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New formulations for the Scheduled Service Network Design Problem with Piecewise Linear Costs

Mike Hewitt, Fabien Léhuédé

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1 Introduction

The planning of consolidation-based transportation networks involves defining the moves of vehicles in a hub network in order to satisfy a given transportation demand. In this context, a set of demands, or *commodities*, have to be transported from their respective origins to their destinations in the network. Each commodity has a quantity. A release date can be provided to specify at what date and time the commodity is available at its origin and a due date can specify the latest date at which it can be delivered to its destination. The network is organized to efficiently serve multiple commodities which are consolidated in vehicles. Network design problems then typically consists in determining one or several paths for each commodity in the network as well as determining the number of vehicles required on each link of the network as well as the time at which each vehicle starts (its *dispatch time*) if

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release dates and due dates are involved, such that the sum of transportation cost is minimized.

Such planning operations involve solving challenging optimization problems to minimize the transportation costs. Different problem variants exist depending, for example, on the type of company taking the decision (typically carriers, industries or third-party logistics companies), on the decision level (tactical or more operational) or even of the type of service (FTL: full-truckload or LTL:less-than-truckload) in the case of motor carriers. The most studied in operations research literature are modeled as Service Network Design Problem (SNDP) or the Scheduled Service Network Design Problem (SSNDP) both reviewed recently in Crainic and Hewitt (2021). In the same book (Crainic et al., 2021), co-edited by Teodor Crainic, Michel Gendreau and Bernard Gendron, the chapter of Bakir et al. (2021) on motor carrier services, underlines that a large number of problems, from flow planning to vehicle routing and scheduling could be classified as network design problems in this domain.

In this chapter, we are particularly interested in variants of the SSNDP that integrates piecewise-linear costs. This type of cost, illustrated in Figure 1, is met in numerous contexts in the planning of transportation networks. One of the first application is the modeling of economies of scale related to the transported quantity when a shipper seeks to optimize its distribution (Balakrishnan and Graves, 1989; Croxton et al., 2003). It is the case in particular in a collaborative context where multiple shippers organize the pooling of their transport operations (Chabot et al., 2018; Tang et al., 2019; Arslan et al., 2020; Lai et al., 2022). Accordingly, numerous models have been proposed to integrate piecewise-linear cost functions in network design application (see e.g. Balakrishnan and Graves (1989); Croxton et al. (2007); Frangioni and Gendron (2009); Gendron and Gouveia (2017); Fortz et al. (2017)). The chapter of Frangioni and Gendron (2021) present a recent synthesis of models and methods for piecewise-linear costs optimization in network design applications.

Considering the SSNDP, it is classically modeled by discretizing time and designing a so-called Time-Expanded Network (TEN). In this case, it can be modeled as a Fixed-Charge Multicommodity Network Flow Problem and benefit of all advances proposed to model piecewise-linear costs in this problem. Nevertheless, solving large scale formulations on TEN is challenging for commercial solvers and advanced techniques such as the dynamic discretization discovery algorithm are needed to generate the network dynamically (Boland et al., 2017; Hewitt, 2019). Recently, the consolidation based model was proposed as an alternative to the TEN formulation (Hewitt and Lehuédé, 2022, 2023). In particular, Hewitt and Lehuédé (2023) shows that realistic size instances of the SSNDP can be solved by a MIP solver without having to implement an elaborate decomposition method. The paper also states that piecewise-linear cost functions can be easily integrated in the model but no comparison of this model to the literature or experiments of this model has been performed yet.

As a result, this chapter presents the TEN and consolidation based models for the SSNDP with piecewise linear costs. It proposes a comparison of the performance of the two models. This comparison is performed on instances derived from a real carrier network and demand. We show that, on these instances, the new consolidation based formulation outperforms the classical TEN model in terms of number of instances

