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Flavio d'Ambrosio, Eric Chatelet, Julien Ravoux, Georges Jacquet-Richardet. Forced response of shrouded bladed disc assemblies: a jointed experimental numerical approach. ASME Turbo Expo 2004, Power for Land, Sea and Air, IGTI 2004, Jun 2004, Vienne, Austria. pp.337-347, 10.1115/GT2004-53705 . hal-04696311

**HAL Id: hal-04696311**

**<https://hal.science/hal-04696311v1>**

Submitted on 26 Sep 2024

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# FORCED RESPONSE OF SHROUDED BLADED DISC ASSEMBLIES: A JOINTED EXPERIMENTAL NUMERICAL APPROACH

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## ABSTRACT

*The objective of the proposed study is to progress towards a better modeling of bladed assemblies dynamic. Coupling devices are introduced in bladed stages to increase frequencies of resonance above the range of possible excitations. Commonly they are not only used to stiffen flexible structures but also to increase damping by dry friction, reducing the amplitude of vibration. Consequently, the resulting dynamic behavior is complex and highly non linear. In order to improve numerical capabilities used for the design of part span shrouds and to gain a better understanding of the dynamic behavior of shrouded assemblies, a jointed experimental and numerical approach has been conducted.*

*A first experimental test based on a cantilever beam is considered. The beam, excited by a controlled electrodynamic shaker, is associated to a dry friction damper at its free end. A numerical finite element analysis and a macroslip model of contact has been developed. The solution method in time domain allows accurate computations of response levels and gives the main harmonics of the steady state response. Comparison between numerical and experimental results is very good.*

*A second experimental set up is constituted by an assembly of 13 beams, cyclically mounted around a common disk and linked by geometrically simplified shrouds. The set up allows controlling the resultant forces in the contact and is able to exhibit all states of contact from fully slipping to fully stuck. The first results obtained are associated to a single couple of blades brought into contact.*

## 1 INTRODUCTION

One of the most common failure modes for turbomachinery wheels is high-cycle fatigue of blades. To avoid high amplitudes and extend the working life of machines, natural frequencies should be distinguished from excitation frequencies. However, in practice, real structures are characterized by so many natural frequencies that it is impossible to be free of resonance and vibratory stresses have to be lowered by increasing structural damping. Griffin [1] describes the most common techniques used for increasing damping by use of dry friction. Many kinds of devices can be distinguished like Tie -wire, Cottage roof damper or Shrouds. Tie-wire dampers can simply dissipate energy from dry friction due to a relative motion between surfaces in contact. Shrouds can be welded to form a single ring at the end of all blades. In this case, blades are stiffer and resonance frequencies can be pushed above the range of possible excitation. Usually shrouds are not welded but simply pressed together by centrifugal forces when running. The relative displacement between shrouds of each blade causes energy dissipation by dry friction and reduces risks of aerodynamic instability [2]. Placed between 65% and 80% of blade length, shrouds have a great influence on the dynamical behavior of the assembly and shall be designed for optimized efficiency. Until now, due to the complexity of the associated modeling and the high level of computer resources needed, the design of shrouded assemblies still widely depends on empirical knowledge of turbomachinery constructors.

When dealing with the dynamical behavior of structures involving highly non linear dry friction elements, both experimental and numerical studies can be found in the literature. One of

the main controlling parameter is the ratio between excitation force and normal load in the contact. Changing this ratio can lead to three different contact states [3,4] : completely stuck, completely slip and stick/slip. Experimental tests are often conducted without controlling the excitation force during acquisition in swept sine mode. This can interfere with the accuracy of results [5] and good agreements between experimental and numerical results are not always obtained [6]. Applications can be found within the field of forced vibrations of shrouded turbine blades [7] or forced responses of mistuned bladed disk assemblies without friction contacts [8]. In few cases [9], an instrumented support with strain gages gives the tangential force in the contact and the relative displacement between adjacent points. Finally, an estimation of the amount of energy dissipated by friction and its relation with the normal load on the contact surface can be obtained from hysteresis loops.

Several adapted numerical techniques have been developed and mainly applied to simplified test cases. The reference for validation of new methods is often based on time integration techniques. Harmonic Balance Method [10], Alternative Frequency Time domain method [11] or Incremental Harmonic Method [12] are mostly used and are generally shown to be faster than direct time integrations. However, the interest of those techniques decreases when high harmonic orders have to be considered to predict response levels with enough accuracy. In the case of linear systems, different modal reductions may be used [13,14]. For non linear systems, the Craig-Bampton reduction technique is powerful [15], but the associated number of degrees of freedom obtained after reduction remains generally high. Recently, researches based on non linear modes have shown good efficiencies and allow high reduction levels [16,17]. However, published results concern systems with relatively small initial numbers of dof and generally the expression of non linear forces has to be continue and analytically known. Finally, the choice of contact models is obviously very important and those proposed by Cattaneo, Masing or Mindlin [18] are currently used. Recently, Menq [19,20] proposed more precise models for frictional sliding between shroud connections. These models are adapted to microslip behavior of the contact and they allow variable normal loads.

In the present study two different applications are considered and, for both cases, a jointed experimental/numerical approach is conducted. The first part concerns an academic application and is focussed on a simple case of vibration in presence of dry friction. The second part is focussed on a more realistic application based on two shrouded beams with frictional constraints at shrouds interface. This second application represents a first step toward the study of the dynamic behavior of cyclic symmetrical structures linked by non linear elements. Results obtained also show the interest of the jointed progression and the complementarity of experimental and numerical approaches.

