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Tooth friction in spur gear transmission. From local to mean tooth friction loss

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Abstract

In a gear transmission, power losses come from a variety of sources, classified as load-dependent and no-load-dependent losses. No-load-dependent losses are the ones that remain constant regardless of the applied load, such as losses due to drag in rolling element bearings, seals, churning and/or windage for gears. Load-dependent losses include friction losses due to sliding and rolling between gear teeth. These losses are particularly important in some gear transmissions because of their direct impact on overall efficiency and thermal behaviour. Consequently, knowledge of the friction coefficient associated to this loss is of major interest for optimising gears (geometry, surface finish, lubrication, etc.).

In this article, a new method is proposed for predicting the mechanical efficiency losses associated with the mean coefficient of friction for pairs of cylindrical gears. It is based on a model developed previously to estimate the instantaneous tooth friction along the plane of action in gears. This model takes into account the lubricant shear and the friction on contacting asperities. The proposed mean friction coefficient is deduced from the instantaneous friction coefficient using specific calculations, particularly at the pitch point. To verify the method, the estimated results are compared with those in the literature calculated for gears and operating conditions common on FZG test rigs.

Keywords: tooth friction, power loss, friction coefficient, cylindrical gears, gear transmission

1. Introduction

Power losses in gear transmissions are becoming an increasingly important research topic due to stricter environmental regulations and rising fuel prices. They can be categorised into no-load dependent losses and load-dependent losses. No-load dependent losses are due to drag losses in rolling element bearings, windage and/or churning in gears or seals for example. They therefore occur even if no load is transmitted. Load-dependent losses are generated in gears and bearings when torque is transferred and are due to friction between contacting surfaces. For transmissions that operate in low or moderate speed conditions such as the ones used in passenger vehicles or agricultural tractors, power loss is mainly caused by tooth friction.

To accurately predict the load-dependent losses generated by a pair of gears in mesh, the friction coefficient must be correctly evaluated. Numerous studies have been conducted on the load-dependent losses and propose different methods for calculating the friction coefficient [1].

A group of models [2], [3] investigated the efficiency of a spur gear pair by assuming a uniform μ (friction coefficient) along the entire contact surface. A tangential friction force along the sliding direction was computed by using a given constant friction coefficient μ , and the geometric and kinematic parameters of the spur gears. As a result, the amount of reduction of torque transmitted to the driven gear was used to calculate the mechanical efficiency of the gear pair. These models were useful in bringing a qualitative understanding to the role of spur gear geometry on efficiency. But they fell short in terms of the definition of μ , as a user defined constant value must be used for every contacting point on the tooth surface. However, the published experiments on sliding/rolling contacts indicate that many parameters might influence μ [4], [5]. In addition, these studies were limited to spur gears and many complicating effects of the tooth bending and contact deformations, tooth profile modifications and manufacturing errors were not included.

Several searchers have proposed advanced models using an EHL model to predict instantaneous friction coefficient [6], [7]. Among these, EHL models for smooth surfaces are used to determine the surface shear stress distribution caused by the fluid film and, consequently, the instantaneous friction coefficient at the contact [8], [9]. In addition, a model developed for lightly loaded spur gears under hydrodynamic lubrication conditions without deflection and local deformation is presented in [10]. The lubrication of involute spur gears has been investigated using a transient thermal-EHL model with smooth surfaces [12], [13]. All these models deal only with smooth surfaces, but for gear contact, the impact of the surface roughness cannot be ignored. Thus, a friction model based on mixed-EHL contacts was proposed in [14] and used to calculate the frictional power losses of two spur gears, taking into account roughness. The model was designed so that each asperity has the same radius of curvature and the asperity heights follow a Gaussian distribution. The influence of the asperity contacts on the friction coefficient and therefore on efficiency has also been studied and presented in [7]. Although they have been relatively improved in the EHL aspects of the problem, applications are limited to simple spur gears with ideal load distributions and no tooth bending deformations.

Several others searchers have measured the friction coefficient, using twin-disk machines under conditions simulating a gear pair, so that the friction coefficient can be used to predict the efficiency of a gear pair [5], [12], [13], [14]. Some of these studies [4], [15] have resulted in well-known and widely used empirical formulae for the friction coefficient. These empirical formulae indicate that μ is a function of a list of parameters such as sliding and rolling velocities, radii of curvature of the contacting surfaces, contact load or contact pressure, surface roughness and lubricant viscosity. In addition, some models provide an approximative mean friction coefficient which requires a few number of input parameters and computational effort. It can be noticed that these models are very different from each other because they take into account different parameters and/or operating ranges [1], [15]. To overcome these limitations and to avoid the complexity of calculations of EHL models, a semi-empirical model of instantaneous sliding friction coefficient of non-conformal rolling-sliding contacts was developed by Diab *et al.* [16], [17]. This model, which is simpler than the EHL method, has the advantage of including a complete set of equations to simulate a mixed lubricated contact.

In this paper, a new method based on Diab [16], [17] is proposed for predicting the mean friction coefficient during meshing. Instead of calculating the instantaneous friction coefficient along the plane of action, which involves considerable complexity and calculation time, the originality of the proposed method consists in taking the previously developed model [16], [17] and performing the calculation at a single point (pitch point). Having identified the method for calculating the mean friction coefficient from the estimated instantaneous losses during meshing, a formulation of this coefficient is determined. Finally, this new formulation is compared with results from the literature.

