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## **► To cite this version:**

Matteo Porrino. The Garabit viaduct as the apogee of classical French railway overpass design, 1880–1884. 8th International Congress on Construction History, Institute for Preservation and Construction History, ETH, Jun 2024, Zurich, Switzerland. p. 659-666, <10.3218/4166-8>. <hal-04690997>

**HAL Id: hal-04690997**

**<https://hal.science/hal-04690997v1>**

Submitted on 31 Oct 2024

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# The Garabit viaduct as the apogee of classical French railway overpass design, 1880–1884

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**Abstract:** Designed on the basis of the Maria Pia bridge, the first railway viaduct to cross the Douro river near Porto, the Garabit viaduct, while exceeding the span of the Portuguese structure, was technically an improved version. After its completion, the viaduct therefore appears as a virtually perfect solution to a given problem. Its beauty, still recognized today, is the result of its overall proportions and the rationality of its forms, which are intuitively apparent. However, despite the exemplary nature of this achievement, the viaduct quickly ceased to be a model, and bridges modeled on the Garabit viaduct are few and far between. This observation leads us to question the actual place of this structure in the history of construction. We would like to place the Garabit viaduct more precisely in relation to other earlier and later railway bridges, and more broadly in the history of structural analysis. This is the aim of this study.

## Introduction

It is widely accepted that the Garabit viaduct represents the achievement of a new degree of rapprochement and unification between structural science and bridge design. Its overall design is based on a virtually unprecedented scheme, and its sizing depends on highly sophisticated calculation methods. Its execution required extreme precision in the manufacture of the various elements and in their assembly, while the adoption of advanced techniques for lifting and launching the spans enabled the project to be completed in a very short time.

It should also be noted that the Garabit viaduct received significant media coverage before and after its completion, in the form of articles, mainly in the French and English press, or through the publication of memoirs and explanatory notes. Eiffel himself communicated on the Garabit project on a number of occasions, in presentations and specialized conferences, the transcripts of which were published in the form of excerpts and minutes of several learned societies.

### 1. General information on the Garabit viaduct

The viaduct spans the Truyère valley at Garabit (Fig. 1). The structure is made entirely of rolled puddle iron, mostly riveted together, first in a factory, then on site. On-site work began in January 1880 and was completed in November 1884. The loading tests were carried out much later, between April 9 and 13, 1888. The viaduct went into service on May 27, 1888, with the opening of the Saint-Chély-d'Apcher – Saint-Flour section of the line. The line was fully opened on November 10, 1888. The completion of this project brought Eiffel international renown.

The steel structure of the bridge comprises an upper deck subdivided into three sections, five truncated-pyramidal piers—four supporting the deck on the Marvejols side, one supporting the deck on the Neussargues side—and a large parabolic arch on which the central section of the deck rests thanks to two bents.

The metal section of the structure is 448.3 m long. The central section of the deck, between the two arch bents, comprises three spans of 24.64 m; the deck on the Marvejols side, between the Marvejols abutment and the first bent, comprises five spans, three intermediate spans of 55.5 m and two end spans of 51.8 m; the deck on the Neussargues side, between the second bent and the Neussargues abutment, comprises two equal spans of 51.8 m each.

The viaduct has a single traffic lane. The height of the rails is 122.2 m above the river (835.5 m above sea level).

The main arch is made up of two main trusses located symmetrically in relation to the median plane of the structure, in two planes that are slightly inclined with respect to the median plane. As a result, their spacing varies between a maximum of 20 m at the arch springings and a minimum of 6.281 m at the crown, measured at the extrados. This arrangement protects the arch, which is primarily designed to support vertical loads, from the strongest winds.

The span of the arch, measures 165 meters. The rise from the lower surface and the depth of cross-section at crown are 51.858 m and 10 m respectively.

The two main trusses are crescent-shaped in elevation, with a parabolic axis. The height of the trusses decreases as one gets closer to the springings, where it becomes negligible. Indeed the trusses are designed as pinned at the springings (acting like a cylindrical bearing). Their height varies between this minimum and a maximum of 10 m measured at the crown.

The shape of these trusses gives the arch a very high degree of bending stiffness, particularly with regard to deformations resulting from non-symmetrical load distribution, and has the added advantage of eliminating any indeterminacy as to the exact point where the resultant of the internal forces passes through the springings. This point always coincides with the axis/contact point of the pinned connections.

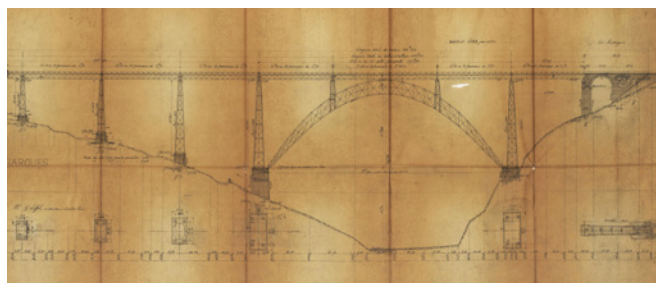


Figure 1. Garabit viaduct, elevation and plan, 1883 (152 AQ 151-1 031, Eiffel, Archives nationales du monde du travail, Roubaix).