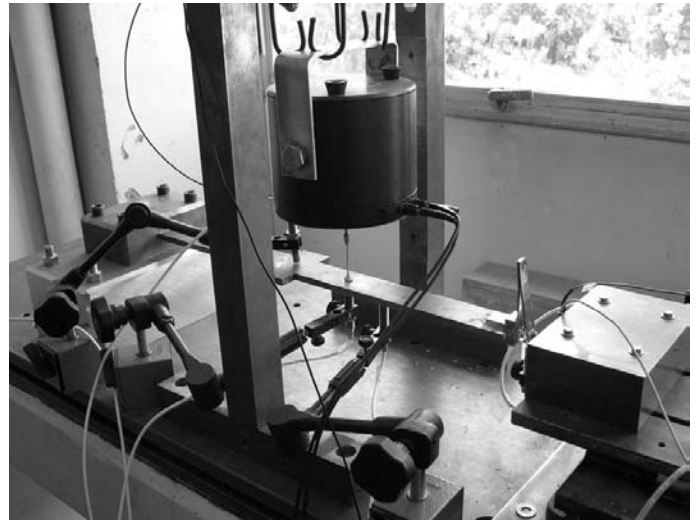


Figure 1. First experimental set up - simple beam

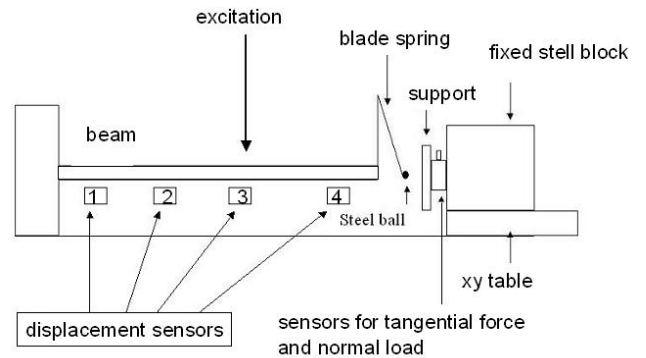


Figure 2. Description of the experiment

## 2 VIBRATION ANALYSIS OF A CANTILEVER BEAM WITH DRY FRICTION CONTACT

A first simple application has been developed in order to build and validate the numerical and experimental tools needed to study bladed assemblies vibration in presence of dry friction effects. The system considered is constituted by a cantilever beam with a rubbing element mounted at its free end. Instrumentation gives displacements and excitation forces, but also the normal load and the transversal force with respect to the contact surface. The contact model is obtained from hysteresis loops and measured Frequency Response Function (FRF). Both experimental and numerical procedures are described and results obtained are compared and discussed.

## 2.1 Description of the set up

The experimental set up, shown in figure 1 and schematized in figure 2, is based on a steel beam with the following characteristics :  $0.50\text{ m}$  in length and rectangular cross section of  $0.008\text{ m}$  by  $0.04\text{ m}$ . Different features are considered:

- A support with a blade spring is mounted at the free end of the beam
- The spring stiffness in direction of contact is low and gives a quite high sensitivity to normal load.
- A steel ball taken from a ball bearing, stuck to the blade spring, rubs on the support
- The beam is notched in a steel block to minimise structural damping brought by boundary conditions
- To charge the contact, a steel block fixed on a controlled XY table is used. A second support is screwed on this block. Friction occurs between the rolling element of the blade spring and this second support
- The base of the table is fixed on an experimental bench

Four eddy current sensors are used to measure beam displacements and to identify mode shapes. The excitation is obtained from an electrodynamic shaker (  $90\text{ N}$  ) placed approximately at the middle of the beam :  $0.30\text{ m}$  from the clamped end. A quartz force sensor, inserted between the support and the steel block, measures the two orthogonal components of the dynamic force (transversal force) and the quasi-static force (normal force). The synoptic scheme of the experimental procedure is shown in figure 3. Displacement measures give the dynamic behavior of the structure in the linear case (clamped-free beam). In this case, the force is not controlled and doesn't rest constant during acquisition but decreases at resonance. However, the ratio between displacement and excitation force is constant at each frequency. When considering non linear dynamic systems, the force is controlled using an adapted acquisition device (LMS CADA-X) and remains constant during all swept sine. The positions of the four eddy sensors are at  $0.1$ ,  $0.22$ ,  $0.30$  and  $0.35\text{ m}$  respectively from the clamped end.

## 2.2 Experimental results

For the clamped-free beam, the resonance frequency of the first mode without dry friction is  $24.0\text{ Hz}$ . Normal load is applied by moving the XY table and the beam is excited with a swept sine  $F_{ex} \sin(\omega t)$  where  $\omega$  is the angular frequency and magnitude  $F_{ex}$  is controlled for all measurements. The normal load  $F_n$  is considered within the range  $0.5$  to  $40\text{ N}$  and the amplitude of the excitation force  $F_{ex}$  is  $2\text{ N}$ . Figure 4 shows the constant level of excitation force obtained during acquisition. Frequency Response Functions given by the second sensor, are reported in figure 5.

