



**HAL**  
open science

# Fair burden-sharing for climate change mitigation: an axiomatic approach

Emma Jagu Schippers, David Lowing

► **To cite this version:**

Emma Jagu Schippers, David Lowing. Fair burden-sharing for climate change mitigation: an axiomatic approach. 2024. hal-04685868

**HAL Id: hal-04685868**

**<https://hal.science/hal-04685868v1>**

Preprint submitted on 3 Sep 2024

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Fair burden-sharing for climate change mitigation: an axiomatic approach

Emma Jagu Schippers \*      David Lowing \*

September 3, 2024

## Abstract

A significant challenge in climate change negotiations is establishing a burden-sharing method that all or most governments find fair. Two key fairness principles are emphasized by the United Nations Framework Convention on Climate Change in allocating mitigation efforts: the Polluter-Pays principle (“common but differentiated responsibilities”), suggesting that the countries with the highest greenhouse gas emissions should contribute more, and the Ability-to-Pay principle (“respective capabilities”), suggesting that economically advantaged countries should contribute more. This paper proposes a new burden-sharing method that integrates the Polluter-Pays and Ability-to-Pay principles without resorting to weighted indicators. We provide an algorithmic procedure to implement the method in polynomial time and conduct an axiomatic study to emphasize the significance of our approach. Finally, we apply our method using worldwide data.

**Keywords:** Climate mitigation; Burden-sharing; Axiomatic study

**JEL code:** Q54; C71

## 1 Introduction

Climate change, driven by the increasing concentrations of greenhouse gases (GHGs) in the atmosphere, is considered one of the major challenges of the 21st century. To avoid catastrophic long-term

---

\*Université Paris-Saclay, CentraleSupélec, Industrial Engineering Research Department, 3 rue Joliot-Curie, 91190 Gif-Sur-Yvette, France. Email: emma.jagu@centralesupelec.fr; david.lowing@centralesupelec.fr

consequences, the Paris Agreement aims to cap the global average temperature increase at no more than 2°C, with efforts to limit it to 1.5°C (UNFCCC (2015)). According to the IPCC (2022), to limit global warming to 1.5°C above pre-industrial levels, global CO<sub>2</sub> emissions need to reach net zero around mid-century. Consequently, there is a strong political need to allocate emissions reduction efforts to stay within the remaining carbon budgets. The issue of burden-sharing has remained central and contentious throughout the development of climate change agreements, including the Paris Agreement.

As a result, there has been a continuous research interest in examining the issue of how to share the burden of climate change mitigation since the 1990s, leading to a substantial number of publications utilizing different methodological approaches (Zhou and Wang (2016)). Typically grounded in microeconomic theory, these studies employ various models: for example, general equilibrium theory to allocate CO<sub>2</sub> emission permits (Jensen and Rasmussen (2000)), Nash-Cournot models to analyze cost pass-through rates (Wang and Zhou (2017)), and game-theoretical approaches to incentivize climate-change mitigation (Doncaster et al. (2017)). Cooperative game theory has also been used, with the Shapley value (Filar and Gaertner (1997)) and the Core (Eyckmans and Tulkens (2003)), addressing burden-sharing solutions and stability issues. Other approaches include climate policy based on climate liabilities (de Villemeur and Leroux (2019)) and applying bankruptcy theory to reallocate emissions (Algaba et al. (2023)). Additionally, the indicators on which burden-sharing methods are built upon include population, emissions, energy, GDP, or emission intensity, which can be used in a single indicator approach (Rose (1990); Rose and Stevens (1993); Torvanger (2023)), or in a composite indicator approach (Ringius et al. (2002); Holz et al. (2018); Pozo et al. (2020); van den Berg et al. (2020)) through a weighted sum or product.

Regardless of the methodological approach, the fairness of burden-sharing methods remains a central and contentious issue in climate mitigation efforts. This debate began with the creation of the United Nations Framework Convention on Climate Change (UNFCCC) in 1992, which established the principle of "common but differentiated responsibilities and respective capabilities (UNFCCC (1992))." As a result, developed countries often emphasize the need for major emerging economies to take on more significant roles in climate mitigation, whereas developing countries highlight developed nations' historical responsibility and greater capacity to act. Thus, a key issue in designing a burden-sharing method that aligns with the UNFCCC is incorporating the Responsibility (Polluter-Pays) principle and the Capability (Ability-to-Pay) principle. The **Ability-to-Pay principle** argues that countries with greater economic capacity should bear a larger share of the burden, with vari-

ables like GDP, GDP per capita, and revenue distribution often used to assess this principle. The **Polluter-Pays principle** posits that countries with the highest historical contribution to greenhouse gas emissions should bear a greater share of the burden<sup>1</sup>. The issue of merging the two principles has been addressed in the literature by designing weighted indicators, such as the Responsibility-Capability indicator (Baer et al. (2009); Holz et al. (2018)). These studies have effectively modeled balanced burden-sharing scenarios, demonstrating that wealthier countries' fair-share allocations usually demand mitigation efforts far exceeding their domestic potential, while poorer countries' fair-share allocations typically remain below their mitigation capacity. However, weighted indicators inherently require determining the appropriate weight for each principle, necessitating arbitration between Responsibility and Capability.

In this paper, we propose an alternative to weighted indicators by designing a burden-sharing method grounded in microeconomic theory that reconciles the Polluter-Pays and Ability-to-Pay principles while avoiding the need for arbitration between the two.

To achieve this, we begin by defining **burden-sharing problems** as a domain of problems characterized by four inputs: a set of countries, a real-valued vector representing the population size of each country, a classification of the countries based on their level of development, and real-valued vector representing the environmental burden of each country. The environmental burden of a country reflects the damage this country has inflicted on the environment, depicted by annual or historic GHG emissions. On this domain of problems, a **burden-sharing method** is a function that assigns, to each country, a share of the global environmental burden - i.e. the sum of all the countries' environmental burdens. We then characterize our burden-sharing method by adopting an axiomatic approach<sup>2</sup>. Specifically, we translate the Polluter-Pays and Ability-to-Pay principles into formal axioms for burden-sharing methods. One of this paper's main results is demonstrating a unique burden-sharing method that satisfies these axioms. We show that this method is easily implementable

---

<sup>1</sup>It is important to note that the choice of the reference date for calculating national contributions to greenhouse gas emissions remains contentious. If the Polluter-Pays principle only considers recent annual emissions, emerging or newly developed countries like China or India will bear a larger burden, whereas historic emissions since the Industrial Revolution would result in earlier-developed countries carrying a larger burden.

<sup>2</sup>In economics, the axiomatic approach is a normative procedure for problem-solving. Its goal is to identify suitable solutions based on mathematical properties known as axioms, defined according to fairness principles. By examining solutions satisfying these axioms, we can identify a family or a unique solution that meets all the criteria.

with numerical applications and effectively balances the Polluter-Pays and Ability-to-Pay principles without resorting to weighted indicators. In Appendix 5.2, we also demonstrate that our method is strongly related to recognized solution concepts from cooperative game theory. Finally, we apply our method to world data and compare it with burden-sharing methods that either apply the Polluter-Pays principle or the Ability-to-Pay principle. We confirm that the national burden shares obtained with our method are consistent with the axioms, with slight variations compared with methods that only verify the Polluter-Pays principle or the Ability-to-Pay principles.

In the following, Section 2 formalizes burden-sharing problems and presents our proposed method. It also presents a normative study of our proposition. Section 3 applies and discusses our burden-sharing method using GDP, population and GHG emissions data for 189 countries. Finally, we conclude in Section 4.

## 2 Methodology

This section formally describes our model and introduces our burden-sharing method. We provide an algorithm to compute this method, whose complexity is polynomial. Additionally, we propose a characterization of the method by invoking axioms grounded in the Polluter-Pays Principle and the Ability-to-Pay Principle. To properly describe our model, we must define a few mathematical notion beforehand.

Let  $N \subset \mathbb{N}$  be a finite and countable set of objects. A partition of  $N$  is a grouping of its elements into non-empty subsets, in such a way that every element of  $N$  is included in exactly one subset. Formally, a family of sub-sets  $\mathcal{C} = \{C_1, \dots, C_K\}$  is a partition of  $N$  if and only if it verifies all the following conditions:

1.  $\emptyset \notin \mathcal{C}$ ;
2.  $\bigcup_{1 \leq k \leq K} C_k = N$ ;
3. and  $C_k \cap C_{k'} = \emptyset$  for any two  $C_k, C_{k'} \in \mathcal{C}$ .

