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Reducing Intuitive-Physics Prediction Error through Playing

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Abstract. We present a mobile robot that autonomously generates behaviors to calibrate its intuitive-physics engine, also known as the "Game Engine in the Head" (GEITH). Most POMDP and Active Inference learning techniques operate in a closed world in which the set of states is defined a priori. However, implementing an "innate" GEITH and a set of interactive behaviors allowed us to avoid these limitations and design a mechanism for information search and learning in an open world. The results show that over a few tens of interaction cycles, the robot's prediction errors decrease, which shows an improvement in the GEITH calibration. Moreover, the robot generates behaviors that human observers describe as playful.

Keywords: Active inference \cdot developmental learning \cdot enaction \cdot intrinsic motivation \cdot robotics \cdot core knowledge.

1 Introduction

In their book "The pragmatic Turn", Engel, Friston, and Kragic [2] advocate a shift from a representation-centered perspective to an "action-oriented" perspective on cognition. Aligned with this shift, we present a robotics implementation to study the intricacy between action and perception. Closely related to pragmatic philosophy, Whitehead's process philosophy proposes useful concepts to describe perception from entities having unconscious experience of connection to the world, which he calls enduring objects [23]. He distinguishes between two modes of perception: the mode of presentational immediacy and the mode of causal efficiency [17]. We apply these concepts to robotics: sensory input is provided to the robot in the mode of presentational immediacy, and the robot learns spatio-sequential patterns of actions and outcomes that account for the causal-efficiency mode of perception.

Karl Friston and his research group have proposed Active Inference as a method to interactively learn the causes of sensory signals by minimizing free

energy, or equivalently, prediction error [3]. Active inference has been used in the framework of Partially Observable Markov Decision Processes (POMDPs) to allow artificial agents to learn a causal model of the environment that they can only partially observe [18]. The causal model uses the distribution of the probability of each possible state of the world. In essence, at each instant, the agent estimates which states are the most or least likely to be the actual state of the world. The expected information gained to enhance this estimate is involved when selecting the next action. Active inference has been used in robotics [11] but generally under closed-world settings in the sense that the set of possible states is known a priori—a requirement for most of the mathematical apparatus of active inference.

When the robot is thrown into an open world, the problem of learning the cause of sensory input remains open. The POMDP and active inference literature suggest that the robot needs prior assumptions about the world to cope with complexity [7]. The present study examines how the "Game Engine In The head" (GEITH) can work as a suitable prior assumption that an autonomous robot could use to maintain a causal model of perception in the open world and reduce prediction errors.

Joshua Tenenbaum and his research group have proposed GEITH [21] as the capacity of cognitive beings to simulate the basic dynamics of physics and interactions. In the brain, the GEITH rests on structures that are partially predefined by genes and then completed through ontogenetic development. Similarly, it is possible to endow artificial agents and robots with a predefined software game engine and expect them to refine the parameters of their game engine and modify their predictions through ongoing interaction. The refinement of the game engine is assessed through the prediction error of sensory signals. The decrease in prediction errors shows an improvement of the game engine.

Compared with general Bayesian models classically used in active inference, GEITH adds the assumption that all the sensorimotor experiences can be localized in the 3D Euclidean space, at least approximately. It implements predefined linear-algebra functions to compute spatial transformations (rotations and translations) between frames of reference.

2 Our hypothesis

We comply with active inference theory in several regards. Firstly, we do not assume that sensory signals are representational of the state of the world. The world is hidden to the agent so that a given state may return contrary sensory signals when acted on differently by the agent. This implies a "conceptual inversion" of the interaction cycle in which action comes first and the sensory signal comes second as an outcome of action. Secondly, we do not provide the agent with presupposed ontological knowledge about entities in the world. The agent must infer the presence of causes in the world through patterns of interactive experience. Thirdly, no extrinsic goal is encoded in the agent in the form of goal states that the agent should search based on reward or other criteria. However,

we may associate some *prior preference* with interactions. In short, the agent has no *rewarding world states* but has *rewarding interactions*. For a deeper examination of these principles in relation to the active inference literature, we refer the reader to [7].

We also adopt prediction error as a measure of the quality of the agent's world model. However, we are not using the gradient descent of the prediction error as a motivational principle to drive the learning process. As we develop, our agent is not always driven by a value optimization process; it may also enact behaviors that we call disinterested. Disinterested behaviors can consist of aimless innate tendencies, or individual habits taken through the agent's lifetime. In this implementation, the reduction of prediction errors is not a means of improving the world model, but a consequence of its improvement.