When the normal load is  $40\text{ N}$  contact is completely stuck and the frequency of resonance is at  $87\text{ Hz}$ . In this case, the

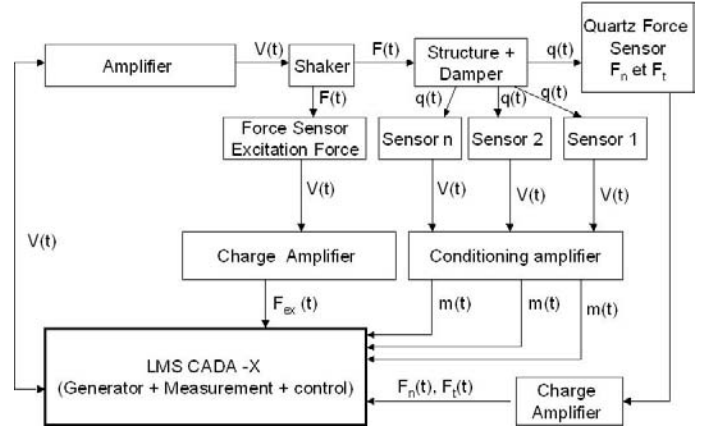


Figure 3. Synoptic scheme of the measuring chain

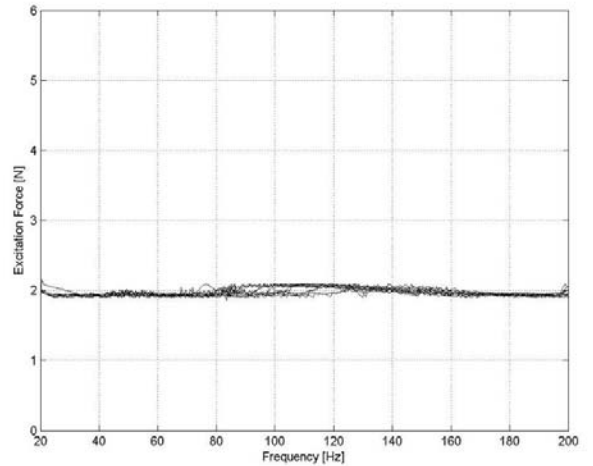


Figure 4. Variation of the controlled excitation force during acquisition

contact between the steel ball and the support acts as a simple spring and the transversal force,  $F_t$  has a sinusoidal shape.  $F_t$  is proportional to the displacement of the contact point as far as its value remains lower than  $\mu \cdot F_n$  ( $\mu$  friction coefficient). For lower values of normal loads, peaks of resonance remain at the same value of frequency but become flat. In this case, transversal forces reach  $\mu \cdot F_n$  and during a cycle of vibration contact is stuck when  $F_t \leq \mu \cdot F_n$  and slips when  $F_t = \pm \mu \cdot F_n$  (see figure 6). Then, these permanent changes in boundary conditions, between stuck and slipping states, do not allow any resonance. Finally, when the ratio between normal load and excitation force is very low, contact remains always in a slipping state and the boundary conditions become stationary again. In this case  $F_t$  jumps from  $\mu \cdot F_n$  for  $\dot{x}_c > 0$  to  $-\mu \cdot F_n$  for  $\dot{x}_c < 0$ , where  $\dot{x}_c$  is the speed at the

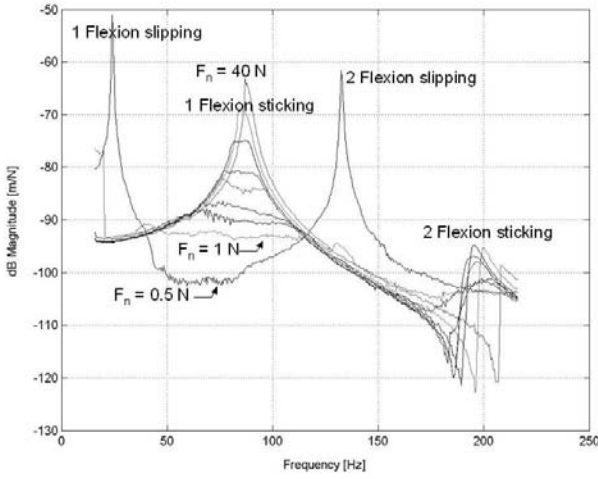


Figure 5. FRFs for the 2<sup>nd</sup> sensor with different values of normal load  $F_n = 0.5 - 1 - 2 - 3 - 4 - 5 - 10 - 20 - 40$  N and excitation force  $F_{ex} = 2$  N

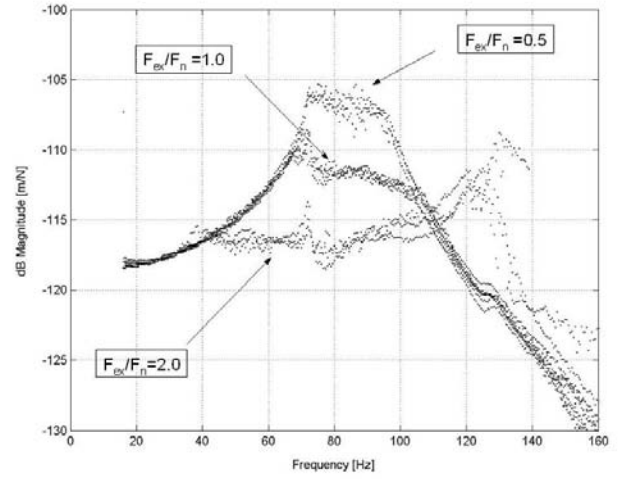


Figure 7. FRF for  $F_{ex}/F_n = 0.5 - 1.0 - 2.0$  ( $F_n = 2.5 - 3 - 5 - 10$  N)

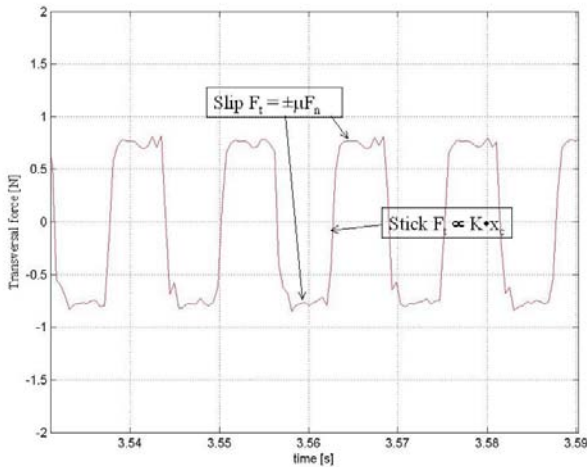


Figure 6. Transversal Force ( $F_t$ ) in stick/slip state

contact point, and the structure exhibits a first peak of resonance at 24 Hz.

