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# Epistemology, Mathematical Formalism and Appropriate Mathematical Formalism

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## Abstract

Suppose a *secular academic discipline* (AD) endeavors to develop an epistemology. This study's formal theory shows efforts, which in aggregate, are not inclusive of any mathematical formalism result in epistemology that are vacuously formulated; *mathematical formalism* (*mf*) is, as such, not a luxury, rather is a necessary parameter of any epistemology that is robustly (non-vacuously) formulated. Suppose a concept or principle (COPR) is solely rooted in language, that is, is not natively mathematical. Only *mf* that are *non-parametrically formulated* (*mfn*), such as, *Set Theory* are robust to a formalization of the COPR. Suppose, contrarily that a COPR is natively mathematical; either *mf* that are parametrically formulated (*mfp*), such as, *Differential Calculus*, or *mfn* are robust to a parameterization of the COPR.

Applying the foregoing, any and all applications of *mfp* to a core concern of Economics, namely "*what is rationality* (WIR)?" are inherently vacuous. The rationale is straightforward, namely ideally, "*how exactly to implement rationality* (HIR)?" which neoclassical theory parameterizes as, 'value maximization' ought to be derived from a parameterization, a priori, of the concept of rationality. In the skipping of the WIR question to the HIR question, rationality, which first and foremost is a parameter of man - non-satiation with respect to 'wealth' is, first and foremost, a parameter of man - is itself not parameterized. The vacuousness of the resulting neoclassical characterization of rationality is evident in the implication that there does not exist any person or time-invariant parameterization of rationality. With respect, as such, to rationality, asymptotically, the neoclassical standard is, '*anything goes*'. In presence of the evidence, there is arrival at epistemological and paradigm grounds for an alternative to each of neoclassical theory and neoclassical mathematics. For concreteness of the necessity of new mathematical formalism, with calculus shown, in this study, to be inappropriate to a parameterization of rationality - because only *mfn* are appropriate to the objective - noting that the concept of rationality is foundational to all of the Social Sciences and Humanities (SSH), neoclassical theory lacks mathematical formalism that are appropriate to the

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development of the epistemology of either economics or any of the other disciplines that make up the SSH.

Since Philosophy refines language, all truly philosophical questions are rooted in language, are not natively mathematically specified. In presence of the foregoing, and additional evidence in support of the axiom that logic is non-robust to the development or progression of epistemology - logic refines existing epistemology, points out gaps or cracks in existing epistemology, and incentivizes the direction of, or extensions to epistemology, but is unable itself to actualize the changes that it points out to be desirable or necessary - ideally Philosophy journals become receptive to studies which, having applied logic arguments towards critiques of epistemology, additionally, apply solely *mfn* towards the development or progression of an epistemology. As is historically implored by Frege, if the response is, as such, there is arrival at a robust and appropriate progression to the relevance of Philosophy to the welfare of man.

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# 1 Introduction

First, no sharp boundary can be drawn between logic and arithmetic. Considered from a scientific point of view, both together constitute a unified science. Of course, the division of the entire field of knowledge into the various sciences is determined not merely by theoretical but also by pragmatic considerations; and by the preceding I do not mean to say anything against a certain pragmatic division; only it must not become a schism, as is at present the case to the detriment of all sides concerned - Frege (1984, pgs. 112-113).

The self conceptions and dividing lines among the behavioral disciplines make no scientific sense. How can there be three separate fields, sociology, anthropology, social psychology, for instance, studying social behavior and organization? How can the basic conceptual frameworks for the three fields, as outlined by their respective Great Masters and as taught to PhD candidates, have almost nothing in common? In the name of science, these arbitrarinesses must be abolished - Gintis (2014, pg. xiii).

Economic theory has been particularly compromised by its neglect of the facts concerning human behavior - Gintis (2014, pg. xiv).

In the first quote above, Gottlob Frege, one of the founding fathers of Philosophy (see for example, Dummett 2001) affirms the pragmatism of a segmentation of academic research into various disciplines. Simultaneously, focusing on the erroneous perception that logic and mathematics are dichotomous tools, Frege posits that the self same pragmatism requires that, toolwise the academic disciplines not become silos, some emphasizing logic (e.g. Philosophy, Sociology, Psychology), others mathematics (e.g. Economics, Finance). In Gintis (2014), Herbert Gintis posits that the existence of various academic disciplines, all of whom study man, but which yet do not have any epistemology in common is neither ideal nor pragmatic. In stated respect, since all such disciplines study man, ideally there exist some propositional truths that cut across all such disciplines, propositional truths which facilitate cooperation across disciplines. In particular, Gintis laments economics' abstraction away from human behavior, as such, hitherto it, economics' non-robustness to cooperation with disciplines, such as, Philosophy, Sociology, or Psychology.