2. Local gear friction loss

Load and kinematics vary along gear mesh. For cylindrical gears, those evolutions can be computed analytically along a plane of action which corresponds to the imaginary plane where tooth contacts occur. For spur gear, this plane can be reduced to a line, due to axial symmetry along tooth width, when considering rigid bodies and no tooth reliefs. Those hypotheses are kept for the rest of this investigation. The local sliding friction power loss can then be computed using equation 1:

$$P(x) = F_n(x) \cdot \mu(x) \cdot v_s(x) \quad (1)$$

with x the position on the path of contact, $P(x)$ the local gear friction loss, $F_n(x)$ the local force normal to the tooth profile, $\mu(x)$ the local coefficient of friction and $v_s(x)$ the local sliding speed. The local friction coefficient $\mu(x)$ in equation 1 is calculated using Diab model [16], [17]. $F_n(x)$ is calculated taking into account the modified tooth profiles. Those parameters are presented on figure 1. In this figure, the sliding speed decreases up to zero at the pitch point and then increases; the local normal force evolves with the number of teeth in contact. As a result of

these variations, the local friction coefficient and the associated gear friction loss vary along the action line: Figure 1 shows that maximum losses occur at the tooth tips and minimum losses at the pitch point. In actual operation, the spur gear pair may mesh with more than one tooth pair. Therefore, to estimate the total local gear friction loss, the local gear friction loss of each gear pair must be combined according to meshing order (figure 2).

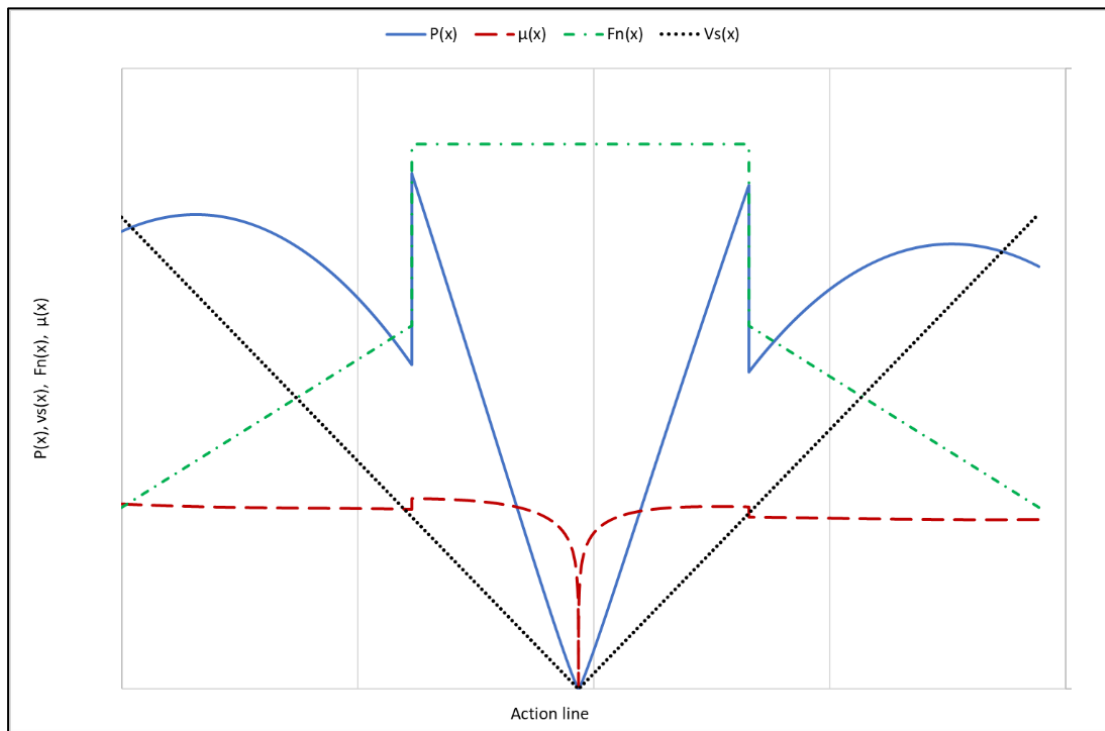


Figure 1 instantaneous friction coefficient $\mu(x)$, instantaneous normal load $F_n(x)$, instantaneous sliding speed $v_s(x)$,