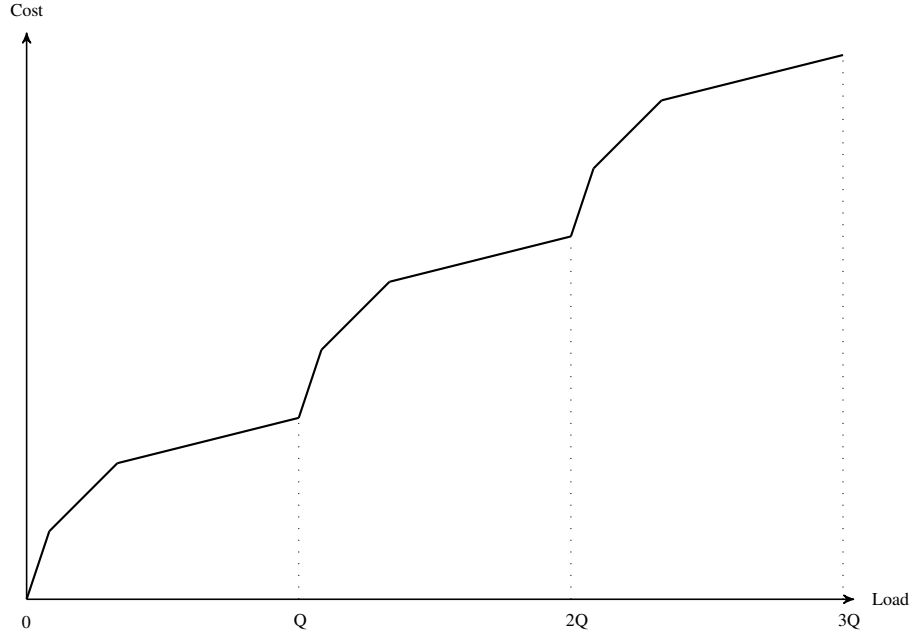


Fig. 1: Typical LTL rate on a transportation network link for a homogeneous fleet of vehicles of capacity Q

solved, solving time and optimality gap for instances that are not solved within a two hours time limit.

2 The Piecewise-Linear Scheduled Service Network Design Problem: Time-Expanded Network formulation

Let us consider a set of consolidation terminals, modeled by a set of nodes \mathcal{N} , and the transportation links between these terminals, modeled by a set of arcs \mathcal{A} . Each arc $(i, j) \in \mathcal{A}$ in this network is associated with a travel duration τ_{ij} for the vehicles that can be dispatched on this link. The corresponding network is called the *flat network* and it is denoted by $\mathcal{D} = (\mathcal{N}, \mathcal{A})$.

The transportation demand is modeled by a set of shipments $k \in \mathcal{K}$, each associated with a size d_k (in the same unit as vehicle capacity), an origin $o_k \in \mathcal{N}$ and a destination $d_k \in \mathcal{N}$. The date at which the shipment is available at its origin is the *release date*, denoted by e_k , while the latest delivery date is the *due date*, denoted by l_k . In addition, we assume that a set P_k is given to model the possible paths from o_k to d_k of a shipment k in the network.

Related to these definitions: we denote by \mathcal{K}_{ij} , the set of shipments k for which there exist a path $p \in P_k$ that contains the arc (i, j) ; we let $P(i, j)_k \subseteq P_k$ denote the set of paths that use arc (i, j) ; and we denote by $[\alpha_{ij}^{kp}, \beta_{ij}^{kp}]$ the time window in which a shipment $k \in \mathcal{K}$ can dispatch on arc (i, j) if it takes path p according to the shipment release date and due dates as well as the travel times on the path. Here, *dispatch time* denotes the time at which the shipment leaves the origin of the related arc. It can also refer to the time at which the vehicle that transports this shipment starts travelling on this arc. Unloading, sorting or loading times can be integrated on the arc travel times.

For each arc $(i, j) \in \mathcal{A}$, a piecewise-linear cost function $g_{ij}(u_{ij})$ can be defined according to the load u_{ij} on the arc. This function is determined by a finite set of segments $S_{ij} = \{1, \dots, |S_{ij}|\}$. Each segment $s \in S_{ij}$ is defined by a lower and upper bound $[b_{ij}^{s-1}, b_{ij}^s]$ related to the load u_{ij} , an intercept f_{ij}^s and a slope c_{ij}^s , such that $g_{ij}(u_{ij}) = f_{ij}^s + c_{ij}^s u_{ij}$ if $u_{ij} \in [b_{ij}^{s-1}, b_{ij}^s]$. This cost structure and notation is illustrated on Figure 2.

