or outdoors conditions. It has been successfully used to predict indoor dynamic thermal comfort of individuals exposed to transient conditions, or in outdoor spaces (Nazarian et al., 2019), (Vellei and Le Dréau, 2020). It relies on a relatively simple mathematical model of the human body, consisting of two differential equations that provide the skin and body core temperatures. These temperatures allow for the determination of an "effective" or "equivalent" temperature of a reference environment much resembling an indoor office, that provokes the same physiological response as the actual environment. A comfort scale was derived from ranges of temperatures of this reference environment – see (Gagge et al., 1986).

Based on the same representation of the human body and developed at the turn of the 1990's (Höppe, 1989), (Höppe, 1993), the *Physiological Equivalent Temperature* (PET). The physiolo-

gical model on which the PET is based is called "Instationary Munich Energy balance Model" (IMEM). The comfort index derived from the IMEM also relies on an equivalent temperature, determined in similar indoors conditions. The PET comfort index was integrated in the German VDI norm (VDI, 2008) and has been regularly used in the last decade (Lau et al., 2019), (Kataoutas et al., 2015), (Krüger et al., 2012). It remains however somewhat mysterious : the first exhaustive description of the steady PET model can be found in (Walther and Goestchel, 2018).

As per its transient version, the works by (Höppe, 1993), (Höppe, 2002), (VDI, 2008) partially provide the details about the model. More explanations could potentially be found in (Höppe, 1994), (Höppe, 1989) and (Höppe, 1984), however to date those manuscripts are not accessible by modern means of communication. As a result, the existing literature does not cover critical aspects of the model, noticeably a clear description of the equations used, how they are numerically solved and a validation against experimental data.

The work presented here thus aims at describing thoroughly the "Instationary Munich Energy Model of the Individual", comparing its result with openly available experimental data and evaluating the impact of the numerical procedure used to compute the body thermophysiological reaction.

2. DESCRIPTION OF HÖPPE'S TWO-NODE MODEL

The illustration of Höppe's "two-node model", is shown on Figure 1, where the human consists in a cylinder composed of three layers : the body core, the skin, and a clothing layer that partially covers the cylinder. In essence the model is much alike Gagge's, except that convection and radiation are split here between the bare and clothed part of the body.

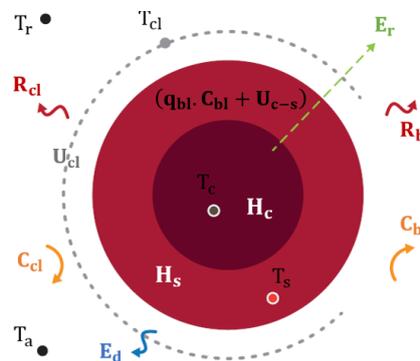


Figure 1 : Illustration of the two-node model by Höppe, considering separately bare and clothed skin.

The diagram on Figure 1 is also a representation of the heat fluxes between the body layers and the external environment. They can be divided into two main groups :

- Sensible heat transfer including conductive, convective and radiation exchanges : U_{c-s} is the skin conductance, U_{cl} the clothing conductance, $q_{bl}c_{bl}$ is heat transport by blood flow, C_b convection on bare skin, C_{cl} convection on clothed skin, R_b and finally R_{cl} the radiant exchange on bare and clothed skin ;
- Latent heat transfer including perspiration through skin (E_d) and exchange of humidity by breathing (E_r). Sweating may also occur but is not represented on Figure 1.