The relationship between response levels and the ratio excitation force / normal load has been fully examined. As shown above, ratios below 0.05 and above 4 lead to the quasi-linear cases of fully stuck and fully slipping contact states. From 0.05 to 2 the contact is mainly stuck with an increasing amount of slipping component during each cycle, while from 2 to 4 the dominant state is slipping. Three different ratios  $F_{ex}/F_n$  are first considered : 0.5, 1 and 2 respectively. Each of these ratios are

obtained from various values of normal loads such as, for example, 2.5 - 3 - 5 - 10 N. Figure 7 gives the frequency response functions associated to the first sensor and obtained from the 12 configurations tested. These results show that, when the dominant state in the contact is stuck, response levels can reasonably be considered as only dependent upon the ratio  $F_{ex}/F_n$ .

Ratios above 2 lead to much more unpredictable behaviors. In figure 8 three cases are reported where  $F_{ex}/F_n$  is 2.5 and  $F_{ex} = 2.5 - 5 - 7.5$  N respectively. During this experiment, contact was stuck at the beginning of the swept sine process and evolved toward a slipping state. As shown in figure 8, this evolution is very dependant upon the value of the normal load applied and occurs suddenly. The amount of energy needed to initiate slipping shall increase when normal load increases and the local contact characteristic has a major influence.

Therefore, the response magnitude appears to be dependent upon the ratio  $F_{ex}/F_n$  when this value is smaller than 2. For  $F_{ex}/F_n$  greater than 2, near slid resonance peak, instabilities in the contact nature due to boundary conditions and to the state of the contact surface, produce a discontinuity in the dynamical behavior and jumps from stick to slip branches is observed in the frequency response function.

Values of displacements at the contact point and associated tangential forces are measured and allow drawing hysteresis loops. Two different cases are considered. First, the analysis is performed at resonance of the sliding configuration and ratios  $F_{ex}/F_n$  are considered within the range 2.5 to 3. In this case, as classically shown in the literature [9], hysteresis loops are well approximated by a parallelogram and may be reduced to a straight line when contact is stuck due to insufficient energy brought to initiate sliding (figure 9). The second analysis

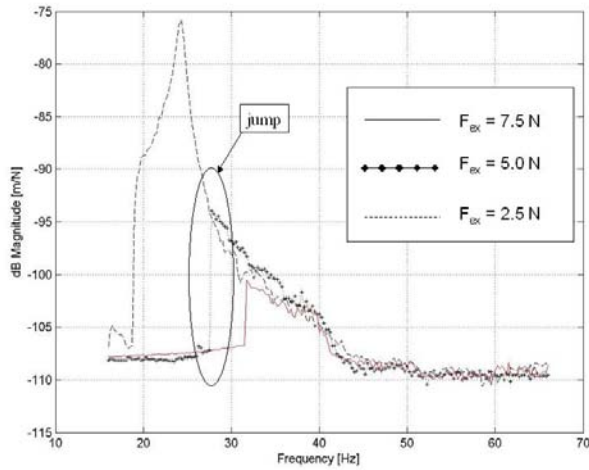


Figure 8. FRFs for  $F_{ex}/F_n=2.5$  from lower to higher frequencies

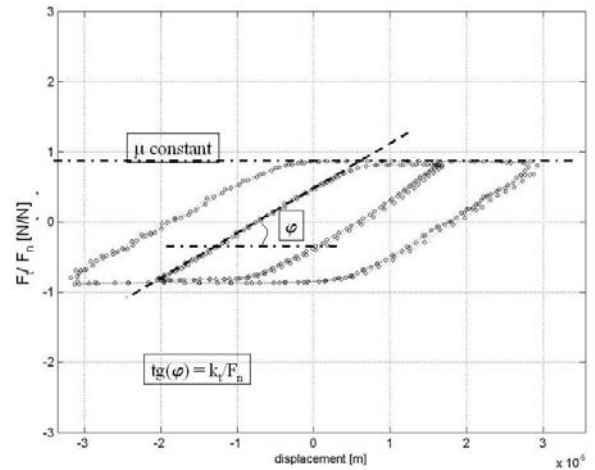


Figure 10. Hysteresis cycles for  $F_{ex}/F_n=0.2$  Excitation frequency 80 Hz and 100 Hz

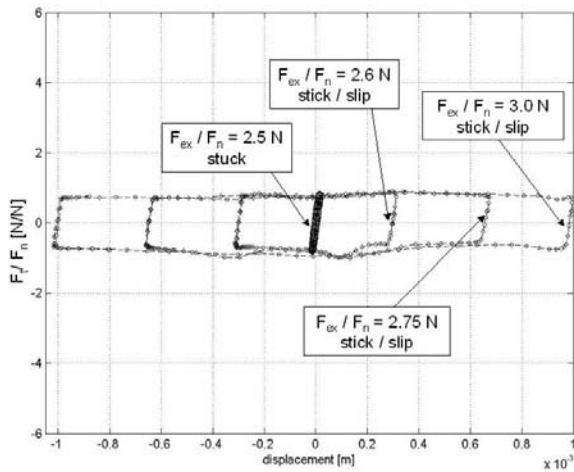


Figure 9. Hysteresis cycles for  $F_{ex}/F_n = 2.5 - 2.6 - 2.75 - 3.0$  Excitation frequency = 25 Hz

is performed at the resonance of the sticking configuration with a ratio  $F_{ex}/F_n$  equal to 0.2 (figure 10). The shapes of the loops is here also very well approximated by a parallelogram. Consequently, for the application considered, a single contact model can be chosen for the numerical analysis.

