



**HAL**  
open science

# Extreme softening of QCD phase transition under weak acceleration: first principle Monte Carlo results for gluon plasma

M.N. Chernodub, Vladimir Goy, Alexander Molochkov, Daniil Stepanov,  
Arina Pochinok

## ► To cite this version:

M.N. Chernodub, Vladimir Goy, Alexander Molochkov, Daniil Stepanov, Arina Pochinok. Extreme softening of QCD phase transition under weak acceleration: first principle Monte Carlo results for gluon plasma. 2024. hal-04690809

**HAL Id: hal-04690809**

**<https://hal.science/hal-04690809>**

Preprint submitted on 6 Sep 2024

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Extreme softening of QCD phase transition under weak acceleration: first principle Monte Carlo results for gluon plasma

M. N. Chernodub,<sup>1</sup> V. A. Goy,<sup>2</sup> A. V. Molochkov,<sup>2,3</sup> D. V. Stepanov,<sup>2</sup> and A. S. Pochinok<sup>2</sup>

<sup>1</sup>*Institut Denis Poisson UMR 7013, Université de Tours, 37200 Tours, France*

<sup>2</sup>*Pacific Quantum Center, Far Eastern Federal University, 690922 Vladivostok, Russia*

<sup>3</sup>*Beijing Institute of Mathematical Sciences and Applications, Tsinghua University, 101408, Beijing, China*

(Dated: September 4, 2024)

We study the properties of gluon plasma subjected to a weak acceleration using first-principle numerical Monte Carlo simulations. We use the Luttinger (Tolman-Ehrenfest) correspondence between temperature gradient and gravitational field to impose acceleration in imaginary time formalism. Under acceleration, the system resides in global thermal equilibrium. Our results indicate that even the weakest acceleration up to  $a \simeq 27$  MeV drastically softens the deconfinement phase transition, converting the first-order phase transition of a static system to a soft crossover for accelerating gluons. The accelerating environment can be relevant to the first moments of the early Universe and the initial glasma regime of relativistic heavy ion collisions. In particular, our results imply that the acceleration, if present, may also inhibit the detection of the thermodynamic phase transition from quark-gluon plasma to the hadronic phase.

**Introduction.** Quark-gluon plasma is a state of matter believed to have existed in the Early Universe up to a microsecond after the Big Bang [1]. In this phase, the fundamental constituents of matter, quarks and gluons, are not confined within hadrons. Relativistic collisions of heavy ions recreate this state in a laboratory setting, allowing experimental access to the fascinating properties of quark-gluon plasma [2].

When ions collide, they transfer their kinetic energy to a rapidly expanding plasma fireball, thus experiencing a rapid deceleration [3]. The deceleration is caused by the strong longitudinal chromoelectric fields that mediate the interaction between the ions. The strength of these fields is of the order of  $E \sim Q_s^2/g$ , where  $g$  is the strong coupling constant. The typical deceleration achieved in a collision is argued to be of the order of the gluon saturation scale,  $|a| \simeq Q_s \simeq 1$  GeV [4].

A uniformly accelerated system possesses the Rindler event horizon [5] beyond which the events do not influence the accelerating particles [6]. The presence of the event horizon fosters the Unruh effect, where an observer, uniformly accelerated in a zero-temperature Minkowski vacuum, detects a thermal bath of particles with the Unruh temperature: [7]

$$T_U = \frac{|a|}{2\pi}. \quad (1)$$

The Rindler horizon of an accelerated system has a profound similarity with the event horizon of a black hole, which also separates causally disconnected regions of spacetime. [8, 9] For black holes, the presence of the event horizon leads to a very intriguing quantum effect: black holes evaporate via the Hawking production of particle pairs, where one of the particles falls to the black hole while the other one escapes to the spatial infinity. [10, 11] The particle radiation, which can be interpreted as a tunneling process through the event horizon [12], is perceived as thermal radiation with the Hawking temperature  $T_H = \kappa/(2\pi)$ , where  $\kappa = 1/(4M)$  is the acceleration

due to gravity at the event horizon of a black hole with mass  $M$ . The similarity of the Hawking and Unruh temperatures (1) suggests that the thermal character of both phenomena originates from the presence of the appropriate event horizons.

In application to colliding ions, deceleration was suggested to lead to a rapid thermalization of the gluon matter through the Unruh effect, which was argued to produce a final thermal gluon state via quantum tunneling through the emerging event horizon. The effect should appear at the short time scale of  $t \simeq 2\pi/Q_s \simeq 1$  fm with the resulting heat bath temperature (1) reaching a typical QCD scale of  $T = |a|/(2\pi) \sim 200$  MeV. [4] The rapid thermalization leads to the subsequent emergence of quark-gluon plasma, which achieves thermal equilibrium at a later stage.

Besides the thermalization process, the acceleration of interacting particle systems can cause changes in their phase structure. The restoration of the chiral symmetry for interacting fermions has been considered in Ref. [13] within the Nambu–Jona-Lasinio model [14, 15]. This effect seems to be a natural consequence of the observation that the acceleration is associated with the Unruh temperature (1), while thermal effects usually lead to the restoration of a spontaneously broken symmetry. [16] The symmetry restoration due to acceleration has been questioned in Ref. [17], which concluded that the acceleration of a vacuum itself cannot cause phase transitions. While we will not delve into this question, we mention that one should distinguish two physical scenarios of acceleration: (i) a vacuum of interacting particles viewed from the point of the acceleration observer and (ii) a physically accelerated object from the points of view of the observer that accelerates together with the object. While Ref. [17] deals with the former question, here we consider an accelerated thermal state of hot gluons at  $T \geq T_U$ , which corresponds to the thermally equilibrated system of accelerated particles. For an equilibrium system, the region with  $T < T_U$  is usually considered to be forbidden [18–

21] (see, however, a recent discussion in Refs. [22, 23]). Restoration of a spontaneously broken symmetry by acceleration has also been addressed in various physical scenarios [24–29]. A very recent critical assessment of the restoration of broken symmetry due to acceleration in an interacting field theory can be found in Ref. [30].