The arch is therefore a variable-height structure, both in plan and in elevation. In plan view, it corresponds to the static diagram of a fixed-end beam, and in elevation, to that of a two-hinged arch.

## 2. Previous metal arch bridges and viaducts—Maria Pia bridge over the Douro river

In May 1875, Eiffel won an international competition launched by the Royal Portuguese Railway Company for the construction of a railroad viaduct in Porto. The winning design, drawn up by Théophile Seyrig, a graduate of the Ecole Centrale and Eiffel's partner since 1868 in the firm *Gustave Eiffel et Cie*, had been chosen over projects submitted by three other companies: Gouin, Fives-Lille and the English firm Head, Wrightson & Co (Fig. 2).

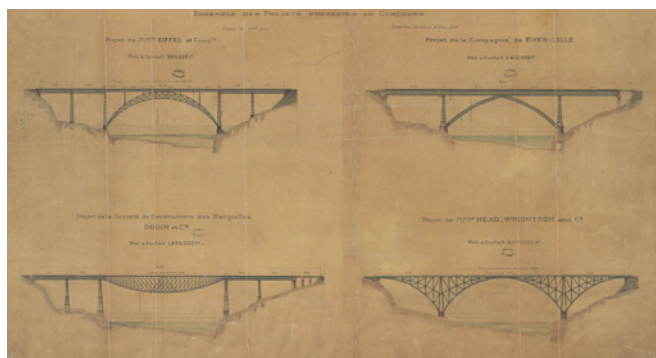


Figure 2. Bridge over the Douro, projects presented in the competition, 1875 (152 AQ 304 001, Eiffel, Archives nationales du monde du travail, Roubaix).

The new bridge over the Douro, characterized by its clear overall configuration, was ultimately chosen because of the cost savings its construction allowed. The structure featured a two-hinged crescent arch and a straight deck resting on lattice piers. The lump-sum price requested for its construction was 965,000 F, corresponding to a cost price three times lower than the most expensive of the competing projects.

Noting the discrepancy between the cost prices of the various projects, the sponsoring company asked for

a commission to be set up, in which the engineers Jean-Baptiste Krantz, Léon Molinos and Henri de Dion agreed to participate. The commission was tasked with examining in depth the project presented by the Eiffel company, with the aim of ensuring that the most economically attractive project was not considerably inferior to the others in terms of strength and stability. As part of this study, de Dion was also able to carry out the first application of a calculation method he had recently developed, the basis of which he had just unveiled to the *Société des ingénieurs civils*. The commission's final report was entirely favorable to the adoption of the project, for which de Dion soon became co-responsible for finalizing the sizing calculations.

Assembly of the arch, with its 160-meter opening, was carried out directly from the river banks, without scaffolding. All components were manufactured in the company's own workshops in Levallois-Perret before being transported to the site. The lifting and assembly phases, facilitated by the geometric accuracy of the parts, proceeded at a steady pace, and construction, which had started in January 1876 and lasted until the end of October 1877, was completed for the metal part in just six months of activity. (Work was interrupted by the harsh winter weather, heavy rains and floods, and resumed in March 1877. The loading tests were carried out between November 1 and 2 1877, and the bridge was inaugurated on November 4 1877), (Fig. 3).

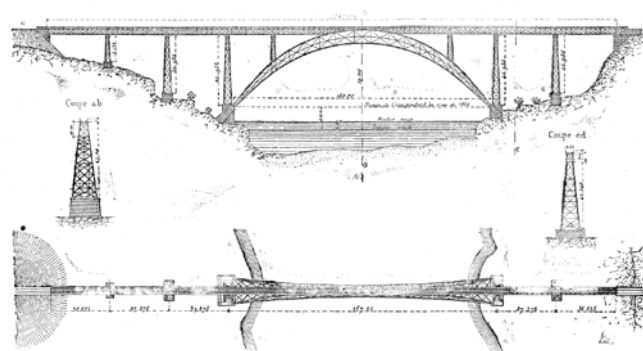


Figure 3. Bridge over the Douro, project completed ("Revue technique de l'Exposition universelle de 1878", *Le Constructeur*, n. 340, 24/2/1879, p. 147).

The origins of wooden or cast-iron bridges in the simple form of an arch supporting a straight deck can be traced back to the debates that took place in Europe as early as the 18<sup>th</sup> century concerning the behavior of spandrels in stone bridges. The interpretation of the behavior of this area of the structure, the heaviness of which actually consolidated stone bridges, remained highly uncertain for framework bridges. By the turn of the century, most cast-iron bridges were modeled on stone bridges, the volume occupied by the masonry being copied with an arrangement of frames and braces that could be adapted to a variety of construction principles. Examples from the first half of the 19<sup>th</sup> century offer a panorama of these technical choices.