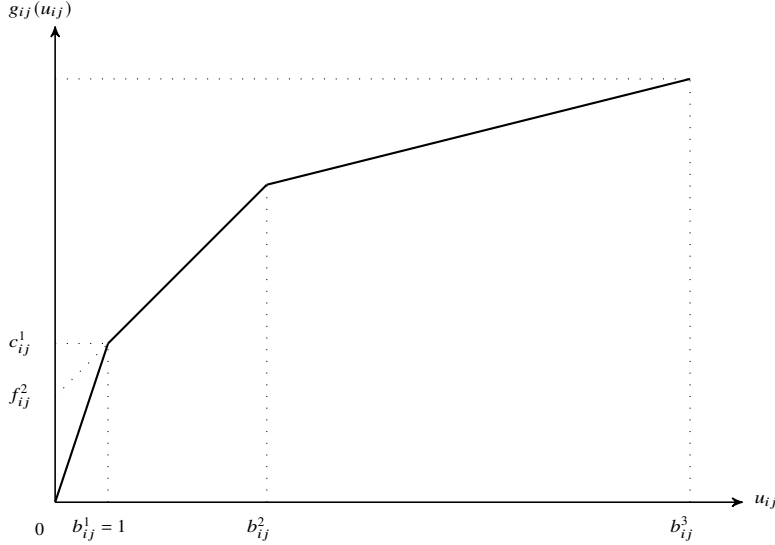


Fig. 2: Piecewise-linear cost function notation

We define the PLSSNDP as the problem of selecting one path for each shipment, determining a feasible dispatch time for each arc on this path and determining the number of necessary vehicles on the network to perform the shipments transportation such that the overall transportation cost is minimized.

To model this problem, the TEN formulation is based on a discretization of the time horizon in T periods, denoted by $\mathcal{T} = \{1, \dots, T\}$. Then the TEN $\mathcal{D}_{\mathcal{T}} = (\mathcal{N}_{\mathcal{T}}, \mathcal{A}_{\mathcal{T}})$ is defined with: $\mathcal{N}_{\mathcal{T}}$ denoting the node set $\{(i, t), i \in \mathcal{N}, t \in \mathcal{T}\}$ that

models the activities at terminal i during period t , and the set $\mathcal{A}_{\mathcal{T}}$ which models traveling and waiting operations in the network. More explicitly, $\mathcal{A}_{\mathcal{T}}$ is composed of traveling arcs of the form $((i, t), (j, t')), (i, t), (j, t') \in \mathcal{N}_{\mathcal{T}}, i \neq j, t' = \min(t'' \in \tau | t'' - t \geq \tau_{ij})$, which model the travel from a terminal i at time t to a terminal j at time t' , and waiting arcs of the form $((i, t), (i, t+1)), (i, t), (i, t+1) \in \mathcal{N}_{\mathcal{T}}$, which model a shipment that is waiting at a terminal i from period t to period $t+1$.

Based on this TEN, we define four sets of decision variables. First, for all shipments and paths, we define a variable v_p^k , equal to one if the path $p \in \mathcal{P}_k$ is selected for shipment $k \in \mathcal{K}$. Second, the binary variables $x_{ij}^{ktt'}$, $k \in \mathcal{K}, ((i, t), (j, t')) \in \mathcal{A}_{\mathcal{T}}$ denote whether shipment k takes arc $((i, t), (j, t'))$. Third, for each arc $((i, t), (j, t')) \in \mathcal{A}_{\mathcal{T}}$, we define the binary variable $y_{ij}^{stt'}$ which is equal to one if the load on $((i, t), (j, t'))$ falls within $[b_{ij}^{s-1}, b_{ij}^s]$, $s \in S_{ij}$ and last, $z_{ij}^{stt'} \in \mathfrak{R}^+$, which models the load on this arc if it falls within segment $s \in S_{ij}$.

Given these decision variables, the SSNDP can be formulated as follows. We label this formulation $TEN(\mathcal{D})$.

$$z_{TEN(\mathcal{D})} = \text{minimize} \quad \sum_{((i,t),(j,t')) \in \mathcal{A}_{\mathcal{T}}} \sum_{s \in S_{ij}} (f_{ij}^s y_{ij}^{stt'} + c_{ij}^s z_{ij}^{stt'}) \quad (1)$$

subject to

$$\sum_{p \in \mathcal{P}_k} v_p^k = 1 \quad \forall k \in \mathcal{K}, \quad (2)$$

$$\sum_{p \in P(i,j)_k} v_p^k = \sum_{(t,t'): ((i,t),(j,t')) \in \mathcal{A}_{\mathcal{T}}} x_{ij}^{ktt'} \quad \forall (i,j) \in \mathcal{A}, k \in \mathcal{K}_{ij}, \quad (3)$$

$$\sum_{((i,t),(j,t')) \in \mathcal{A}_{\mathcal{T}}} x_{ij}^{ktt'} - \sum_{((j,t'),(i,t)) \in \mathcal{A}_{\mathcal{T}}} x_{ji}^{kt't} = \begin{cases} 1 & (i,t) = (o_k, e_k) \\ -1 & (i,t) = (d_k, l_k) \\ 0 & \text{o.w.} \end{cases} \quad (4)$$