Absent any prejudice to the enumerated quotes, this study opines that a wedge which has contributed to the siloing of the disciplines of the *Social Sciences and Humanities (SSH)* is the concern that mathematical formalism will, as has been the case in Economics, reduce a behavioral discipline to be no more than an extension of mathematics (for expressions of the concern, see Lawson 2016, 2019). Samuelson (1952) addresses the highlighted concern, posits that ideally Mathematics serves a discipline, that is, facilitates rigor of exposition, does not take over a discipline. Romer (2015) concurs, posits Economics has been mathematized at the expense of the principles of economics, resulting in formal theoretical models which yield propositions that are solely mathematical, propositions that do not have expression, simultaneously as behaviors. With no pun intended, the mathematization of economics has illustration as follows. Seminally, Holmstrom (1999) infers that, feasibly agents' efforts do not coincide with their ability. Feasibly, as such, the same socioeconomic agents are parameterized by efforts that do not

coincide with their ability. Clearly, there is arrival at a behavioral, as opposed to a mathematical phenomenon, but yet a phenomenon that, feasibly is amenable to mathematical formalization. Holmstrom (1999) has spanned lots of follow up studies, many of which attempt to infer the conditions under which effort coincides with ability. Consider then the following two studies. In Netzer and Scheuer (2010), ability and effort coincide whenever agents' expectations are weakly decreasing. In Garrett and Pavan (2012), under alternate mathematical conditions, ability coincides with effort whenever agents' expectations functions are strictly convex. With neither of the two enumerated results having any behavioral content, there is arrival at the import of *the joke* in Samuelson (1952b, pg. 678), to wit, in his, *Paul Samuelson's* opinion, only *Leonard Savage*, the author of Savage (1972) has the capacity to robustly translate such purely mathematical results into rational behavior.

The discussion to follow introduces a propositional truth that undergirds this study's formal theoretical effort. The term, Ph.D. has interpretation as, '*Doctor of Philosophy*', that is, asserts Ph.D. holders are the philosophers of their respective disciplines. Logic is, as such, a tool that is common to all academic disciplines. Consider, however, that seminally, *Bertrand Russell* admitted that his effort towards a demonstration that "all mathematics is symbolic logic" - all mathematics is logic in symbolic form - had been wrongheaded (see Baldwin 2001, pg. 37). In stated respect, contrary to Russell's effort, whereas set theory embeds the use of logic, set theory is considered mathematics, that is, is a component of *Analysis*, is not considered logic (ibid). With logic then a tool of mathematics, as opposed to vice versa, it has constitution as a strict subset of mathematics, such that whereas ideally all mathematics are reducible to logic, the function is non-invertible (see for example, Frege 1984, pg. 113). For concreteness, whereas logic is the tool for philosophizing, regardless relative to mathematics, it is non-robust to some objectives; the feasibility, as such of the usefulness of an introduction of mathematical formalism into efforts for the development of an epistemology (see for example, Hodgson 2007; Gintis 2014). In stated respect, seminally, with mathematics as the benchmark, Wittgenstein sensitized as to the weakness of logic, namely that whereas logic facilitates a more rigorous understanding of, and use of language - a more rigorous understanding and use of concepts or principles - it is not robust to the establishment of propositional truths (see for example, Hacker 2001, pgs. 81, 89-90). If then concepts or principles are not rooted in mathematical formalism, applications of logic to discussions of the concepts or principles reduce to '*philosophical testimony*', as such, are vacuous (see for example, García 2024). In essence, mathematical formalism facilitate descriptions of concepts or principles in terms of alternate elements (observational terms), native to language, which have existence independent of the concepts or principles (Dusek 2008 concurs). In presence of the foregoing, if logic arguments are to be most useful to the disciplines of the *SSH*, the concepts or principles (the epistemology) of the *SSH* are formulated as propositional truths, as such, are mathematically - non-vacuously - formulated (for a similar dichotomization, see Bigo and Negru 2008). The evidence seems to concur, namely whereas economics can be asserted to have arrived at a 'non-vacuous' epistemology, which regardless is, for the most part, non-robust to a modeling of agents' behaviors, by the same token, each of sociology or psychology has yet to arrive at an epistemology that can be asserted to be non-vacuous. In aggregate, either the non-incorporation of mathematical formalism or the

incorporation of mathematical formalism that are non-robust to their epistemological objectives militate against the development of epistemology that are robustly formulated.

Applying logic arguments, this study reiterates Wittgenstein's seminal insight, namely that whereas logic arguments are able to robustly infer the desirability or necessity of an epistemology, simultaneously logic arguments are non-robust to the development or progression of an epistemology. In presence of the evidence, it is not the case that the disciplines of the *SSH* ought to, rather condescendingly be considerate of mathematical formalism, but rather that it is imperative that the disciplines of the *SSH* seek out, encourage, and imbibe mathematical formalism that is appropriate to the development and progression of their epistemology. Simultaneously, the formal theoretical effort infers the boundaries, necessary to be imposed, which ensure that mathematical formalism serve solely to facilitate a non-vacuous epistemology, do not become an end in-of-themselves. For concreteness, inclusive of the behavioral sciences, there is arrival at inferences in respect of mathematical formalism that are either appropriate to or excluded from efforts for the formalization of the concepts or principles of any scientific discipline.