2.1. GOVERNING EQUATIONS

The set of governing equations consists in the following heat balances for the core, skin and clothing temperatures :

$$H_c = mC_p(1 - \alpha) \frac{dT_c}{dt} = S_d \cdot [M + E_r - (q_{bl}c_{bl} + U_{c-s})(T_c - T_s)] \quad (1)$$

$$H_s = mC_p\alpha \frac{dT_s}{dt} = S_d [R_b + C_b + E_d + (q_{bl}c_{bl} + U_{c-s})(T_c - T_s) - U_{cl}(T_s - T_{cl})] \quad (2)$$

$$H_{cl} = 0 = S_d [C_{cl} + R_{cl} + U_{cl}(T_s - T_{cl})] \quad (3)$$

With m the body mass, α the fraction of skin of body mass, C_p the calorific capacity of the body, M the metabolic activity and S_d the surface of the human body calculated by the Dubois' formula. The central point of the present work being the comparison with experimental data and a numerical analysis of the model, we refer the reader to the details of each term presented in (Walther and Goestchel, 2018). One point can nonetheless be outlined : the coefficients in Equations (1), (2) and (3) are non-linear : they evolve with the skin, core and clothing temperatures.

2.2. TEMPERATURE CONTROL MECHANISMS

The human body is considered as a thermally controlled system evolving in response to the temperature difference with "set temperatures" of the skin, core and body. The temperature control actions for the two-node model used by Höppe are provided hereinafter.

Vasomotricity – The blood flow rate q_{bl} is ruled by the difference of both skin and core temperatures with their set values T_s^{set} , T_c^{set} as per equation (4) :

$$q_{bl} = \frac{q_{bl}^n + C_d \cdot (T_c - T_c^{set})}{1 + C_s \cdot (T_s^{set} - T_s)} \text{ [L/m}^2\text{/h]} \quad (4)$$

The coefficients C_d and C_s relate to the dilation and constriction phenomena. The control mechanism is such that the temperature differences are set to zero in equation (4) when they are negative. The blood flow q_{bl} varies more than tenfolds between its extreme values.

Sweating – The body temperature is the weighted average of skin and core temperature – see equation (5). Its value triggers sweating and the evaporative cooling.

$$T_b = \alpha \cdot T_s + (1 - \alpha) \cdot T_c \text{ [}^\circ\text{C]} \quad (5)$$

where α is the fraction of skin mass of the body. Furthermore, α depends on the blood flow q_b since the proportion of the skin compared to the core is greater if it contains more blood.

Sweating is determined by the difference between body and body set temperature and the sweating coefficient $C_{sw} = 304.94 \text{ [g/(m}^2\text{.h.K)]}$ as per equation (6).

$$q_m^{sw} = C_{sw} \cdot (T_b - T_b^{set}) \text{ [kg/m}^2\text{/h]} \quad (6)$$

Evaporation E_{sw} at skin surface depends on the production of sweat and on the latent heat :

$$E_{sw} = L_v \cdot \frac{q_m^{sw}}{3600} \text{ [W/m}^2\text{]} \quad (7)$$

Shivering – Contractions of muscles in very cold environments result in heat generation. This translates into a source term M_{sh} as follows :

$$M_{sh} = C_{sh} \cdot (T_s^{set} - T_s) \cdot (T_c^{set} - T_c) \text{ [W/m}^2\text{]} \quad (8)$$

with $C_{sh} = 19.4 \text{ [W/m}^2\text{/K}^2\text{]}$ the shivering factor. Note that in Equation (8) temperature differences are considered zero when they become negative, as a part of the control model.

3. COMPARISON AGAINST THE EXISTING LITERATURE

In this section, the model is compared to the experimental and numerical literature and possible improvements are explored.

3.1. COMPARISON WITH EXPERIMENTAL DATA

The data used in the experiment of (Munir et al., 2009) serves as a means of comparison : a group of lightly clothed, sedentary male individuals was sequentially exposed to different ambient conditions and their average core and skin temperatures were measured. The duration of each stage as well as the corresponding temperature, relative humidity and air velocity are summarised in Table 1.