$$\sum_{k \in \mathcal{K}_{ij}} q_k x_{ij}^{ktt'} = \sum_{s \in S_{ij}} z_{ij}^{stt'} \quad \forall ((i,t), (j,t')) \in \mathcal{A}_{\mathcal{T}}, \quad (5)$$

$$b_{ij}^{s-1} y_{ij}^{stt'} \leq z_{ij}^{stt'} \leq b_{ij}^s y_{ij}^{stt'} \quad \forall ((i,t), (j,t')) \in \mathcal{A}_{\mathcal{T}}, s \in S_{ij}, \quad (6)$$

$$\sum_{s \in S_{ij}} y_{ij}^{stt'} \leq 1 \quad \forall ((i,t), (j,t')) \in \mathcal{A}_{\mathcal{T}}, \quad (7)$$

$$v_p^k \in \{0, 1\} \quad \forall k \in \mathcal{K}, p \in P(i,j)_k, \quad (8)$$

$$x_{ij}^{ktt'} \in \{0, 1\} \quad \forall ((i,t), (j,t')) \in \mathcal{A}_{\mathcal{T}}, k \in \mathcal{K}. \quad (9)$$

$$y_{ij}^{stt'} \in \{0, 1\} \quad \forall ((i,t), (j,t')) \in \mathcal{A}_{\mathcal{T}}, s \in S_{ij} \quad (10)$$

$$z_{ij}^{stt'} \geq 0 \quad \forall ((i,t), (j,t')) \in \mathcal{A}_{\mathcal{T}}, s \in S_{ij} \quad (11)$$

The objective function (1) minimizes the sum of piecewise-linear costs. Constraints (2) state that one path should be assigned to each shipment. Constraints (3) tie the chosen path to the arcs of the network. Constraints (4) model the balancing of shipment flows at their respective origin, destination and intermediate terminals. Constraints (5) connect the flow variables of the piecewise-linear cost model to the SSNDP load variables. Constraints (6) state that a segment s is used for an arc $((i, t)(j, t'))$ if the load on this arc falls within the segment bounds. Constraints (7) state that only one segment can be selected for the piecewise-linear cost function on one arc. Constraints (8) – (11) define the decision variable definition domains.

This model is referred to as the *Multiple Choice Model* in the literature (Balakrishnan and Graves, 1989; Vielma et al., 2010). Like network design problems in general, this formulation can be strengthened through disaggregation. Namely, we have the inequalities

$$x_{ij}^{ktt'} \leq \sum_{s \in S_{ij}: b_{ij}^s \geq q_k} y_{ij}^{stt'} \quad \forall ((i, t), (j, t')) \in \mathcal{A}_T, k \in \mathcal{K}. \quad (12)$$

These constraints reflect the observation that if shipment k flows on arc $((i, t), (j, t'))$ then because of the binary nature of the x variables, a segment must be chosen for that arc that has an upper bound at least as great as the demand of shipment k .

3 The consolidation based formulation

The consolidation based formulation has been recently proposed in (Hewitt and Lehu  d  , 2022, 2023) for the SSNDP and the service network scheduling problem, a variant where the path of each commodity is known. This model presumes an *a priori* enumeration of consolidations on each arc $(i, j) \in \mathcal{A}$, wherein a *consolidation* is defined as a set of shipments that dispatch on the same physical move at the same time. Regarding notation for this formulation, we let Ω_{ij} define the set of feasible shipment consolidations on arc (i, j) . Thus, an element $\omega \in \Omega_{ij}$ is a subset of commodities (i.e. $\omega \subseteq \mathcal{K}$) that have a non-zero intersection of their time windows on arc $(i, j) \in \mathcal{A}$. For $(i, j) \in \mathcal{A}, \omega \in \Omega_{ij}$, we let ϕ_ω^k indicate whether shipment $k \in \mathcal{K}$ is contained in consolidation ω .

Since all shipment consolidations are enumerated, the consolidation based formulation allows us to calculate the cost associated with transporting consolidation $\omega \in \Omega_{ij}$ on $(i, j) \in \mathcal{A}$ independently of the dispatch time of the consolidation. This cost is denoted by c_ω for all $\omega \in \Omega_{ij}$ and defined as $c_\omega = g_{ij}(\sum_{k \in \omega} q_k)$. Hence, the formulation integrates the piecewise linear cost function before optimization.