In the second half of the century, the Saint Louis Bridge over the Mississippi, built by James B. Eads between 1867 and 1874, was one of the most remarkable examples of this constructive form. The structure, made of three steel arches 152 m, 158 m and 152 m wide respectively, featured a double deck, with the lower level open to rail traffic.

While the assembly of the Maria Pia bridge from the river banks was indeed based on a technique inaugurated by Eads for his bridge over the Mississippi—which had recently been put into service and was seen as an architectural success at the time—observation of the arches with spandrels of this particular example led Seyrig to depart radically from it for the general layout of his bridge over the Douro.

The switch to a large arch supporting a more or less rigid straight deck was closely linked to the impossibility of subjecting the spandrel zone to scientifically based analytical study, which led to the elimination of these zones in most long-span bridges, especially in Europe.

The words spoken by Henri de Dion in his speech in January 1877, on the occasion of his appointment as President of the *Société des ingénieurs civils*, are illuminating: “When we wanted to tackle these problems, we gave in to the tendency of subordinating the form of the construction to the formula, to make each part calculable independently of its neighbor; or if we didn’t give up the benefit of linking the different parts, we started by supposing the construction reduced to a geometric skeleton using hypotheses so that the calculated form is considerably different from the one we propose to execute, in order to be able to apply rigorous formulas. These formulas are so long and arduous to calculate, that an engineer can only rely on their results after verifications of all kinds, which he often doesn’t have the time to carry out” (de Dion 1877, 78).

Seyrig’s 1878 memoir on the Douro bridge sheds light on his approach to the project: “The arch, which had to support a straight deck at several points, which can be considered as the girder, could obviously not have spandrels. A glance at the design of the bridge is enough to convince us that the use of any kind of infill between the arch and the girder would have inevitably led to a considerable use of material, the aesthetic effect and working conditions of which would have been nothing short of satisfactory. However, it was necessary to foresee the effect of rolling loads, which is to give, at all points of the arch, bending moments varying within very wide limits, and whose effect is added to that of tangential compression. This condition leads to the use of a very high arch, offering a maximum moment of inertia, with the use of a quantity of metal little greater than that required by compression alone” (Seyrig 1878, 754).

Eiffel’s text illustrating the Maria Pia bridge project exhibited at the 1878 Universal Exhibition is also worth quoting here: “Indeed, the tympanum pieces by which the arches are usually stiffened would, in the present case, have had lengths of up to 35 or 40 meters. Their calculation, always very uncertain, would have become almost impracticable with these dimensions, if the effects of expansion and those resulting from the mobility of loads had had to be taken into account. – Out of caution, the extent of which was unknown, we would have had to use considerable quantities of metal for accessory parts such as spandrels, yet without achieving absolute safety. It therefore seemed preferable to do away with the spandrels altogether, by giving the arch sufficient stiffness to withstand by itself the deforming forces resulting from the uneven distribution of loads. To this end, the arch was given a very great height, set at 10 meters at the keystone” (Eiffel 1878, 4–5).

These thoughts on the design of complex structures, formulated by de Dion, Seyrig and Eiffel, highlight two characteristic aspects of an approach that could be described as French.

On the one hand, they show the intention of carrying out an analysis restricted to the elastic behavior that can provide an almost exact result for each part of the structure. As a result, structures are rationalized and simplified, and the hierarchy and interdependence of elements become clearer. Spandrels are eliminated, if they cannot be studied, and the structure is reduced to a beam-like structure resting on an arch.

In addition, they reveal a marked preference for schemes with little static indeterminacy, in other words, a tendency to view complex structures as the sum of statically determined, or isostatic, systems whenever possible. The deck is placed on the arch, but the two parts of the structure remain independent of each other, they are not interdependent or integral; and the deck itself is not considered as a long continuous beam but as a sequence of several short spans.

### 3. Improvements to the Garabit viaduct compared with the Maria Pia bridge

The new project included several improvements over the bridge already built over the Douro. In his presentations of the Garabit viaduct, Eiffel placed particular emphasis on four of them:

- New provisions used in metal piers
- Track position in the upper deck
- Parabolic shape of the arch / approach of intermediate supports towards the crown
- Cutting the horizontal decks at the arch supports

Although little mentioned by Eiffel in his publications on the viaduct, one of the most obvious changes between the two structures is the way in which the relationship between the deck and the arch is resolved in the middle of the span.

It should be noted that these two elements must be considered as independent of each other. This condition, inherent to the modeling on which the calculations were based, had to be respected in the final drawings and during the execution phase, in order to obtain the greatest possible concordance between physical reality and the model. In particular, the constructive arrangements had to allow for any relative horizontal movement between the deck and the lower structures, whether arches or piers.