Duration (min)	Temperature (°C)	Relative humidity (%)	Air velocity (m/s)
30	29.4	46.8	0.09
20	19.5	52.8	0.21
60	29.3	47.1	0.09
20	38.9	46.7	0.22
20	29.5	47.1	0.1

Tableau 1 : Summary of the ambient conditions in the experiment of (Munir et al., 2009)

On Figure 2 a), the results for both Gagge’s and Höppe’s model are plotted. The skin and core temperatures of Gagge’s model were determined with the validated procedure available in (Tartarini and Schiavon, 2020) with initial conditions being the skin and core set values. One can observe that both models reproduce the dynamics and amplitude of the skin temperatures for varying ambient conditions. They exhibit similar features in terms of dynamics, although Höppe’s model is subject to larger discrepancies in amplitudes : Höppe’s model skin temperature overshoot at about 120 min exposure seems unlikely, as the skin temperature reaches the core temperature. In terms of root mean square error (RMSE) *versus* the data, Gagge’s reaches 0.75 [K] whereas Höppe scores 1.69 [K].

3.2. OBSERVATIONS AND POSSIBLE IMPROVEMENTS

The observation made by (Melnikov et al., 2018) for Gagge’s original two-node model is that heat transfer between core and skin appears to be insufficient for the rapid variations of skin temperatures observed experimentally in (Munir et al., 2009). (Melnikov et al., 2018) hence performed a global minimisation procedure on the three physiological parameters affecting the skin temperature in Equation (4) (*i.e.* dilation and striction coefficients, and basal blood flow rate). Provided that physical safeguards for parameters values are respected, such a procedure is a means of finding the appropriate thermophysiological constants for the cohort studied. They

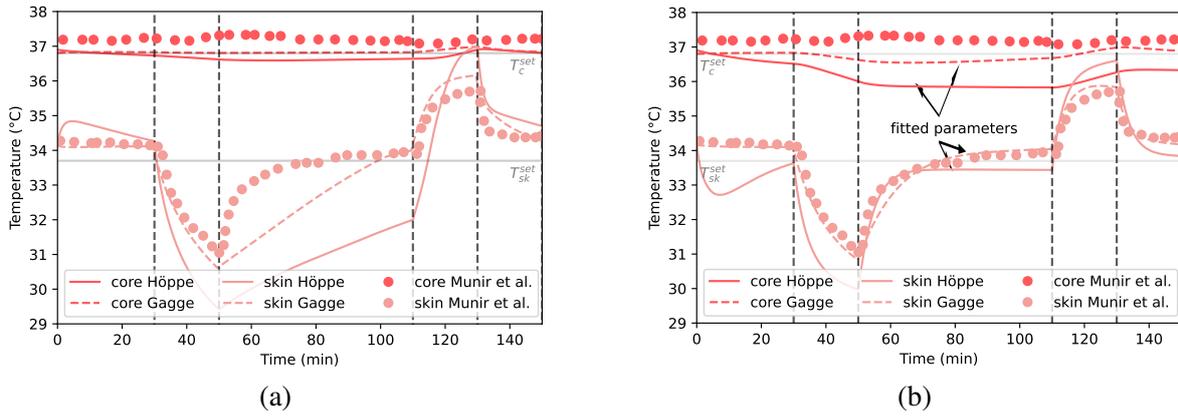


Figure 2 : a) Comparison of the skin and core temperatures with field data from (Munir et al., 2009) and b) Comparison after a parametric fit on Höppe's and Gagge's model constants after (Melnikov et al., 2018).

found that a dilation coefficient $C_d = 50$ [$\text{L}\cdot\text{m}^{-2}\cdot\text{h}^{-1}\cdot\text{K}^{-1}$], a basal skin blood flow $q_{bl}^n = 10.7$ [$\text{L}\cdot\text{m}^{-2}\cdot\text{h}^{-1}$] and the dilation coefficient $C_s = 0.1$ [K^{-1}] minimize the discrepancy with the available measurements.