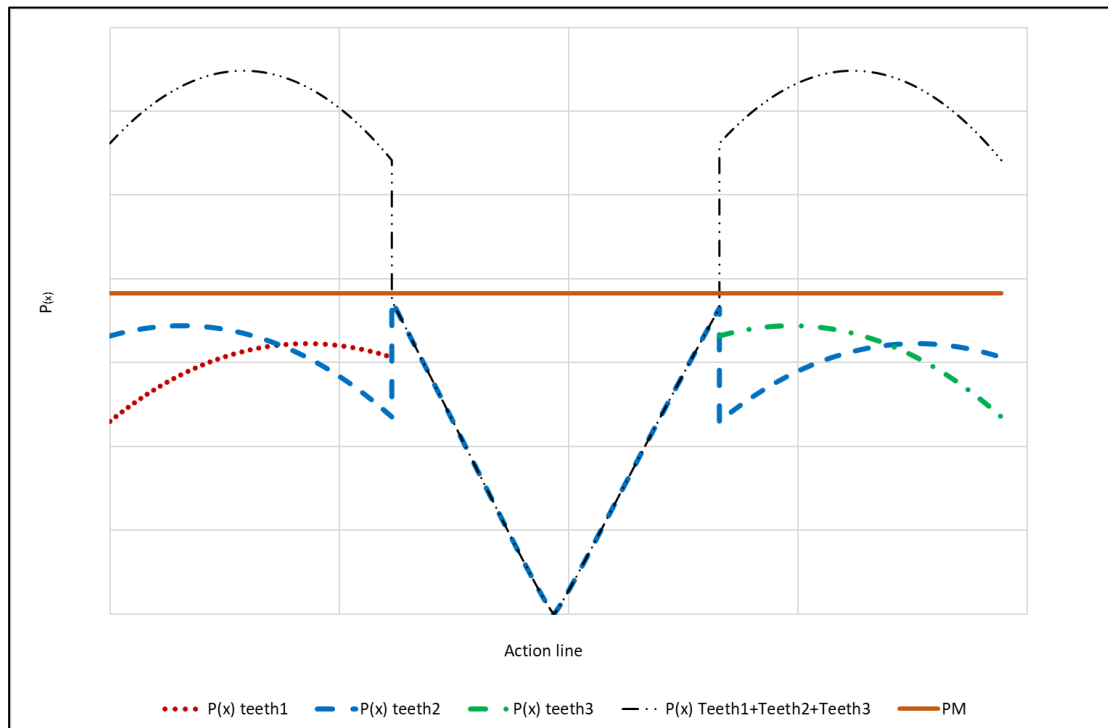


Figure 2 Instantaneous friction losses $P(x)$ and average friction losses P_M , along the meshing line of FZG C gear

Finally, the average value of the total local gear friction loss, P_M , can be determined by equation 2:

$$P_M = \frac{1}{p_{bn}} \int_A^E F_n(x) \cdot \mu(x) \cdot v_s(x) dx \quad (2)$$

with p_{bn} the normal base pitch, A beginning of contact and E end of gear mesh.

ISO 14179-2 Technical Report [18] proposes a simple method based on the definition of efficiency to calculate the average gear friction loss:

$$P_M = \mu_{mz} H_v P_{in} \quad (3)$$

with μ_{mz} the average friction coefficient of the gear mesh, H_v the tooth loss factor which is independent of operating conditions (torque, speed, oil temperature, etc.) but only of the gear geometry, P_{in} the transmitted power.

Fernandes *et al.* [19] have shown that the simple formula for H_v proposed by ISO 14179-2 [18] (appendix 1) gives good results when rigid bodies and no tooth profile modifications are considered. For gears with tooth profile modification, Vexel *et al.* [20] have proposed a more complete formula that takes into account tip and face end reliefs.

The average friction coefficient μ_{mz} results from multiple parameters (loads, speeds, surfaces roughness, solids and fluid physical properties, etc.). Its estimation is therefore complex. Many papers deal with this subject and the question remains open. The ISO 14179-2 Technical Report [18] relies on Schlenk's relationship [21] to calculate a value of μ_{mz} (appendix 1, equation 13) which is developed on the basis of experimental data from a gear back-to-back test rig [19].

On the other hand, the local friction coefficients $\mu(x)$ obtained from Diab model [16], [17] cannot be used directly in equation 2 as it requires a mean value (μ_{mz}) on gear mesh. By using equations 1 and 2, the average friction coefficient μ_{mz} can be deduced from the instantaneous friction coefficient:

$$\mu_{mz} = \frac{\frac{1}{p_b} \int_A^E F_n(x) \cdot \mu(x) \cdot v_s(x) dx}{H_v P_{in}} \quad (4)$$

A comparison of the two methods for obtaining a mean friction coefficient is proposed on the basis of FZG C14 carburised steel gears (table 1) and the ISO VG 320 mineral oil (tribological parameters in [17]).

As the FZG tests are widely used and known, certain representative operating conditions were chosen to compare the coefficients of friction numerically.

Table 1: FZG C14 gears

Gear type	FZG C14	
	Pinion	Wheel
Number of teeth	16	24
Module (mm)	4.5	
Centre distance (mm)	91.5	
Pressure angle (°)	20	
Helix angle (°)	0	
Face width (mm)	14	
Addendum modification	+0.1817	+0.1715
Mean surface roughness Ra (µm)	0.8	0.8

RMS of height surface roughness Rq (μm)	1	1
RMS of slope surface roughness Rdq (-)	0.042	0.048

Table 2 shows the estimated mean friction coefficient calculated from Diab model [16], [17] using equation 4, μ_{mz-D} , that using Schlenk formula [18], [21], equation 13, μ_{mz-S} . Comparison was made with the value of the lubricant factor X_L proposed by Schlenk [18], [21] for a mineral oil, $X_{L-S}=1$ and that proposed by Frenandes and al. [19], $X_{L-F}=0.846$.

