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# Spatial Precoding in Frequency Domain for Multi-User MIMO Affine Frequency Division Multiplexing

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**Abstract**—This paper deals with the potential of affine frequency division multiplexing (AFDM) to operate within a multi-user multi-input multi-output (MU-MIMO) system, facilitated by spatial precoding performed in frequency domain. To this end, we take advantage of the fact that AFDM based on discrete affine Fourier transform (DAFT) can be rewritten as a discrete Fourier transform (DFT)-based precoded orthogonal frequency division multiplexing (OFDM) waveform. By considering the well-known zero-forcing (ZF) precoder and white noise, we show that MU-MIMO AFDM achieves the same performance as MU-MIMO OFDM in terms of both sum rate and bit error rate (BER).

**Index Terms**—AFDM, 6G, multi-user, MIMO.

## I. INTRODUCTION

The affine frequency division multiplexing (AFDM) modulation scheme [1], [2] emerges as a promising candidate for sixth generation (6G) communications. It inherently exhibits robustness against both time and frequency selective channels [3]–[5], and fulfills certain requirements of 6G including integrated sensing and communications [3]–[6]. Based on discrete affine Fourier transform (DAFT), this approach can be viewed as a generalization of orthogonal chirp division multiplexing (OCDM) [7]. However, unlike OCDM, it allows for the adjustment of the number of chirps per symbol to address channel selectivity. In this regard, AFDM demonstrates robustness similar to that of OTFS [8] against multipath and mobility. However, AFDM operates on a "per symbol" modulation basis, while OTFS utilizes a "block of symbols" modulation scheme. Consequently, OTFS inherently introduces higher latency compared to AFDM. Nonetheless, recent research has focused more on OTFS, leading to the resolution of several technical challenges that still remain unaddressed in AFDM.

Among other aspects, the sensing capability of AFDM against OFDM has been explored in [9]. Additionally, the authors of [1], [2], [10]–[12] address pilot design, channel estimation, and equalization for the AFDM modulation scheme in the affine domain, *i.e.* specifically after DAFT demodulation. In [13], [14], the issue of channel estimation and equalization is addressed in the context of multi-input multi-output-AFDM (MIMO-AFDM), which extends the single-input single-output AFDM (SISO-AFDM) system, namely specifically in the affine domain. However, to the best of our knowledge, no multi-user (MU)-MIMO AFDM has been suggested in the literature to date. Recently in [15], it has been shown that the AFDM modulation based on I (DAFT) can be reformulated as a discrete Fourier transform (DFT)-based precoded OFDM waveform, subject to certain conditions on the chirps parameters. This modulation, claimed to be backward compatible with OCDM and OFDM, offers the simplicity of implementation and allows for generating AFDM signals at any desired sampling rate. Additionally, the solution proposed in [15] allows AFDM signal samples to be represented in the frequency domain as precoded data from the affine domain. Consequently, frequency processing employed in OFDM could be seamlessly applied in AFDM.

In this paper, we focus on the spatial precoding in the frequency domain to enable MU-MIMO AFDM, such as illustrated in Fig. 1, where a base station (BS) transmits signals to two user equipments (UEs). To this end, we reinterpret and summarize the DFT-based SISO-AFDM suggested in [15]. This highlights the modulation as a transformation of data from the affine domain through frequency domain, and ultimately, to time domain. Then we show that a spatial precoder in the frequency domain enables the digital separation of streams dedicated to different UEs. Furthermore, we analytically establish that

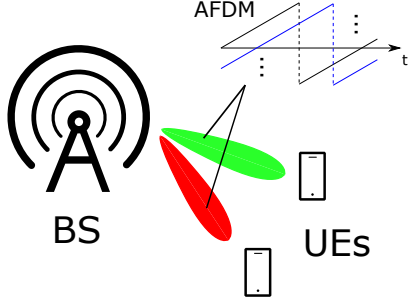


Fig. 1. MU-MIMO AFDM downlink transmission.