Using the same method, the physiological parameters that minimize the discrepancy for Höppe's model are, *i.e.* $q_{bl}^n = 15$ [$\text{L}\cdot\text{m}^{-2}\cdot\text{h}^{-1}$], $C_s = 0.05$ [K^{-1}] and $C_d = 134.8$ [$\text{L}\cdot\text{m}^{-2}\cdot\text{h}^{-1}\cdot\text{K}^{-1}$]. The results in terms of skin and core temperatures for both models are presented on Figure 2 b). One can conclude that :

- The improvement of the prediction of skin temperature is pronounced, as the root mean square error (RMSE) for Gagge's model is divided by three (0.08 K versus 0.75 K initially). For Höppe's model the RMSE drops from 1.69 K to 0.83 K, at the cost of an unrealistic behaviour.
- The core temperature in both models is also affected by the change of the coefficient and the temperature changes with a different dynamic than the data, exhibiting a drop in core temperature that does not reflect the measurements. What is more, in Höppe's model, the skin temperature exceeds the core one, which appears unrealistic in such conditions.

4. NUMERICAL CONSIDERATIONS

The original versions of the two-node models involve the explicit forward Euler integration scheme – see *Addendum b* to ANSI/ASHRAE Standard 55-2013 for Gagge's model and the Fortan program in (VDI, 2008) for Höppe's. This numerical scheme had the advantage of simplicity in times when scientific computing was not as user-friendly as it may be nowadays. However, the conditional stability of explicit schemes is put at risk by the thermophysiological model and the 60 seconds default time step used in the reference models may be questioned with regard to the non-linear coefficients of the model.

4.1. BRIEF DESCRIPTION OF THE θ -SCHEME

In the sequel, following notation will be used, where T^+ is the temperature at the time step $t + dt$ such that $\frac{dT}{dt} = \frac{T(t+dt) - T(t)}{dt} \simeq \frac{T^+ - T}{\Delta t}$. At a given time step where the temperature array $[T] = T_c, T_s, T_{cl}$ is known (*e.g.* the initial conditions), discretising the system of equations (1),(2),(3) leads to solving simultaneously for the unknown array $[T^+] = T_c^+, T_s^+, T_{cl}^+$ as a linear

combination of the explicit (known) and implicit temperatures (unknown) :

$$T_c^+ = T_c + \frac{\Delta t}{mC_p} \cdot \left((1 - \theta) \cdot \frac{H_c([T])}{1 - \alpha([T])} + \theta \cdot \frac{H_c([T^+])}{1 - \alpha([T^+])} \right) \quad (9)$$

$$T_s^+ = T_s + \frac{\Delta t}{mC_p} \cdot \left((1 - \theta) \cdot \frac{H_s([T])}{\alpha([T])} + \theta \cdot \frac{H_s([T^+])}{\alpha([T^+])} \right) \quad (10)$$

$$0 = H_{cl}(T_c^+, T_s^+, T_{cl}^+) \quad (11)$$

The value of θ in Equations (9) and (10) determines the type of numerical scheme : $\theta = 0$ yields Euler's explicit scheme, $\theta = 1$ is the full implicit scheme and $\theta = \frac{1}{2}$ represents Crank-Nicolson's semi-implicit scheme, which error is of second order in time. The effect of the choice of θ will be explored in the following section.

4.2. INFLUENCE OF THE INTEGRATION SCHEME

In order to evaluate the influence of the integration scheme, an numerical experience is led in this section, in order to compare the behaviour of the time integration for T_c and T_s with a full explicit, full implicit and Crank-Nicolson scheme. The exposure to three type of ambient conditions are simulated :

- "Office" is a typical indoor with operative temperature conditions, average relative humidity and low air velocity.
- "High Mean Radiant Temperature (MRT)" is a hot outdoor with a radiant temperature 30 [K] higher than the dry bulb temperature, 40 [%] relative humidity and low air velocity.
- "Winter" represents cold outdoor conditions with 0.5 [m/s] air velocity.

The conditions evaluated are summarised in the following table, with an exposure time of one hour, starting from the initial conditions $[T] = 36.8, 33.8, (T_a + T_r)/2$.