We are using a cognitive architecture designed previously based on sensorimotor and enactive principles [7]. The present article reports the integration of the new GEITH module within this cognitive architecture, as illustrated in Figure 1. The GEITH supports the simulation of behaviors before their selection by the cognitive architecture and their enaction by the robot. At the beginning of each interaction cycle, the simulation computes the *predicted outcome*. At the end of the interaction cycle, the predicted outcome is compared with the *actual* outcome to calculate the prediction error. We investigate the core elements of the GEITH that are needed for the agent to reduce prediction error.

We draw inspiration from studies on *core knowledge* in the brains of animals and human infants. For example, Elizabeth Spelke and her colleagues argued for the existence of two core geometry systems that "evolved before the emergence of the human species": "The *core navigation system* captures absolute distance and sense [..] but not relative length or angle; the *core form analysis system* does the reverse" [19, p. 2789]. We start by implementing the minimal requirements she deems necessary for both systems, namely the ability to handle points in spatial memory, the foundational elements of Euclidean geometry.

Our cognitive architecture encodes behaviors as composite interactions which are sequences of primitive interactions. A primitive interaction is a control loop that involves actuator commands, expected sensory feedback, spatio-temporal attributes, termination conditions, termination outcome, and prior preference. Examples are given in Section 3. GEITH may consider some of the outcomes as the result of interaction with "something" in the environment. In this case, the GEITH instantiates a data structure called a phenomenon⁵ and localizes this phenomenon at the position of the interaction in spatio-sequential memory. Next, GEITH simulates subsequent interactions with phenomena to predict future outcomes.

We seeded the cognitive architecture with "innate" composite interactions that cause the robot to explore the environment and interact with points encountered on the floor (Fig. 1, top center). In other studies, we implemented the

⁵ Common-sense usage of the term *phenomenon*: "something" that a cognitive being perceives in the environment. Technically: "any useful grouping of a subset of spatiotemporal patterns experienced by an agent in an environment" [20, p. 8].

Fig. 1. The game engine within the cognitive architecture (derived from [7], Fig. 6). Bottom: the history of interactions enacted over time. Rightward triangles: forward. Leftward triangles: backward. Squares: swipe. Circles: turn. shade: outcome white or black. Center: the GEITH. Red hexagon: the focus of attention localized at the position of the phenomenon. Magenta hexagon: the prompt is the localization of the next selected interaction's destination: swipe to the right. Left: the types of phenomena inferred through interactive experience. Solid objects, walls, and other robots can be detected by the echo-localization sensor, but are not present in this experiment. Top center: predefined composite interactions and GEITH parameters. Top right: three-dimensional emotional state based on dopamine (DA), serotonin (5-HT), and nor-adrenaline (NA). Right: the cognitive architecture selects the next behavior based on the emotional state and the expected outcome predicted by the GEITH.

learning of new composite interactions [8], but here we only examine the refinement of the GEITH parameters allowing the tuning of primitive interactions to reduce the prediction error.

The cognitive architecture uses variables that represent the robot's emotional state to select composite interactions to try to enact. We use Hugo Lövheim's "cube of emotions" [12] as a basic emotional model based on three neurotransmitters: dopamine (DA), serotonin (5-HT), and nor-adrenaline (NA) (Fig. 1, top right). This model associates dopamine with pleasure and reward seeking behavior, serotonin with well-being and playful behavior, and nor-adrenaline with responses to arousal and stress. It has been used successfully for simple emotional robotics. Our robot visually indicates its predominant neurotransmitter level using an intuitive color code studied by Max Talanov and his team:

green for dopamine, white for serotonin, red for noradrenaline, and blue when all three neurotransmitter levels are low [1].

The GEITH implements two levels of spatio-sequential working memory: egocentric and allo-phenomenon-centric (Fig. 1, center). The interactions and displacements are received in the egocentric reference frame based on the position of sensors and translation speed given as GEITH parameters, and the yaw measured by the Inertial Measurement Unit (IMU) which plays a similar role as the vestibular system (Fig. 2, top right). When the robot encounters a new phenomenon, the GEITH instantiates a new allocentric reference frame centered on this phenomenon to track the displacement of the robot relative to this phenomenon (Fig. 2, bottom right). Comparable mechanisms of coordinate conversion have recently been found in the brain [22] and implemented in other software cognitive architectures [15]. Phenomenon-centric frames relate to Jeff Hawkins' thousand brain hypothesis [9], according to which the brain records thousands of small spatio-temporal models to memorize interactions with different kinds of object.