In our work, we study the non-perturbative properties of gluon plasma subjected to weak acceleration using first-principle numerical Monte Carlo simulations. Throughout this article, we use the units  $c = \hbar = 1$ .

### Global thermal equilibrium under acceleration.

Under a uniform acceleration, a generic particle system resides in global thermal equilibrium characterized by inhomogeneous temperature  $T(x)$ . In a classical approach, it is convenient to describe the corresponding physical environment by the inverse temperature four-vector  $\beta^\mu(x) \equiv u^\mu(x)/T(x)$ , which is associated with the local fluid velocity  $u^\mu = u^\mu(x)$ .

In thermal equilibrium, the inverse temperature  $\beta^\mu$  satisfies the Killing equation,  $\partial_\mu \beta_\nu + \partial_\nu \beta_\mu = 0$  [31, 32]. For a thermalized system that accelerates uniformly in the direction  $z$ , one gets the appropriate solution of this equation:  $\beta^\mu(x) \partial_\mu = (1/T_0)[(1 + a_0 z) \partial_t + a_0 t \partial_z]$ . Then, the local temperature  $T(x)$ , the local fluid velocity  $u^\mu(x)$ , and the local proper acceleration  $a^\mu(x) \equiv u^\nu \partial_\nu u^\mu$  of the accelerated particle fluid are respectively (see, e.g., [33]):

$$T(t, z) = \frac{T_0}{\sqrt{(1 + a_0 z)^2 - (a_0 t)^2}}, \quad (2a)$$

$$u^\mu(t, z) \partial_\mu = \frac{T(x)}{T_0} [(1 + a_0 z) \partial_t + a_0 t \partial_z], \quad (2b)$$

$$a^\mu(t, z) \partial_\mu = a_0 \frac{T^2(x)}{T_0^2} [a_0 t \partial_t + (1 + a_0 z) \partial_z], \quad (2c)$$

where  $T_0 = T(t = z = 0)$  and  $a_0 = a(t = z = 0)$  are interpreted as the reference quantities taken at a  $z = 0$  plane at time  $t = 0$ . The proper acceleration,  $\alpha^\mu = a^\mu(x)/T(x)$ , has a constant magnitude,  $\alpha^\mu \alpha_\mu = -a_0^2/T_0^2$ .

For definiteness, we take  $a_0 > 0$ . Then, an accelerating particle follows a hyperbolic trajectory in spacetime with the entire worldline confined within the right Rindler wedge,  $z > |t| - 1/a_0$ . The boundary of the Rindler wedge corresponds to the Rindler horizon,

$$z_R(t) = |t| - 1/a_0, \quad (3)$$

which sets the physical causal boundary of the system. Any events that appear beyond the Rindler horizon, at  $z < z_R(t)$ , cannot affect the particle motion because the light signals from those events will never reach the particle subjected to a constant acceleration. Consequently, the thermodynamics of a uniformly accelerating particle system is defined only within the right Rindler wedge. At the Rindler horizon (3), all quantities (2) diverge.

It is convenient to rewrite Eqs. (2) at  $t = 0$ , which give the local temperature and acceleration,  $a^\mu(z) = a(z) \delta^{\mu,3}$ ,

$$T(z) = \frac{T_0}{1 + a_0 z}, \quad a(z) = \frac{a_0}{1 + a_0 z}, \quad (4)$$

for a locally static fluid  $u^\mu = \delta^{\mu,0}$ . Equations (4) show that acceleration is tied to a temperature gradient,

$$a(z) = -\frac{1}{T(z)} \frac{\partial T(z)}{\partial z}, \quad (5)$$

in accordance with the Tolman-Ehrenfest [34, 35] and Luttinger [36], relations. The Rindler horizon (3) then becomes simply  $z_R = -1/a_0$ .

We study the effect of a weak acceleration on the gluon plasma in the first-principle approach within the lattice gauge theory. We implement the  $T = T(z)$  temperature profile (4) at a fixed  $z$ -independent spatial volume, noticing that our  $t = 0$  results can be extended to any  $t$  by replacing the  $a(z) \rightarrow a(t, z)$  in calculated expectation values [37–39]. The path-integral formalism justifying our setup has been carefully elaborated in Ref. [39].

**No sign problem for an accelerating system.** In sharp contrast to vorticity and finite baryonic chemical potential, the accelerating environment can be directly implemented in the imaginary time formalism without the need to work in a complex plane and subsequently perform an analytical continuation to the real values of acceleration. Indeed, the acceleration is the rate at which the velocity  $\mathbf{v} = \partial \mathbf{x} / \partial t$  of a body changes over time,  $\mathbf{a} = \partial \mathbf{v} / \partial t \equiv \partial^2 \mathbf{x} / \partial t^2$ . The Wick rotation from the real to imaginary time,  $t \rightarrow -i\tau$ , amounts to simply flipping the sign of the acceleration vector,  $\mathbf{a} \rightarrow \mathbf{a}_E = -\mathbf{a}$ . Thus, we set the temperature gradient (4) in the imaginary-time formalism as  $T(z) = T_0 / (1 - a_E z)$ . Hereafter, we denote  $a = -a_E \equiv a_0$  to simplify our notations.

We also notice that the acceleration in the imaginary time formalism can be implemented, respecting the Kubo–Martin–Schwinger (KMS) condition [40, 41], at least in two other ways: (i) as a motion along the imaginary circle in the Euclidean spacetime and (ii) as a double-periodicity condition along imaginary time direction in the Euclidean-Rindler spacetime [33]. Here, we use a third method of utilizing Tolman-Ehrenfest (Luttinger) correspondence [34–36] set by the inhomogeneous temperature profile (4).