The partition  $\mathcal{C} = \{N\}$  is called the trivial partition. The partition  $\mathcal{C} = \{\{i\} \mid i \in N\}$  is called the finest partition. The set of all partitions of  $N$  is denoted by  $\mathbb{P}(N)$ . Take any  $i \in N$ . The element of  $\mathcal{C}$  to which  $i$  belongs may be denoted by  $C(i)$ . The cardinality of  $N$ , i.e., the number of elements

of a finite set of objects  $N$  is denoted by  $|N|$ . For each real-valued vector  $x \in \mathbb{R}^{|N|}$ , the sum of the coordinates of  $x$  is denoted by the norm  $\|x\| = \sum_{i \in N} x_i$ .

## 2.1 Model

Consider a finite and fixed set of **countries**  $N = \{1, \dots, n\}$ , with  $|N| = n$ . Each country  $i \in N$  is assigned an **environmental burden**  $b_i \in \mathbb{R}_+$ , representing, for example, its cumulated greenhouse gas emissions. The vector of environmental burdens is denoted by  $b \in \mathbb{R}_+^n$ . In addition, each country  $i \in N$  is assigned a fixed **population**  $p_i \in \mathbb{R}_{++}$ , representing, for example, its number of inhabitants. The vector of populations is denoted by  $p \in \mathbb{R}_{++}^n$ .

Let  $C = \{C_1, \dots, C_K\} \in \mathbb{P}(N)$  be a partition of  $N$  into  $K$  elements. Each element of  $C$  represents a class of countries with a similar level of development (typically measured by GDP). In addition, the elements of  $C$  are linearly ranked based on their indices. This ranking reflects the varying level of development among the countries. A country in  $C_k$  has a higher level than countries in  $C_{k-1}$  and a lower level than countries in  $C_{k+1}$ . Hereafter, we refer to a partition  $C$  of  $N$  as a **classification** of  $N$ .

**Remark 2.1.** Crucially, the parametrization of inputs affects how fairness principles are embodied in the resulting burden-shares (as discussed in Section 3.4). In particular, the classification of countries has a profound impact on the representation of the Ability-to-Pay principle. To enhance the clarity of our results, we narrowed down the scope of relevant country classifications in Section 3.2.

The environmental burden per class (1) and global environmental burden (2) are respectively defined as

$$\forall C_k \in C, \quad B_k = \sum_{i \in C_k} b_i, \quad (1)$$

$$\text{and} \quad B = \sum_{1 \leq k \leq K} B_k. \quad (2)$$

**Definition 2.2.** A tuple  $(N, b, p, C)$  is called a **Burden-sharing problem**. It represents a set of countries  $N$  which are asymmetric with respect to their environmental burden, their population and their level of development. The problem is to determine how to share the global environment burden (2) fairly among the countries based on these asymmetries. The set of all burden-sharing problems is denoted by  $\mathbb{B}$ .

## 2.2 Burden-sharing method

The main issue addressed in this paper is to determine how much should each country contribute to mitigating climate change. To that end, we seek a fair and relevant method to share the global environmental burden among the countries. A burden-sharing vector for some problem  $(N, b, p, C) \in \mathbb{B}$  is a vector  $x \in \mathbb{R}_+^n$  verifying  $\|x\| = B$ . A **burden-sharing method**  $f : \mathbb{B} \rightarrow \mathbb{R}_+^n$  is a map that associates a burden-sharing vector  $f(N, b, p, C) \in \mathbb{R}_+^n$  to each burden-sharing problem  $(N, b, p, C) \in \mathbb{B}$ . This map should take into account all the inputs specified by the problem.

---

### Algorithm 1: Computation of $\varphi$

---

**Data:** Countries  $N$ , environmental burden  $b$ , population  $p$ , classification  $C$ .

$\varphi \leftarrow (0, \dots, 0)$

**for**  $1 \leq k \leq K$  **do**

**for**  $j \in C_k$  **do**

**for**  $i \in N$  **do**

**if**  $i = j$  **then**

$$\varphi_i \leftarrow \varphi_i + \frac{b_j}{K - k + 1}$$

**else if**  $i \in \bigcup_{l > k} C_l$  **then**

$$\varphi_i \leftarrow \varphi_i + \frac{p_i}{\sum_{i' \in C(i)} p_{i'}} \times \frac{b_j}{K - k + 1}$$

**end**

**else**

$$\varphi_i \leftarrow \varphi_i$$

**end**

**end**

**end**

**end**

**Result:**  $\varphi$

---

In this paper, we propose an original burden-sharing method  $\varphi$ , which can be defined by the following procedure. The method divides the environmental burden of each country  $i \in N$  into  $K - k + 1$  identical shares, where  $k$  is the index of the class  $C_k$  to which country  $i$  belongs, i.e.,  $C(i) = C_k$ . Country  $i$  is then assigned one of these shares, and the remaining shares are assigned to the  $K - k$  classes ranked above  $i$ . Within each class, each share is fairly divided among the countries in that class proportionally to their population size. This procedure is applied to the environmental

burden of each country. It can be implemented according to Algorithm 1. Observe that the complexity of this procedure is polynomial.

In addition to this procedure, we provide an analytical representation of our burden-sharing method  $\varphi$ . According to this representation, each country  $i \in N$  is assigned two components. First, they bear a share of their own burden,  $\frac{b_i}{K-k+1}$ , where  $k$  is the index of the class to which they belong. The remainder of their burden is supported by higher-ranked countries. Second, they support a share of the burden of lower-ranked countries,  $\sum_{l < k} \frac{B_l}{K-l+1}$ . This share is proportional to their relative population compared to other countries within the same class.

**Definition 2.3.** The burden-sharing method  $\varphi$  is defined, for each  $(N, b, p, C) \in \mathbb{B}$ , as

$$\forall C_k \in C, \forall i \in C_k, \quad \varphi_i(N, b, p, C) = \frac{b_i}{K-k+1} + \frac{p_i}{\sum_{i' \in C_k} p_{i'}} \times \sum_{l < k} \frac{B_l}{K-l+1}.$$

From Algorithm 1 and Definition 2.3, it is clear that our method ensures that each country bears a certain share of its own environmental burden (Polluter-Pays principle) while also supporting lower-ranked peers by endorsing a share of their burden (Ability-to-Pay principle). Our burden-sharing method strikes a balance between the Ability-to-Pay principle and the Polluter-Pays principle. It applies these two principles cumulatively without any exogenous arbitration, distinguishing it from existing approaches in the literature. To better understand how  $\varphi$  balances the Ability-to-Pay principle and the Polluter-Pays principle, we conduct an axiomatic study in Section 2.3 highlighting key properties characterizing our burden-sharing method.

**Example 2.4.** Consider an illustrative burden-sharing problem with only seven countries, i.e.  $N = \{1, \dots, 7\}$ , partitioned into three classes. To illustrate how  $\varphi$  works, consider  $b_1$  the environmental burden held by country 1. Let us follow the procedure depicted by Algorithm 1 and divide  $b_1$  among the countries (see Figure 1). First,  $b_1$  is divided into three equal shares, which corresponds to the number of classes no lower than the class to which 1 belongs. Then, 1 is assigned one of these shares. The remaining shares are assigned to member countries of class  $C_2$  and class  $C_3$ . Within each class,  $c_1/3$  is fairly divided among its member countries in proportion to their population. For instance, country 4 is assigned a portion of  $c_1/3$  that is determined by its relative population compared to country 5, i.e.,  $\frac{p_4}{p_4+p_5} \frac{b_1}{3}$ . Similarly, country 7 is assigned a portion of  $c_1/3$  that is determined by its relative population compared to country 6, i.e.,  $\frac{p_7}{p_6+p_7} \frac{b_1}{3}$ . Observe that neither 2 or 3 have to bear a share of 1's burden.

**Remark 2.5.** Observe that if the classification is the trivial partition, i.e.,  $C = \{N\}$ , then our method will assign, to each country, the entirety of their environmental burden, i.e.,  $\varphi_i(N, b, p, \{N\}) = b_i$