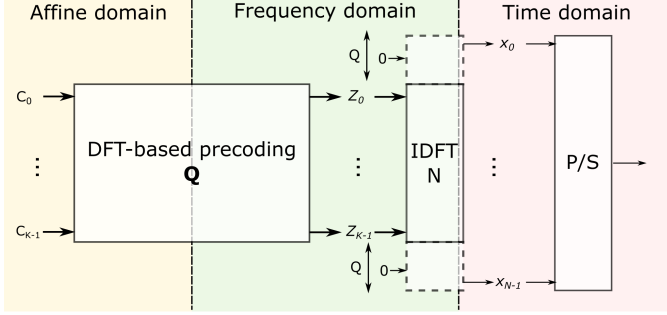


Fig. 2. DFT-based SISO AFDM transmission chain.

the achievable sum rate of UEs in MU-MIMO AFDM equals that of OFDM, in presence of white Gaussian noise. Numerical results also show that in the presence of colored noise, the sum rate of OFDM achieves a higher sum rate compared to AFDM. This improvement in the sum rate of OFDM can be attributed to the whitening effect of AFDM demodulation on the noise. Conversely, AFDM significantly outperforms OFDM in terms of BER in the presence of colored noise.

The rest of the paper is organized as follows: Section II summarized the DFT-based AFDM modulation in SISO system, and Section III introduces the spatial precoding in frequency domain applied to AFDM to allows for MU-MIMO AFDM. Subsequently, the corresponding demodulation is described in Section IV, where the theoretical sum rate is derived as well. Numerical results are provided in Section V and Section VI concludes this paper.

## II. SISO AFDM SYSTEM MODEL

The AFDM modulation scheme is originally based on DAFT [1], [10], such that the AFDM signal vector  $\mathbf{x} \in \mathbb{C}^{K \times 1}$  ( $K$  being the number of carrier chirps) can be expressed as

$$\mathbf{x} = \mathbf{\Lambda}_q \mathcal{F}_K^H \mathbf{\Lambda}_\alpha \mathbf{C}, \quad (1)$$

where  $\mathbf{C}$  of size  $K \times 1$  is the vector containing the data symbol  $C_m$  (the indices  $m = 0, 1, \dots, K - 1$  refer to the affine domain) randomly taken from a constellation, and  $\mathcal{F}_K^H$  is the IDFT matrix of size  $K \times K$  containing the element  $\frac{e^{2j\pi \frac{mn}{K}}}{\sqrt{K}}$  at position  $(n, m)$ ,  $n = 0, 1, \dots, K - 1$  ( $n$  refers to the time domain indices). The  $K \times K$  diagonal matrices  $\mathbf{\Lambda}_\alpha$  and  $\mathbf{\Lambda}_q$  contain the elements  $e^{\frac{j\pi \alpha m^2}{K}}$  and  $e^{\frac{j\pi q n^2}{K}}$  ( $n = 0, 1, \dots, K - 1$ ) on their diagonal, respectively. In turn,  $q$  and  $\alpha$  are two real-valued chirp parameters, where  $|q|$  indicates the number of chirps per symbol duration. It is important to note that the definition of (1) leads to a signal  $\mathbf{x}$  sampled at the Nyquist rate, transforming the vector of data  $\mathbf{C}$  from the affine domain to the time domain.

It is interesting to notice that recent findings in [15] demonstrate that AFDM can be characterized as precoded OFDM using a DFT-based modulation. To this end, both  $q$  and  $\frac{N}{q}$  must be integers. Furthermore, if  $\frac{N}{q}$  is even and  $q < 0$  (similar results can be obtained for odd  $\frac{N}{q}$  value and  $q > 0$ , but not tackled in this paper for conciseness matter), then the  $N \times 1$  AFDM signal vector  $\mathbf{x}$  ( $N \geq K$ ) can be rewritten as

$$\mathbf{x} = \mathcal{F}_N^H \mathbf{O} \underbrace{\mathbf{D}_c \mathbf{T}_{|q|K} \mathbf{D}_J \mathbf{A}}_{\mathbf{z}} \mathbf{C}, \quad (2)$$

where  $\mathbf{A}$  is a  $|q|K \times K$  complex matrix defined as

$$\mathbf{A} = \underbrace{[1, \dots, 1]}^{|q|} \otimes \text{diag}([A_0, \dots, A_{K-1}]), \quad (3)$$

with

$$A_m = \sqrt{|q|} e^{-j\frac{\pi}{4}} \exp\left(\frac{j\pi m^2 (\alpha - \frac{1}{q})}{K}\right). \quad (4)$$