We define the binary decision variable δ_ω to indicate whether the consolidation $\omega \in \Omega_{ij}$ is chosen. Like the formulation $TEN(\mathcal{D})$, the choice of physical routing of shipments is done with the binary variables $v_p^k \in \{0, 1\}, p \in P_k, k \in \mathcal{K}$. The decision variable γ_{ij}^k represents the time at which shipment k dispatches on arc $(i, j) \in \mathcal{A}$.

The consolidation-based formulation we present, which we label $CONS(\mathcal{D})$, is as follows.

$$z_{CONS(\mathcal{D})} = \text{minimize} \quad \sum_{(i,j) \in \mathcal{A}} \sum_{\omega \in \Omega_{ij}} c_{\omega} \delta_{\omega} \quad (13)$$

subject to

$$\sum_{p \in P_k} v_p^k = 1 \quad \forall k \in \mathcal{K}, \quad (14)$$

$$\sum_{\omega \in \Omega_{ij}} \phi_{\omega}^k \delta_{\omega} = \sum_{p \in P(i,j)_k} v_p^k \quad \forall (i,j) \in \mathcal{A}, k \in \mathcal{K}_{ij}, \quad (15)$$

$$\gamma_{ij}^k - \gamma_{ij}^{k'} \leq T(1 - \sum_{\omega \in \Omega_{ij}} \phi_{\omega}^k \phi_{\omega}^{k'} \delta_{\omega}) \quad \forall (i,j) \in \mathcal{A}, k, k' \in \mathcal{K}_{ij}, \quad (16)$$

$$\sum_{p \in P(i,j)_k} \alpha_{ij}^{kp} v_p^k \leq \gamma_{ij}^k \quad \forall (i,j) \in \mathcal{A}, k \in \mathcal{K}_{ij}, \quad (17)$$

$$\gamma_{ij}^k \leq \sum_{p \in P(i,j)_k} \beta_{ij}^{kp} v_p^k \quad \forall (i,j) \in \mathcal{A}, k \in \mathcal{K}_{ij}, \quad (18)$$

$$\sum_{(o_k, j) \in \mathcal{A}} \gamma_{o_k j}^k \geq \ell_k \quad \forall k \in \mathcal{K}, \quad (19)$$

$$\sum_{(i, d_k) \in \mathcal{A}} (\gamma_{id_k}^k + \tau_{id_k} (\sum_{p \in P(i, d_k)_k} v_p^k)) \leq l_k \quad \forall k \in \mathcal{K}, \quad (20)$$

$$\sum_{(i,j) \in \mathcal{A}} (\gamma_{ij}^k + \tau_{ij} (\sum_{p \in P(i,j)_k} v_p^k)) \leq \sum_{(j,i) \in \mathcal{A}} \gamma_{ji}^k \quad \forall j \in N, k \in \mathcal{K}, \quad (21)$$

$$v_p^k \in \{0, 1\} \quad \forall k \in \mathcal{K}, p \in P(i, j)_k, \quad (22)$$

$$\delta_{\omega} \in \{0, 1\} \quad \forall \omega \in \Omega_{ij}, (i, j) \in \mathcal{A}, \quad (23)$$

$$\gamma_{ij}^k \geq 0 \quad \forall k \in \mathcal{K}, (i, j) \in \mathcal{A}. \quad (24)$$

The objective function (13) minimizes the sum of transportation costs. Constraints (14) state that a path should be assigned to each commodity. Constraints (15) indicates that if the path assigned to a commodity k uses an arc $(i, j) \in \mathcal{A}$, then this commodity must be part of a selected consolidation on (i, j) . Constraints (16) state that two commodities that belong to the same selected consolidation on an arc $(i, j) \in \mathcal{A}$ are dispatched at the same time on (i, j) . Constraints (17) and Constraints (18) state that a commodity cannot be dispatched before the opening and after the end of its time window on each arc of its selected path. Constraints (19) enforce the departure of a commodity from its origin after its release date and constraints (20) make sure each commodity is delivered at its destination by its due date. Precedences on a commodity path, integrating travel times, is modeled by Constraints (21). Constraints (22) and (23) define the domains of the variables.

Consolidation-pruning: Hewitt and Lehu  d   (2023) present a condition under which consolidations can be removed from $CONS(\mathcal{D})$ without rendering all optimal solutions infeasible. The logic behind that observation can be adapted to a similar observation for the case of non-linear costs. Namely, that when a consolidation ω can be partitioned into subsets, $\omega_i, i = 1, \dots, m$ such that the $c_\omega \geq \sum_{i=1}^m c_{\omega_i}$ then consolidation ω can be removed from $CONS(\mathcal{D})$ without rendering all optimal solutions infeasible.