Table 2: mean friction coefficients μ_{mz-D} and μ_{mz-S} ,

Wheel Rotational Speed	Load Stage 6 – 135.3 Nm on the pinion			Load Stage 12 - 534.5 Nm on the pinion		
	μ_{mz-D} %	μ_{mz-S} %		μ_{mz-D} %	μ_{mz-S} %	
		X_{L-S}	X_{L-F}		X_{L-S}	X_{L-F}
1500rpm	5.06	5.29	4.48	5.21	6.96	5.89
3000rpm	4.53	4.60	3.89	4.84	6.06	5.13

For load stage 6, there is good agreement between Diab estimate and Schlenk one with $X_L = 1$ for both speeds (between 2 and 5% of relative difference). On the one hand, Diab's model is based on a physical approach at the scale of tooth contact. On the other hand, Schlenk's model is obtained from an empirical analysis of FZG tests. Part of the difference may therefore be due to changes in the distribution of power losses in the transmission (for example, the reduction in viscous friction in the bearings). If we consider the increase in speed at load stage 6, the difference between the two models is reduced because the proportion of power losses due to the bearings decreases compared with that due to the meshing. However, when $X_L = 0.846$, the relative difference rises to 11-14%.

In contrast, for load stage 12, there is good agreement between Diab estimate and Schlenk one with $X_L = 0.846$ for both speeds (between 6 and 13% of relative difference). However, when $X_L = 1$, the relative difference increases to 25-34%.

These comparisons clearly demonstrate the need to determine the mean friction coefficient from tribological evaluations such as that proposed in [16], [17]. Indeed, a 10% difference in its estimation could result in a 10 % difference in the estimated losses for gear meshing.

3. Determination of the mean friction coefficient from the local friction coefficient

To calculate the average gear friction loss, P_M , in equation 3, the value of the mean friction coefficient must be known. Because many parameters such as rolling and sliding speeds, lubricant viscosity, surface roughness, contact load, etc. significantly affect the friction coefficient, its value must be carefully adapted to these parameters.

The mean friction coefficient formulation μ_{mz} according to ISO (equation 13) requires a unique value for each input parameters (speeds, load, etc.). As these parameters vary along the path of contact, these values must be judiciously chosen so that: (i) they represent on average the set of values varying along the path of contact (Figure 2) and (ii) the value of the average tooth friction loss calculated from the average formulation must correspond to that obtained from instantaneous calculation along the path of contact. This second point is very important because the mean value of the integral of a product is not necessarily equal to the product of mean values (see

Table 3, in which $\mu(x)$ is calculated using Diab model):

$$\frac{1}{p_b} \int_A^D F_n(x) \cdot \mu(x) \cdot v_s(x) dx \neq \text{mean}(F_n(x)) \cdot \text{mean}(\mu(x)) \cdot \text{mean}(v_s(x)) \quad (5)$$

Table 3 Power losses for FZG tests (table 1)

Rotational speed	$\frac{1}{p_b} \int_A^D F_n(x) \cdot \mu(x) \cdot v_s(x) dx, (W)$		$mean(F_n(x)) \cdot mean(\mu(x)) \cdot mean(v_s(x)), (W)$	
	Load stage 6	Load stage 12	Load stage 6	Load stage 12
1500rpm	302	1210	247	1061
3000rpm	541	2287	444	1913

The next section focuses on a simple method for moving from the local friction coefficient to the mean one ($\mu(x) \rightarrow \mu_{mz}$).

First, as the sliding speed varies along the path of contact (figure 1), a simple analytical formula is given to obtain a single value, v_s , based on the RMS value of instantaneous sliding speed $v_s(x)$ as a function of the sliding speeds of the approach (A), recess (E) and pitch point (C) ($v_s^{(A)}, v_s^{(C)}, v_s^{(E)}$):

$$v_s = \frac{1}{2} (RMS(v_s^{(A)} - v_s^{(C)}) + RMS(v_s^{(E)} - v_s^{(C)})) \quad (6)$$

At pitch point $v_s^{(C)} = 0$

$$RMS(v_s^{(A)} - 0) = \frac{1}{\sqrt{3}} v_s^{(A)} \quad (7)$$

$$RMS(v_s^{(E)} - 0) = \frac{1}{\sqrt{3}} v_s^{(E)} \quad (8)$$

$$v_s = \frac{1}{2} \left(\frac{1}{\sqrt{3}} v_s^{(A)} + \frac{1}{\sqrt{3}} v_s^{(E)} \right) = \frac{1}{2\sqrt{3}} (v_s^{(A)} + v_s^{(E)}) \quad (9)$$

Secondly, the local normal load applied varies depending on whether one or two pairs of teeth are in contact (Figure 2). The single applied normal load is assumed to be constant and equal to:

$$F_{n\varepsilon} = \frac{F_n}{\varepsilon_\alpha} \quad (10)$$

where F_n is the normal load for a pair of teeth in contact and ε_α , the transverse contact ratio.

Thirdly, a simple analytical formula to calculate the average value of other contact parameters such as tooth profile radii of curvature, pressure and film thickness is rather complicated because of their non-linear evolution. In this specific case, a single value is taken at a representative point on the contact path. The pitch point (C) is of interest because it is generally located towards the middle of the contact path. The contact parameters are therefore calculated at this point (appendix 2). Finally, the mean friction coefficient deduced from Diab model [16], [17] is expressed as:

$$\mu_{mz} = \frac{1}{p} \left[\frac{A_a}{A_0} \tau_a + \left(1 - \frac{A_a}{A_0} \right) \tau_f \right] \quad (11)$$

with the following parameters defined in appendix 2: p , single mean pressure, $\frac{A_a}{A_0}$, the single ratio between actual contact area and apparent one, τ_a , the single shear stress from asperities and τ_f , the single shear stress from the fluid.