**Lattice setup.** We consider SU(3) gauge theory with the anisotropic Wilson action [42]:

$$S = \sum_x \sum_{i>j=1}^3 \beta_\sigma(x_3) (1 - \mathcal{P}_{x,ij}) + \sum_x \sum_{i=1}^3 \beta_\tau(x_3) (1 - \mathcal{P}_{x,4i}), \quad (6)$$

where  $x = \{x_1, x_2; x_3 \equiv z; x_4 \equiv \tau\}$  is the Euclidean coordinate of the lattice sites,  $\mathcal{P}_P = \frac{1}{3} \text{Re Tr } U_P$ , with  $U_{x,\mu\nu} = U_{x,\mu} U_{x+\hat{\mu},\nu} U_{x+\hat{\nu},\mu}^\dagger U_{x,\nu}^\dagger$ , is the lattice action on an elementary plaquette  $P \equiv P_{n,\mu\nu} = \{n, \mu\nu\}$ , and  $\mu, \nu = 1, \dots, 4$  are Euclidean directions. We consider the lattices  $N_\tau \times N_x \times N_y \times N_z$  with the size  $N_\tau = 8$  along the imaginary time  $\tau$ , the transverse spatial size  $N_s \equiv N_x = N_y =$

42 and the longitudinal sizes  $N_z = 104, 126, 148, 170$  along the acceleration direction  $z \equiv x_3$ .

The Wilson action (6) with two couplings  $\beta_\sigma$  and  $\beta_\tau$  allows us to fix separately the spacial,  $\mathbf{a}_\sigma = \mathbf{a}_\sigma(\beta_\sigma, \beta_\tau)$ , and temporal,  $\mathbf{a}_\tau = \mathbf{a}_\tau(\beta_\sigma, \beta_\tau)$ , lattice spacings that stand for the physical length of the elementary lattice links in the appropriate directions. [43] We use an improved procedure following Ref. [42] to simulate the theory on the asymmetric lattices. The technical details of our simulations are described in the Appendix.

We impose periodic boundary conditions along  $x, y, \tau$  directions and keep open boundaries along the acceleration direction  $z$ . The physical length of the imaginary time circle,  $L_\tau \equiv N_\tau \mathbf{a}_\tau(z) = 1/T(z)$  is chosen to match the temperature profile (4) corresponding to a constant proper acceleration. In the  $z$  direction, temperature varies only on a central segment that occupies half of the volume. We keep two  $\delta L_z = \mathbf{a}_\sigma N_z/4 \sim (1 \dots 4)$  fm thick slices at both sites of the central segment to mitigate the effects of the open boundary conditions along the  $z$  direction. It is important to stress that we made the lengths of the transverse spatial directions  $L_i = N_i \mathbf{a}_\sigma(z)$  with  $i = x, y$  are made  $z$ -independent by imposing a particular  $z$  dependence for the  $\beta_\sigma$  and  $\beta_\tau$  couplings in the lattice action (6) (the details are given in the Appendix).

In order to probe the phase structure of the model, we calculate the order parameter of confinement, the Polyakov loop,  $L(z)$ , and its susceptibility  $\chi(z)$ ,

$$L(z) = \langle |L_z| \rangle, \quad L_z = \frac{1}{N_s^2} \sum_{x,y=0}^{N_s-1} \frac{1}{3} \text{Tr} \prod_{\tau=0}^{N_\tau-1} U_{x,\tau}, \quad (7)$$

$$\chi(z) = \langle |L_z|^2 \rangle - \langle |L_z| \rangle^2, \quad (8)$$

where  $\mathbf{x} = (x, y, z)$  is the spatial coordinate. The notation  $\langle \mathcal{O} \rangle$  represents a statistical averaging of the quantity  $\mathcal{O}$  over the whole ensemble of the field configurations. For each lattice size and acceleration, the central temperature at  $z = 0$  is chosen to be equal, within good numerical accuracy, to the critical temperature of the non-accelerating system,  $T_0 \equiv T(z = 0) = T_c(a = 0)$ .

**Results.** Figure 1 shows the Polyakov loop in an accelerating gluon fluid. The results agree qualitatively with our expectations given the explicit form of the temperature profile (4): the  $a > 0$  acceleration enhances the deconfining phase at  $z < 0$  and drives the system deeper into the confining phase at  $z > 0$ .

To estimate the quantitative effect of acceleration on gluon medium, we compute the excess of the local free energy of an accelerating heavy quark at the position  $z$  with respect to the energy of a static ( $a = 0$ ) heavy quark that resides at the same temperature  $T_{\text{bulk}} = T(z, a)$  but now in the whole space:

$$\Delta F(z) = -T_{\text{bulk}} \ln \frac{L_{\text{acc}}(z, a)}{L_{\text{bulk}}(T_{\text{bulk}})} \Big|_{T_{\text{bulk}}=T(z,a)}, \quad (9)$$

with  $L_{\text{bulk}} = \langle |L_{\mathbf{x}}| \rangle_{\mathbf{x}}$  averaged over all  $\mathbf{x}$  coordinates.

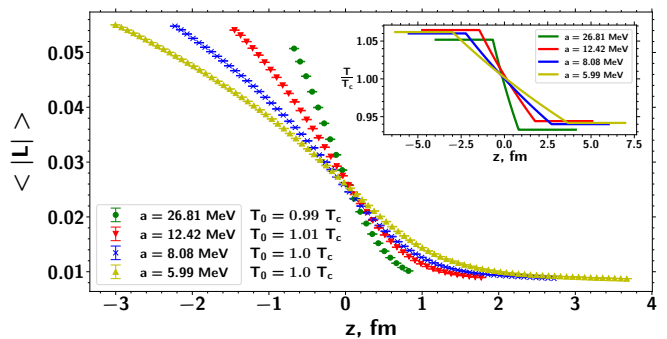


Figure 1. The local expectation value of the Polyakov loop,  $L = L(z)$ , at various accelerations  $a \equiv a_0 = -a_E > 0$  along the  $z$  direction. Shown is a central region, where the temperature is inhomogeneous (*cf.* the inset). The temperature in the central plane,  $z = 0$ , is chosen to be close to the critical deconfinement temperature  $T_c$  without acceleration,  $a = 0$ . The confining (deconfining) regions are located at the right,  $z \gtrsim 0$  (left,  $z \lesssim 0$ ) semi-volumes.