The matrix  $\mathbf{D}_J$  of size  $|q|K \times |q|K$  is diagonal and contains the elements

$$D_{J,r,m} = \exp\left(-\frac{2j\pi r}{q} (m \bmod |q|)\right), \quad (5)$$

at diagonal index  $rK + m$ ,  $r = 0, 1, \dots, |q| - 1$ . Then,  $\mathcal{F}_{|q|K}$  is the DFT matrix of size  $|q|K \times |q|K$ ,  $\mathbf{T} = [\mathbf{I}_{K}, \mathbf{0}_{K \times (|q|-1)K}]$  is a  $K \times |q|K$  truncation matrix where  $\mathbf{I}_K$  is the  $K \times K$  identity matrix, and  $\mathbf{0}_{K \times (|q|-1)K}$  is the  $K \times (|q| - 1)K$  zero matrix. In addition,  $\mathbf{D}_c$  is the  $K \times K$  diagonal matrix containing the elements  $e^{-j\pi \frac{k^2}{qK}}$ ,  $k = 0, 1, \dots, K - 1$  on its diagonal ( $k$  refers to the frequency domain indices), and  $\mathbf{O} = [\mathbf{0}_{Q \times K}; \mathbf{I}_K; \mathbf{0}_{Q \times K}]$  represents a matrix of size  $N \times K$ , corresponding to the

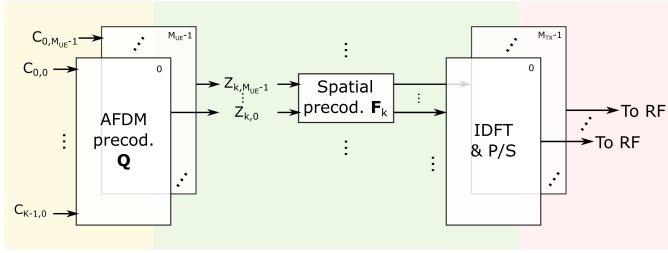


Fig. 3. Transmission chain of spatially precoded MU-MIMO AFDM transmitter.

null carrier concatenation at the edge of the band, where  $Q = \frac{N-K}{2}$ . Note that the matrix  $\mathbf{Q}$  allows to adjust the sampling rate of  $\mathbf{x}$  at any desired value according to the oversampling ratio  $\frac{N}{K}$ . Further details about DFT-based AFDM modulation can be found in [15].

The DFT-based AFDM modulation (2) is illustrated in Fig. 2, where the precoding corresponds to the matrix multiplication  $\mathbf{Q} = \mathbf{D}_c \mathbf{T} \mathcal{F}_{|q|K} \mathbf{D}_J \mathbf{A}$ ,  $\mathbf{Q}$  being of size  $K \times K$ . In addition, note that  $\mathbf{Z}$  is the  $K \times 1$  vector containing the elements  $Z_k$  of the precoded data in the frequency domain. Consequently, it is worth highlighting that AFDM, such defined in (2), can be actually viewed as precoded-OFDM, rendering AFDM backward compatible with OFDM [15]. In particular, it is noteworthy that when  $\mathbf{Q} = \mathbf{I}_K$ , AFDM corresponds to OFDM. Moreover, as the AFDM signal can be expressed in the frequency domain through  $\mathbf{X}$ , spatial precoding can be applied to extend SISO AFDM to MU-MIMO AFDM, such as presented herein.