In the bridge over the Douro river, deck and arch intertwine at the key, where only one arrangement of bars can be seen, that of the arch, behind which is the deck. As a result, it is no longer possible to distinguish between a simply supported deck and a perfectly independent arch in the structure as a whole. Here’s how Eiffel justifies this choice: “For the upper deck, given the minimum surface area to be exposed to the wind, the maximum rise to be given to the arch and the general connection of the construction, it was best to have the deck go through the arch; this arrangement was preferable to that which consisted in superimposing the deck on the arch and establishing contact by a single support point, as in the preliminary project. The first solution was therefore adopted” (Eiffel 1878, 6).

The morphology of the Garabit site demanded an increase in the height of the structure, in order to connect a route located at the level of the high plateaus bordering the river. The rail level had to reach 835.50 m, i.e. 122.20 m above the deepest part of the valley. In addition, to ensure that the project progressed quickly and economically, the chosen solution had to keep the dimensions of the arch close to those of the structures already built. In the final design for the Garabit viaduct, the deck passes over the arch. The two elements do not intersect. In the project elevation, one is visually separated from the other, so that the deck barely seems to touch the arch. This superimposition makes it easy to increase the height of the structure, not to mention the fact that here, as seen, the arch has a parabolic shape and is, therefore, higher—compared to a circular arch with the same opening. Both elements are clearly visible in the overall structure, and entirely independent of each other.

#### 4. Notes on the sizing of the Maria Pia bridge and the Garabit viaduct

The initial calculations for the bridge over the Douro were drawn up by Théophile Seyrig, who was also responsible for the overall design, and later verified by Henri de Dion. Henri de Dion finalized the sizing of the structure using the method he had developed in 1875. This method was indeed designed to calculate the resistance and deformations of a straight or curved part, while retaining in the differential equations all the variables resulting from the shape and the dimensions of the part. These equations were calculated by means of a graphical construction, in which a surface gave the angular movement, and a volume traced on this surface gave the translational movement of one point in relation to another, taking into account suitable heights for this volume (de Dion 1877, 797).

De Dion used the method to solve various case studies, including the 160-meter arch of the Douro bridge, which features a cross section of variable height, but rests on two hinged joints, thus simplifying the problem. Shortly afterwards, de Dion used the method to calculate the truss for the Galerie des Machines at the 1878 Universal Exhibition.

The calculations accompanying the initial version of the Garabit viaduct submitted to the Conseil Général des Ponts et Chaussées for approval were based on the method adopted for the Maria Pia bridge. However, when the final project was drawn up, the dimensions of the structure were entirely revised by Maurice Kœchlin, hired by Eiffel in the autumn of 1879 and entrusted with the key position of head of the design office (Émile Nougier, an engineer from the École des Mines, recruited in 1876, was in charge of technical studies and assemblies). Compared to the preliminary project, this new version included a reduction in the rise of the arch. All the construction drawings and resistance diagrams, as well as the technical report, were submitted to the State engineers on June 1<sup>st</sup>, 1881, and subsequently checked by engineer Léon Boyer. The masonry project was drawn up entirely by the State engineers.

The strength calculations for the metal parts were therefore drawn up by Kœchlin, probably seen by Eiffel and then checked by engineer Boyer.

The permissible working stresses for the sections were 6 kg/mm<sup>2</sup>, under the combined effect of loads and wind. The

overload was determined on the basis of the weight of the heaviest locomotives used by the *Compagnie du Midi*. The wind action used corresponded to 150 kg/m<sup>2</sup> while trains circulated, and 270 kg/m<sup>2</sup> when trains were not circulating, and circulation was suspended in the event of strong wind. In the calculations, the wind is represented as a pressure uniformly distributed over the surface of the wall facing the wind, but acting only on the surface of the bars of the second wall. It was also assumed that the surfaces exposed to the wind for each panel were concentrated on the vertical posts that frame them, and that the force borne by these posts was shared equally between the intrados and extrados. To these actions were added those affecting the supports and the deck. For the deck, as for the arch, it was assumed that the first wall was impacted over its entire surface, while the second was impacted only over the lattice surface. These evaluation criteria were in fact those used to calculate the viaducts for the Orléans line, designed by Nordling.

In view of the loads applied to the structure, the arch bars work at an average of 2 kg/mm<sup>2</sup> under their own weight, 2 kg/mm<sup>2</sup> under operating loads alone, and 2 kg/mm<sup>2</sup> under the effect of wind alone. Temperature variations have very little influence compared to the effects of these loads.

In 1889, the year his tower was inaugurated, Gustave Eiffel published a memoir in support of the final project for the Garabit viaduct. This was, in fact, the written version of his presentation to the *Société des ingénieurs civils*, held a year earlier, shortly after the viaduct had been commissioned. The document describes the various parts of the structural study, including the deck, pier and central arch calculations. This last part, the most extensive, also includes a study of the effects of temperature variations and wind on the arch (Eiffel 1889).

Several calculation methods were adopted for sizing the structure. The deck spans and lattice piers were calculated using methods that had already become commonplace among structural steelwork specialists.