4. Method validation

The method of predicting the mean friction coefficient μ_{mz} is evaluated from calculations applied to carburised gears of type C14 from FZG machine (table 1). Six operating conditions (three wheel speeds: 400, 1500, 3000 rpm and two static torques on the pinion shaft: load stage 6, moderate load level and load stage 12, higher one) were calculated with ISO VG 320 mineral oil at 80°C. The tribological parameters associated to the lubricant, the steel and the surface finish of contacting parts are those obtained from the tests on a twin-disk machine in [17]. The surface finish of the gears is similar to that of discs in [17].

In what follows, for each study, a comparison is given between the proposed method (equation 11 and parameters in appendix 2) and the average instantaneous friction coefficients calculated from the Diab model (equation 4), μ_{mz} -D, and Schlenk formulae (appendix 1, equation 13 and two values of X_L : $X_{L-S}=1$ and $X_{L-F}=0.846$), μ_{mz} -S.

Figure 3 shows these comparisons for the reference cases versus the relative film thickness (central film thickness at the pitch point over RMS roughness).

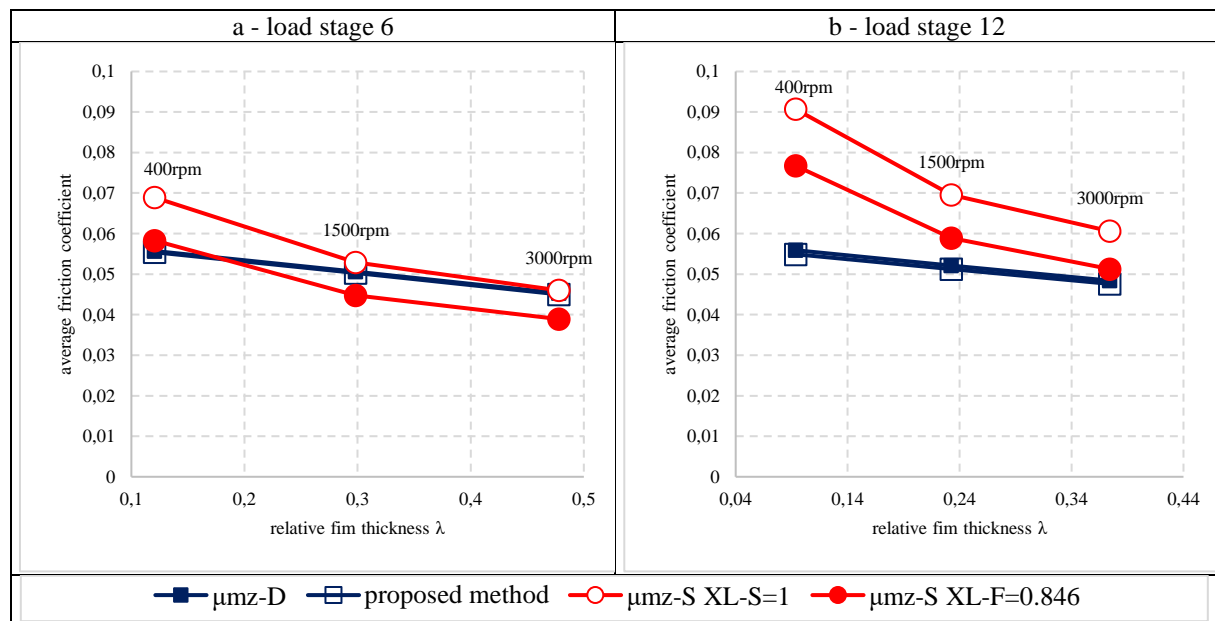


Figure 3: mean friction coefficient from proposed method, Diab model and Schlenk formulae for ISO VG 320 mineral oil-carburised steels and surface finishing similar to [17]

It appears that there is good agreement, whatever the relative film thickness, λ , and the load stages, between the average friction coefficients obtained by the proposed method and those obtained by instantaneous calculations using Diab model. For load stage 6 (figure 3-a), good agreement is also observed between the proposed method and Schlenk formula for $X_{L-S} = 1$ as λ increases (from 23% to 3% relative difference). Conversely, with $X_{L-S} = 0.846$, the Schlenk formula underestimates the average friction coefficient when λ increases compared with the proposed method (from 5% to 14% relative difference). Furthermore, unlike the proposed model, Schlenk's model does not take into account thermal effects in the contact. However, at this load level, the thermal effects in the contact area are low and therefore a good agreement between the two models is observed. For load stage 12 (figure 3-b), all the average friction coefficients estimated with Schlenk are higher than those calculated with the proposed method. (more than 25% minimum with $X_{L-S} = 1$ relative difference and 8% with $X_{L-F}=0.846$).