### III. SPATIAL PRECODING FOR MU-MIMO AFDM

We consider a MU-MIMO system consisting of  $M_{\text{TX}}$  transmit (TX) antennas at the base station and  $M_{\text{UE}}$  user equipments (UEs), each equipped with a single antenna. We assume that each TX antenna is fed by one radio frequency (RF) chain. The MU-MIMO AFDM transmission chain is illustrated in Fig. 3. Subsequently, the input-output relation between the transmitted/received symbols in the frequency domain at subcarrier index  $k$  is expressed as

$$\mathbf{Y}_k = \mathbf{H}_k \mathbf{P}_k^{1/2} \mathbf{F}_k \mathbf{Z}_k + \mathbf{W}_k, \quad (6)$$

where  $\mathbf{Y}_k$  is the  $M_{\text{UE}} \times 1$  vector of the received signal in the frequency domain (prior to AFDM decoding), and  $\mathbf{Z}_k$  is the corresponding  $M_{\text{UE}} \times 1$  vector of the transmitted signal. Subsequently, the  $M_{\text{TX}} \times M_{\text{TX}}$  matrix  $\mathbf{P}_k = \text{diag}(p_0, \dots, p_{M_{\text{TX}}-1})$  contains the transmit power on each antenna. Then,  $\mathbf{W}_k$  is the  $M_{\text{UE}} \times 1$  vector

of the additive white Gaussian noise (AWGN) such that  $\mathbf{W}_k \sim \mathcal{NC}(\mathbf{0}_{M_{\text{UE}}}, \sigma^2 \mathbf{I}_{M_{\text{UE}}})$ . Moreover,  $\mathbf{H}_k$  is the  $M_{\text{UE}} \times M_{\text{TX}}$  matrix containing the coefficients of the channel frequency responses between the  $M_{\text{TX}}$  TX antennas and  $M_{\text{UE}}$  UEs at frequency index  $k$ . Alternatively,  $\mathbf{F}_k$  is the  $M_{\text{TX}} \times M_{\text{UE}}$  matrix of spatial precoding. The expression of  $\mathbf{F}_k$  depends on the chosen method. In this context, we particularly investigate the well known zero-forcing (ZF) precoder, for which the corresponding matrix is provided in [16] and written as follows:

$$\mathbf{F}_k^{\text{ZF}} = \mathbf{P}_k^{1/2} \mathbf{H}_k^H (\mathbf{H}_k \mathbf{P}_k \mathbf{H}_k^H)^{-1}. \quad (7)$$

The main advantage of the ZF precoder is to directly invert the matrix  $\mathbf{H}_k \mathbf{P}_k^{1/2}$ , therefore avoiding interferences between UEs. However, the problem must be well conditioned such that the matrix  $\mathbf{H}_k \mathbf{P}_k \mathbf{H}_k^H$  is actually invertible.

### IV. AFDM RECEIVER

We denote by  $\mathbf{Y}_i$  the  $K \times 1$  signal vector (in the frequency domain) received by the  $i$ th UE,  $i = 0, \dots, M_{\text{UE}} - 1$ . In turn, the element  $Y_{k,i}$  corresponds to the symbol received at the  $k$ th subcarrier by the  $i$ th UE, and is expressed as

$$Y_{k,i} = R_{k,i} + W_{k,i}, \quad (8)$$

where  $R_{k,i}$  corresponds to the received signal at the  $k$ th subcarrier by the  $i$ th UE when the noise is not considered. The  $K \times 1$  received data symbol vector is then expressed as

$$\begin{aligned} \hat{\mathbf{C}}_i &= \mathbf{Q}^{-1} \mathbf{Y}_i \\ &= \mathbf{Q}^{-1} \mathbf{R}_i + \mathbf{Q}^{-1} \mathbf{W}_i \\ &= \bar{\mathbf{R}}_i + \bar{\mathbf{W}}_i, \end{aligned} \quad (9)$$

where  $\mathbf{R}_i$  is the  $K \times 1$  vector that contains  $R_{k,i}$ . Moreover,  $\bar{\mathbf{R}}_i$  and  $\bar{\mathbf{W}}_i$  have been defined for clarity purpose. The sum rate (in bit/s/Hz) of the considered transmission is defined as the overall achievable spectral efficiency (SE) for all UEs, and can be expressed as