### 2.3 Numerical model

Experimental results show that responses are all lying on the linear response cone of completely stuck case till the contact is fully slipping (figure 5). As shown for example in [10], when contact exhibits a microslip behavior, response peaks move to-

ward the left of the stuck resonance peak because the equivalent tangential stiffness does not remain constant but depends on the tangential force value. Consequently a macroslip model shall fully capture the behavior obtained here and has been chosen for the numerical analysis.

The global dynamic behavior of the contact is modeled using Masing's approach. The principal difference with the more classical Coulomb's model, is an additional spring considered in series with the friction element to model elastic deformations [18]. This tangential stiffness modifies the rectangular hysteresis loops of a Coulomb's model toward a parallelogram. According to Masing's model, asperities within contact are linearly and elastically deformed so that the global interaction force is expressed as an elastic force acting as a spring in tangential direction. Then, when contact is completely stuck from a dynamical point of view, the system is modeled by a clamped beam simply supported on a spring at its free end. The associated stiffness value is obtained from the tangent to the hysteresis cycles when relative motion inverts direction : value obtained  $3.0e5$  N/m (figure 10). The friction coefficient is defined as the ratio between tangential force and normal load in sliding conditions and its value is constant (0.9). Surface roughness has a strong effect on friction coefficients [21] and a poor quality of the surface shall explain the high value obtained as compared to classical values (between 0.5 and 0.8).

Solution methods used for frictionally damped systems are usually in the frequency domain and modal techniques such as Craig-Bampton reductions are commonly used. However, considering both cases, an incorrect choice or an insufficient number of elements kept in the reduction base may lead to poor approximations. Then, a time domain solution method is retained here to

validate the procedure with as few hypotheses as possible. Obviously induced computational time is very high and other cautions have to be taken. To maintain a low number of dof, a beam element (2 nodes with 3 dof) is chosen to represent the structure and the whole system is modeled using about 40 dof.

## 2.4 Numerical results and discussion

The beam is first considered in a clamped-free configuration to adjust steel material properties. Numerical and experimental results obtained from a simple harmonic analysis are in good agreement until the fifth natural frequency with : Modulus of elasticity  $E = 2.02e11N/m^2$ , density  $\rho = 7800Kg/m^3$  and poisson coefficient  $\nu = 0.3$  (see figure 11). A comparison based on the other linear limit case, where the free end of the beam is simply supported by a spring, is also reported in figure 11. This case represents the completely stuck contact and the stiffness value of spring is given by the tangential stiffness of contact previously found. The very good agreement between those results validate the model associated to the stuck contact.

For stick/slip configurations a time domain analysis is needed and the non linear transversal force depends on displacement and speed of the contact point as follows:

$$F_t(x_c, \dot{x}_c) = \begin{cases} \text{sign}(\dot{x}_c) \cdot \mu F_n & \text{slid contact} \\ \text{sign}(\dot{x}_c) \cdot \mu F_n + k_t \cdot (x_c - x_0) & \text{stuck contact} \end{cases} \quad (1)$$

where  $x_c$  is the contact displacement and  $x_0$  the value of displacement associated to the transition from slid to stuck configurations. The slid and stick conditions are verified by the following equations:

$$\text{slid conditions} \begin{cases} x_c > 2 \cdot \mu F_n / k_t + x_0 & \text{if } \dot{x}_c > 0 \\ x_c < -2 \cdot \mu F_n / k_t + x_0 & \text{if } \dot{x}_c < 0 \end{cases} \quad (2)$$

$$\text{stick conditions} \begin{cases} x_c < 2 \cdot \mu F_n / k_t + x_0 & \text{if } \dot{x}_c > 0 \\ x_c > -2 \cdot \mu F_n / k_t + x_0 & \text{if } \dot{x}_c < 0 \end{cases} \quad (3)$$

The motion equation of the whole system takes the classical form:

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = \{F_{ex}\} - \{F_{nl}\} \quad (4)$$

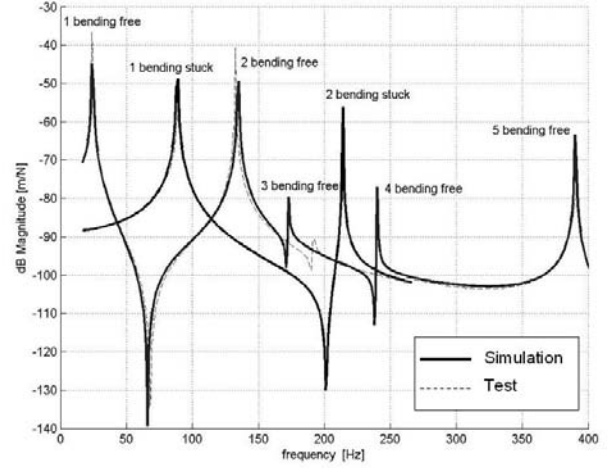


Figure 11. Linear case - Comparison between experimental and numerical results for the free and completely stuck configuration

where the non linear vector force is given by

$$\{F_{nl}\}^t = \{0 \dots F_n F_t 0\}$$

$[M][C][K]$  are the assembled matrices of mass , damping and stiffness,  $\{F_{ex}\}$  is the excitation force vector and  $\{x\}$  the displacement vector. A classical constant-average acceleration method (Newmark  $\beta = 1 \setminus 4$ ) with a time step  $dt = 1/(270 * frequency)$  is used to solve equation 4. Four FRFs are computed with the following ratios  $F_{ex}/F_n$  : 0.2-1-2-5 N. In the first case, contact is always stuck except near the resonance point at 87 Hz. When increasing the ratio, response curves become flatter but a peak can still be identified until the ratio reaches 2. Here the peak disappears and consequently the efficiency of the damper is maximum. Numerically, contact state swings between two configurations: slip and stick. Boundary conditions [5] during a period of excitation change between the two limit cases of beam clamped-Free and beam clamped-simply supported . For the intermediate conditions there is no stationary state and consequently no resonance can occur. In the last case where  $F_{ex}/F_n=5$  contact always slips and the system does not change its boundary conditions. Consequently the response exhibits a peak of resonance at 24 Hz and the damper loses its efficiency. A good agreement between experimental and numerical results is obtained (figure 12).