Once the robot has selected an object in the environment, its serotonin level increases, which triggers behaviors of interaction with this object to calibrate its GEITH parameters. Phenomenon-centric memory is discretized into a hexagonal grid inspired by grid cells in the entorhinal cortex [13]. The cognitive architecture uses this grid as a small finite discrete model in which to search for information and optimize it.

3 Experiment

We designed a mobile robotic platform called Petitcat⁶ based on the *Osoyoo robot car* [14]. The experiment reported here uses only two sensors. The *floor luminosity sensor* is a bar of 5 infrared-reflective sensors directed to the floor. From this bar of sensors, we retrieve 4 possible signals: none, left, front, or right signaling the absence or relative position of a black tape present beneath them. The IMU measures the yaw during the enaction of interactions. Note that Petitcat cannot see the black tape from a distance. He has no camera, lidar, or odometer. What looks like eyes on his head is an ultrasonic echo-localization sensor not exploited in this experiment. The emotion indicator is an RGB LED (Fig. 2).

The C++ software running on the robot's Arduino board controls the enaction of primitive interactions. A personal computer implements the GEITH and the cognitive architecture that remote controls the robot via Wi-Fi. The cognitive architecture selects the primitive interaction to try to enact and sends it to the robot. The robot tries to enact it and sends the outcome back to the PC. The code is open source and shared online [6].

For this experiment, we defined four possible commands: forward, backward, swipe, and turn. Forward and backward are longitudinal translations. Their

⁶ Sections 3 and 4 personalize the robot by name and pronoun to enhance readability. We do not claim that he has a psychology or gender.

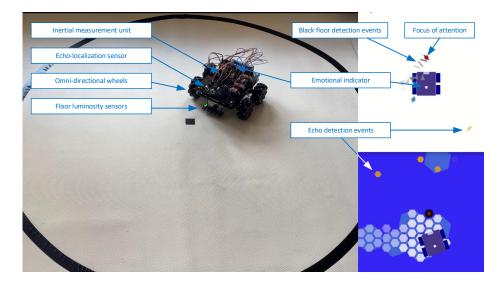


Fig. 2. Screenshot of a video example run [4]. Left: Petitcat playing with a point made of a piece of black tape on the floor. Top right: Petitcat's egocentric memory. Black segments: black tape detection events. Bottom right: phenomenon-centric memory. Black hexagon: the point phenomenon used as origin of the allocentric reference frame. Yellow hexagons: echo measured with the sonar. Red hexagon: focus of attention.

spatio-temporal attribute is the target duration (float). Swipe is a lateral translation. Its spatio-temporal attributes are the direction (left or right) and target duration (float). Turn consists in turning in place. Its spatio-temporal attribute is the target yaw (float), negative when counter-trigonometric.

The control loop monitors the elapsed time, yaw, and floor luminosity. The termination conditions are reaching the target duration or yaw, or detecting the black tape, making two possible outcomes: white or black. This gives eight primitive interactions identified by their tuple (command, outcome): 4 commands × 2 outcomes. All interactions are given a zero prior preference except (forward, white) which has a positive one. Additionally, the robot returns the measured spatio-temporal attributes: measured duration (float), measured yaw (float), and black tape detection (none, left, front, right).

When the black tape is detected, the movement is interrupted and a "reflex" movement is performed to withdraw from the tape by a few centimeters. When detection is on the side, this withdrawal includes a rotation to the opposite side, which tends to bring the robot back into a position perpendicular to the tape. This behavior was implemented to prevent the robot from falling off a table or exiting the arena.