Suppose then the introduction of mathematical formalism into a discipline, *D* of the *SSH*, conveniently termed a *DSSH*. Necessarily, the *DSSH* has characterization as a conglomeration of concepts and/or principles. Applying a canon of philosophical thought, ideally the concepts or principles of the *DSSH* already have formalization as analytical statements, that is, as the combination of *definitional statements* and *descriptive (observational) statements* which link the definitions to words or terms that already have meaning in language (see for example, the discussion of Quine's work in Hylton 2001; the discussion of *Quine*, in Burge 1992, pg. 7; or the discussion of Carnap, in Sarkar 2001, pg. 99). The formal theory infers that either the defining or the observational statements are required to be derived from mathematical formalism, because otherwise an epistemology consists, in entirety, of philosophical testimony, as such, is vacuous. Since observational statements are required to be linked to language - terminology with which other agents already have familiarity - there is arrival at the following benchmark, namely that the definitions of either concepts or principles are derived using mathematical formalism. With the definitional and observational statements then having statement in language, logic arguments serve to affirm, challenge or refine the outcomes of mathematical formalism (see for example, Glock 2004, pgs. 430-431). Relatedly, study outcomes address the concern, in Dow (2008) as to whether mathematical formalism can be deemed to be more robust than, say, empirical formalism, namely restrict the development and progression of epistemology to the introduction of mathematical formalism. For concreteness, if empirical formalism do not implement a specific mathematical formalism, they solely facilitate logic arguments, not epistemological arguments. For supporting arguments, see Lucas (1976) or Roll (1977). For an elaboration, see the discussion in Section 2.3.

Suppose then that a mathematical tool - pre-existing or new - is applied to a question of interest that has domicile in a *DSSH*. Suppose the tool results in some new insights. If the tool is robust to the *DSSH*, equivalently if the tool is commensurable to the concepts or principles of the *DSSH*, it's insights can be presented as the combination of new *definitional statements* and new *descriptive (observational) statements* that link the definitions to either words or terms that already have meaning in language, or to a pre-existing combination of *definitional statements* and *descriptive (observational) statements*. Importantly, if a new

mathematical formalism is disruptively true, the formal theory shows it's outcomes only can be linked directly to words or terms that already have meaning in language, that is, ought not to be expected or required to be linked to some pre-existing combination of definitional and observational statements. The notion, as such, that it is methodology that links a literature is shown to be erroneous. In stated respect, there is arrival at a negation of the logically-derived recommendation in Larue (2022) that a model ought to be accepted in economics only whenever a school of thought has already emerged around a new methodology. To see this, whereas the recommendation in Larue (2022) can simply reflect network strength and requires some a priori coordination that transpires behind the scenes - coordination which feasibly is oligopolist - this study's recommendation is rooted in the merits of any new mathematical formalism. For concreteness, if the Larue (2022) recommendation were to be applied to *Isaac Newton's Principia*, which single-handedly introduced a new mathematical formalism, namely *Calculus* to Physics, the new formalism is considered too much of an outlier to the then standard, which was Geometry. For a discussion of how this study's outcomes relate to recommendations in respect of the management of model pluralism in Rodrik (2015), see the discussion in Section 2.3.

An illustration of study findings is, perhaps helpful. In neoclassical economics, rationality has characterization as *value maximization*; clearly the stated characterization is solely mathematical, does not connote any specific behavior. Further, since the characterization lacks a supporting *descriptive (observational) statement* that links directly to language, *it is both mathematically and epistemologically vacuous*. Consider, however, the parameterization of rationality in Obrimah (2022), which as far as this study is aware is the sole effort that *parameterizes rationality itself*, as opposed to, say, "*how to be rational*" (see for example, Schwartz 1972; Kreps and Wilson 1982); for concreteness of the distinction, it is matter of fact that efforts that parameterize *how to be rational* leave the rationality concept itself bereft of any analytical definition; for instance, whereas Kreps and Wilson (1982) recommend that agents engage with *dynamically evolving rational beliefs*, rationality itself is not parameterized. Using formal theoretical (mathematical) proofs, Obrimah (2022) arrives at a parameterization of rationality as, "*non-satiation with respect to the unraveling of unawareness*" and arrives at the following *descriptive (observational) statement*, namely that rationality has implementation as, "*progressiveness of the effort for the refinement of any existing awareness' for arrival, feasibly at new awareness'.*" Clearly, the terms, *progressiveness*, *effort*, *refinement*, *existing*, and *awareness* all are native to each of economics and language. Further, absent the observational statement, the meaning of the terms, '*non-satiation*' or '*unraveling*' that are incorporated into the definitional statement are ambiguous, hence the importance of the observational statement. Lastly, the finding that every pair consisting of an existing rational awareness and an unawareness that has yet to be unraveled subsist in a *probability measure space*, such that it is efforts for the refinement of existing awareness' that facilitate the unraveling of existing unawareness' furnishes the analogous mathematical parameterization of rationality. Clearly, the effort in Obrimah (2022) passes this study's test of mathematical formalism that is appropriately introduced into the *SSH*. In stated respect, whereas, qualitatively all agents are *identically rational*, regardless they are *independently*, that is, heterogeneously rational; the parameterization satisfies, as such, the canon that the best distributions are independently, yet identically realized, that is, are *iid*. It is straightforward that a parameterization of