4 Computational analysis

In this section, we computationally evaluate the solvability of the two formulations. Before presenting the results of our computational study, we describe the instances on which it is based as well as the computational environment in which experiments were run.

4.1 Instances

We consider a subset of the set of instances used in Hewitt and Lehu  d   (2023). That set consists of randomly generated instances that model a portion of the network of a United States-based LTL carrier network that consists of 25 terminals (e.g. $|\mathcal{N}| = 25$) and 530 physical moves between terminals (e.g. $|\mathcal{A}| = 530$).

We consider instances that vary in the number of shipments, or, the size of the set \mathcal{K} . In these instances, all commodities have at most ten potential paths (i.e. $|\mathcal{P}_k| \leq 10, \forall k \in \mathcal{K}$). To identify the set \mathcal{P}_k for shipment k , we enumerated all paths from o_k to d_k to identify a set of candidate paths. We then removed from this candidate set all paths for which the total time required to travel that path exceeded the time available to deliver the shipment. Formally, we removed path p for commodity k if $\sum_{(i,j) \in p} \tau_{ij} > l_k - e_k$. Next, we removed from the candidate set, paths that exhibited one of two criteria, both of which were inspired by discussions with practitioners. The first criteria is paths that visit more than three terminals other than the origin or destination. The second factor is paths with distance greater than 150% the distance between origin and destination terminals, when such a direct move is possible. Then, from the remaining set of candidate paths, we included in \mathcal{P}_k the ten shortest with respect to travel time. If the candidate set did not contain ten paths then we included in \mathcal{P}_k all candidate paths.

Regarding transportation costs on arc (i, j) , we define piecewise linear costs based on a given vehicle capacity of Q and carrier-provided transportation cost C_{ij}^{FTL} for a full truckload on arc (i, j) . To represent cost functions with increasing number of segments in our experiments, we model that the capacity of an individual vehicle, Q , is divided into M segments of identical lengths. Hence, $b_{ij}^1 = Q/M, b_{ij}^2 = 2Q/M, b_{ij}^M = Q$. To model economies of scales, we define a cost F_{ij} as the maximal cost of the first

segment: $g_{ij}(b_{ij}^1) = F_{ij}$. Hence, the first segment has a slope $c_{ij}^1 = F_{ij}/b_{ij}^1 = MF_{ij}/Q$. Then the second segment has a slope $c_{ij}^2 = F_{ij}/b_{ij}^2 = MF_{ij}/2Q$, the third segment has a slope $c_{ij}^3 = MF_{ij}/3Q, \dots$, the M^{th} segment has a slope $c_{ij}^M = F_{ij}/Q$. The intersect of the first segment is $f_{ij}^1 = 0$. Then to ensure that the function is continuous, the intersect of each segment $s \in \{2, \dots, M\}$ is defined as $f_{ij}^s = F_{ij} \times \sum_{k=2}^s 1/k$. To calibrate the cost function such that $g_{ij}(Q) = C_{ij}^{FTL}$, we define $F_{ij} = C_{ij}^{FTL} / \sum_{k=1}^M 1/k$, depending on the number of required segments M . This function is illustrated on Figure 3. It is repeated for loads that are greater than one truck with the same M slopes.

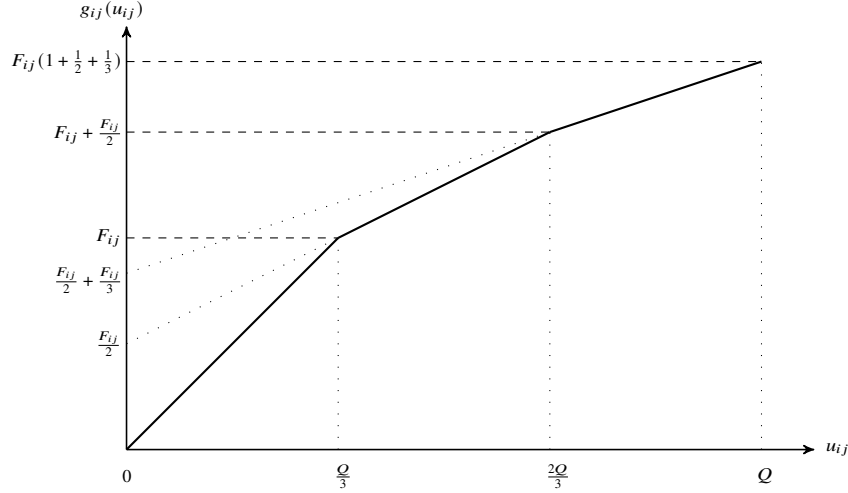


Fig. 3: First M segments of a piecewise-linear cost function defined for our experiments for vehicles of capacity Q .