We seeded the cognitive architecture with the four composite interactions below, which constitute "innate" ways for Petitcat to interact with points. The GEITH tries to simulate them, computes their spatio-temporal attributes according to the position of the phenomenon in memory, and proposes those that are feasible in the current context.

```
1. \langle\langle \text{forward, black}\rangle\rangle
2. \langle\langle \text{turn, white}\rangle, \langle \text{forward, black}\rangle\rangle
3. \langle\langle \text{swipe, white}\rangle, \langle \text{forward, black}\rangle\rangle
4. \langle\langle \text{swipe, white}\rangle, \langle \text{turn, white}\rangle, \langle \text{forward, black}\rangle\rangle
```

Neurotransmitter levels can vary from 0 to 100 and are initialized at 50. DA prevails in case of equality. The prevalence of DA makes Petitcat initially select the \(\forward, white \) interaction because it has a positive prior preference. When he detects a point (by surprise), 5-HT increases to its max. The prevalence of 5-HT and the presence of a point phenomenon in memory trigger the selection of innate interactions with the point. If the prediction errors do not decrease (that is, the prediction is not longer improving), 5-HT decreases. When 5-HT drops below or equal to DA, the \(\forward, white \) interaction is again selected, causing Petitcat to explore new destinations.

Prediction errors may concern both the outcome of primitive interactions and the spatio-temporal measures. Prediction errors on the outcome (black predicted but white occurred, or the reverse) mean that the selected primitive interaction failed and another interaction was actually enacted instead. Failing primitive interactions cause the composite interaction to which they belong to abort, and NA to rise to its max.

The GEITH uses a focus of attention point and a prompt point to compute the spatio-temporal attributes of interactions (Fig. 1, center). When Petitcat interacts with a point, the GEITH places the focus of attention at the place of the phenomenon. A failure to interact with the point means that the localization of the phenomenon in memory is erroneous. The high NA level that occurs in case of failure causes the GEITH to move the focus of attention to another cell in phenomenon-centric memory in search of the point. Cells compete to catch the focus with preference given to those closer to the last detected position of the phenomenon but having gone the longest period since last being visited. The interactions with the point continue afterward based on the focus in different cells. NA is reset to 50 if Petitcat finds the lost point; otherwise, it progressively decreases until it drops below 50 causing Petitcat to abandon the search.

In addition to the number of failed interactions, we also expect the prediction errors of the yaw and of the forward duration to decrease as the GEITH adjusts its parameters. Our GEITH has about 20 parameters, but this experiment only involves FLOOR_SENSOR_POSITION, FORWARD_SPEED, WITHDRAWAL_DISTANCE, and SIDE_WITHDRAWAL_YAW. Note that GEITH has no means to infer the absolute values of these parameters but can only adjust them in relation to each other. The GEITH cannot either predict that the point will be detected on the side of the floor luminosity sensor, which will cause a withdrawal with rotation. The cognitive architecture makes the robot aim straight at the point. The GEITH thus always predicts a straight withdrawal.

4 Results

Several videos of experiment runs are available online [5]. Here we analyze the representative run recorded in [4]. In this run, Petitcat encountered the point in Step 1 and interacted with it up to Step 60. In Step 17, it missed the point, but found it again in Step 20. This is shown in the *outcome code prediction error plot* in Fig. 3. The fact that Peticat did not miss the point after Step 20 shows an improvement of the GEITH parameters.

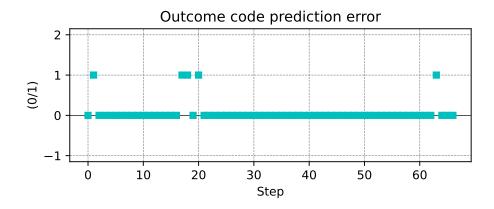


Fig. 3. Outcome prediction error plot: 0 if predicted outcome = actual outcome (successful interaction), 1 otherwise (failed interaction). Step 0: Petitcat moved forward. Step 1: he unexpectedly detected the point. Step 17: he expected to detect the point while translating forward but missed it. Step 18: he expected to not detect the point while turning but detected it. Step 20: he did not predict detecting the point but did. Step 63: As he moved away from the point, he did not expect to detect the arena border.

As explained above, the GEITH cannot predict when Peticat will detect the point on the side. This can cause large yaw prediction errors because the robot unexpectedly turned during withdrawal. Fig. 4 shows these prediction errors that do not improve over time.

The GEITH simulates turning while withdrawing based on the SIDE_WITH-DRAWAL_YAW parameter. To adjust this parameter, the GEITH must compute the *yaw residual error* that is left when knowing on which side the point was detected. Fig. 5 shows that the residual yaw error decreases as the robot adjusts the SIDE_WITHDRAWAL_YAW parameter.