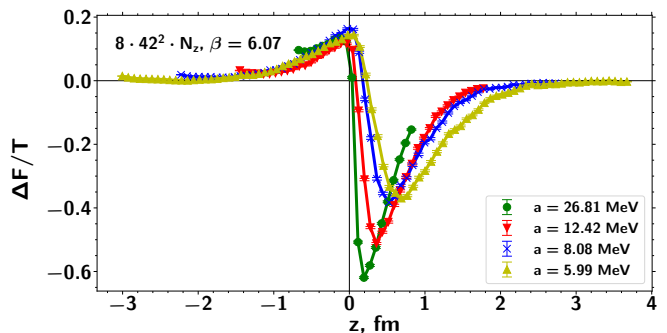


Figure 2. The effect of the acceleration on the free energy of a free heavy quark (9) with the notations of Fig. 1.

The free energy, shown in Fig. 2, has a clear tendency to increase at the deconfining side ( $z \lesssim 0$ ) of the accelerating gluon fluid. In other words, the acceleration tends to drive the deconfining plasma towards the confinement region. A reciprocal effect is observed in the deconfining side ( $z \gtrsim 0$ ), where the acceleration makes the free energy lower, thus driving the system toward deconfinement.

In Figure 3, we show the susceptibility of the Polyakov loop at various accelerations. To compare this quantity at the non-accelerating case, we introduce the matching susceptibility, which is measured for the homogeneous,  $z$ -independent gluon matter at temperature  $T_{\text{bulk}}$  fixed to match  $T = T(z, a)$  for chosen  $z$  and  $a$  of the accelerating gluon matter. For the matching acceleration, we took the moderate value  $a \simeq 8$  MeV. Figure 3 shows that a nonzero acceleration makes the phase susceptibility curve (i) wider and (ii) lower, as compared to the matching  $a = 0$  case, where the bulk variation of temperature is only taken into account. Both properties coherently imply that the accelerating gluon fluid experiences a broad crossover transition to the deconfining phase instead of the first-order phase transition that is expected in the

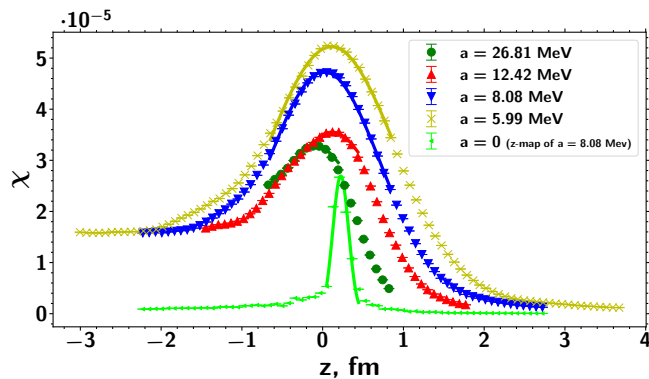


Figure 3. Susceptibility of the Polyakov loop (8) for the accelerating gluon medium (in notations of Fig. 1). The matching  $a = 0$  curve shows the susceptibility  $\chi = \chi(z)$  found at a globally homogeneous temperature  $T_{\text{bulk}}$  that matches the local temperature  $T = T(z, a \simeq 8 \text{ MeV})$  for each  $z$ .

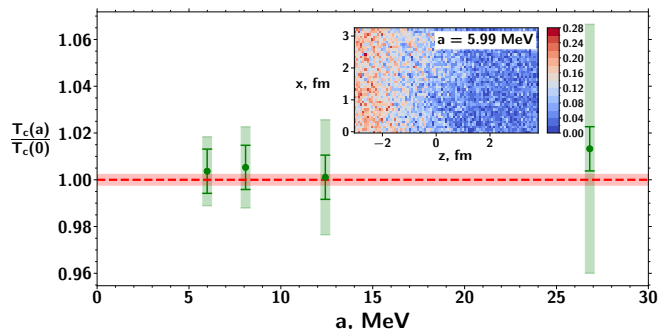


Figure 4. Phase diagram of the hot gluon matter under acceleration in the  $(a, T)$  plane. The error bars, given by the vertical lines, are shown together with the widths of the crossovers given by the semi-transparent bands. The inset depicts the density plot of the Polyakov line in the  $(x, z)$  plane at a typical gluon configuration, averaged over the  $y$  direction.

non-accelerating case.

To quantify the effect of acceleration on gluon matter, we obtain the critical temperature and the width of the deconfining transition by fitting the Polyakov loop susceptibility by a Gaussian function at each  $a$ . The fits are shown in Fig. 3 by the solid lines. The obtained phase diagram of the accelerating gluon matter is demonstrated in Fig. 4.

Despite the physical value of acceleration  $a$  being much smaller than the QCD mass scale,  $a \ll \Lambda_{\text{QCD}} \simeq 100 \text{ MeV}$ , the effect of the acceleration on the phase diagram is unexpectedly substantial. Our results show that even the weakest studied acceleration,  $a \simeq 6 \text{ MeV}$ , converts the first-order thermal phase transition in SU(3) Yang-Mills theory into a smooth crossover. As the acceleration increases towards the biggest studied values

$a \simeq 27 \text{ MeV}$ , the width of the crossover becomes larger. On the other hand, the acceleration of gluon plasma does not affect the position of the (pseudo)critical temperature of the deconfinement crossover transition, which remains approximately equal, within a few percent accuracy, to the critical temperature of the phase transition the non-accelerating system,  $T^{\text{crossover}}(a > 0) \simeq T_c^{\text{1st order}}(a = 0)$ .