A last consideration concerns the harmonic content at steady state responses. For the stuck configuration there is no relative displacement between contact surfaces and the beam vibrates at the same frequency as the excitation. In the case of stick/slip configurations, a power spectrum analysis of the



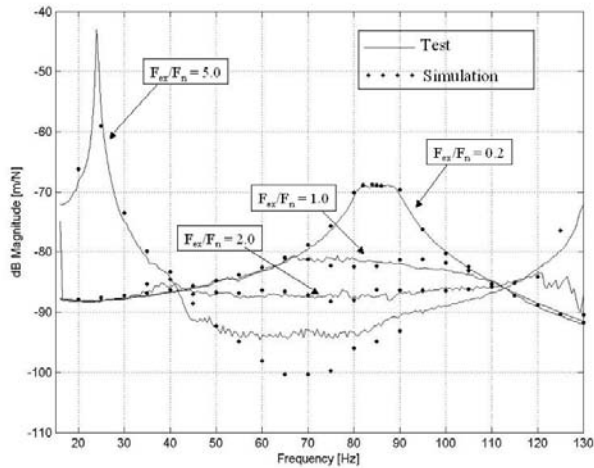


Figure 12. Comparison between experimental and numerical results for selected values of  $F_{ex}/F_n$

steady response, exhibits only odd multi-harmonic components (figure 13). This result is classical and, for example, Pierre [12] shows that for macroslip cases even harmonics do not give any participation. The first harmonic represents about 90% of the total amplitude.

A jointed numerical experimental approach has been used and applied to a simple case constituted by a beam with a friction damper to get a better understanding about the main phenomena involved and the adapted numerical modeling techniques in presence of dry friction. A good agreement between experimental and numerical results has been obtained after a careful choice of the contact model. Contact characteristics have been identified from experimental results. The tangential stiffness, measured from hysteresis loops, is a major parameter for frequency of resonance prediction in sticking case. Obviously, friction coefficient also plays a major role. Finally, it is very important to control the excitation force to avoid changes in the ratio  $F_{ex}/F_n$  and consequently changes in the dynamic behavior during swept sine. Without force control, an excitation at fixed sinus in time is needed. This procedure is much slower than FRF but allows considering all harmonics at steady state response.

The jointed numerical experimental approach allows a reciprocal understanding and validation of methodologies: the interest of experimental tests within numerical developments is obvious. More originally, numerical development have been intensively used to validate acquisition techniques within the non linear experimental process. The interest of this jointed approach appears also clearly when considering the more realistic application considered above.

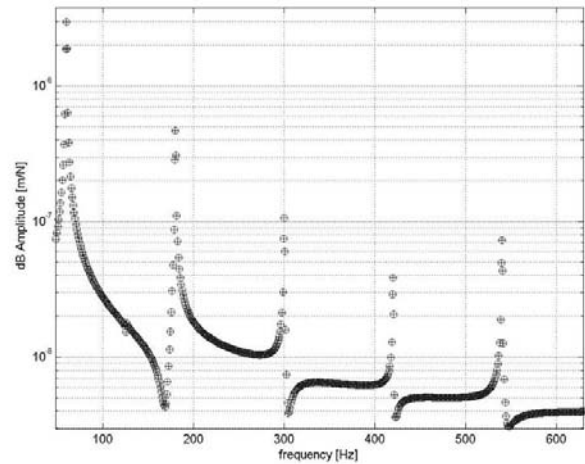


Figure 13. Fast Fourier Transform of steady response for  $F_{ex}/F_n = 2$  at 60 Hz

### 3 CASE OF TWO SIMPLE BLADES LINKED BY SHROUDS

A device has been designed in collaboration with Thermodyn (Le Creusot France), Nuovo Pignone / General Electric's division. The goal of the initiated study is to consider a system which remain as simple as possible but which is able to exhibit the main vibrational features of shrouded bladed assemblies. In addition to the requirement of representability of real assemblies, the principal specifications considered were as follows :

- Geometrically simple configuration. Parallelepiped blades linked by simplified shrouds (dimensions: 8 x 30 x 145 mm)
- Shrouds clamped on blades and instrumented with strain gages (dimensions: 5x 20 x 45 mm)
- Frequencies of the two first bending modes lower than 2500 Hz for practical experimental requirements.
- System used for loading contacts fast and simple.
- Normal loads in the contact up to 1500 N
- The two surfaces of contact shall remain parallel during excitation

The choice of the number of blades (prototype composed of 13 beams and 26 shrouds) has been conditioned by clamping and interface charging requirements. The main difficulty encountered lies in the design of the contact loading device. Two screws are acting anti symmetrically on a pin which crosses the part of the beam placed inside the hub (figure 14). In that way, blades turn around their symmetrical axis and shrouds are brought into contact and charged. The maximal rotation is about 2.8 degrees and this value allows a normal load in the contact of 1800 N. The whole device showing beam clamping, loading screws and instrumented shrouds is presented in figure 15.