rationality as, *value maximization*, a parameterization which leaves the definition of *value* up to the individual agents, fails the test of an *identically* realized property.

In Hodgson (2007) and Gintis (2014), the path to an alternative formal theoretical structure to neoclassical economics commences with the development of a more viable formal theoretical structure for the rationality of socioeconomic man (see also, Derksen 2005; Holcombe 2021). In similar vein, as can be deemed to be acknowledged in Heise (2020), absent a robust parameterization of agents' rationality that is invariant both across agents and over time, yet supportive of the heterogeneity of agents and agents' activities, as such inherently robust to pluralism, economics can hardly claim to be a science that ought to be taken seriously; epistemology, as such, is the path to the establishment of any paradigm that can be held in common across all of the *SSH* (for a similar recommendation, see Dow 2008 or Bigo and Negru 2008). This study opines that Obrimah (2022, 2023, 2023b) are robust to the enumerated objective. To see this, building on Obrimah (2022), Obrimah (2023) shows either preferences or behaviors, such as prudence or adventuresomeness are tools that are applied towards either the maintenance or progression of rationality, are not substitutes for rationality. Obrimah (2023b) addresses the three commonalities of heterodox theory that are enumerated in Lee (2008) or Ramazzotti (2022), namely a structure of production that does not allow for scarcity; a wage-price-tax structure; and applying Lucas (1976) and Kydland and Prescott (1977), a fiscal, as opposed to a monetary policy structure that straddles the first two structures; all within one grand overarching framework that robustly deviates from neoclassical formalism. For illustration, in the formal theory, government influences prices in the private sector solely via *transfer payments* and *incentives* that are premised on it's very own structure of *fiscal prices*, the structure and components of which are derived in that study.

In essence, this study's formal theory infers that it is not methodologies that connect literature either across studies or across time; that rather it is the evolution of the analytical statements that parameterize an academic discipline's concepts and principles that, ideally connects literature either across studies or across time. To see this, with each restricted to be appropriate to the objective, the inference in this study, that the severity of the incommensurability that is inferred between a new and existing mathematical formalism predicts the progressiveness of the new mathematical formalism explicitly rules out either uniformity or congruence of formalism as rules of thumb for assessing the appropriateness, or otherwise of any new mathematical formalism. Concurring, whereas individual researchers might act differently, each of Norgaard (1989), Bigo (2010), and Dobusch and Kapeller (2012) posit that, ideally it is research questions that determine methodologies, not vice versa. Also concurring, in Lisciandra and Korbmacher (2021), it is possible to reconcile two seemingly inconsistent formalisms that model the exact same concept. In aggregate, there is arrival at the inference that the introduction of appropriate mathematical formalism into a discipline ought not to be regarded as either an option or a novelty, rather is a necessary condition if a *DSSH* is to arrive at an epistemology that is both non-vacuous and robust. Consistent, as such, with the intuition in either Frege (1984) or Gintis (2014), with the caveat in tow that any and all introductions of mathematical formalism are restricted to be appropriate, there is arrival at the insight that there ought not exist any methodological walls either within or across the disciplines of the *SSH*. The rest of the study is organized as follows. With the exposition rooted in a



discussion of related literature, Section 2 engages with logical arguments which demonstrate that mathematical formalism is a necessary parameter of any epistemology that is non-vacuously formulated. The criteria for assessing the appropriateness of any mathematical formalism to any scientifically formulated discipline are formally and theoretically established in Section 3. Section 4 concludes.