Ambient condition	Office	High MRT	Winter
Air T_a [°C]	20	30	0
Radiant T_r [°C]	20	60	0
Humidity φ [%]	50	40	90
Velocity v [m/s]	0.15	0.15	0.5
Activity met [-]	1	1	1
Clothing clo [-]	0.6	0.3	1

Tableau 2 : Summary of the ambient parameters for the conditions tested

The core temperature being less prone to rapid variations, we will examine the impact of the numerical schemes in terms of skin temperature, the latter being one of the components of thermal sensation (Vellei and Le Dréau, 2020), (Fanger et al., 1970). Figure 3 shows the evolution of the skin temperature over the first 600 [s] in the three ambient conditions described hereinabove. One can observe a similar skin temperature for 600 [s] exposure in the "High MRT" environment for the three integration schemes, however a discrepancy of more than 2 [K] is noticeable between the "Office" and "Winter" conditions between the full implicit and explicit schemes. It takes ~ 1 h exposure for the three schemes to reach the same temperature.

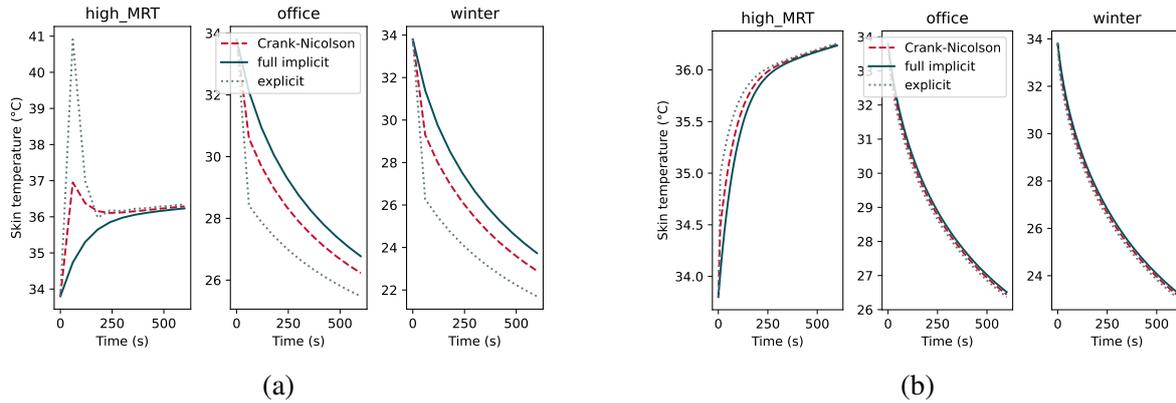


Figure 3 : a) Skin temperature with $\theta = 0, 0.5, 1$ for three ambient conditions with $\Delta t = 60$ [s] and b) Comparison between the explicit, implicit and Crank-Nicolson for various ambient conditions with $\Delta t = 10$ [s]

4.3. INFLUENCE OF THE TIME STEP ON THE PET

When choosing the time discretisation of a control problem, a particular care must be given to the amplitude of the time step : control models add up numerical stability and choice of the sampling rate (Walther, 2021)¹. Indeed, the time interval at which the difference between the actual value and the set value is measured matters : Large time steps may lead to increased numerical or "control" instabilities. In other words, the controller actions modify the behaviour of the system depending on the measured error, and the latter varies with different sizes of time step. The influence of the time discretization on the PET is exposed in the following paragraphs.

Figure 4 a) shows the evolution of the PET for $\Delta t = 10$ s for the explicit, semi-implicit and full implicit schemes. Comparing with the results observed in Figure 3, the results are more homogeneous and no overshoot can be observed for the "high MRT" ambient conditions. The discrepancy between schemes is also smaller.

As an illustration, Figure 4 a) shows the evolution of the PET over time in the same ambient conditions as presented in the previous Section, however at a lower $\Delta = 10$ [s]. One can observe that the temperature profiles exhibit more similarity than for $\Delta t = 60$ [s], which is the value used in the standards for the comfort indexes SET and PET (Gagge et al., 1971), (VDI, 2008). The explicit scheme performs noticeably better than with 60 seconds time step.