The deck sections are calculated using the Clapeyron formulas, widely used in straight bridge design at the time. Indeed they allow the calculation of the bending moment of continuous beams on several supports. The upper deck is interrupted twice, above the two arch supports. Each of the three consecutive sections thus generated constitutes a multi-support continuous beam. The design of the piers is based on graphic statics. The external actions are determined by tracing funicular polygons and decomposing them using the method developed by Karl Culmann, described in his main work published in 1864–66.

The central arch was sized in relation to the loads applied, based on the Bresse formulas (Bresse 1859, 89–91).

#### 5. Wilhelm Ritter's fundamental contribution and the post-Garabit bridge and viaduct project

The first chapter of *Vers une architecture*, published in Paris in 1923, opens with a photo showing the full-length elevation of the Garabit viaduct. In fact, Le Corbusier devotes the early part of his text to the *aesthetics of engineering*, which he believes achieves harmony of form through economy and calculation (Fig. 4).





Figure 4. Le Corbusier, *Vers une architecture*, Paris, G. Crès, 1923. Frontispiece and page 1.

However, Eiffel's viaduct was now some forty years old, and while it was still a visually stunning achievement which could impress observers, it had technically become almost obsolete. A number of viaducts of similar size were built in other European countries—Switzerland, Italy, Greece, Hungary and Romania, for example—from 1875 until the turn of the century, but few of them were inspired by the viaduct's form.

Despite the exemplary nature of this project, the viaduct soon ceased to be considered a model, and bridges based on the Garabit viaduct are virtually non-existent, especially when compared with those that adopt certain construction schemes, mainly disseminated through the projects of a few practicing engineers, most of them former students of Culmann or Ritter.

Moreover, Ritter's interpretation of Culmann's work, and his popularization endeavors through the publication of at least three volumes devoted to the graphical analysis of structures—not to mention the various scientific articles on specific subjects published in the specialized press—offered engineers then working on steel structures and bridge projects calculation procedures and verification tools that were clearly illustrated and ready to use. These publications made a substantial contribution to the spread of graphic statics in common engineering practice at the end of the century (e.g. Ritter 1886, Kœchlin 1889).

Between 1873 and 1881, Ritter held a professorship at the Riga Polytechnic. After Culmann's death, the Swiss Education Council asked him to continue his work by taking up the newly created professorship of graphic statics and bridge construction at ETH Zurich.

At this point, Ritter began work on his *Anwendungen der graphischen Statik*, published in several volumes between 1888 and 1906. An extension of Culmann's work, these publications constitute Ritter's most significant contribution to the science of structures, as well as the most exhaustive and influential text on the graphical analysis of structures in German.

However, Ritter was also active in other fields of engineering. His contribution to the theory of suspension bridges, notably through the publication of *Statische*

*Berechnung der Versteifungsfachwerke der Hängebrücken*, published in 1883, is particularly noteworthy.

A few years earlier, during his years at the Riga Polytechnic, Ritter had already published another text that is fundamental for the advancement of knowledge on the behavior of arch and suspension bridges, which is particularly significant for the subject at hand. Published in 1877 in the *Zeitschrift für Bauwesen*, one of the two German professional journals on civil and building engineering at the time, under the title *Versteifungsfachwerke bei Bogen- und Hängebrücken*, this article opened the way to a new approach to bridge and viaduct design based on a novel analogy between arch and suspension bridges. Robert Maillart (1872–1940), one of Ritter's most brilliant and inventive students, was considerably influenced by this work, which led him, among other things, to define a new typology of concrete bridges with a three-hinged arch stiffened by the deck.

While the Douro bridge and Garabit viaduct were making headlines in the specialized press, engineering structures of a different type were being inaugurated in several European countries.

As far as large constructions are concerned, it is above all attempts to take second-order effects into account, i.e. abandoning the assumption of geometric linearization, and considering the ductility characteristics of the material which drove new approaches to design.

The article *Versteifungsfachwerke bei Bogen- und Hängebrücken* (Support structures for arch and suspension bridges) published by Ritter in 1877 is particularly significant in this respect. In it, Ritter presents his study of the behavior of a structure consisting of a deck and an arch linked together by vertical struts and also presents an analogy between arch bridges and suspension bridges, which can be interpreted as statically inverted solutions of each other (Fig. 5).

"In many suspension bridges, in order to reduce deleterious or inconvenient variations due to asymmetrical loads, the chain or wire rope was connected to a reinforcing trellis, for which the railing, necessary in any case, often offered a favorable opportunity. In the case of arch bridges, notably certain American wooden bridges, a lattice was sometimes connected to the arch, but it is not always clear whether the lattice should reinforce the arch or the arch reinforce the lattice.

A correct theory of these stiffening lattices has not been studied to date [...].

#### 1 - Prerequisites

We assume that the construction to be studied is formed by an arch AB (figure 1) and a straight beam (or truss) A'B', which is only connected to the arch by vertical posts. If this combined construction is subjected to any kind of variable load, not only the arch but also the beam will participate in taking up the load; both parts will therefore get deformed; the deflections they undergo depend on each other's deflections due to the presence of the vertical posts, so that we must first study this dependency.