## 2 Some Preliminary Results

The concreteness of the inference that logic is non-robust to the formulation of propositional truth is, at the very least, a component of the philosophical legacy of Ludwig Wittgenstein (see for example, Hacker 2001). For concreteness, Wittgenstein establishes that every robustly formulated proposition creates two truths, the truth that is propositioned and a truth that is the direct negation of the stated proposition. Logic, however, tends to fail the enumerated test, namely that it span, simultaneously via a single proposition, two dichotomous truths. To see this, consider the brilliant arguments that are put forth in Badhwar (1993) in respect of the proposition that altruism can be a component of agents' subjectively formulated utility, an argument that is validated using the tools of logic and illustrations from the willingness of the French to help Jews at the risk of their very own lives in the course of the *Second World War*. Consider, however, that, simultaneously Badhwar (1993) is unable to establish that absence of altruism in an agent's preferences results in a diminution to either rationality or utility. In stated respect, in the very same data, some Frenchmen derived utility from an elimination of the risk of harm via an unwillingness to help out Jews. Whereas then the study establishes a truth, simultaneously it is unable to establish that the negation of said truth results in a dichotomous truth. In presence of Wittgenstein's insight, as is the case in Badhwar (1993), logic serves primarily to facilitate a more rigorous understanding of language, that is, of concepts or principles, is not robust to the establishment of propositional truths (see for example, Hacker 2001, pgs. 81, 89-90). To further see this, suppose the existence of a robust parameterization of rationality. Necessarily, the proposition - "*rationality consists in such and such behaviors*" - spans, endogenously the alternate proposition, "*irrationality consists in behaviors that are dichotomous to such and such*". Logic, however, only can help refine an understanding of rationality, cannot itself be the source of a parameterization of rationality (see also Hintikka 1998; Preston 2004). The logical propositions to follow demonstrate that the characterization of Philosophy as '*a refiner or critic of language, as such, a refiner or critic of concepts and/or principles*' is not vacuous; in stated respect, whereas Philosophy itself does not progress knowledge, the reality that it can direct or redirect agents' efforts implies Philosophy is non-vacuous, rather is a substantive field of inquiry - the essence, it seems to me of a study that I admit is susceptible to alternate interpretations, namely Ranalli (2020).

### 2.1 Mathematical Formalism: A necessary condition if epistemology is to be non-vacuous

**Axiom 1** *Logic is non-robust to the development of propositional truth.*

**Logical Proposition 1** *Whereas the objective of philosophizing in academic research is the refinement of language, that is, the refinement of concepts or principles, equivalently the refinement of epistemology, with logic non-robust to the development of propositional truths, the development of epistemology - propositional truths - is itself solely facilitated by either language or mathematical formalism.*

**Proof.** Follows directly from the foregoing, that is, Axiom 1. ■

**Logical Proposition 2** *Since feasibly epistemology that is developed using solely language turns out vacuous, there is arrival at the beneficence of mathematical formalism, namely the avoidance of vacuous definitions; equivalently the avoidance of epistemology that are vacuously formulated.*

**Proof.** Evidence for a material conformity, namely an epistemology that is solely rooted in language which can be shown to be vacuously formulated, suffices.

The material conformity follows directly from the admission of non-arrival at a robustly formulated epistemology for virtue ethics in Nussbaum and Hursthouse (1984); the resort, temporarily to a vacuous formulation of ethics in Hursthouse (1995, 1999, 2013) or Battaly (2010) as (I paraphrase), “*a virtue is whatever any reasonable, good, or moral man would consider to be a virtue*” (see for example, Hursthouse 1995, 1999; Everitt 2007); and the admission that virtue ethics theory remains vacuous in Annas (2013) or Nussbaum (2013). For a discussion of the non-robustness of ‘*philosophical testimony*’, such as in Hursthouse (1995, 1999, 2013) to the development of propositional truths, see García (2024). In similar vein, whereas Nussbaum and Hursthouse (1984) apply philosophical testimony towards a motivation of the *commensurability of virtues* as the path that facilitates a non-vacuous epistemology for virtue ethics, simultaneously Nussbaum admits the insufficiency of her philosophical testimony as the tool that facilitates the propositional evidence. Each of Crisp (2010) or Das (2020) discuss the absence of a non-vacuous epistemology in virtue ethics theory. Consider then the response by Hursthouse to Nussbaum’s proposition in respect of the desirability of an exploration of the commensurability of virtues, in Nussbaum and Hursthouse (1984, pg. 81, *italics* supplied):

“Plato claims that anything worthy of the name of episteme must involve numbering and measuring; given ethical knowledge is clearly for him worthy of such names, it may seem that he must have envisaged a *mathematical ethics* while unfortunately leaving it quite unclear what he thought it would look like. He would not have been alone in so doing - *Locke and Leibniz also claim that ethical knowledge, when deserving of the name, is or would be mathematical*, and leave us with no helpful examples of such knowledge.” ■

**Logical Proposition 3** *All definitions that are non-vacuous are supported by descriptive, equivalently observational statements or sentences. If each of a definition and its supporting observational statement are rooted in language, there is arrival, regardless at a vacuous definition; the importance, as such of mathematical formalism, namely that a definition is analytically formulated, is not solely derived from language.*

**Proof.** The definition, “*a virtue is whatever any reasonable, good, or moral man would consider to be a virtue*” contains a *definition* and an attempt at an *observational* term, yet

is vacuous. To see this, whereas all agents understand what the term, *morality* connotes, regardless all do not agree on it's contents, that is, the *meaning of morality*. Regardless then of the presence of a seeming observational term, namely morality, with the meaning of morality itself unclarified, the linkage of virtue to morality remains vacuous - the essence of the critique of virtue ethics in Everitt (2007). Ditto the linkage of virtue to either *reasonability* or *goodness*.