4.4. NUMERICAL OSCILLATIONS AT ELEVATED TEMPERATURES

For some extreme conditions of heat exposure, similar to the "high MRT" conditions except with $T_a = 41$ [°C], the explicit scheme exhibits an oscillatory behaviour, as presented on Figure 4 b). Owing to the thermophysiological response, numerical oscillations appear for elevated air and radiant temperatures. Such conditions trigger high sweating rates, as per Equation (6), provoking large values of latent heat flux that destabilise the explicit scheme. The latter is indeed stable, but not necessarily convergent (Patankar, 1980).

1. See the example provided in Chapter 2 - Section 3

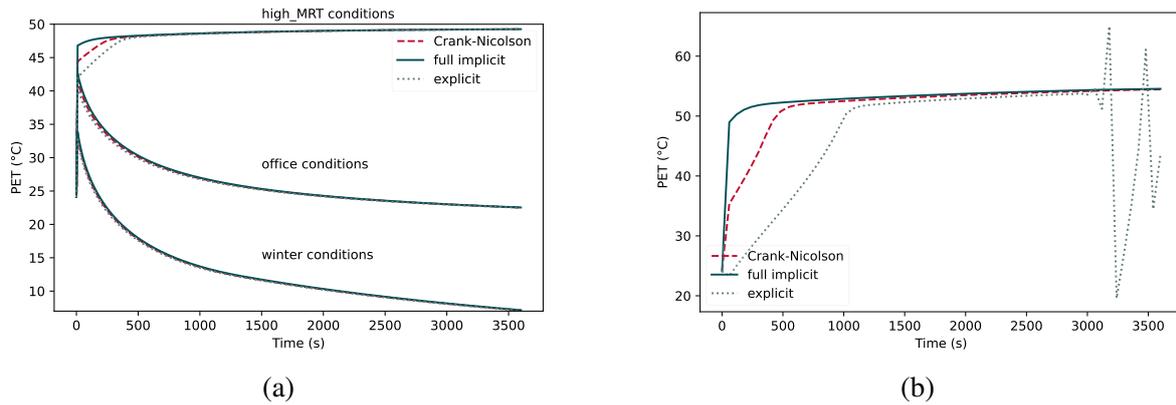


Figure 4 : a) Comparison between the explicit, implicit and Crank-Nicolson for various ambient conditions with $\Delta t = 10 [s]$ and b). Comparison between the explicit, implicit and Crank-Nicolson scheme for high MRT conditions and $T_a = 41 [^{\circ}\text{C}]$ with $\Delta t = 60 [s]$

5. CONCLUSION & PERSPECTIVES

The present work proposes a presentation of Höppe’s transient two-node model and explores both its ability to compare with field data and the aptitude of the numerical procedure used to compute the physiological reaction.

The parameters of Höppe’s physiological model can be tuned so that the skin temperature exhibits a lower error in comparison with measured data, however at the cost of an unrealistic behaviour, *i.e.* the skin temperature reaching the core temperature.

The practical implications of the results on the numerical side of the problem are the following :

- A time step $\Delta t < 60 [s]$ is required in order to capture the dynamics for all schemes.
- Given the likeliness of occurrence of oscillations, using the explicit scheme appears unsafe. An implicit or semi-implicit one would allow solving this issue.
- The implicit integration schemes (either full implicit or Crank-Nicolson) offer a better representation of the non-linearity of coefficients in the system of differential equations.

The stability issue with of the explicit scheme rises probably also applies to Gagge’s two-node model. As a perspective, the influence of the numerical schemes of Gagge’s two-node model could be explored.

The code of the transient PET model and SET model are available in following repository :
https://github.com/eddes/AREP/blob/master/PET_transient.py
https://github.com/eddes/AREP/blob/master/SET_theta_scheme.py

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