Surface roughness has a strong influence on the friction coefficient. For example, the friction coefficient decreases with increasing surface finish quality, particularly for low relative film thicknesses. The impact of surface roughness on the proposed method is evaluated for different surface roughness qualities and compared again with Diab and Schlenk (figure 4). Once more, the proposed method shows very good agreement with the average instantaneous friction coefficients calculated from the Diab model

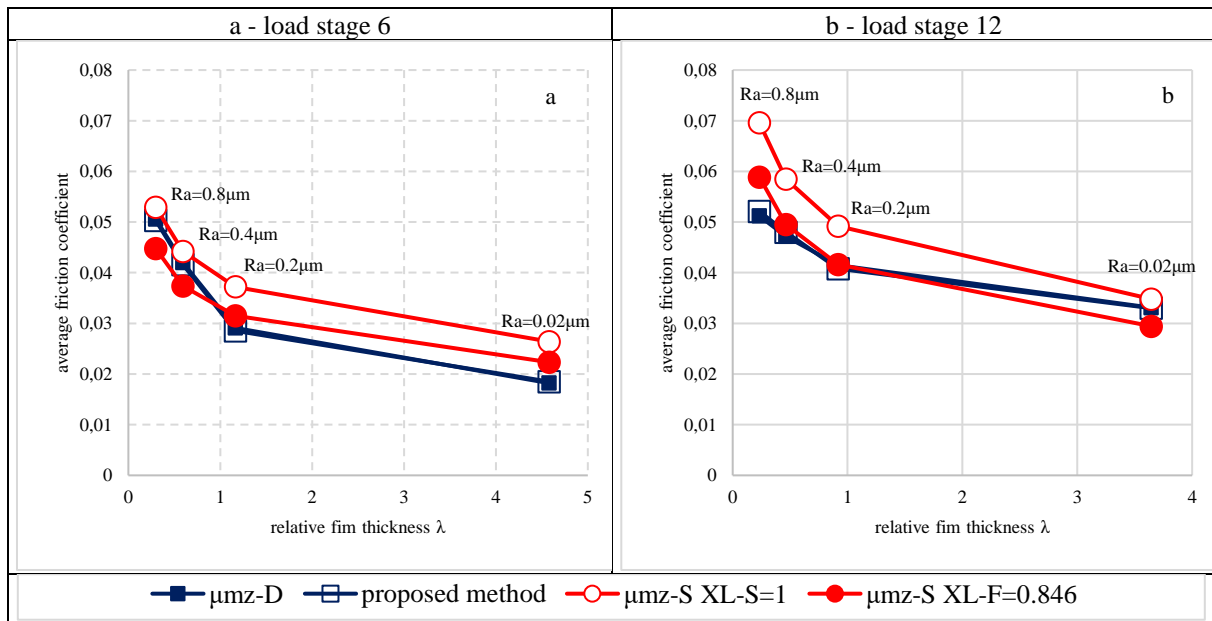


Figure 4: average friction coefficients from proposed method, Diab model and Schlenk formulae for ISO VG 320 mineral oil-carburised steels similar to [11], 1500 rpm and 4 different surface finishes

For load stage 6 and a relative film thickness of less than 0.59 (boundary lubrication), Schlenk formula with $X_{L-S}=1$ gives good agreement with proposed method (less than 5% relative difference). Conversely, with $X_{L-S}=0.846$, Schlenk underestimates the average friction coefficient compared with the proposed method (more than 12%). On the other hand, for mixed lubrication and full film lubrication, Schlenk overestimates the average friction coefficient (more than 27% with $X_{L-S}=1$ and 10% with $X_{L-F}=0.846$). For load stage 12, Schlenk with $X_{L-S}=1$ overestimates the average friction coefficient when the relative film thickness is less than 1 (more than 27%) but good agreement is noted with $X_{L-F}=0.846$. (around of 12%).

Oil viscosity plays a crucial role in determining the oil film thickness, which is an important parameter affecting the friction coefficient. As viscosity increases, film thickness tends to increase, reducing metal-to-metal contact and friction. Conversely, lower viscosity results in a thinner lubricant film, which can increase metal-to-metal contact and friction. To study the impact of viscosity on friction, four temperatures (40, 60, 90 and 120°C) were chosen because viscosity decreases with temperature. Figure 5 shows the viscosity impact and that the proposed method still correctly estimates the average instantaneous friction coefficients.

For load stage 6, Schlenk formula with $X_{L-S}=1$ overestimates the average friction coefficient at 40°C and 60°C, while good agreement is observed at 90°C and 120°C. On the other hand, with $X_{L-S}=0.846$, good agreement is observed at 40°C and 60°C, and an underestimation is observed at 90°C and 120°C. For load stage 12, whatever X_{L-S} , Schlenk significantly overestimates the average friction coefficient.

From a general point of view, most of the differences observed can be explained as follows. The Diab model is based on mathematical and physical equations that take into account all the parameters related to lubrication phenomena, such as the lubrication regime (boundary, mixed, full film lubrication), the oil properties, the properties of the solids, etc. The Schlenk model is an empirical formula developed on the basis of experimental data from an FZG back-to-back machine. It does not take into account the thermal effects of high loads and sliding speeds. Thermal effects lead to a localised increase in temperature (in the contact zone) which, in turn, leads to a reduction in the viscosity of the lubricant and, consequently, a reduction in the coefficient of friction.