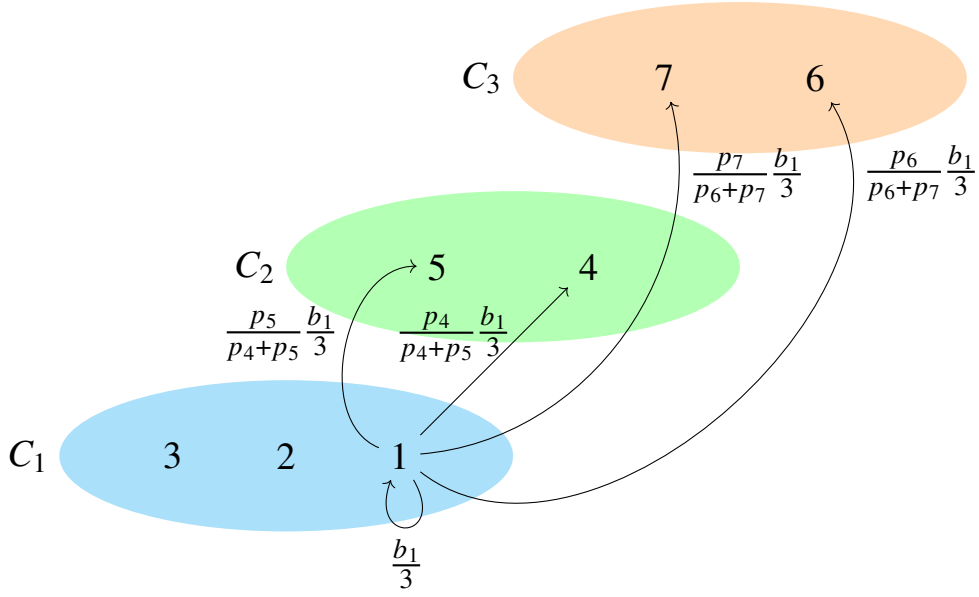


Figure 1: Redistribution of  $b_1$  according to  $\varphi$

for each  $i \in N$ . Such burden-sharing can be viewed as completely aligned with the Polluter-Pays principle. Conversely, if the classification is the finest partition, i.e.,  $C = \{\{i\} \mid i \in N\}$ , then our method departs from the Polluter-Pays principle and is philosophically closer to the Ability-to-Pay principle. However, it cannot be viewed as completely aligned with the Ability-to-Pay principle as defined in the current methods employed in the literature.

**Remark 2.6.** The burden-sharing method  $\varphi$  is strongly related to cooperative game theory. In Appendix 5.2, we show that it is possible to derive a cooperative game from any burden-sharing problem. Then, we demonstrate that the burden-sharing method  $\varphi$  coincides with a particular solution for such cooperative games. This solution is inspired by the Owen value and the Permission value for cooperative games, respectively introduced by Owen (1977) and Gilles et al. (1992). These two values are well-known in the economic literature and have been applied to a broad spectrum of research questions, including polluted river problems (see Ni and Wang, 2007; Dong et al., 2012; van den Brink et al., 2018), liability sharing problems (see Dehez and Ferey, 2013; Ferey and Dehez, 2016; Oishi et al., 2023), and fees allocation (see Vazquez-Brage et al., 1997). The Permission and the Owen value both generalize the well-known Shapley value introduced by Shapley (1953). They also have strong axiomatic foundations (see van den Brink, 2017; Khmel'nitskaya and Yanovskaya, 2007; Albizuri, 2008) that support their appeal. Technical details and proofs are provided in Appendix 5.2.

### 2.3 Axiomatic foundation

We now introduce mathematical properties for burden-sharing methods, referred to as **axioms**. These axioms are new and they align with the Ability-to-Pay and Polluter-Pays principles. They are all based on the thought experiment that one country increases (or decreases) its environmental burden. This increase will have consequences on the prescription of the method. Each axiom describes different consequences with their own interpretation.

Our first axiom ensures that a country is not obligated to contribute to easing the burden of more developed countries nor the burden of countries at a similar level of development. Essentially, this axiom establishes independence, protecting countries from the increase in environmental burdens of their more developed or equally developed peers. It also implies that a country's burden share may depend on its own environmental burden (Ability-to-Pay principle) and that of less developed countries (Polluter-Pays principle).

**Axiom 1** (Higher Ranking Independence (**HRI**)). Pick any two  $(N, b, p, C), (N, b', p, C) \in \mathbb{B}$ , any  $C_k \leq C$  and any  $i \in C_k$ . If  $b_j = b'_j$  for any  $j \neq i$ , then

$$\forall j \in \bigcup_{l \leq k} C_l \setminus \{i\}, \quad f_j(N, b, p, C) = f_j(N, b', p, C).$$

The next axiom advocates that countries at a comparable level of development ought to contribute equitably to alleviating the burden of less developed countries. Formally, if a country increases its environmental burden, any two other countries within an higher-ranked class should endorse an share of that increase proportional to their population. This axiom is consistent with the Ability-to-Pay principle, emphasizing the idea that countries with similar economic capacities should share the responsibility of mitigating the challenges faced by their less developed peers.

**Axiom 2** (Intra-Fairness (**IAF**)). Pick any two  $(N, b, p, C), (N, b', p, C) \in \mathbb{B}$ , any  $C_k \leq C$  and any  $i \in C_k$ . If  $b_j = b'_j$  for any  $j \neq i$ , then for any  $l > k$  and any two  $j, j' \in C_l$ ,

$$\frac{1}{p_j} \left( f_j(N, b, p, C) - f_j(N, b', p, C) \right) = \frac{1}{p_{j'}} \left( f_{j'}(N, b, p, C) - f_{j'}(N, b', p, C) \right).$$

The next axiom addresses the aggregated burden assigned to a class. If a country increases its environmental burden, the axiom advocates that this will have an impact on the classes of countries with higher levels of development. This impact should be identical on each class, regardless of their indices. This axiom is consistent with the Ability-to-Pay principle, emphasizing the idea that classes of countries should be equally responsible for less developed countries.

**Axiom 3** (Inter-Fairness (**IEF**)). Pick any two  $(N, b, p, C), (N, b', p, C) \in \mathbb{B}$ , any  $C_k \leq C$  and any  $i \in C_k$ . If  $b_j = b'_j$  for any  $j \neq i$ , then

$$\forall l, l' \geq k, \quad \sum_{j \in C_l} [f_j(N, b, p, C) - f_j(N, b', p, C)] = \sum_{j \in C_{l'}} [f_j(N, b, p, C) - f_j(N, b', p, C)].$$

Combining these three axioms results in a characterization of the burden-sharing method  $\varphi$ . This implies that  $\varphi$  stands as the unique method that satisfies all three axioms, with each axiom playing a crucial role in defining the method.

**Theorem 2.7.** A burden-sharing method  $f$  satisfies (HRI), (IAF) and (IEF) on  $\mathbb{B}$  if and only if  $f = \varphi$ .

**Proof.** See Appendix 5.1. □

Our methodology strikes a balance between the Polluter-Pays and the Ability-to-Pay principles (see Figure 2). By definition, it is partially in line with the Polluter-Pays principle since it relies on input  $b$  representing the environmental burden attributable to the countries. However, our burden-sharing method to redistribute  $b$  is characterized by axioms that are either completely or partially in line with the Ability-to-Pay principle. Consequently, the entire methodology can be viewed as a balance between the Polluter-Pays and the Ability-to-Pay principles.

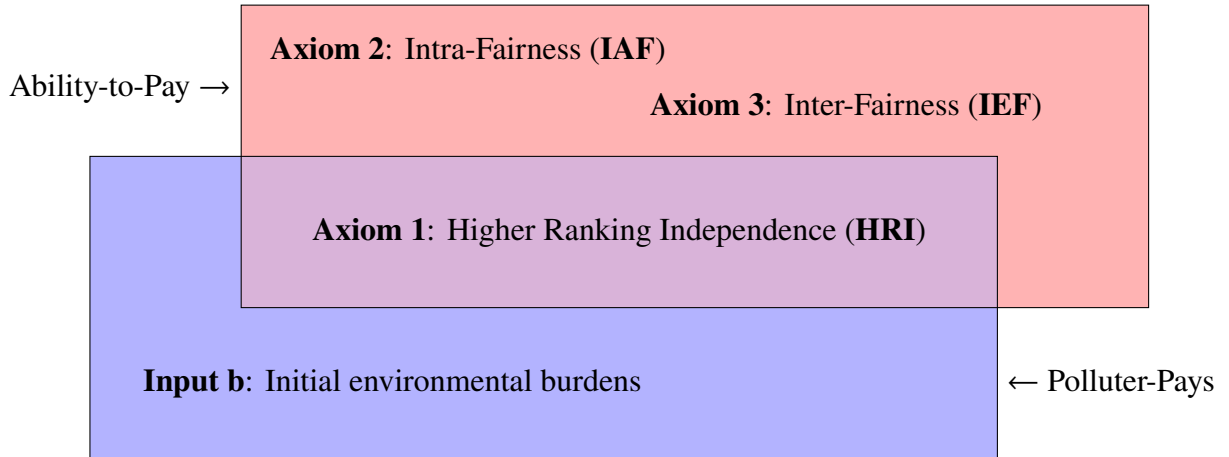


Figure 2: Balancing the Polluter-Pays and the Ability-to-Pay principles

### 3 Application

This section applies our burden-sharing method using worldwide data following a three-step process: data collection (Section 3.1), classification of countries by development levels (Section 3.2), and implementation/comparison of the burden-sharing methods (Section 3.3).

### 3.1 Step 1: Data collection

We collect data from two sources: the World Development Indicators from Bank (2023) and Historic GHG Emissions from Watch (2024). After merging the two datasets and excluding countries with incomplete data, we have a remaining total of 189 countries.