We make two assumptions here, which simplify the study: 1. we neglect the elastic shortening or elongation of the vertical posts, and 2. we assume that the horizontal displacements of the arch points are equal to zero. [...]

As a result of these two conditions, it immediately follows that the fundamental condition is that two corresponding points in the beam and in the arch must always move in the vertical direction by the same distance.” (Ritter 1877, 189–190).

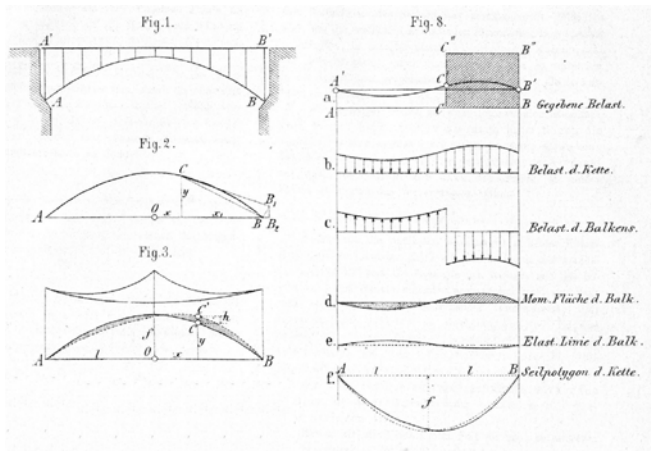


Figure 5. W. Ritter, “Versteifungsfachwerke bei Bogen- und Hängebrücken”, *Zeitschrift für Bauwesen*, XXVII-1877, fig. 1–3, 8.

This approach allows the beam to be seen as a stiffening element that can relieve the arch of bending forces. As a result, the arch is transformed into a simply compressed load-bearing element. In bridges, this generally translates into a greater deck height, hence greater bending stiffness and a smaller arch thickness, if this dimension is otherwise kept constant. (This is the same typology developed later for reinforced concrete bridges by Robert Maillart, also a Ritter student, and known as the *stiffened arch*). This is therefore the additional combination of a funicular arch and a rigid deck acting as a stiffening girder for the arch. It’s also a perfect inversion of the suspension bridge principle.

This specificity, combined with a marked preference for hyperstatic designs, became the dominant and defining characteristics of a number of bridges built between the early 1980s and the late 1990s, the vast majority designed by a handful of engineers trained at ETH Zurich.

It should be noted that this approach also tacitly corresponds to an application of the *lower bound theorem* of plasticity. Or better still, it prefigures the validity of such a proposition, before the theorem was formulated. In this structural typology, we can see the presence of two distinct and effective elementary systems, within a structure we might call a combined structure. The structure is of course statically indeterminate, and it was not possible with the theoretical bases known and the calculation procedures available at the time to determine the state of stress in any section for a given load.

In terms of advances in plasticity theory, it was only in the fifties of the 20<sup>th</sup> century that the fundamental concept of the *plastic hinge* appeared, and towards the end of the decade that the *lower* and *upper bound theorems* were formulated. These two theorems of plasticity theory provide a framework for the preliminary study of structures and their analysis.

The lower limit principle, in particular, states that if it is possible to find a distribution of moments in equilibrium with the applied loads, the value of which is everywhere less than

or equal to the value of the plastic moment, the structure will either not collapse or will be precisely at the point of collapse.

The lower limit theorem expresses the fact that the structure will adapt or reorganize itself, so to speak, in order to support the applied load if possible. It provides lower limit values for the collapse load, which can be considered as safety values. The maximum lower limit is the collapse load itself.

Historically, a number of researchers have pointed out that there are many cases where designers, listening to their intuition and anticipating the consequences of these theorems, came up with reliable structural solutions, which could then be rigorously validated using limit analysis. These engineers and architects understood that it was in fact illusory to imagine being able to describe the real state of stress of a structure, and were content to prove that there was at least one possible state of equilibrium for the structure under the action of loads (Ochsendorf 2005, 353, 364).

If there is at least one safety condition for the structure under the applied loads, then the structure will adapt to this or a similar condition; indeed, the actual end state may also not correspond to that presumed by the designer. Several solutions exist for a hyperstatic structure, and each of them, according to the lower limit theorem, can be seen as valid (all these possible solutions correspond to valid design solutions for this problem). As long as the designer can discern a possible equilibrium solution and ensure that the material is capable of withstanding the internal forces without occurrence of instability problems, the structure will resist (Ochsendorf 2005, 354, 364).

## 6. Later metal arch bridges and viaducts—The Kirchenfeld bridge over the Aare and the Paderno viaduct over the Adda

At a time when certain major achievements were the result of approaches aimed at determining the exact distribution of stress in structures, through purely elastic calculations that were often very laborious, a number of engineers approached things differently, long before a theory of plasticity was developed. Their approach is based on the use of equilibrium equations, seen as a primitive and simplified design procedure, and relies on a new way of looking at the mechanical behavior of materials.