We presume a vehicle capacity of $Q = 22,000$ pounds and consider divisions of that capacity into three, four, and five segments of identical lengths. We classify an instance by the number of shipments, $|\mathcal{K}|$, and capacity division. We consider classes with 100, 150, or 200 numbers of shipments and five instances per class, leaving a total of 45 instances in our experimental study.

4.2 Computational setting

We next describe the computational environment and settings used in our experimental study. We first note that each experiment was run on the same computer, which is equipped with 200 GB of RAM and 64 Intel Xeon Gold 6130 CPU processors operating at 2.10GHz. This computer runs the Ubuntu distribution of the Linux

operating system. All formulations and code to instantiate them were implemented in Python 3.7 (VanRossum and Drake, 2010). All code was run on a single thread for two hours, or, 7,200 seconds. All optimization models were also solved with CPLEX 22.10 (Studio-CPLEX, 2022) to a 1% tolerance. In all experiments CPLEX 22.10 was configured to use a single thread. All CPLEX parameters not mentioned were left at their default values.

4.3 Analysis

In our computational analysis we seek to understand the following issues. The first is which of the two proposed formulations is more amenable to solution by a MIP solver. The second is to what extent the characteristics of an instance affects the performance of a MIP solver when attempting to solve instances of the two formulations. We focus on two instance characteristics in this analysis: the number of shipments, $|\mathcal{K}|$, and the capacity division underlying the cost structure. The third is the effectiveness of the consolidation pruning speed-up technique proposed in Section 3. The fourth and final issue we consider is a comparison of the difficulty of solving instances of $CONS(\mathcal{D})$ when there are piecewise linear costs, as considered in this paper, as opposed to the traditional cost structure considered in the SSNDP.

4.4 Performance of formulations

To compare the performance of the different formulations and speed-up techniques we focus on three statistics: (1) the number of instances solved, (2) the average time to solve an instance, and, (3) the average optimality gap of instances not solved. We report these statistics aggregated over instances with same number of shipments, $|\mathcal{K}|$. Table 1a, 1b, and 1c report these statistics for each of the three capacity divisions.

We see that the MIP solver is capable of solving all but one instance of the consolidation-based formulation and only two of the time expanded network-based formulation. Focusing on the consolidation-based formulation, $DVCONS$, we see that instances become harder to solve as the number of shipments in the instance increases. The correlation between the number of segments in the cost function and the difficulty of an instance is not clear. Theoretically speaking, the number of segments in the cost function has no impact on the number of variables or constraints in this model. But we see that the three segments instances take longer to solve. This is in contrast to the time expanded network-based formulation in which the number of variables and constraints increases in the number of segments. The impact of this can be seen by noting that the average gap of unsolved instances of the TEN formulation increases in the number of segments. Finally, we also solved instances of TEN strengthened with the valid inequalities (12) added in an *a priori* manner. We note that with those valid inequalities, CPLEX still solved only two instances of

Table 1: Performance statistics, 15 instances per group

\mathcal{K}	<i>DVCONS</i>			<i>TEN</i>		
	% Solved	Time	Gap unsolved	% Solved	Time	Gap unsolved
100	5	8.45	N/A	0	N/A	1.69%
150	5	929.46	N/A	0	N/A	5.82%
200	4	2,683.19	5.37%	0	N/A	6.92%
Summary	14	1,207.04	5.37%	0	N/A	4.81%

(a) Capacity division into 3 segments

\mathcal{K}	<i>DVCONS</i>			<i>TEN</i>		
	% Solved	Time	Gap unsolved	% Solved	Time	Gap unsolved
100	5	5.31	N/A	2	5,905.48	5.32%
150	5	35.54	N/A	0	N/A	12.80%
200	5	392.66	N/A	0	N/A	33.16%
Summary	15	144.50	N/A	2	5,905.48	17.09%

(b) Capacity division into 4 segments

\mathcal{K}	<i>DVCONS</i>			<i>TEN</i>		
	# Solved	Time	Gap unsolved	# Solved	Time	Gap unsolved
100	5	6.06	N/A	0	N/A	5.10%
150	5	37.75	N/A	0	N/A	15.29%
200	5	538.36	N/A	0	N/A	35.93%
Summary	15	194.06	N/A	0	N/A	18.77%

(c) Capacity division into 5 segments

the *TEN* and the average gap of those unsolved was larger. Adding such inequalities in a cutting-plane type manner may yield better results. However, we still conclude that the consolidation-based formulation is easier to solve.