### 3.2 Step 2: Classification

The classification of countries into classes involves two critical decisions: the number of classes and the allocation of countries to each class. Let us start with the latter. We aim to organize the categories by their level of development, commonly measured by GDP per capita. Thus, countries are placed into the classes in ascending order based on their GDP per capita. But, the question remains: how many countries should each class contain? Drawing on the axiom (IEF) from Section 2.3, we propose the following principle as a guideline for the classification:

**Per capita Inter-fairness:** Within a country, each individual endorses a share of the burden of less developed countries, according to our burden-sharing method. On the other hand, the (IEF) axiom advocates that two classes of countries should equally support lower-ranked ones. By extending the idea of (IEF) to individuals, we can define a per capita Inter-fairness principle. This principle asserts that two individuals, regardless of the development level of their respective countries, should bear an equal share of the burden for supporting a less developed country. Formally, pick any two  $(N, b, p, C), (N, b', p, C) \in \mathbb{B}$ , any  $C_k \leq C$  and any  $i \in C_k$ . If  $b_j = b'_j$  for any  $j \neq i$ , then, for any two  $l, l' \geq k$ ,

$$\forall i \in C_l, \forall i' \in C_{l'}, \quad f_i(N, b, p, C) - f_i(N, b', p, C) = f_{i'}(N, b, p, C) - f_{i'}(N, b', p, C). \quad (3)$$

Given the nature of our burden-sharing method, one key requirement for (3) to hold is that the population size of each class should ideally be equal. However, achieving equal population sizes may not always be feasible. As a result, we may need to deviate from strictly adhering to (3). Instead, our goal is to develop a classification system that closely approximates (3) by minimizing the disparity in support among individuals from different countries and classes. This approach aligns with the per capita Inter-fairness principle as closely as possible.

Turning now to the first consideration in our classification decision — the determination of the number of classes — it is important to note that a larger number of classes tends to skew the burden

on the most developed countries. However, given the global data constraints and the presence of populous nations such as China and India, our application must accommodate outliers. Consequently, to maintain classes of equal population sizes, we define five classes, with China and India each forming unique classes due to their substantial populations. Figure 3 presents key information on the five classes: number of countries, total population, cumulative greenhouse gas emissions, and total GDP.

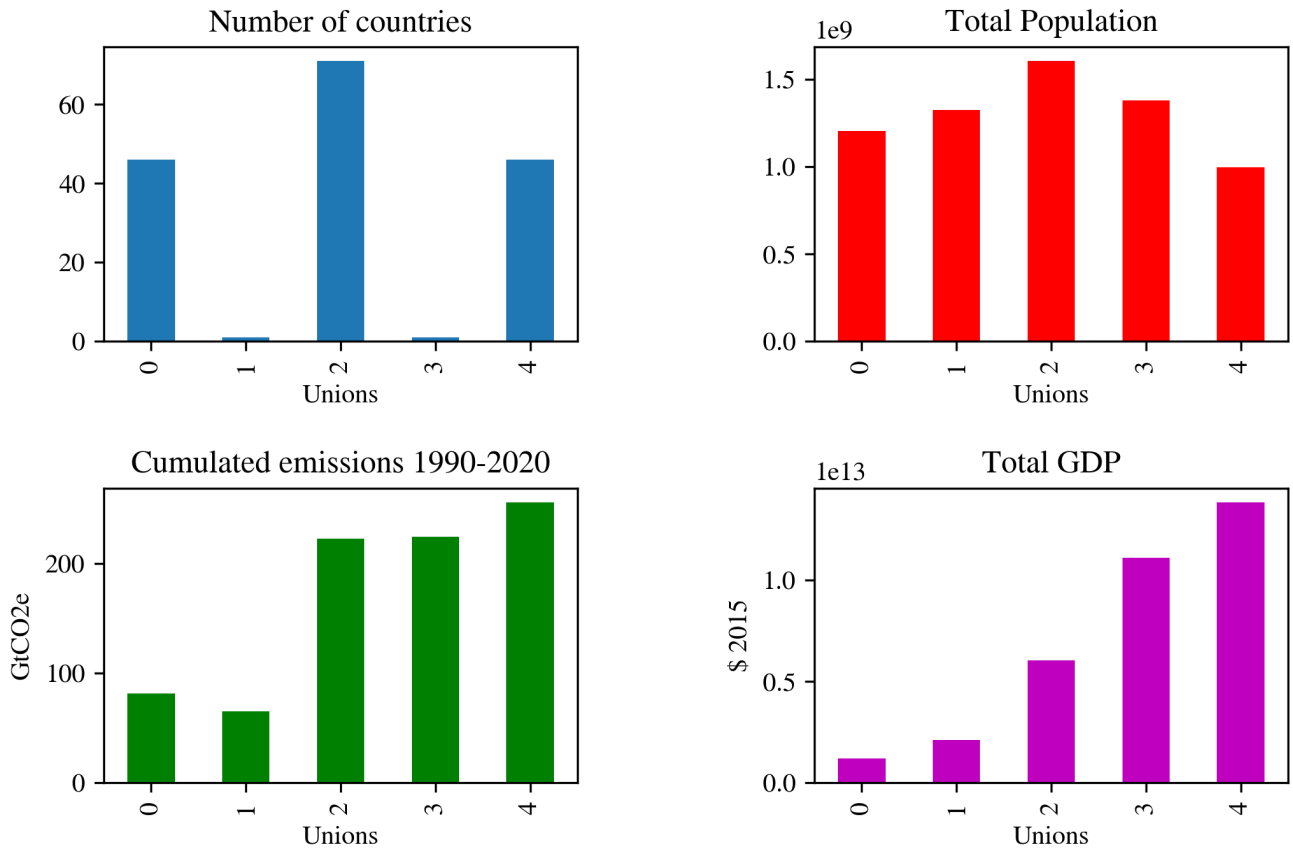


Figure 3: Key information on the five classes.

### 3.3 Step 3: Implementation/comparison of the burden-sharing methods

In this section, we compare our burden-sharing approach with methods that rely on simple indicators, specifically focusing on Polluter-Pays and Ability-to-Pay indicators.

#### 3.3.1 Polluter-Pays indicators

The selection of an appropriate indicator to embody the Polluter-Pays principle presents significant challenges, primarily concerning the choice of proxy variables (such as GHG emissions, CO<sub>2</sub> emis-

sions, or energy consumption) and the determination of relevant reference dates. The implications for emerging countries can differ substantially depending on whether emissions are considered for a single year (e.g., current annual emissions) or accumulated over a historical period (from 1850 to the present, or from 1990 to the present).

In our analysis, we explore two scenarios to illustrate these variances: one where the burden is allocated based on GHG emissions from the year 2020, and another where it is based on the cumulative GHG emissions from the period 1990 to 2020. This approach allows us to illustrate how different temporal frames can influence the distribution of responsibilities under the Polluter-Pays principle.

### **3.3.2 Ability-to-Pay indicator**

Indicators to represent the Ability to Pay principle in climate burden sharing typically involve metrics that reflect a country's economic capacity and wealth. These indicators are used to determine how much each country can afford to contribute to climate change mitigation and adaptation efforts. Beyond GDP and GDP per capita, some analyses also incorporate national revenue distribution as a key indicator.

Given the data available in our current database, we have selected 2015 GDP as the representative indicator for demonstrating the Ability-to-Pay principle.

### **3.3.3 Polluter-Pays/Ability-to-Pay method**

Our method integrates the principles of Polluter-Pays and Ability-to-Pay without relying on composite indicators. Initially, we employ a Polluter-Pays indicator, choosing either 2020 GHG emissions or cumulative GHG emissions from 1990 to 2020, to establish the initial environmental burden attributed to each country. Subsequently, we apply the Ability-to-Pay principle by redistributing these environmental burdens based on a development-level classification. The specifics of this classification are detailed in Section 3.2.

## **3.4 Discussion**

Figure 4 illustrates our numerical application by showing the burdens resulting from the three approaches (Polluter-Pays only, Ability-to-Pay only, and the Polluter-Pays/Ability-to-Pay method) in a selection of countries. The figure shows that our Polluter-Pays/Ability-to-Pay method produces burden-shares that can fall between, above, or below those generated by Polluter-Pays only and

Ability-to-Pay only methods. For example, burden-shares for the USA, the UK, and Russia fall between the two, while Brazil's is higher and India's is lower. This can be explained by our method's design, which operates as a sequence of both principles rather than a weighted average. Initially, the Polluter-Pays principle is applied to allocate environmental burdens. Subsequently, the Ability-to-Pay principle redistributes these burdens according to the three axioms detailed in Section 2.3. Thus, the resulting burden does not need to lie within the range defined by the Polluter-Pays only and Ability-to-Pay only methods.