Some outstanding examples of this trend are provided by the achievements of Swiss engineer Jules Röthlisberger with the Bern-based construction company *Gottlieb Ott & Co*, and later with the *Società nazionale officine di Savigliano – SNOS*.

Early examples include the Javroz bridge near Charmey (span 85.786 m, rise 19.706 m, height 56.10 m, designed with Moritz Probst), built between 1878 and 1880—which, along with the Maria Pia bridge over the Douro, is one of the first metal arch bridges without spandrels—, and the bridge over the Schwarzwasser between Bern and Schwarzenburg (span 114.01 m, rise 21.48 m, height 63 m, designed with Paul Simons), built between 1881 and 1882 (Fig. 6). Both structures adopt a parabolic trussed arch, fixed at the springings.

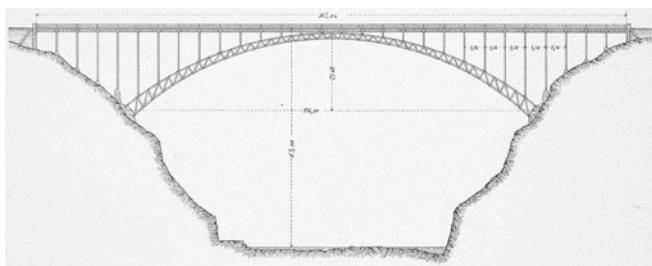


Figure 6. Bridge over the Schwarzwasser, J. Röthlisberger, P. Simons, 1881–82 (*Schweizerische Bauzeitung*, vol. IV, 1884, n. 23, 6/12/1884).

The same design was used for the best-known Kirchenfeld bridge, built over the river Aare in Bern between 1881 and 1883. Set in an urban context and measuring a total length of 229 m, this road bridge features two similar arches with a 78 m opening and a 22 m rise, and a height of around 37 m. The bridge was designed by engineers Probst and Röthlisberger for *Gottlieb Ott & Co*, and completed in twenty-one months as a puddled iron, welded and riveted structure. This was the company's last project, and the third bridge of this type built in Switzerland.

In 1885, Röthlisberger became head of the technical department of the *Società nazionale officine di Savigliano*, and during his twenty-five years with *SNOS*, was responsible for several projects of significant technical value. Among them, the bridges over the river Po at Casalmaggiore (1884–87) and Cremona (1887–92), as well as the road bridge with tramway line at Piacenza, and the two bridges over the river Adda at Trezzo and Paderno, which are certainly the most noteworthy. The Paderno viaduct over the river Adda, known as the San Michele bridge, built between 1887 and 1889, represents the culmination of Röthlisberger's work and a masterpiece of late 19<sup>th</sup> century engineering.

The Paderno viaduct has a total length of 266 m for the structural parts, rising to 304 m including the masonry parts (Fig. 7). The arch has a span of 150 m, a rise of 37.5 m and a height of 74 m, and is 85 m high including the height of the deck. The structure features a double deck, with the lower level open to rail traffic. The structure is entirely made of riveted wrought iron. The reference static scheme features both a high deck and a parabolic arch fixed at the springings.

When Simons and Röthlisberger designed the bridge over the Schwarzwasser in the early eighties, they opted for a fixed-end parabolic-arch design. This choice was not made immediately, but was in fact the result of Culmann's comments on an earlier project drawn up as part of a competition published by the City of Berne. As reported by the cantonal chief engineer Émile Ganguillet in an article published in the *Schweizerische Bauzeitung* in 1884, Culmann had in fact demonstrated that the project presented by the firm *Gottlieb Ott & Co*, which comprised a two-hinged arch of constant thickness, could not constitute a satisfactory solution from a technical point of view, but that it would be preferable to adopt a fixed-end arch of slightly increasing thickness between the crown and the springings (Ganguillet 1884).

In Switzerland, more than anywhere else, the structural system of a trussed girder resting on a fixed-end arch had been widely used since the first local steelwork projects. It was adopted by Röthlisberger for the Paderno viaduct over

the river Adda, his last work, as it had been thirty years earlier for the bridge over the river Aare at Olten, on the Olten-Basel railway line, one of Switzerland's very first railway bridges, built by Etzel and Riggenbach between 1854 and 1856. Last but not least, all the bridges built by *Gottlieb Ott & Co* of Berne featured a fixed-end arch.

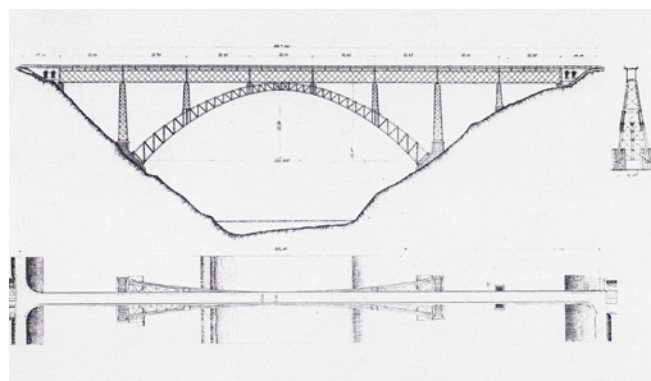


Figure 7. Paderno viaduct on the Adda, elevation, cross-section and plan, J. Röthlisberger, 1887–89 (*Il viadotto di Paderno sull'Adda*, SNOS, Torino, Camilla e Bertolero, 1889).