4.5 Impact of consolidation pruning

We next consider in Table 2 the impact of consolidation pruning. We report the same summary statistics as above, again aggregated over instances with the same number of shipments. We report results separately by the number of segments in the piecewise linear cost function. We see that the consolidation pruning technique

\mathcal{K}	3 segments			4 segments			5 segments		
	# Solved	Time	Gap unsolved	# Solved	Time	Gap unsolved	# Solved	Time	Gap unsolved
100	5	5.78	N/A	5	2.70	N/A	5	2.82	N/A
150	5	480.90	N/A	5	13.72	N/A	5	8.92	N/A
200	5	2,006.51	N/A	5	364.62	N/A	5	773.96	N/A
Summary	15	831.06	N/A	15	127.01	N/A	15	261.90	N/A

Table 2: *DVCONS* with consolidation pruning

enables the solver to solve every instance and often in less time, on average.

4.6 Comparison to the classical model

Finally, we seek to assess whether the SSNDP with Piecewise Linear costs is easier or harder to solve than the variant of the SSNDP that is traditionally studied wherein transportation costs are paid on a per-vehicle basis. To understand this, we formulate and solve the consolidation-based SSNDP formulation of the instances considered in this experimental study. We report performance statistics for these instances in Table 3.

\mathcal{K}	$CONS(\mathcal{D})$			$CONS(\mathcal{D})$ with pruning		
	# Solved	Time	Gap unsolved	# Solved	Time	Gap unsolved
100	5	108.22	N/A	5	81.16	N/A
150	1	1,404.58	2.42%	1	654.21	2.08%
200	0	N/A	6.43%	0	N/A	3.59%
Summary	6	756.40	4.43%	6	367.68	2.84%

Table 3: SSNDP with per vehicle transportation costs

We see that the variant with piecewise linear costs is easier to solve than the per-vehicle cost structure traditionally modeled and solved. We hypothesize that this may be due to the traditional cost structure requiring a set of integer decision variables that model the number of vehicles used on arc $(i, j) \in \mathcal{A}$.

5 Conclusions and future work

Piecewise linear cost structures often appear in transportation and logistics contexts wherein the per unit price of a good or service exhibits economies of scale. Such a structure is often seen in the context of Less-than-truckload freight transportation wherein a carrier will charge a shipper a per unit of volume price that decreases as the total volume transported increases. The planning of transportation operations for such a carrier can be assisted with the solution of the Scheduled Service Network Design Problem. In this chapter, we considered such a context and cost structure and studied the Scheduled Service Network Design Problem with Piecewise Linear Costs. One practical setting in which the solution of this problem is relevant is when a shipper outsources its freight transportation to a dedicated carrier and the goods the shipper seeks to have transported are small relative to vehicle capacity. While such cost structures are prevalent in practice and modeling them is fairly straightforward, the resulting mathematical programs are notoriously difficult to solve.

Recently, a new formulation was proposed for the Scheduled Service Network Design Problem (SSNDP) in which transportation costs are paid on a per-vehicle basis. This new formulation is based on enumerations of shipments that will be transported together and are referred to as “consolidations.” It has been shown that this new formulation is much easier to solve, computationally-speaking, than the classical formulation of the SSNDP that is based on a time expanded network. Thus, in this chapter we studied whether adapting the ideas underlying that formulation to the case wherein costs are piecewise linear yields a formulation that is similarly easier to solve than a formulation based on a time-expanded network. We proposed such a formulation and with a computational study demonstrated that the consolidation-based formulation was easier to solve. We also adapted a speed-up technique for solving a consolidation-based formulation of the SSNDP with per-vehicle transportation costs to the case of piecewise linear costs.

Regarding future work, we see avenues for both algorithmic and modeling work. While we observed that instances of the proposed consolidation-based formulation are easier to solve than those of the time expanded network-based formulation, its enumerative nature likely necessitates a solution method that generates consolidations dynamically. The piecewise linear cost structure will likely present algorithmic challenges not present in a comparable method for a consolidation-based formulation of the SSNDP with the traditional cost structure. From a modeling perspective, we considered a shipper-oriented model wherein the transportation costs on each arc have a piecewise linear structure. However, taking the perspective of a carrier will likely yield a different model as a carrier with its own fleet may rely on both owned and outsourced transportation. In this case, the cost structure would involve a combination of paying for the use of owned vehicles, as in the classical SSNDP, and outsourcing transportation to a third party, as in the model considered in this paper.

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