**Conclusions.** In our article, we studied the effects of acceleration on the phase diagram of gluon matter using first-principle Monte Carlo simulations. The thermal equilibrium of the gluon matter is ensured by the particular form of the spatial inhomogeneity (4) of local temperature dictated by the Killing equation. The temperature inhomogeneity satisfies the Luttinger (Tolman-Ehrenfest) correspondence (5) between temperature gradient and gravitational field. On a practical side, we notice that the imposing of acceleration in the imaginary time formalism is free from the sign problem. Therefore, our approach allows us to calculate properties of accelerating systems directly in lattice Monte Carlo simulations.

We show that even the weakest acceleration of the order of  $a \simeq 6 \text{ MeV}$  drastically softens the deconfinement phase transition, converting the first-order phase transition of a static hot gluon system to a very soft and broad crossover without noticeable (with an accuracy within a few percent) shifting the position of the corresponding pseudocritical temperature. This unexpected property, which persists up to the maximal studied acceleration  $a \simeq 27 \text{ MeV}$ , is correlated coherently with our other observations: (i) the acceleration appears to drive the hot gluon matter residing in the deconfinement phase towards the confining phase; (ii) correspondingly, hot but still confining gluon matter is driven by acceleration closer to the deconfining domain.

The softening effect on the phase transition may have some consequences in the physical environments where (quark) gluon plasma experiences acceleration. One immediate application could be the early Universe. The softening acceleration effects could also hinder the search for the critical line in ongoing experiments in relativistic heavy-ion collisions.

## ACKNOWLEDGMENTS

VAG and AVM have been supported by RSF (Project No. 23-12-00072, <https://rscf.ru/project/23-12-00072/>). DVS and ASP were supported by Grant No. FZNS-2024-0002 of the Ministry of Science and Higher Education of Russia. The numerical simulations were performed at the computing cluster of Far Eastern Federal University and the equipment of Shared Resource Center "Far Eastern Computing Resource" IACP FEB RAS (<https://cc.dvo.ru>).

[1] Johann Rafelski, "Connecting QGP-Heavy Ion Physics to the Early Universe," *Nuclear Physics B - Proceedings*