It should be noted that the widespread adoption of this structural scheme is in line with the tendency of Swiss builders to give their structures a high overall rigidity. This can be seen in a number of other aspects that characterize many projects, particularly in the field of railway infrastructure. In addition to fixed-end arches, we note the frequent adoption of deck designs in the form of continuous beams, as well as a marked preference for riveted connections—at a time when this technology was gradually being abandoned in favor of bolted connections.

To this we can add the designers' predilection for the use of highly rigid bracing devices, obtained by arranging bars with particularly complex geometries—no doubt a construction method following from the rich local tradition of timber construction.

As we have seen, Culmann had always favored hyperstatic designs, and his observations had had a certain impact. The influence of Culmann's views on the Swiss technical community was also the reason why the few two-hinged arch bridges in Switzerland were only built in the second half of the 1890s.

There were two other reasons why the designers opted for a hyperstatic structure: the increased stiffness offered by a fixed-end arch meant that the structure behaved much more satisfactorily under dynamic action such as wind and rail traffic; the assembly technique most commonly used in Switzerland was still that based on the use of a few isolated fixed scaffolds, rather than that involving the cantilevered launching of voussoirs or spans, a technique perfectly mastered by Eiffel.

With regard to the sizing process, both Culmann's and Ritter's students tended to use a mixed method. What we see is a combination of a purely graphical approach followed by an analytical development; in particular, this allowed the results obtained through graphical constructions to be verified by analytical results derived from the application of Bresse's theory, a trend initiated by Ritter, which would later be coined *eclectic* (Bertolini 1989, 64).



In this context, one of R  thlisberger's contributions was the use of the elasticity theorems, and in particular the elasticity ellipse theory. The Trezzo and Paderno bridges were the first two trussed-arch structures to be designed and sized using this theory (Bertolini 1989, 60–61).

Finally, as Ritter had done before him, R  thlisberger turned his attention to the interaction between the two elements forming a typical viaduct—the girder and the arch—and sought to describe their mutual relationship. He developed a method for assessing the state of stress of the combined girder and arch system, a problem that was to remain relevant for decades to come.

As the method suggests, the beam is first considered as an elastic structure resting on a perfectly rigid arch, and the reactions coming from the supporting posts are evaluated; after changing sign, these reactions are then applied to the arch, considered as an elastic structure supporting a perfectly flexible beam. After determining the displacements of the beam's support points, the beam calculation is repeated considering it as a continuous beam on flexible supports (Bertolini 1989, 65).

It should be noted that this is the same procedure still used today to subdivide a complex system into two sub-systems of simpler resolution.

## Conclusion

In the case of the two structures designed by Eiffel, there is a tendency to see the structure as the sum of several simple subsystems that have already been studied. The hierarchy and interdependence of parts and elements become easily legible, and the static reference schemes show little static indeterminacy, or result from the arrangement of several isostatic systems. These features make it possible to carry out a highly accurate analysis, albeit time-consuming in terms of mathematical operations, of elastic behavior, providing a practically exact result for every part of the structure. Several calculation methods were adopted for sizing the structure. The deck spans were sized using Clapeyron's formulas, while the trussed piers were sized using Culmann's graphic statics procedures. Finally, the central trussed arch was sized using Bresse's formulas.

The Garabit viaduct, as an upgrade to the Douro bridge, indeed represents the culmination of this approach, aimed at determining stress values in relation to the elastic behavior for the cross sections most exposed to stress. The evaluation of the displacements and distortions observed during the loading tests, and their comparison with the values predicted in the calculation phase, was notably intended to validate the approach as a whole. Eiffel and his engineers used a mathematical approximation and rely on a clear sense of the beauty of lattice structures defining a volume in space. They developed bold, economical structures, and built them in a cost-effective way thanks to an assembly technique based on cranes and steel cables allowing the cantilever launching of the arch segments and deck spans, and to efficient site and labor management.

R  thlisberger's vision, while not opposed to the approach of de Dion, Seyrig or Eiffel, is rational and very modern. The conceptualization that his approach to the Paderno bridge reveals, far from referring to the arrangement of several

simple systems already known, refers to a single complex system, the analysis of which takes into account second-order effects and, even if implicitly, the material's own ductile characteristics. R  thlisberger's projects are open to the innovative physical interpretations of the turn of the century and adopt the most recent mathematical formulations. In this sense, his approach anticipates the end of the classical phase (1875–1900) of theory of structures.

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