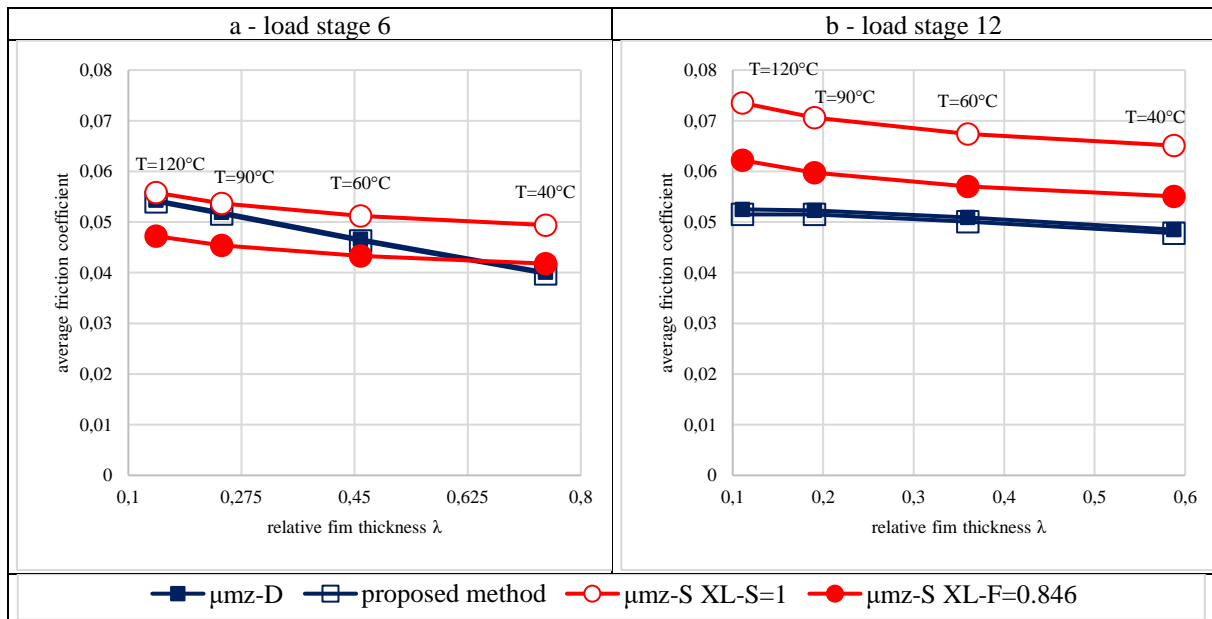


Figure 5: average friction coefficients from proposed method, Diab model and Schlenk formulae for ISO VG 320 mineral oil-carburised steels similar to [11], 1500 rpm and 4 different temperatures

5. Conclusion

A new method for predicting the average friction coefficient needed to estimate the tooth friction power loss, which is used in the ISO standard, is proposed for cylindrical gears. The influence of different operating conditions (rotational speed, load, surface condition and lubricant temperature) has been quantified. For all the cases treated, good agreement was observed between the new method and the average coefficient of friction calculated using the Diab model [16], [17]. On the other hand, the Schlenk formula (used in ISO calculations) [18], [21] with two different values of X_L found in the literature sometimes overestimates or underestimates the average coefficient of friction and sometimes shows good agreement over the different operating conditions analysed.

Future work will involve optimising the friction coefficient related to tooth power losses by including lubrication and thermal analyses. A reduction in tooth power loss needs to be assessed at system level to measure its real impact, and the quality of lubrication needs to be evaluated both for the separation of contacting parts and for cooling.

Declaration of Conflicting Interests

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Nomenclature

Roman letters

A_0 :	Apparent or hertzian contact area, m^2
A_a :	Actual contact area, m^2
A_f :	fluid contact area, m^2
b :	Effective tooth width, m
F_n :	Force normal to tooth profile, N
$F_{n\varepsilon}$:	Single applied normal load, N
$F_{n\varepsilon}^{(f)}$:	Part of single applied normal load supported by supported by fluid, N
$F_{n\varepsilon}^{(a)}$:	Part of single applied normal load supported by surface asperities, N
Hv :	Tooth loss factor
p_{bn} :	Normal base pitch, m
p :	Single mean pressure, Pa
P :	Local gear friction loss, W
P_M :	Average value of the total local gear friction loss, W
P_{in} :	Transmitted power, W
RMS :	Root mean square
Ra :	Arithmetic average roughness, m
Rq :	RMS roughness, m
Rdq :	Root mean square slope., -
v_1, v_2 :	Gear profile velocity ($= \omega_{1,2} \rho_{1,2}$) (pinion, wheel), m/s
v :	Average speed ($(v_1 + v_2)/2$), m/s
v_s :	Sliding speed ($= v_1 - v_2$), m/s
$v_s^{(A)}$:	Sliding speed at starting meshing point, m/s
$v_s^{(E)}$:	Sliding speed at end meshing point, m/s
$v_s^{(C)}$:	Sliding speed at pitch point, m/s
x :	Position along the path of contact, m
X_L :	Lubricant factor

Greek letters

ε_α :	Transverse contact ratio
η_{oil} :	Absolute oil dynamic viscosity at contact inlet Temperature, Pa.s
η_z :	Gear efficiency
λ :	Reduced film thickness
μ :	local coefficient of friction
μ_{mz} :	Average friction coefficient of gear mesh
$\rho_{red,C}$:	Equivalent radius of curvature at pitch point, m
ΣC :	Sum rolling speed ($v_1 + v_2$) at pitch point, m/s
ω :	Rotational speed, rad/s
τ_a :	Shear stress due to asperity contact, Pa
τ_f :	Shear stress from the fluid, Pa

Indices

A :	Beginning of contactor approach
C :	Pitch point of contact
D :	End of single-pair mesh
E :	End of gear mesh or recess
D :	Diab
F :	Fernandes
S :	Schlenk

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Appendix 1: Gear friction loss in the ISO standard 14179-2 [18]

The efficiency, η_z , is defined as the ratio between the useful output power and the transmitted one, P_{in} . Equation 12 considers a gear generating only tooth friction loss P_M :

$$\eta_z = 1 - \frac{P_M}{P_{in}} = 1 - \mu_{mz} H v \quad (12)$$

with: μ_{mz} the mean friction coefficient between the gear teeth along contact path and Hv , given in the ISO Technical report 14179-2 [11], the tooth loss factor which describes gear kinematics, load distribution, and depends only on gear geometry.