Suppose, however, the rhetorical assumption that the content of morality is already formalized mathematically. Then morality can be asserted to be  $x$ . Whereas then the assertion that "*virtue is  $x$* " is non-vacuous, regardless there is arrival at the following snafu, namely that *virtue coincides with morality*, as such the necessity of antecedent philosophizing which excludes either virtue or morality, resulting solely in one or the other, as such a refinement of language. If, otherwise, virtue is deemed not to coincide with morality, necessarily "*virtue is not  $x$* " and there is arrival at the importance of a mathematical formalism that parameterizes virtue independent of morality, and vice versa. For additional concreteness of the foregoing, it is well established that, ideally, analytic philosophy draws it's propositional truths from a scientific basis, then applies logic to refinements of language in respect of the propositional truths (see for example, Peijnenburg 2000; Preston 2004).

More generally, let  $x$  be a term whose meaning is not rooted in language, e.g. morality. Necessarily,  $x$  is an analytic, as opposed to an observational term. If  $y$  is asserted to be  $x$ , there is arrival at, ' $x$  is  $x$ ', that is, a vacuous definition. If it is asserted that,  $y \in x$ , regardless  $y$  remains undefined. If, conversely, it is asserted that,  $x \in y$ , with the meaning of  $x$  as yet unclear, the properties of  $y$  remain unclear, unknown.

Suppose, otherwise, namely that  $y$  is deemed to be operationalized by  $v$ , and that  $v$  is an observational term, to wit, a term that is rooted in language. Since  $y = v$  is excluded, because then,  $y$  is an observational term, as such a contradiction, necessarily, there exists some evidence,  $\Lambda$ , which establishes that, ' $v$  describes  $y$ ', equivalently, that  $v \in y$ ; the equivalence,  $\Lambda \equiv$  *mathematical formalism* completes the proof. ■

Obrimah (2023) provides a rich illustration of *Logical Proposition 3 (LP3)*. Let  $y$  denote *rationality*;  $x$ , agents' *preferences*;  $\hat{v}$ , *consistency over time*;  $\check{v}$ , *inconsistency over time*; and  $U$ , *utility (satisfaction)*. The parameters,  $U$ ,  $\hat{v}$ , and  $\check{v}$  are observational terms whose meaning are rooted in language. The parameters,  $y$  and  $x$  are analytic terms. The study asks, "*what is the relation that ought to subsist between  $y$  and  $x$ ?*" The study's answer asserts, "either  $\hat{v} \circ x$  or  $\check{v} \circ x$ , but not both simultaneously, reside in, that is, are properties of  $y$ ; each, as such is a source of  $U$ ". Clearly, it is  $U$ ,  $\hat{v}$ , and  $\check{v}$  that link the inferences to language, such that the inferences are also conceptually behavioral, not merely mathematically behavioral.

Consistent with *LP3*, Marqués and Wiseman (2008) propose a two-statement rendition of the recommendations in Caldwell (1988) for the acceptability of new mathematical formalism; however, neither study specifies the criteria that govern any such two-statement combination. Relatedly, ascertaining the absence of governing conditions, Marqués and Wiseman (2008) opine that such two-statement combinations do not seem to care about the truthfulness or otherwise of the underlying mathematical formalism. Contrarily, this study arrives at governing criteria that are well defined, namely the

commensurability of a mathematical formalism with the essence of the concept or principle that is modeled - is the concept or principle natively mathematical, or otherwise? - and the requirement that the two-statement combination consist of an analytic definitional statement and an observational statement. Rather than an engagement with notions of falsehood or truthfulness, which can tend to be problematic, this study arrives at the pragmatic requirement that the introduction of mathematical formalism into the development or progression of an epistemology ought to be restricted not to either compromise or contradict the essence of things, a requirement this study believes sufficiently approximates a demand for truth (for concurrence, see Reinke 2021).

**Logical Proposition 4** *An academic discipline arrives at propositional truths, equivalently arrives at a robustly formulated epistemology, equivalently arrives at a non-vacuous epistemology ‘if and only if’ the epistemology is rooted in some appropriately formulated mathematical formalism.*

**Proof.** Suppose otherwise. Then an epistemology is rooted in language, equivalently in logic. But applying Propositions 1 through 3, logic lacks the capacity for propositional truth. If then an epistemology embeds propositional truth, the propositional truth is not derived from logic, rather is the outcome of some mathematical formalism that is appropriate to the objective. ■

**Logical Proposition 5** *A mathematical formalism that is inappropriate to the objective can be introduced into an academic discipline.*

**Proof.** In either logic or mathematics, a material exception to an assertion suffices as evidence that the assertion can be falsified. Suppose then that any mathematical formalism is appropriate to the objectives of an academic discipline.