$$\Sigma = \frac{1}{K} \sum_{i=0}^{M_{\text{UE}}-1} \sum_{k=0}^{K-1} \log_2 \left( 1 + \frac{\mathbb{E}\{|\bar{R}_{k,i}|^2\}}{\mathbb{E}\{|\bar{W}_{k,i}|^2\}} \right), \quad (10)$$

where  $\frac{\mathbb{E}\{|\bar{R}_{k,i}|^2\}}{\mathbb{E}\{|\bar{W}_{k,i}|^2\}}$  is the per subcarrier and per UE signal-to-noise ratio (SNR). It is worth emphasizing that the additive noise is assumed to be white on the one hand,

and since the precoding matrix  $\mathbf{Q}$  (and then  $\mathbf{Q}^{-1}$ ) is orthonormal because it only involves phase shift and one DFT on the other hand, then  $\mathbb{E}\{|\bar{W}_{k,i}|^2\} = \sigma^2$  for any  $k$ . It results that the sum rate of AFDM should be the same as OFDM (*i.e.* when  $\mathbf{Q} = \mathbf{Id}_K$ ), and (10) reduces to:

$$\Sigma = \sum_{i=0}^{M_{\text{UE}}-1} \log_2(1 + \text{SNR}_i), \quad (11)$$

where  $\text{SNR}_i$  is the SNR of the  $i$ th user. Interestingly, if the additive noise is colored, indicating possible effects at the receiver side or the presence of interferers, the property  $\mathbb{E}\{|\bar{W}_{k,i}|^2\} = \sigma^2$  for any  $k$  still holds. This is because the matrix  $\mathbf{Q}^{-1}$  whitens the noise in the affine domain. However, this feature does not hold in OFDM, where  $\mathbf{Q}^{-1} = \mathbf{Id}_K$ . Therefore, OFDM should outperform AFDM in terms of sum rate under conditions of colored noise, as the capacity is minimal in the presence of additive white Gaussian noise. Mathematically, this result is due to the fact that the function  $\log_2(1 + \frac{1}{x})$  is convex for  $x \in \mathbb{R}^*$ , and then

$$\sum_{k=0}^{K-1} \log_2 \left( 1 + \frac{\mathbb{E}\{|\bar{R}_{k,i}|^2\}}{\mathbb{E}\{|\bar{W}_{k,i}|^2\}} \right) \leq \log_2(1 + \text{SNR}_i).$$

In contrast, AFDM should outperform OFDM in terms of bit error rate (BER), because the capability of a receiver to properly decode a noisy signal is maximized in the presence of white noise. This will be numerically verified through simulations in the next section.

## V. NUMERICAL RESULTS

### A. Simulations Parameters

Simulations have been carried out using Matlab to show the performance of the AFDM signal in a MU-MIMO context. A simple  $2 \times 2$  system is considered, whose general parameters are given in Table I. The values  $(x, y, z)$  refers to the Cartesian coordinates of the TX and RX, while  $f$  and  $P_t$  are the center frequency and the transmission power of the BS. A simple power allocation scheme is considered where the same power is allocated to all UEs. The channel has been generated by means of a ray tracing method. Four paths per link are simulated, with scatterers that are randomly positioned within the volume between the BS and the UEs. In order to generate the colored noise, a white Gaussian noise has been filtered by the four taps filter  $\frac{1}{2}[1, 1, 1, 1]$ . To obtain the BER performance, a 16-QAM constellation has been used, and the results have been averaged over 1000 independent simulations runs.

TABLE I  
SIMULATION PARAMETERS

Symbol	Value
$(x, y, z)$ TX0	(0.5,0,25) m
$(x, y, z)$ TX1	(-0.5,0,25) m
$(x, y, z)$ UE0	(5,10,2) m
$(x, y, z)$ UE1	(-5,10,2) m
$f$	6GHz
$P_t$	1W
$K, N$	256, 512
$q, \alpha$	-2,2