As with all burden-sharing methods, choosing proxy variables representing each fairness principle influences the resulting burdens. This is evident for the Polluter-Pays principle, where the choice of the reference year for GHG emissions calculations significantly impacts the burden distribution. In Figure 4, we compare the impact of representing the Polluter-Pays principle with 2020 annual GHG emissions versus historic GHG emissions from 1990 to 2020. As expected, emerging countries like India and China bear a higher burden with 2020 emissions, while developed countries bear a larger burden with historic emissions.

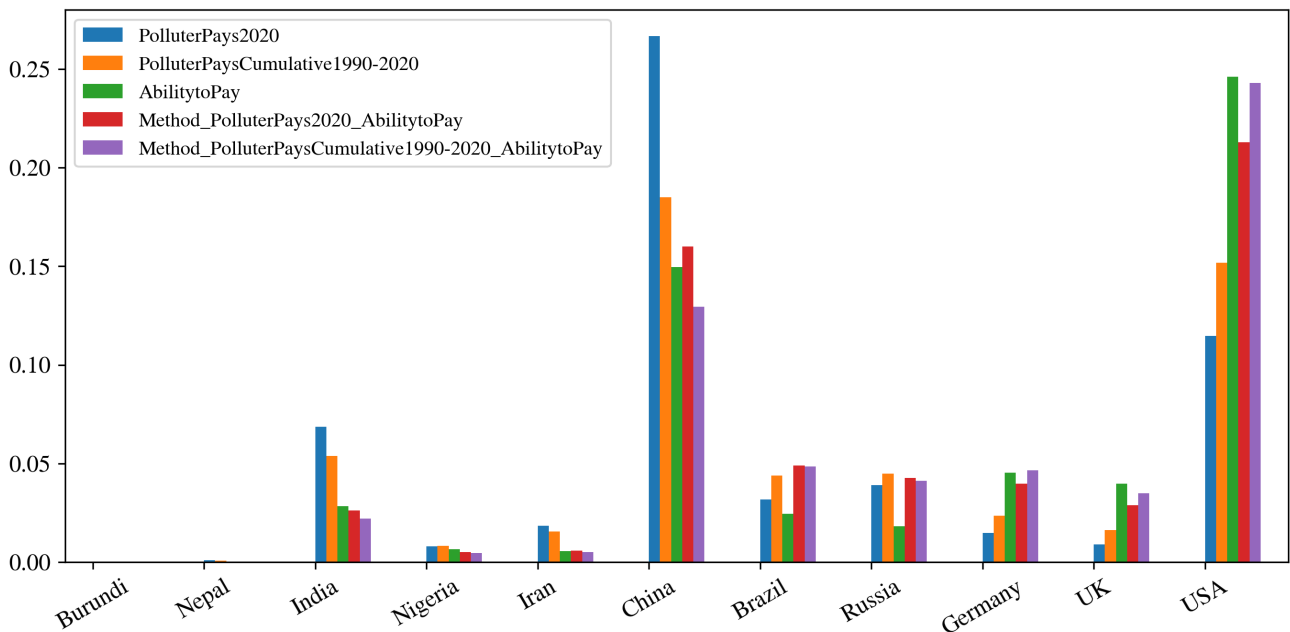


Figure 4: Comparison of different burden-sharing methods.

Although not shown here, the Ability-to-Pay principle is also sensitive to the choice of proxy variables. In this study, we used GDP per capita in 2015, but using a different reference year would alter the results. Moreover, some studies have used a more detailed representation of the Ability-to-Pay principle by decomposing countries into income deciles (Holz et al. (2018)). Such a decomposition

could be an interesting extension of the present application.

Finally, our burden-sharing method relies on parameters that require careful consideration, particularly the classification of countries by development level. The number of classes and the metrics used to allocate countries into each class significantly influence the resulting burden distribution. In one extreme, having only one class equates to the Polluter-Pays only method, as there is no redistribution. In the other extreme, where each country is its own class, the country with the highest GDP per capita (Liechtenstein in our dataset) would bear a disproportionate burden, shouldering solidarity for all other countries. Generally, the more classes there are, the greater the burden is on the most developed classes. In this study, we created classes based on GDP per capita, ensuring each class had a similar population size. This approach led us to form five classes due to the large populations of India and China.

## 4 Conclusion

The fairness of burden-sharing methods - determining the allocation of emissions reduction efforts among countries to limit global warming - is a crucial and contentious issue in climate negotiations. This debate dates back to the establishment of the United Nations Framework Convention on Climate Change (UNFCCC) in 1992, which introduced the principle of "common but differentiated responsibilities and respective capabilities" (UNFCCC (1992)). Therefore, a significant challenge in formulating a fair burden-sharing approach in line with the UNFCCC principles is balancing the Responsibility (Polluter-Pays) principle with the Capability (Ability-to-Pay) principle. Typically, this is done by creating weighted indicators for both principles. However, such an indicator requires deciding the relative importance to be given to responsibility (Polluter-Pays) versus capacity (Ability-to-Pay). That decision is a contentious aspect of international climate negotiations, as countries often diverge based on their past emissions and present economic conditions. Developed nations, which typically have substantial historical emissions, may favor placing more importance on economic capacity. Conversely, developing nations with relatively lower historical emissions are likely to highlight the significance of historical responsibility.

This paper introduces a methodological innovation that offers an alternative to using weighted indicators, thereby avoiding the trade-off between the Polluter-Pays and Ability-to-Pay principles. Instead, we apply these two principles through an axiomatic approach to ensure the fairness of our solution. Initially, the Polluter-Pays principle is upheld by attributing burdens in proportion to



GHG emissions. Then, the burdens are partially redistributed in accordance with the Ability-to-Pay principle, guided by three axioms: Higher Ranking Independence, Inter-Fairness, and Intra-Fairness. Notably, the burden-sharing method that satisfies these three axioms is unique and can be computed efficiently in polynomial time.

The numerical application illustrates the operability of our method, and we compare our results to burden-sharing methods that verify only the Polluter-Pays or only the Ability-to-Pay principles. Like all burden-sharing methods, we observe that the choice of proxy variables (in our case, population, national GHG emissions, and GDP per capita) significantly influences the resulting national burdens. Furthermore, our method specifically requires classifying countries according to development levels. The number of classes and the metric used to allocate countries to each class importantly influence the resulting national burdens. In this study, we narrowed down the possible classifications by applying a principle of "Per capita Inter-fairness". This principle asserts that two individuals, regardless of their country's development level, should bear an equal share of the burden for supporting a less developed country. To achieve this, we ensured that each class, ordered by GDP per capita, had a similar population size.

An interesting future research avenue could further explore how classifying countries by development level influences the resulting national burden shares. For example, by providing alternatives to the "Per capita Inter-fairness" principle or the GDP per capita indicator.

## **Acknowledgment**

We thank Albert Banal-Estanol, Cyril Chambefort, Léa Munich, Jean Christophe Péreau, Romain Presty, Sandrine Spaeter-Loehrer, for useful comments. We thank the participants of the LGI Working Paper days at CentraleSupélec, the LERN seminar at Rouen University, the JMA conference in Lille, the AFSE conference in Bordeaux, the EAERE conference in Leuven, and the FAERE conference in Strasbourg for their insightful remarks. Emma Jagu Schippers is grateful for financial support provided by the *Openlab Economie du Carbone (Stellantis)*. David Lowing is grateful for financial support provided by the *Economics of Gas chair*.

## 5 Appendix

### 5.1 Proof of Theorem 2.7

This section contains the proof of our axiomatic result. In addition, we provide counter-examples to show that the result is tight, i.e., all axioms are necessary.

**Proof.** It is direct to show that the method  $\varphi$  satisfies (HRI), (IAF) and (IEF). Next, consider a method  $f$  satisfying (HRI), (IAF) and (IEF). Let us show that  $f(N, b, p, C)$  is uniquely determined for each  $(N, b, p, C) \in \mathbb{B}$ . We show by induction on  $0(b)$ , the number of null coordinates of  $b$ , that  $f(N, b, p, C)$  is uniquely determined.

**Initialization:** If  $0(b) = n$ , i.e.,  $b = (0, \dots, 0)$ , then, by definition of a burden-sharing method,  $f_i(N, b, p, C) = 0$  for each  $i \in N$ . Thus,  $f(N, b, p, C)$  is uniquely determined.