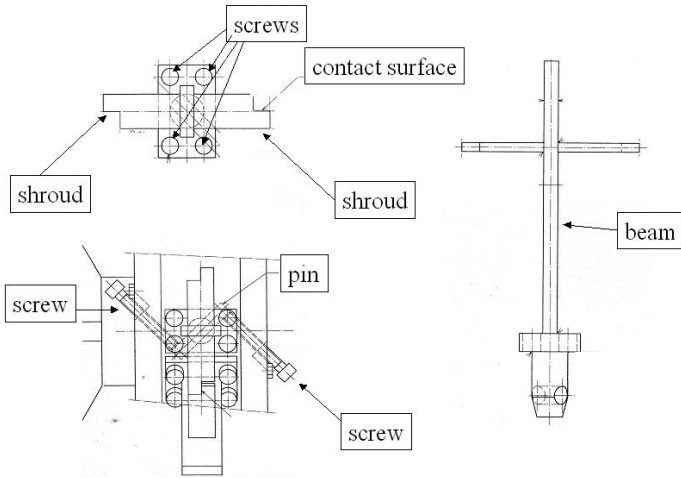


Figure 14. Contact charging device

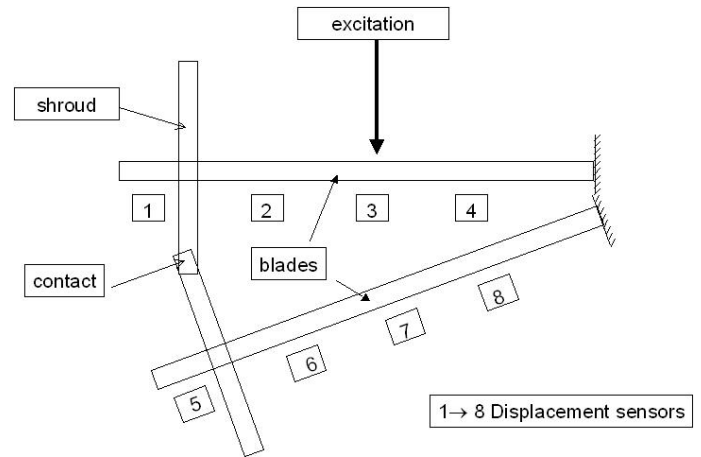


Figure 16. Detail of the two bladed system



Figure 15. Whole real set up

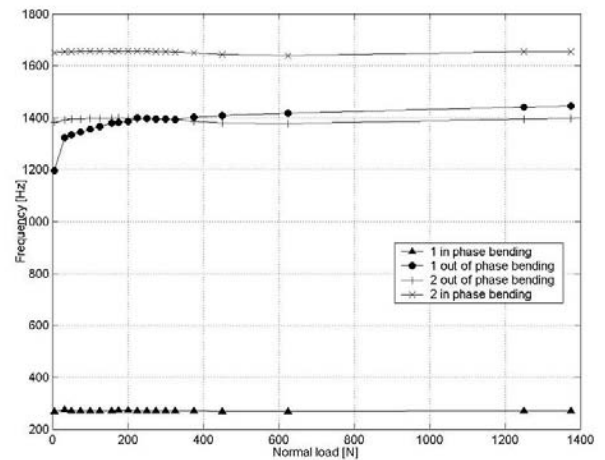


Figure 17. Two bladed system: first and second resonance frequencies for different normal loads

The mean experimental natural frequency obtained for the isolated beams is 250 Hz for the first bending mode and 1415 Hz for the second bending mode. The standard deviations for those modes is lower than 0.5% and such a value allows considering the system as tuned. At this stage the study has been intentionally limited to a single couple of blades in contact. Eight eddy sensors are used to measure displacements while only one blade is excited using an electrodynamic shaker (see figure 16). The two-bladed assembly classically exhibits two types of modes where blades vibrate in or out of phase. Only the first four modes will be considered here.

Figure 17 shows the frequency of resonance obtained for dif-

ferent normal loads. Only the out of phase type modes are affected by differences in normal loads, because for the in phase modes there is no significant relative displacements between blades and the contact does not work. The first and second in phase mode remain respectively at 290 Hz and 1650 Hz. The second out of phase bending frequency remain also at about 1400 Hz because the contact stay always stuck. A first examination of results reveals some apparent inconsistencies. For example, when the excitation force is  $F_{ex} = 5$  N and the normal load in the contact is  $F_n = 1380$  N, the first out-of-phase bending is at 1467 Hz, higher than the second out of phase bending which is at 1396 Hz. Furthermore, when normal load decreases, ratios  $F_{ex}/F_n$  remain always very low (from 0.0035 to 0.17) and within this range

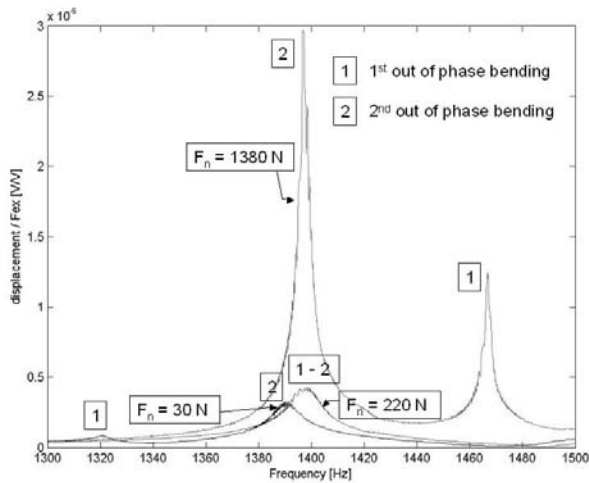


Figure 18. Two bladed system: first and second out of phase bending for different normal loads

the contact should intuitively stay in a stuck state, implying very low resonant frequency variations. However, it appears that for  $F_n = 220$  N the first and second out-of-phase bending frequencies are coincident at 1396 Hz and for  $F_n = 30$  N the first out of phase frequency falls at 1347 Hz. The associated FRFs obtained are shown in figure 18.