Suppose the question, “what is rationality?” The answer that is furnished by neoclassical mathematics? Rationality is evident in utility, equivalently value maximization. But then what is utility or value? Well, *utility is a cardinal number; value is whatever gives an agent utility, that is, numbers*; as such, *an agent maximizes numbers*, clearly a vacuous definition, because utility is not linked to any behavior, equivalently is not linked to any concept. So then utility, equivalently rationality can be different things to different agents; to shareholders, *profit*; to environmentalists, spending on the environment which, feasibly *decreases profits*; to labor, fairer wages, which feasibly embeds a demand that shareholders settle for *smaller profits*; clearly, there is arrival at a formulation of rationality in the context of which feasibly one agent’s exhibition of rationality results, explicitly in the demand that another agent abandon rationality. In presence of the conflicts inherent in its’ parameterization of rationality, neoclassical formalism is inappropriate to a parameterization of the concept of rationality.

Suppose, contrary to the foregoing, that we assume all agents characterize rationality as income (number) maximization. Suppose also that, somehow, the adopted characterization does not generate any conflicts between labor, environmentalists, and shareholders. Applying the adopted agreement, only agents who actively exert effort towards non-increase to their incomes act irrationally. There is arrival, however, simultaneously at

absence of any room for either *ethics* or *morality* in a parameterization of rationality (not that there is any room in the preceding paragraph). To see this, suppose we replace ‘*man*’ with ‘*ethical man*’, yet maintain that rationality has characterization as *income maximization*. Necessarily, the adjective, ‘*ethical*’ is rendered redundant. If then, ethical man is substituted for man, necessarily it is whatever value that ethics connotes that is maximized, not income. But then, ethical man is an analytical term, which as yet does not have any analytical definition. Further, since ethics solely connotes a concept or principle, the notion that it can be maximized, which requires a cardinal property, is lacking in robustness. Yet again, there is arrival at the non-robustness of neoclassical mathematics to a parameterization of agents’ rationality.

Lastly, by its very definition, utility or value maximization is parametric. Rationality is, however, a property of man; as such, as is seminally established in Samuelson (1938), ideally it is primarily ordinal, then only feasibly cardinal. For evidence that first order maximization is only one of several feasible interpretations of value maximization, see Milgrom and Stokey (1982). ■

**Logical Proposition 6** *A parameterization of rationality is of core importance to all of the disciplines of the SSH. A parameterization of rationality that is robust to all of the SSH is inherently behaviorally specified, yet has rendition ‘analogously’, as opposed to ‘natively’ in mathematical parlance.*

**Proof.** If, as in Smith (1776), self interest is the essence of man and the source of market equilibriums, but yet is feasibly perverted by either greed or unawareness as to the adverse macroeconomic implications of a certain course of action, a determination of the constraints that restrict self interest to be rational is core to studies of man, as such is core to all of the disciplines of the *SSH*. If a parameterization of rationality is robust to all of the disciplines of the *SSH* (the recommendation, for instance, in Gintis 2014), it is behaviorally specified and is non-vacuous, that is, is derived from mathematical formalism, as such, has rendition, ‘*analogously*’ in mathematical parlance. If, otherwise, a parameterization of rationality is natively mathematical, rationality is inherently mathematical, not inherently behavioral, clearly an assertion that is lacking in robustness, because neoclassical theory agrees that, conceptually, rationality has characterization as, ‘*non-satiation with respect to wealth*’, clearly a qualitative parameter of man which feasibly is as robustly demonstrated in the closed interval, [\$20,000, \$60,000] as in the closed interval, [\$60,001, \$1 million]. ■

## 2.2 Mathematical Formalism, Neoclassical Theory, and Heterodox Theory

Applying Propositions 4 and 5, there is arrival at an important question, namely how exactly to ensure that it is mathematical formalism that are appropriate that are introduced into a discipline of the *SSH*? The stated question is the objective of the mathematical formalism that is developed in Section 3. In the mean time, it is important to discuss Propositions 4 and 5 in the context of the current status quo in respect of the use of mathematical formalism in the *SSH*. Since the use of mathematical formalism is, at

the present time, mostly restricted to the disciplines of economics and finance, necessarily the discussion revolves around the status quo in those two related disciplines.