The technical report also proposes a formula for calculating the mean friction coefficient (μ_{mz}), based on experimental data from an FZG back-to-back gear test rig [6]:

$$\mu_{mz} = 0.048 \left(\frac{F_n/b}{v \cdot \Sigma c \cdot \rho_{red,C}} \right)^{0.2} \cdot \eta_{oil}^{-0.05} \left(\frac{Ra_1 + Ra_2}{2} \right)^{0.25} \cdot X_L \quad (13)$$

The technical report proposes different values for the lubricant factor X_L as a function of the base oil [18]. Fernandes *et al.* [13] have proposed new values for X_L (Table 4). When considering different gear oil formulations, the lubricant factor X_L must be adjusted to take into account the influence of different base oils and/or additive packages.

Table 4: lubricant factor X_L

	X_{L-S} [18]	X_{L-F} [18].
Mineral oils	1	0.846
Ester	0.8	0.63
Polyalphaolefins	0.8	0.666
Polyglycols	$0.75 \cdot \left(\frac{b}{v \cdot \Sigma c} \right)^{0.2}$	0.585
Phosphoric esters	1.3	-
Traction fluids	1.5	-

Appendix 2: new method based on Diab et al. model [16], [17]

The Diab model for calculating the instantaneous tooth friction coefficient is described in [16], [17]. It is based on a tribological description of tooth contact and induced friction with parameters obtained on twin-disk machines. The new proposed method consists of finding an average value of this friction coefficient. The contact parameters described in the model along the contact path must be averaged.

Thus, the single applied normal load (equation 10) is broken down into two parts. One part is supported by surface asperities, called the asperity part with superscript ‘a’, and the other one by oil film, called the fluid part with superscript ‘f’:

$$F_{n\varepsilon} = F_{n\varepsilon}^{(a)} + F_{n\varepsilon}^{(f)} \quad (14)$$

In what follows, all the contact parameters are considered to be constant along the contact path and refer to contact at the pitch point.

A_0 , the apparent contact area is equal to the apparent contact area calculated at pitch contact and is divided into two parts, one for the actual contact area (A_a) and the other for the complementary fluid area (A_f):

$$A_0 = A_a + A_f \quad (15)$$

The mean friction coefficient is calculated as follows:

$$\mu_{mz} = \frac{1}{p} \left[\frac{A_a}{A_0} \tau_a + \left(1 - \frac{A_a}{A_0}\right) \tau_f \right] \quad (16)$$

With τ_f and τ_a calculated using [16], [17] and contact parameters at the pitch point C.

Where:

A_0 : Apparent contact area [m²]

A_a : Real contact area [m²]

$$\frac{A_a}{A_0} = 0.25 \left(1 - \operatorname{erf}\left(\frac{h_c}{\sqrt{2m_0}}\right)\right) \quad (17)$$

p : Mean contact pressure ($=F_{n\epsilon}/A_0$), Pa

m_0 : Zeroth spectral moment of roughness ($=R_q^2$), [m²]

h_c : Fully flooded isothermal central film thickness, [m]

1. asperity Contribution

The shear stress on asperities is given by:

$$\tau_a = P_a \mu_a \left(1 - e^{-0.5.a. |SSR| \sqrt{\frac{m_2}{m_0}}}\right) \quad (18)$$

Where:

μ_a : Friction coefficient on asperities that depends on surface and lubricant physicochemical properties

a : Semi-minor width of Hertzian contact, [m]

m_2 : Second spectral moment of roughness ($=R_{dq}^2$), [rad²]

SSR : Slide-to-roll Ratio ($=\frac{1}{\sqrt{3}} \left(\frac{v_s^{(A)}}{v_s^{(A)}} + \frac{v_s^{(E)}}{v_s^{(E)}}\right)$)

P_a : Pressure supported by asperities, [Pa]

$$P_a = 0.1E' \sqrt{\frac{m_2}{\pi}} \quad (19)$$

1. Fluid contribution

The shearing stress due to fluid friction is given by:

$$\tau_f = \tau_E \cdot \sinh\left(\frac{\tau_N}{\tau_E}\right) \quad (20)$$

Where

τ_N : Newtonian shear stress, [Pa]

$$\tau_N = \eta_{oil} \frac{v_s}{h_c} \quad (21)$$

v_s : Sliding speed as defined in equation 9 [m/s]

η_{oil} : Absolute oil dynamic viscosity, [Pa.s]

$$\tau_E^{\square} = C_0 \cdot e^{(C_p \cdot p_f^{\square})} e^{C_T \cdot (\frac{1}{T_{40}^{\square}} - \frac{1}{T_{40}^{\square}})}$$
 (22)

C_0 : Eyring shear stress at reference temperature and pressure, [Pa]

C_p : Eyring shear stress parameter for pressure, [1/Pa]

C_T : Eyring shear stress parameter for temperature, [K]

p_f^{\square} : Pressure supported by fluid, [Pa]