- [2] Roman Pasechnik and Michal Šumbera, “Phenomenological Review on Quark–Gluon Plasma: Concepts vs. Observations,” *Universe* **3**, 7 (2017), arXiv:1611.01533 [hep-ph].
- [3] The deceleration is an acceleration with  $a < 0$ .
- [4] Dmitri Kharzeev and Kirill Tuchin, “From color glass condensate to quark gluon plasma through the event horizon,” *Nucl. Phys. A* **753**, 316–334 (2005), arXiv:hep-ph/0501234.
- [5] W. Rindler, “Kruskal Space and the Uniformly Accelerated Frame,” *Am. J. Phys.* **34**, 1174 (1966).
- [6] T.D. Lee, “Are black holes black bodies?” *Nuclear Physics B* **264**, 437–486 (1986).
- [7] W. G. Unruh, “Notes on black hole evaporation,” *Phys. Rev. D* **14**, 870 (1976).
- [8] G. W. Gibbons and M. J. Perry, “Black Holes and Thermal Green’s Functions,” *Proc. Roy. Soc. Lond. A* **358**, 467–494 (1978).
- [9] G. W. Gibbons and M. J. Perry, “Black Holes in Thermal Equilibrium,” *Phys. Rev. Lett.* **36**, 985 (1976).
- [10] S. W. Hawking, “Black hole explosions?” *Nature* **248**, 30–31 (1974).
- [11] S. W. Hawking, “Particle creation by black holes,” *Communications In Mathematical Physics* **43**, 199–220 (1975).
- [12] Maulik K. Parikh and Frank Wilczek, “Hawking radiation as tunneling,” *Phys. Rev. Lett.* **85**, 5042–5045 (2000), arXiv:hep-th/9907001.
- [13] Tadafumi Ohsaku, “Dynamical chiral symmetry breaking and its restoration for an accelerated observer,” *Phys. Lett. B* **599**, 102–110 (2004), arXiv:hep-th/0407067.
- [14] Yoichiro Nambu and G. Jona-Lasinio, “Dynamical Model of Elementary Particles Based on an Analogy with Superconductivity. I.” *Phys. Rev.* **122**, 345–358 (1961).
- [15] Yoichiro Nambu and G. Jona-Lasinio, “Dynamical model of elementary particles based on an analogy with superconductivity. II.” *Phys. Rev.* **124**, 246–254 (1961).
- [16] L. Dolan and R. Jackiw, “Symmetry Behavior at Finite Temperature,” *Phys. Rev. D* **9**, 3320–3341 (1974).
- [17] William G. Unruh and Nathan Weiss, “Acceleration Radiation in Interacting Field Theories,” *Phys. Rev. D* **29**, 1656 (1984).
- [18] F. Becattini, “Thermodynamic equilibrium with acceleration and the Unruh effect,” *Phys. Rev. D* **97**, 085013 (2018), arXiv:1712.08031 [gr-qc].
- [19] George Y. Prokhorov, Oleg V. Teryaev, and Valentin I. Zakharov, “Unruh effect for fermions from the Zubarev density operator,” *Phys. Rev. D* **99**, 071901 (2019), arXiv:1903.09697 [hep-th].
- [20] Georgy Y. Prokhorov, Oleg V. Teryaev, and Valentin I. Zakharov, “Calculation of acceleration effects using the Zubarev density operator,” *Particles* **3**, 1–14 (2020), arXiv:1911.04563 [hep-th].
- [21] Andrea Palermo, Matteo Buzzegoli, and Francesco Becattini, “Exact equilibrium distributions in statistical quantum field theory with rotation and acceleration: Dirac field,” *JHEP* **10**, 077 (2021), arXiv:2106.08340 [hep-th].
- [22] Georgy Yu. Prokhorov, Oleg V. Teryaev, and Valentin I. Zakharov, “Novel phase transition at the Unruh temperature,” (2023), arXiv:2304.13151 [hep-th].
- [23] G. Yu. Prokhorov, O. V. Teryaev, and V. I. Zakharov, “Quantum Phase Transitions in an Accelerated Medium,” *Phys. Part. Nucl.* **55**, 1066–1069 (2024).
- [24] D. Ebert and V. Ch. Zhukovsky, “Restoration of Dynamically Broken Chiral and Color Symmetries for an Accelerated Observer,” *Phys. Lett. B* **645**, 267–274 (2007), arXiv:hep-th/0612009.
- [25] P. Castorina and M. Finocchiaro, “Symmetry Restoration By Acceleration,” *J. Mod. Phys.* **3**, 1703 (2012), arXiv:1207.3677 [hep-th].
- [26] Shingo Takeuchi, “Bose–Einstein condensation in the Rindler space,” *Phys. Lett. B* **750**, 209–217 (2015), arXiv:1501.07471 [hep-th].
- [27] Antonio Dobado, “Brout–Englert–Higgs mechanism for accelerating observers,” *Phys. Rev. D* **96**, 085009 (2017), arXiv:1710.01564 [gr-qc].
- [28] Adrián Casado-Turrión and Antonio Dobado, “Triggering the QCD phase transition through the Unruh effect: chiral symmetry restoration for uniformly accelerated observers,” *Phys. Rev. D* **99**, 125018 (2019), arXiv:1905.11179 [hep-ph].
- [29] Wei Kou and Xurong Chen, “Locating Quark–Antiquark String Breaking in QCD through Chiral Symmetry Restoration and Hawking–Unruh Effect,” (2024), arXiv:2405.18697 [hep-ph].
- [30] Domenico Giuseppe Salluce, Marco Pasini, Antonino Flachi, Antonio Pittelli, and Stefano Ansoldi, “Symmetry restoration and uniformly accelerated observers in Minkowski spacetime,” *JHEP* **05**, 218 (2024), arXiv:2401.16483 [hep-th].
- [31] C. Cercignani and G. M. Kremer, *The Relativistic Boltzmann Equation: Theory and Applications* (Springer, 2002).
- [32] F. Becattini, “Covariant statistical mechanics and the stress-energy tensor,” *Phys. Rev. Lett.* **108**, 244502 (2012), arXiv:1201.5278 [gr-qc].
- [33] Victor E. Ambruş and Maxim N. Chernodub, “Acceleration as a circular motion along an imaginary circle: Kubo–Martin–Schwinger condition for accelerating field theories in imaginary-time formalism,” *Phys. Lett. B* **855**, 138757 (2024), arXiv:2308.03225 [hep-th].
- [34] Richard C. Tolman, “On the Weight of Heat and Thermal Equilibrium in General Relativity,” *Phys. Rev.* **35**, 904–924 (1930).
- [35] Richard Tolman and Paul Ehrenfest, “Temperature Equilibrium in a Static Gravitational Field,” *Phys. Rev.* **36**, 1791–1798 (1930).
- [36] J. M. Luttinger, “Theory of Thermal Transport Coefficients,” *Phys. Rev.* **135**, A1505–A1514 (1964).
- [37] F. Becattini and D. Rindori, “Extensivity, entropy current, area law and Unruh effect,” *Phys. Rev. D* **99**, 125011 (2019), arXiv:1903.05422 [hep-th].
- [38] F. Becattini, M. Buzzegoli, and A. Palermo, “Exact equilibrium distributions in statistical quantum field theory with rotation and acceleration: scalar field,” *JHEP* **02**, 101 (2021), arXiv:2007.08249 [hep-th].
- [39] Maik Selch, Ruslan A. Abramchuk, and M. A. Zubkov, “Effective Lagrangian for the macroscopic motion of fermionic matter,” *Phys. Rev. D* **109**, 016003 (2024), arXiv:2310.02098 [hep-ph].
- [40] Ryogo Kubo, “Statistical-mechanical theory of irreversible processes. i. general theory and simple applications to magnetic and conduction problems,” *Journal of the Physical Society of Japan* **12**, 570–586 (1957).
- [41] Paul C. Martin and Julian Schwinger, “Theory of many-particle systems. i,” *Physical Review* **115**, 1342–1373 (1959).

- [42] F. Karsch, “SU(N) Gauge Theory Couplings on Asymmetric Lattices,” *Nucl. Phys. B* **205**, 285–300 (1982).
- [43] To avoid confusion, we use the Fraktur font  $\mathfrak{a}$  for a lattice spacing and the Italic font  $a$  for an acceleration.
- [44] Ji-Chong Yang, Wen-Wen Li, and Chong-Xing Yue, “Split of the pseudo-critical temperatures of chiral and confine/deconfine transitions by temperature gradient,” (2024), [arXiv:2401.03826 \[hep-lat\]](#).
- [45] Andreas Athenodorou and Michael Teper, “The glueball spectrum of SU(3) gauge theory in 3 + 1 dimensions,” *JHEP* **11**, 172 (2020), [arXiv:2007.06422 \[hep-lat\]](#).