The adjustment of FORWARD_SPEED and WITHDRAWAL_DISTANCE allows for a visible decrease of the forward-duration prediction error shown in Fig. 6.

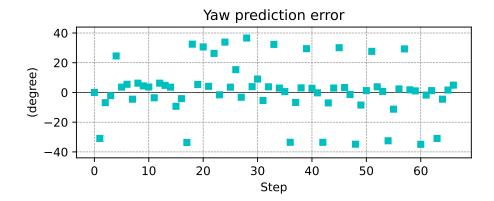


Fig. 4. Yaw prediction error plot. The prediction errors come from different causes which makes the interpretation of the plot difficult. Points above 20 or below -20 are large prediction errors occurring when Petitcat turned while withdrawing because the GEITH did not predict detecting the point on the side.

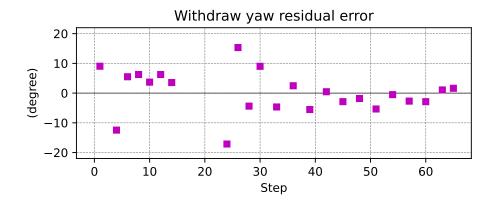


Fig. 5. Yaw residual error of interactions that have a black outcome. It shows a significant decrease as the robot interacts with the point.

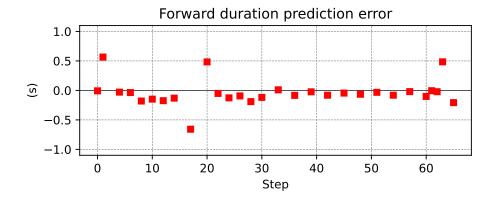


Fig. 6. Duration prediction error for forward interactions. Step 1 and 20: the forward translation was unexpectedly interrupted by the point detection. Step 17: the forward duration was longer than expected because the robot did not detect the point. Except for these events, the plot shows that the forward duration prediction error decreases as the robot is interacting with the point from Step 1 to 62. On Step 63: the robot moves away from the point and a forward duration prediction errors occur as he encounters a new object: the border of the arena.

5 Conclusion

We demonstrated a simple robot that managed to reduce intuitive-physics prediction errors in an open environment. We drew inspiration from theories positing core knowledge in the brain that have innate origins. Compared with our previous studies base on the same robot and earlier iterations of the cognitive architecture [7], the present study adds the model of emotion, the GEITH, and a set of innate behaviors. These elements are hard-coded in the robot, but what is not predefined is the set of world states and the ontology of objects in the world.

When the robot finds an object, it instantiates a small local model in the reference frame of this object. This finite discrete model lends itself to regular active inference techniques. We continue studying how to optimize the process of GEITH refinement in such local models using the active inference python library inferactively-pymdp [10]. This approach, however, remains dependent on the causal structure of the GEITH itself. How the robot could improve the causal structure of the GEITH or find exceptions remains an open question related to explainable AI [20].

This study illustrates Whitehead's two modes of perception. Mode 1, presentational immediacy (in the sense of "seeing a color"), corresponds to the robot's boolean sensory input we label white and black. Mode 2, causal efficiency (in the sense of "seeing an object"), corresponds to the robot maintaining a causal model of the point on the floor. We expect the next step to involve mode-2 perception of lines between points, which could open the way to learning the

compositionality of phenomena. We shall also examine how the robot can perceive the appearance, disappearance, and displacement of objects.

Beyond studying perception, we follow a method that advances through analogies between robots and natural organisms, aiming to enhance our understanding of the potential becoming of agency in robots. This method relates to the method of transduction proposed by another philosopher of process philosophy, Gilbert Simondon [16]. We try to design robots that mimic traits of natural organisms such as perception, surprise, emotions, and preferences. We are not claiming that the robot can actually experience these traits of agency, let alone have sentience. The robot, nonetheless, generates behaviors that human observers easily interpret as lifelike, which could find applications in companion robotics. The robot seems to enjoy exploring for the mere pleasure of movement as it lights up in green (DA prevails); it plays with the point as it does practicing its skills as it lights up in white (5-HT prevails); it seems anxious to search for the lost point as it lights up in red (NA prevails). This interpretation is also reinforced by seeing that the robot also turns its head in search of objects around the arena. In future experiments, we would like to study more precisely to what extent observers assign these subjective traits to the robot.

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