**Hypothesis:** Assume that  $f(N, b, p, C)$  is uniquely determined if  $0(b) = W$ ,  $1 \leq W < n$ .

**Induction:** Let  $0(b) = W - 1$ . Pick any  $i \in N$  such that  $b_i > 0$ . Consider  $b'$  such that  $b'_j = b_j$  for each  $j \neq i$ , and  $b'_i = 0$ . Obviously,  $0(b') = W$ . Therefore,  $f(N, b', p, C)$  is uniquely determined by the Hypothesis. Denote by  $C_k$  the class to which  $i$  belongs. By (HRI),

$$\forall j \in \bigcup_{l \leq k} C_l \setminus \{i\}, \quad f_j(N, b, p, C) = f_j(N, b', p, C) \quad (4)$$

By (IEF) and (4),

$$\begin{aligned} \forall l \geq k, \quad \sum_{j \in C_l} [f_j(N, b', p, C) - f_j(N, b, p, C)] &= \sum_{j \in C_k} [f_j(N, b', p, C) - f_j(N, b, p, C)] \\ &= f_i(N, b', p, C) - f_i(N, b, p, C). \end{aligned} \quad (5)$$

By (IAF), for any  $l > k$  and any two  $j, j' \in C_l$ ,

$$\frac{1}{p_j} (f_j(N, b', p, C) - f_j(N, b, p, C)) = \frac{1}{p_{j'}} (f_{j'}(N, b', p, C) - f_{j'}(N, b, p, C)).$$

Therefore, there exists a  $\lambda \in \mathbb{R}$  such that for each  $l > k$  and each  $j \in C_l$ ,

$$f_j(N, b', p, C) - f_j(N, b, p, C) = \lambda \times p_j. \quad (6)$$

By combining (5) and (6), we obtain, for each  $l > k$ ,

$$\begin{aligned} \lambda \times \sum_{j \in C_l} p_j &= f_i(N, b', p, C) - f_i(N, b, p, C) \\ \iff \lambda &= \frac{1}{\sum_{j \in C_l} p_j} (f_i(N, b', p, C) - f_i(N, b, p, C)). \end{aligned}$$

By replacing  $\lambda$  in (6), we obtain

$$\forall l > k, \forall j \in C_l, \quad f_j(N, b, p, C) - \frac{f_i(N, b, p, C)}{\sum_{j' \in C_l} p_{j'}} = f_j(N, b', p, C) - \frac{f_i(N, b', p, C)}{\sum_{j' \in C_l} p_{j'}}. \quad (7)$$

The right hand side of (7) is uniquely determined by the Hypothesis. By definition of a burden-sharing method and by (4),

$$\begin{aligned} \sum_{j \in N} f_j(N, b, p, C) &= B. \\ \iff f_i(N, b, p, C) + \sum_{l > k} \sum_{j \in C_l} f_j(N, b, p, C) \\ &= B - \sum_{l < k} \sum_{j \in C_l} f_j(N, b', p, C) - \sum_{j \in C_k \setminus \{i\}} f_j(N, b', p, C). \end{aligned} \quad (8)$$

The right hand side of (8) is uniquely determined by the Hypothesis. By combining (7) and (8), we can obtain a solvable system of  $\sum_{l > k} |C_l| + 1$  equations with  $\sum_{l > k} |C_l| + 1$  unknowns. Therefore, there exists a unique solution  $f(N, b, p, C)$  to this system. This shows that  $f(N, b, p, C)$  is uniquely determined if  $0(b) = W - 1$ . This concludes the Induction. We have shown that  $f(N, b, p, C)$  is uniquely determined for each  $(N, b, p, C) \in \mathbb{B}$ . This concludes the proof.  $\square$

To see that all the axioms are necessary, let us consider the following counter examples. Each counter example satisfies two axioms out of the three invoked in Theorem 2.7. Therefore, it is not possible to withdraw an axiom from the characterization of the  $\varphi$  value without consequences on the result. In other word, the counter-examples show that the axioms are non-redundant.

**(HRI)** The method  $g$  defined, for each  $(N, b, p, C) \in \mathbb{B}$ , as

$$\forall k \leq K, \forall i \in C_k, \quad g_i(N, b, p, C) = \frac{1}{|C_k|} \sum_{l \leq K} \frac{B_l}{K},$$

satisfies (IAF) and (IEF) but violates (HRI).

**(IAF)** The method  $h$  defined, for each  $(N, b, p, C) \in \mathbb{B}$ , as

$$\forall k \leq K, \forall i \in C_k, \quad h_i(N, b, p, C) = \frac{b_i}{K - k + 1} + \frac{1}{|C_k|} \sum_{l < k} \frac{B_l}{K - l + 1},$$

satisfies (HRI) and (IEF) but violates (IAF).

**(IEF)** The method  $m$  defined, for each  $(N, b, p, C) \in \mathbb{B}$ , as

$$\forall k \leq K, \forall i \in C_k, \quad m_i(N, b, p, C) = b_i,$$

satisfies (HRI) and (IAF) but violates (IEF).

## 5.2 Relationship with cooperative games

In this section, we introduce some basic notions from cooperative games with transferable utilities. Then, we show how our burden-sharing method relates to well-known solution concepts from cooperative games.

### 5.2.1 Preliminaries on cooperative games

Consider a non-empty and finite set  $N \subset \mathbb{N}$  comprising individuals referred to as **players**. Any subset  $E \in 2^N$  is a **coalition**, representing a group of cooperating players. The grand coalition  $N$  represents a scenario where all players cooperate, while the empty coalition  $\emptyset$  depicts a situation with no player cooperation. A cooperative transferable utility game, abbreviated as a **TU-game**, is defined as a pair  $(N, v)$  consisting of a finite player set  $N \subset \mathbb{N}$  and a **characteristic function**  $v : 2^N \rightarrow \mathbb{R}$ , where  $v(\emptyset) = 0$  is the convention. The real number  $v(E)$  can be interpreted as the worth generated by the players in coalition  $E$  when they collaborate. A characteristic function is additive if

$$\forall E \subseteq N, \quad v(E) = \sum_{i \in E} v(\{i\}).$$

The collection of all TU-games with a finite set of players is symbolized as  $\mathbb{G}$ . Occasionally, there is a need to examine sub-games of some  $(N, v) \in \mathbb{G}$  involving a smaller set of players while preserving the same characteristic function. Pick any  $E \in 2^N$ . The **sub-game**  $(E, v_E) \in \mathbb{G}$  is defined as  $v_E(F) = v(F)$  for each  $F \in 2^E$ . If there is no potential for confusion, we simplify the notation by denoting  $(E, v_E)$  as  $(E, v)$ . A fundamental concern in TU-game theory revolves around the distribution of the grand coalition's worth  $v(N)$  among the players. This concern is often tackled through the utilization of single-valued solutions for TU-games. In a TU-game  $(N, v) \in \mathbb{G}$ , each player  $i \in N$  may obtain a payoff. A payoff vector  $x \in \mathbb{R}^{|N|}$  is a  $|N|$ -dimensional vector assigning a payoff  $x_i \in \mathbb{R}$  to each player  $i \in N$ . A single-valued solution, or a value, is a mapping  $f : \mathbb{G} \rightarrow \mathbb{R}$  that assigns a unique payoff vector  $f(N, v)$  to each  $(N, v) \in \mathbb{G}$ .

The **Shapley value** (see Shapley, 1953) stands out as arguably the most prominent value for TU-games. It is often defined using the concept of marginal contribution. Here, we prefer to introduce its

expression in terms of the Harsanyi dividend, which will prove useful for the rest of this section. The Shapley value can be defined, for each  $(N, v) \in \mathbb{G}$ , as

$$\forall i \in N, \quad Sh_i(N, v) = \sum_{\substack{E \in 2^N \\ E \ni i}} \frac{\Delta_v(E)}{|E|},$$

where  $\Delta_v(E)$  is the **Harsanyi dividend** (see Harsanyi, 1959) of  $E$ . It reflects the net surplus generated by  $E$ , and is formally defined as

$$\Delta_v(E) = v(E) - \sum_{F \subset E} \Delta_v(F).$$

To accommodate asymmetries among players beyond the game's definition, Shapley (1953) proposes a weighted versions of its (symmetric) Shapley value. In this approach, the asymmetries are represented by strictly positive weights on the players, resulting in what is known as the (positively) weighted Shapley values.

**Definition 5.1** (Weighted Shapley values). Pick any  $\omega \in \mathbb{R}_{++}^{|N|}$ . The associated Weighted Shapley value is the value  $Sh^\omega$  on  $\mathbb{G}$  defined, for each  $(N, v) \in \mathbb{G}$ , as

$$\forall i \in N, \quad Sh_i^\omega(N, v) = \sum_{\substack{E \in 2^N \\ E \ni i}} \frac{\omega_i}{\|\omega\|} \Delta_v(E).$$

Next, we introduce games with class structures.<sup>3</sup> Consider any  $N \subset \mathbb{N}$ . A class structure on  $N$  is characterized by a partition  $C = \{C_1, \dots, C_K\}$  of  $N$ . A game  $(N, v) \in \mathbb{G}$  accompanied by a class structure  $C$  on  $N$  is denoted by the tuple  $(N, v, C)$ . Let  $\overline{\mathbb{G}}$  represent the space of all TU-games featuring a finite set of players and a class structure on its player set. One of the most prominent values for TU-games with a class structure is the **Owen value**, as introduced by Owen (1977). It can be delineated in a two-step process.