## APPENDIX

### A. Numerical simulations

The Wilson action (6) with two couplings  $\beta_\sigma$  and  $\beta_\tau$  allows us to fix separately the spacial,  $\mathbf{a}_\sigma = \mathbf{a}_\sigma(\beta_\sigma, \beta_\tau)$ , and temporal,  $\mathbf{a}_\tau = \mathbf{a}_\tau(\beta_\sigma, \beta_\tau)$  lattice spacings. The quantity

$$\xi = \frac{\mathbf{a}_\sigma}{\mathbf{a}_\tau}, \quad (10)$$

gives us the anisotropy of the lattice spacings.

In the continuum limit, where both couplings  $\beta_\sigma$  and  $\beta_\tau$  are large, the lattice spacings vanish,  $\mathbf{a}_\sigma \rightarrow 0$  and  $\mathbf{a}_\tau \rightarrow 0$ . Consequently, the lattice plaquette  $\mathcal{P}_{x,\mu\nu}$  becomes proportional to the square of the continuum field-strength tensor  $F_{\mu\nu}$  times a volume element,  $\mathbf{a}_\sigma^3 \mathbf{a}_\tau$ , and the lattice Wilson action (6) reduces to the continuum action of the Yang-Mills theory [42].

We work with asymmetric lattices,  $N_\tau < N_\sigma$ , corresponding to the case of finite temperature. The local temperature is given by an inverse of the lattice length  $L_\tau = N_\tau \mathbf{a}$  in the imaginary time direction:

$$T(z) = \frac{1}{N_\tau \mathbf{a}_\tau(z)}. \quad (11)$$

We aim to introduce the temperature gradient along the longitudinal spatial direction  $z \equiv x_3$ :

$$T(z) = \frac{T_0}{1 + a_0(z - z_0)}, \quad (12)$$

where  $T_0 \equiv T(z=z_0)$  is the temperature at the  $z = z_0$  plane and  $a_0 > 0$  is the parameter of the temperature inhomogeneity which corresponds, according to the Tolman-Ehrenfest relation (5), to the physical acceleration. We consider the case of a weak acceleration,  $aL_\sigma \ll 1$ , so that the physical temperature (12) is nowhere singular on our lattices. In other words, the position of the Rindler horizon,  $z^R = -1/a$ , resides far away from the spatial region considered in our simulations,  $|z^R| \gg L_\sigma$ .

We should choose the lattice couplings in Eq. (6) in such a way that the spatial lattice spacing remains constant along the whole lattice,

$$\mathbf{a}_\sigma = \mathbf{a}_0 = \frac{1}{N_\tau T_0} = \text{const}, \quad (13)$$

while the physical temperature (11) varies according to Eq. (12), implying, in turn, that the temporal lattice spacing is a linear function of the longitudinal coordinate  $z$ :

$$\mathbf{a}_\tau(z) = \mathbf{a}_0(1 + a_0(z - z_0)), \quad (14)$$

which then corresponds to the anisotropy parameter (10):

$$\xi(z) = \frac{1}{1 + a_0(z - z_0)} = \frac{T(z)}{T_0}. \quad (15)$$

We work at fixed lattice geometry  $N_\tau N_\sigma^3$  so that the variation of temperature in the  $z$  direction is assumed only in the lattice couplings  $\beta_\sigma$  and  $\beta_\tau$  while the lattice extension in the temporal direction,  $N_\tau$ , is obviously independent of  $z$ . For the sake of convenience, we take  $z_0 = 0$  in Eqs. (12), (14), and (15).

A temperature gradient has also been implemented in a recent study of another problem [44] without imposing the fixed-spatial constraint (13) potentially generating unwanted varying-volume effects. In addition, we notice that the non-linear trigonometric behavior of temperature inhomogeneities introduced in Ref. [44] differs from the thermodynamic formula (4) given by the solution of the Killing equation (2). Therefore, the trigonometric temperature inhomogeneities studied in Ref. [44] do not correspond to the global thermal equilibrium of the accelerating medium studied in our article.

### B. Setting up the couplings

One can consider a linear approximation with Eqs. (K.2.4), (K.2.24), and (K.2.25) of Ref. [42], where ‘‘K’’ in front of the equation corresponds to this paper by F. Karsch. It is convenient to parameterize the couplings as follows [42]:

$$\beta_\sigma = \frac{6}{g_\sigma^2(\mathbf{a}, \xi)} \frac{1}{\xi}, \quad \beta_\tau = \frac{6}{g_\tau^2(\mathbf{a}, \xi)} \xi, \quad (16)$$

where at the symmetric value,  $\xi = 1$ , the couplings are equal:

$$g_\tau(\mathbf{a}, 1) = g_\sigma(\mathbf{a}, 1) = g_E(\mathbf{a}). \quad (17)$$

For an asymmetric lattice,  $\xi \neq 1$ , the lattice couplings  $\beta_\tau$  and  $\beta_\sigma$  become different from each other. In the weak coupling limit, relevant to the continuum limit, they can be expanded in terms of the single symmetric coupling  $g_E$ :

$$\frac{1}{g_\sigma^2(\mathbf{a}, \xi)} = \frac{1}{g_E^2(\mathbf{a})} + c_\sigma(\xi) + O(g_E^3), \quad (18a)$$

$$\frac{1}{g_\tau^2(\mathbf{a}, \xi)} = \frac{1}{g_E^2(\mathbf{a})} + c_\tau(\xi) + O(g_E^3), \quad (18b)$$

where the functions  $c_\sigma(\xi)$  and  $c_\tau(\xi)$  are given in Ref. [42]. Thus, one should collect all relevant equations of Ref. [42], neglect  $O(g_E^3)$  terms in Eqs. (18) and use the anisotropy parameter (15) to generate the temperature gradient (12). However, one should do this carefully.