The explanation of those different experimental results has only been made possible after intensive numerical parametric studies. The jointed and incremental approach has been fully needed to explain why the first out of phase mode is higher in frequency than the second one at high normal loads and how different levels of locking may occur.

### 3.1 Two blades - numerical approach

A numerical analysis based on a three dimensional finite element discretisation has been performed. The first natural frequencies computed for the isolated beams are 250 Hz and 1416 Hz. Two different types of linking conditions were chosen for simulating the locking contact between blades. In a first case only two nodes are coupled (one on each contact surface) while in a second case four nodes are coupled. The associated natural frequencies obtained from a linear modal analysis are given in table 1. When coupling concerns a couple of nodes, the first out of phase mode is lower than the second one (respectively at 1352 Hz and 1390 Hz). This explains the experimental results obtained for low normal load values. In this case, the interface is within a first state of locking where relative translations are locked but relative rotations are always possible.

When four nodes are coupled, the first out of phase frequency is higher than the second one. This state corresponds to

Table 1. Numerical bending frequencies for beams with one shroud contact

bending mode	two coupled nodes	four coupled nodes
1 <sup>st</sup> in phase	285 Hz	285 Hz
1 <sup>st</sup> out of phase	1352 Hz	1679 Hz
2 <sup>nd</sup> in phase	1601 Hz	1665 Hz
2 <sup>nd</sup> out of phase	1390 Hz	1390 Hz

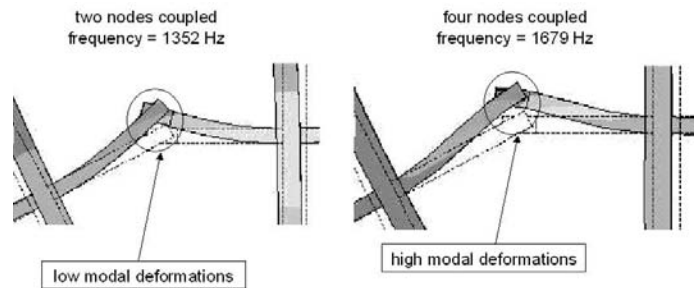


Figure 19. 1<sup>st</sup> out of phase mode with two different locking conditions

an other level of locking where both translations and rotations are locked. The analysis of mode shapes shows that shrouds participation are responsible of this apparent inconsistency. The first out of phase mode shapes associated with the two levels of locking are presented in figure 19. It appears clearly that when locking is high, shrouds deformation has a major influence and the corresponding frequency is higher. On the other hand the second out of phase mode needs low energy amounts at shroud levels because the nodal line is very close to shrouds (figure 20). Numerical results when four nodes are coupled (table 1) are in good agreement with test results associated with high normal loads. In this case the interface cannot be limited to a point to point contact and the whole common interface shall be modeled. Each normal load leads to a different contact surface and consequently a different behavior.

## 4 CONCLUSION

- A jointed experimental numerical procedure, adapted to dynamical studies of systems with non linearities due to dry friction, has been developed and validated. A swept sine with a controlled constant excitation force appeared as the faster experimental technique to obtain steady state responses. The numerical method is based on finite element discretisations and the solution is performed in time domain

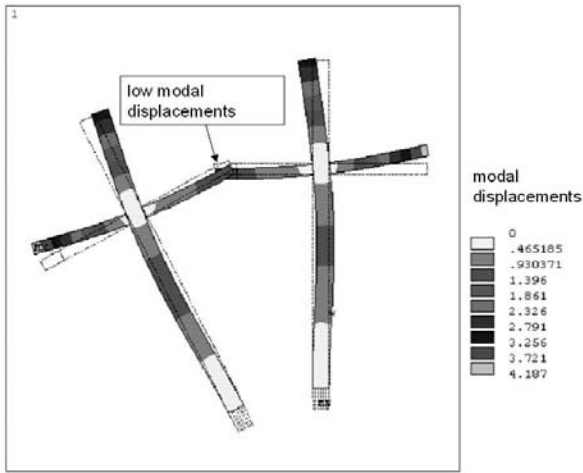


Figure 20. 2<sup>nd</sup> out of phase mode at 1394 Hz

in order to minimize errors of truncation that methods in frequency domain may introduce.

The jointed approach has been used classically in order to validate numerical results and to identify major parameters. This jointed approach has also been needed to explain surprising experimental results.

- The three typical cases of contact have been identified: slip, stick and stick/slip. A first simple test based on a cantilever beam with a rubbing element at its free end has allowed a parametric study and a better understanding of the phenomenon involved. One of the main results obtained lies in the full explanation of the vibrational amplitude decreasing within stick-slip states. Response curves become flat not only because of the induced damping due to rubbing but also because the system swings between two states and disables resonance possibilities. For the cases considered, very good agreements have been obtained between numerical and experimental results.
- An experimental device representative of a real shrouded bladed assembly has been presented. This device shall give a better understanding of the dynamical behavior of cyclically symmetrical structures associated with dry friction elements and shall allow detailed examinations of the effects of mistuning associated to non uniformities within the different contacts. Results presented here are associated to a first step where only a single couple of blades linked by shrouds is considered. The contact behavior appeared much more complex than the one involved within the first test. The contacting zone is a function of normal load and a simple macroslip model may be used only when this normal load is low. Finally, the jointed procedure has been needed to explain some apparent experimental inconsistencies, for ex-

ample when the second out of phase mode appeared lower in frequency than the first one.

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