- (i) First, consider a TU-game  $(C, v^C)$  where the classes act as cooperating players. In this game, each class is allocated its Shapley value.
- (ii) Second, for each class  $C_k \in C$ , consider a TU-game  $(C_k, v^{C_k})$  where the members of that class engage in cooperation. In this game, each member is assigned its Shapley value.

When the class structure (i.e. the partition) is trivial, the Owen value coincides with the Shapley value. The subsequent definition formalizes the Owen value.

---

<sup>3</sup>Class structures are commonly referred to as “coalition structures” or “a priori unions” in the literature. However, for consistency with the rest of this paper, we continue to use the term “class structures”.

**Definition 5.2** (Owen (1977)). The Owen value is defined, for each  $(N, v, C) \in \overline{\mathbb{G}}$ , as

$$\forall C_k \in C, \forall i \in C_k, \quad Ow_i(N, v, C) = Sh_i(C_k, v^{C_k}), \quad (9)$$

where,

(i) the TU-game  $(C, v^C)$ , played by the classes, is defined as

$$\forall F \subseteq (C \setminus C_k) \cup E, \quad v^C(F) = v\left(\bigcup_{H \in F} H\right).$$

(ii) the TU-game  $(C_k, v^{C_k})$ , played by the members of class  $C_k \in C$ , is defined as

$$\forall E \subseteq C_k, \quad v^{C_k}(E) = Sh_E\left((C \setminus C_k) \cup E, v^C\right).$$

In the framework of TU-games, Gilles et al. (1992) explore scenarios where certain players are integrated into a hierarchical (or permission) structure. Formally, a **hierarchical structure** defined on the player set  $N$  is a map  $S : N \rightarrow 2^N$ . The relation  $j \in S(i)$  indicates that player  $i$  holds a hierarchical position above player  $j$ . Alternatively, we denote  $i \in S^{-1}(j)$  if and only if  $j \in S(i)$ . The trivial structure  $S^0$  is defined such that, for each  $i \in N$ ,  $S^0(i) = \emptyset$ . The transitive closure of a hierarchical structure  $S$  is another hierarchical structure  $\hat{S}$  such that, for each  $i \in N$ ,  $j \in \hat{S}(i)$  if and only if there exists a path  $i = h_1, h_2, \dots, h_k = j$  such that  $h_k \in S(h_{k-1}), \dots, h_2 \in S(h_1)$ . The players in  $\hat{S}(i)$  are referred to as the **subordinates** of  $i$  in  $S$ , and those in  $\hat{S}^{-1}(i) := j \in N : i \in \hat{S}(j)$  are known as the **superiors** of  $i$  in  $S$ . A cycle occurs in the structure when there exists a path  $i = h_1, h_2, \dots, h_k = i$  such that  $h_k \in S(h_{k-1}), \dots, h_2 \in S(h_1)$ .

A TU-game  $(N, v)$  endowed with a hierarchical structure (with no cycles)  $S$  is represented as a tuple  $(N, v, S)$ . The class encompassing all games with a hierarchical structure is denoted by  $\tilde{\mathbb{G}}$ . In the context of TU-games with a hierarchical structure, the conjunctive approach to coalition feasibility posits that a player must obtain permission from all of their superiors before taking action. Specifically, a coalition  $E \subset N$  is deemed feasible if and only if it contains all the superiors of its members, i.e.,  $\hat{S}^{-1}(E) \subset E$ , where  $S^{-1}(E) = \cup_{i \in E} S^{-1}(i)$  and  $S(E) = \cup_{i \in E} S(i)$ . For any  $E \subseteq N$ , we define  $\sigma(E) = E \setminus \hat{S}(N \setminus E)$  as the feasible part of  $E$  according to  $S$ . van den Brink and Gilles (1996) introduced the (conjunctive) **Permission value**: a solution for TU-games with a hierarchical structure that is defined as the Shapley value of a restricted TU-game. This restricted game associates, to each coalition, a worth equal to the worth of its feasible part in the original game.

**Definition 5.3** (van den Brink and Gilles (1996)). The Permission value is defined, for each  $(N, v, S) \in \tilde{\mathbb{G}}$ , as

$$\forall i \in N, \quad \rho_i(N, v, S) = Sh_i(N, v \circ \sigma), \quad (10)$$

where  $(v \circ \sigma)(E) = v(\sigma(E))$  for each  $E \subseteq N$ .

### 5.2.2 Burden-sharing method as a value for cooperative games

To each burden-sharing problem  $(N, b, p, C) \in \mathbb{B}$ , it is possible to define a **burden-sharing game**  $(N, v, C, S)$ , where  $N$  is the player set,  $C$  is a class structure,  $S$  hierarchical structure defined over  $C$  formalizing the levels of development, and  $v : 2^N \mapsto \mathbb{R}$  is an additive characteristic function defined as

$$\forall E \subseteq N, \quad v(E) = \sum_{i \in E} b_i.$$

A burden-sharing game can be viewed as a way to measure the burden of each coalition of countries. Denote by  $\mathbb{BG}$  the domain of burden-sharing games. We define a value on  $\mathbb{BG}$  following a two-step process, akin to the approach in Owen (1977). To elaborate, let us introduce specific game definitions. For each  $(N, v, C, S) \in \mathbb{BG}$ , we define the **inter-class game**  $(C, v^C, S)$  as

$$\forall E \subseteq C, \quad v^C(E) = \sum_{C_k \in E} B_k.$$

In an inter-class game  $(C, v^C, S)$ , the classes in  $C$  take on the roles of players and are part of a hierarchical structure  $S$ . The worth of a subset  $E \subseteq C$  is determined by the sum of the environmental burden of the classes in  $E$ . It is noteworthy that  $v^C$  is additive. Select any  $C_k \in C$ . The inter-class game  $\left((C \setminus C_k) \cup H_k, v^C, S\right)$ , where  $H_k \subseteq C_k$ , represents the sub-game of  $(C, v^C, S)$  in which the class  $C_k$  has lost some of its members. Consequently, in this sub-game,  $C_k$  has diminished to a subset of countries  $H_k \subseteq C_k$ .

For each  $(N, v, C, S) \in \mathbb{BG}$ , and for each class  $C_k \in C$ , the associated **intra-class game**  $(C_k, v^{C_k})$  is defined as

$$\forall H_k \subseteq C_k, H_k \neq \emptyset, \quad v^{C_k}(H_k) = \rho_{H_k}\left((C \setminus C_k) \cup H_k, v^C, S\right),$$

with the convention  $v^{C_k}(\emptyset) = 0$ . In an intra-class game  $(C_k, v^{C_k})$ , the player set comprises countries at a similar level of development. The worth of  $H_k \subseteq C_k$  corresponds to its Permission value in the inter-class game  $\left((C \setminus C_k) \cup H_k, v^C, S\right)$ .

We have the material to define a value  $\Phi$  for burden-sharing games. It is defined according to a two-step process mimicking that of the Owen value. In the initial step, each class engages in the inter-class game and obtains a payoff based on the Permission value. Subsequently, in the second step, the payoff acquired by each class is distributed among its member countries. For this purpose, we consider an intra-class game within each class, where every country participating in such a game receives a payoff according to a Weighted Shapley value whose weights are determined by the population vector.

**Definition 5.4.** The value  $\Phi$  is defined, for each  $(N, v, C, S) \in \mathbb{BG}$ , as

$$\forall C_k \in C, \forall i \in C_k, \quad \Phi_i(N, v, C, S) = Sh_i^p(C_k, v^{C_k}).$$

The value generalizes several solution concepts from cooperative games theory, including the Owen value, the Permission value and the (Weighted) Shapley value. The value can also be viewed as a weighted version of the Permission-Owen value introduced by Abe et al. (2024). The Permission-Owen value is a two-step solution concept for TU-games with coalition-wise permission structures. This family of games is a generalization of  $\mathbb{BG}$  in which one allows all TU-games, all class structure and all permission structures defined over the elements of the class structure. The Permission-Owen value is computed similarly to  $\Phi$ , but it does not require a weight vector such as  $p$ . Instead, it applies the Permission value in the first step and the (non-weighted) Shapley value in the second step. Observe that, while we define  $\Phi$  on  $\mathbb{BG}$ , one could define the solution on the whole class of TU-games with coalition-wise permission structures. Consequently,  $\Phi$  generalizes the Permission-Owen value. The next proposition is the main result of this section. We demonstrate that our burden-sharing method  $\varphi$  can be obtained by the value  $\Phi$ , thus showing that  $\varphi$  is strongly grounded in cooperative game theory.