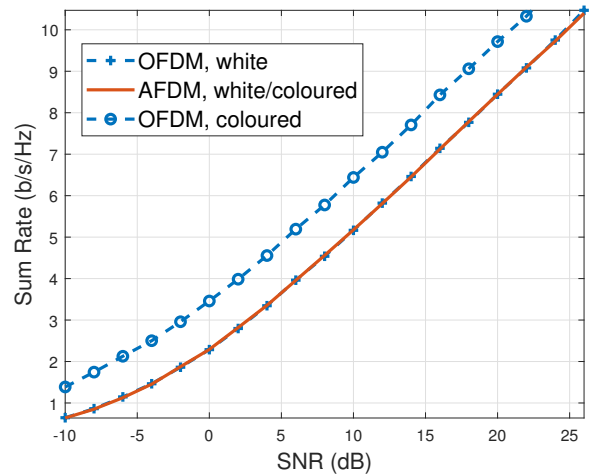


Fig. 4. Sum rate versus SNR (dB) for spatially precoded AFDM compared with OFDM. Comparison of white and colored additive noise.

### B. Simulation Results

Fig. 4 illustrates the sum rate  $\Sigma$ , defined in (10), plotted against SNR (dB) for MU-MIMO AFDM and OFDM, considering both white and colored Gaussian noise cases. It can be observed that the performance of AFDM matches that of OFDM in white noise, therefore numerically, confirming (11). Additionally, AFDM maintains consistent performance in colored noise, whereas OFDM demonstrates superior performance, as mentioned earlier. However, it is important to note that while the sum rate provides a theoretical achievable bound, a more practical metric should be considered for a fair comparison between MU-MIMO OFDM and AFDM.

In Fig. 5, we show the BER versus SNR (dB), using the same parameters as before for MU-MIMO AFDM and OFDM. Similar to previous observations, it can be noticed that the BER trajectories of AFDM and OFDM coincide in the case of white noise. Furthermore, AFDM maintains consistent BER performance in both white and

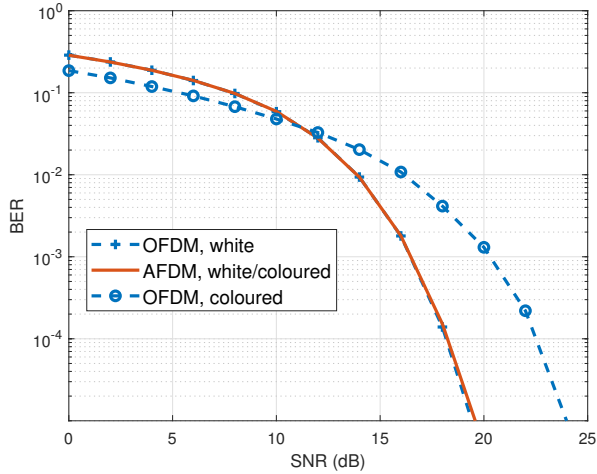


Fig. 5. BER versus SNR (dB) for spatially precoded AFDM compared with OFDM. Comparison of white and colored additive noise.

colored noise scenarios as AFDM decoding effectively whitens the noise. Conversely, AFDM exhibits superior BER performance compared to OFDM in colored noise for  $\text{SNR} > 12$  dB, achieving a gain of 4 dB at  $\text{BER} = 10^{-4}$ . The marginal advantage of OFDM over AFDM in the SNR range  $< 12$  dB is attributed to the unequal distribution of bit errors in the presence of colored noise in OFDM.

## VI. CONCLUSION

This paper presents the spatial precoding in frequency domain enabling AFDM to operate within a MU-MIMO system. To this end, we first summarize the SISO-AFDM as a DFT-based precoded OFDM, from which we express the frequency samples of the AFDM signal. These frequency samples can then undergo spatial precoding using methods such as ZF precoding. We analytically prove that MU-MIMO AFDM achieves the same sum rate as OFDM in white noise scenario. Furthermore, numerical results show that the sum rate of OFDM is improved in colored noise compared to AFDM. In contrast, MU-MIMO AFDM exhibits superior BER performance compared to OFDM in colored noise settings. This paper lays the groundwork for future investigations in (MU)-MIMO AFDM, encompassing areas such as MIMO-AFDM sensing and communication, and full-duplex MIMO-AFDM.

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