We rewrite Eqs. (16) and (18) as follows:

$$\beta_\sigma(\beta, \xi) = \frac{\beta}{\xi} + \frac{6}{\xi} c_\sigma(\xi), \quad (19a)$$

$$\beta_\tau(\beta, \xi) = \beta \xi + 6\xi c_\tau(\xi), \quad (19b)$$

where  $O(g_E^3)$  terms are neglected. We also denoted

$$\beta_\sigma = \frac{6}{g_\sigma^2} \frac{1}{\xi}, \quad \beta_\tau = \frac{6}{g_\tau^2} \xi, \quad \beta = \frac{6}{g_E^2}, \quad (20)$$



where  $\beta$  plays a role of the central value of the lattice coupling according to Eqs. (19). From now on, we will work only in terms of  $\beta$  and  $\xi$ .

We must work with the following conditions:

- (i) one has anisotropy between spatial and temporal lattice spacings (10);
- (ii) the spatial lattice spacing is fixed regardless of the value of the anisotropy.

In other words, we have to find the set of the lattice couplings  $\beta$  and  $\xi$  which fulfill the following conditions:

- (i) The anisotropy (10) is given by the value of  $\xi$ :

$$\xi = \frac{\mathbf{a}_\sigma(\beta, \xi)}{\mathbf{a}_\tau(\beta, \xi)}. \quad (21)$$

- (ii) At a fixed  $\xi$ , the value of  $\beta$  should be chosen in such a way that the spatial lattice spacing (13) is independent of the asymmetry  $\xi$  so that the variations in temperature do not affect the spatial geometry (size of the spatial directions) of the system:

$$\mathbf{a}_\sigma(\beta, \xi) = \mathbf{a}_0. \quad (22)$$

Condition (i) is already ensured by the choice of Eq. (19). However, condition (ii) is less trivial and has to be explicitly cared for.

In the case of the absence of anisotropy,  $\xi = 1$ , Eq. (22) means that a fixed value of  $\mathbf{a}$  is achieved at a fixed value of  $\beta$ . In the presence of a nontrivial anisotropy,  $\xi \neq 1$ , Eq. (22) implies that a fixed  $\mathbf{a}$  constrains a relation  $\beta = \beta_f(\xi, \mathbf{a}_0)$  between  $\beta$  and  $\xi$ , such that  $\mathbf{a}_\sigma(\beta_f(\xi, \mathbf{a}_0), \xi) = \mathbf{a}_0$ . Here the subscript “ $f$ ” in  $\beta_f$  means “fixed”.

Coming back to Ref. [42], we notice from Eq. (K.2.6) that the spatial lattice spacing has the following behavior:

$$\mathbf{a}(\beta, \xi) = \mathbf{a}(\beta) \exp \left[ -\frac{c_\sigma(\xi) + c_\tau(\xi)}{4b_0} \right], \quad (23)$$

where  $\mathbf{a}(\beta)$  is the lattice spacing for the isotropic lattice,  $\mathbf{a}(\beta) \equiv \mathbf{a}(\beta, \xi = 1)$  and

$$b_0 = \frac{11N_c}{48\pi^2} \equiv \frac{11}{16\pi^2} = 0.69\dots \quad \text{for } N_c = 3, \quad (24)$$

is the first coefficient of the beta function. According to Ref. [42], the correction in the spatial lattice spacing (23) due to non-trivial behavior of  $\xi$  is very small, of the order of 1%. The physical values of the isotropic lattice coupling  $\mathbf{a}(\beta)$  in Eq. (23) as functions of  $\beta$  can be found in Table 1 of Ref. [45].

The final functions defining the  $\beta(\xi)$  dependence are presented below:

$$\beta_\sigma(\beta_0, \xi) = \frac{1}{\xi} (\beta(\beta_0, \xi) + 6 \cdot 1.105 \cdot c_\sigma(\xi)), \quad (25a)$$

$$\beta_\tau(\beta_0, \xi) = \xi (\beta(\beta_0, \xi) + 6 \cdot c_\tau(\xi)), \quad (25b)$$

$$\beta(\beta_0, \xi) = \beta \left( \mathbf{a}(\beta_0) \exp \left[ \frac{c_\sigma(\xi) + c_\tau(\xi)}{4b_0} \right] \right), \quad (25c)$$

where function  $\beta(\mathbf{a})$  is the inverse function for  $\mathbf{a}(\beta)$  [45],  $\beta_0$  is the critical  $\beta$ -value for finite temperature. We performed calculations with the lattice size in temporal direction  $N_t = 8$  ( $\beta_0 = 6.07$ ). In function (25a) we add the additional multiplication factor 1.105 for improve stability of the  $\mathbf{a}_\sigma = \text{const}$  in the used  $\xi$ -range. We got the value of the factor by fitting lattice data.

At zero temperature, we explicitly calculated the asymmetry value  $\xi$  from the Creutz ratio shown in Fig. 5. We kept the spatial size of the lattice constant, and the resulting deviations were, again, less than one percent.

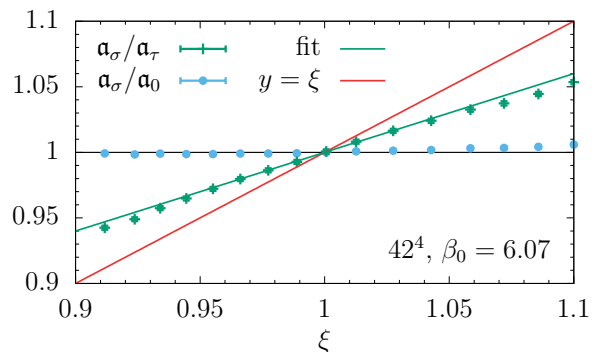


Figure 5. Relations  $\mathbf{a}_\sigma/\mathbf{a}_\tau$  and  $\mathbf{a}_\sigma/\mathbf{a}_0$  are obtained from the analysis of the Creutz ratio for Wilson loops  $5 \times 5$  for  $\beta_0 = 6.07$  on the lattice  $42^4$ . We used the line function for fit:  $y(\xi) = k\xi + (1-k)$  which gives us the following best fit value:  $k = 0.600(16)$ .