**Proposition 5.5.** For each burden-sharing problem  $(N, b, p, C) \in \mathbb{B}$  and its associated burden-sharing game  $(N, v, C, S) \in \mathbb{BG}$ , it holds that

$$\varphi(N, b, p, C) = \Phi(N, v, C, S).$$

**Proof.** Pick any  $(N, b, p, C) \in \mathbb{B}$ . Consider its associated game  $(N, v, C, S) \in \mathbb{BG}$ . Let us compute  $\Phi(N, v, C, S)$ . Consider the inter-class game  $(C, v^C, S)$  whose characteristic function is defined as

$$\forall E \subseteq C, \quad v^C(E) = \sum_{C_k \in E} B_k.$$

Recall that the function  $v^C$  is additive. Consequently, it holds that

$$\forall E \subseteq N, \quad \Delta_{v^C}(E) = \begin{cases} v^C(E) & \text{if } |E| = 1. \\ 0 & \text{otherwise.} \end{cases}$$



By definition of the Permission value, we directly have

$$\forall C_k \in \mathcal{C}, \forall H_k \subseteq C_k, \quad \rho_{H_k}((C \setminus C_k) \cup H_k, v^C, S) = \frac{1}{K - k + 1} \sum_{i \in H_k} b_i + \sum_{l < k} \frac{B_l}{K - l + 1}.$$

Next, consider the intra-class game  $(C_k, v^{C_k})$  whose characteristic function is defined as

$$\forall H_k \subseteq C_k, \quad v^{C_k}(H_k) = \rho_{H_k}((C \setminus C_k) \cup H_k, v^C, S).$$

By definition of the Weighted Shapley value  $Sh^p$ , the desired result follows

$$\begin{aligned} \forall C_k \in \mathcal{C}, \forall i \in C_k, \quad \Phi_i(N, v, C, S) &= Sh_i^p(C_k, v^{C_k}) \\ &= \frac{b_i}{K - k + 1} + \frac{p_i}{\sum_{i' \in C_k} p_{i'}} \sum_{l < k} \frac{B_l}{K - l + 1} \\ &= \varphi_i(N, b, p, C). \end{aligned}$$

□

## Reference

Abe, T., Lowing, D., Nakada, S., Techer, K., 2024. The permission-owen value: Axiomatization and application to liability situations. Available at SSRN .

Albizuri, M.J., 2008. Axiomatizations of the owen value without efficiency. *Mathematical Social Sciences* 55, 78–89.

Algaba, E., Márquez, G., Martínez-Lozano, J., Sánchez-Soriano, J., 2023. A novel methodology for public management of annual greenhouse gas emissions in the european union. *Socio-Economic Planning Sciences* 89, 101697.

Baer, P., Fieldman, G., Athanasiou, T., Kartha, S., 2009. Greenhouse Development Rights: towards an equitable framework for global climate policy, in: *The Politics of Climate Change*. Routledge, pp. 192–212.

Bank, W., 2023. World development indicators. Data retrieved from <https://datacatalog.worldbank.org/search/dataset/0037712/World-Development-Indicator>

van den Berg, N.J., van Soest, H.L., Hof, A.F., den Elzen, M.G.J., van Vuuren, D.P., Chen, W., Drouet, L., Emmerling, J., Fujimori, S., Höhne, N., Köberle, A.C., McCollum, D., Schaeffer, R.,

- Shekhar, S., Vishwanathan, S.S., Vrontisi, Z., Blok, K., 2020. Implications of various effort-sharing approaches for national carbon budgets and emission pathways. *Climatic Change* 162, 1805–1822.
- van den Brink, R., 2017. Games with a permission structure - a survey on generalizations and applications. *TOP* 25, 1–33.
- van den Brink, R., Gilles, R.P., 1996. Axiomatizations of the conjunctive permission value for games with permission structures. *Games and Economic Behavior* 12, 113–126.
- van den Brink, R., He, S., Huang, J.P., 2018. Polluted river problems and games with a permission structure. *Games and Economic Behavior* 108, 182–205.
- Dehez, P., Ferey, S., 2013. How to share joint liability: A cooperative game approach. *Mathematical Social Sciences* 66, 44–50.
- Doncaster, C.P., Tavoni, A., Dyke, J.G., 2017. Using adaptation insurance to incentivize climate-change mitigation. *Ecological Economics* 135, 246–258.
- Dong, B., Ni, D., Wang, Y., 2012. Sharing a polluted river network. *Environmental and Resource Economics* 53, 367–387.
- Eyckmans, J., Tulkens, H., 2003. Simulating coalitionally stable burden sharing agreements for the climate change problem. *Resource and Energy Economics* 25, 299–327.
- Ferey, S., Dehez, P., 2016. Multiple causation, apportionment, and the shapley value. *The Journal of Legal Studies* 45, 143–171.
- Filar, J.A., Gaertner, P.S., 1997. A regional allocation of world CO<sub>2</sub> emission reductions. *Mathematics and Computers in Simulation* 43, 269–275.
- Gilles, R.P., Owen, G., van den Brink, R., 1992. Games with permission structures: the conjunctive approach. *International Journal of Game Theory* 20, 277–293.
- Harsanyi, J., 1959. A bargaining model for cooperative n-person games, in: Tucker, A.W., Luce, R.D. (Eds.), *Contributions to the Theory of Games*. Princeton University Press, Princeton. volume 4 of 40, pp. 325–355.
- Holz, C., Kartha, S., Athanasiou, T., 2018. Fairly sharing 1.5: national fair shares of a 1.5°C-compliant global mitigation effort. *International Environmental Agreements: Politics, Law and Economics* 18, 117–134.

- IPCC, 2022. Climate Change 2022: Mitigation of Climate Change. Contribution of Working Group III to the Sixth Assessment Report of the Intergovernmental Panel on Climate Change. Cambridge University Press, Cambridge, UK and New York, NY, USA. URL: <https://www.ipcc.ch/report/ar6/wg3/about/how-to-cite-this-report/>, doi:10.1017/9781009157926.
- Jensen, J., Rasmussen, T.N., 2000. Allocation of co2 emissions permits: A general equilibrium analysis of policy instruments. *Journal of Environmental Economics and Management* 40, 111–136.
- Khmelnitskaya, A.B., Yanovskaya, E.B., 2007. Owen coalitional value without additivity axiom. *Mathematical Methods of Operations Research* 66, 255–261.
- Ni, D., Wang, Y., 2007. Sharing a polluted river. *Games and Economic Behavior* 60, 176–186.
- Oishi, T., van der Laan, G., van den Brink, R., 2023. Axiomatic analysis of liability problems with rooted-tree networks in tort law. *Economic Theory* , 1–30.
- Owen, G., 1977. Values of games with a priori unions. *Mathematical economics and game theory* , 76–88.
- Pozo, C., Galán-Martín, Á., Reiner, D.M., Mac Dowell, N., Guillén-Gosálbez, G., 2020. Equity in allocating carbon dioxide removal quotas. *Nature Climate Change* 10, 640–646.
- Ringius, L., Torvanger, A., Underdal, A., 2002. Burden Sharing and Fairness Principles in International Climate Policy. *International Environmental Agreements: Politics, Law and Economics* 2, 1–22.
- Rose, A., 1990. Reducing conflict in global warming policy: The potential of equity as a unifying principle. *Energy Policy* 18, 927–935.
- Rose, A., Stevens, B., 1993. The efficiency and equity of marketable permits for CO2 emissions. *Resource and Energy Economics* 15, 117–146.
- Shapley, L.S., 1953. A value for n-person games. *Contributions to the Theory of Games* 2, 307–317.
- Torvanger, A., 2023. What If Country Commitments for CO2 Removal Were Based on Responsibility for Historical Emissions? *Energies* 16, 4350.

UNFCCC, 1992. United Framework Convention on Climate Change. Technical Report. URL: <https://unfccc.int/resource/docs/convkp/conveng.pdf>.

UNFCCC, 2015. Paris Agreement. Technical Report. URL: [https://unfccc.int/files/meetings/paris\\_nov\\_2015/application/pdf/paris\\_agreement\\_engl](https://unfccc.int/files/meetings/paris_nov_2015/application/pdf/paris_agreement_engl)

Vazquez-Brage, M., van den Nouweland, A., Garcia-Jurado, I., 1997. Owen's coalitional value and aircraft landing fees. *Mathematical Social Sciences* 34, 273–286.

de Villemeur, E.B., Leroux, J., 2019. Tradable climate liabilities: a thought experiment. *Ecological Economics* 164, 106355.

Wang, M., Zhou, P., 2017. Does emission permit allocation affect CO<sub>2</sub> cost pass-through? A theoretical analysis. *Energy Economics* 66, 140–146.

Watch, C., 2024. Historic ghg emissions data. Data retrieved from <https://www.climatewatchdata.org/ghg-emissions>.

Zhou, P., Wang, M., 2016. Carbon dioxide emissions allocation: A review. *Ecological economics* 125, 47–59.