Concurring on the relevance of Proposition 5, each of Sugden (2002), Dusek (2008), and Rodrik (2015) discuss the non-realism of much of the mathematical formalism that has been introduced into the discipline of economics. In stated respect, the non-robustness of the neoclassical formulation of rationality to the incorporation of either ethics or morality into behavior is part rationale for the development of heterodox theory (see for example, Lee 2008, 2011). In so far as the attitude to the introduction of new tools into the *SSH* is concerned, Lee (2011) characterizes the status quo as follows. Let the existing clusters of methodologies have characterization as, ‘*churches*’. In Lee (2011), the emergence of a significant deviation from any of the existing churches has characterization as the emergence of an ‘*heresy*’. In Lee (2008), see also, Marglin (2023) or Heise (2023), heretics are sanctioned, are unable to secure jobs in the most prestigious academic departments. In Dobusch and Kapeller (2012), in no particular order, the existing churches have enumeration as:

- Neoclassical Theory or Orthodoxy
- Heterodox Theories
  - The Austrian School
  - Evolutionary Theory
  - Institutionalists
  - Ecological Theory

As is established in Proposition 5, the major flaw of Neoclassical theory resides in a non-robust characterization of rationality that is inferior to the philosophical parameterization of man as boundedly rational in Simon (1976, 1978, 2000), a parameterization that now is mathematically formalized in Obrimah (2022, 2023). With focus on all of the disciplines of the *SSH*, I frame the definition of Heterodox Theory in Lee (2008) as follows.

Heterodox theory is an empirically grounded theoretical explanation of the process of social provisioning within a society, a theory that emphasizes ethics, morals, justice, and institutions.

It is straightforward to infer that rather than a tackling of the major deficiency of Neoclassical theory, namely a parameterization of rationality that is inherently non-robust, heterodox theory sidesteps the issue, rather as is evinced in Proposition 3, redefines value maximizing man as ‘ethical’ man, then focuses on the optimization of any of, ethics, morals, justice, or institutions. But then, yet again, there is arrival at multitudinous specifications of rationality which feasibly conflict, as such, much as is the case with neoclassical theory’s definition, there is arrival at a parameterization of rationality that is not invariant either across agents or across time. Plus, with none of ethics, morals, justice, or institutions having, as yet, any analytical definitions, there is arrival, nonetheless at a parameterization

of rationality that is inherently vacuous. Consider then that neither neoclassical nor heterodox theories consider the availability of an analytically formulated yet behavioral parameterization of rationality to be of any significant consequence, yet regardless the two theories are conflicting theories? Seems more like Orthodox Protestants, Catholics, Charismatics, Anglicans, Contemporary Protestants and all of their respective dichotomous attitudes towards the sanctity of church services. Dobusch and Kapeller (2012) concur on the seeming artifice of the disagreements, assert that most of heterodox theory is of the *combative* sort, that is, is either of the ‘*Selfish*’ (for dominance or survival of the proposers) sort; or the ‘*Disinterested*’ (containment of the status quo) sort; hardly is of the ‘*Interested*’ (ecumenical integration and diversification - neither in this church, or that church) sort. In each of Holcombe (2008) and Marglin (2023), in the absence of an ecumenical effort, much as is the case with neoclassical theory, heterodox theory is premised on some ideology or the other, as such, models partial, as opposed to general equilibriums, resulting, qualitatively in the same sorts of non-robustness’ of which neoclassical theory is guilty.

Suppose then that we take a step back to Adam Smith (Smith 1776) and the seminal insight that a socioeconomy is the outcome of the aggregate of socioeconomic agents’ individual self interests, as such the centrality of agents’ intentionality and the mechanisms which can be deemed to best constrain agents’ self interests to be rational. Well, the central question then in the study of man and man’s activities (Marshall 1895) has constitution as an analytic parameterization of man’s intentionality and rationality; as such, the importance of a theory of rational choice, exactly the effort that is recommended to Sociology in Gintis (2014). Whereas Sociology has engaged with efforts for the development of a theory of rational choice, a theory which Sociology posits ought to be built up from the parameterization of some representative agent’s rationality (Weber 1922; Boudon 2003), as yet, there does not exist any successful peer reviewed effort.

In Dobusch and Kapeller (2012), four different instruments have been proposed towards a common platform for the integration of intellectual efforts in the *SSH*. The four feasible instruments that have been proposed are, respectively, *ontological reflection* (see for example, Nelson 2003; Lawson 2010); *methodological similarities* (Dow 2008 observes, but argues against); *opposition to mainstream economics* (Colander et al. 2004; Dequech 2007-2008); or *capitalism* (Lee 2010). Yet again, note the stark omission of the intentionality and rationality of man as a common platform for the integration of efforts in the *SSH*. Contrarily, as is the case in this study, in Fales and Markovsky (1997) and Mearman (2011), the *concepts and principles* of a discipline are the anchor instruments of the discipline. However, Mearman (2011) is unable to decipher any agreement on either concepts or principles among proponents of heterodox theory. Consistent, as such, with the foregoing, heterodox theory is not arranged around concepts or principles, rather as is inferred in Dobusch and Kapeller (2012), is arranged into *social (affiliation)*, as opposed to *logical (conceptual)* constructs. With the quote taken from Dobusch and Kapeller (2012, pg. 1040), Dequech (2007-2008, pg. 279) concurs with Mearman (2011) as follows:

Heterodox economics can be defined negatively, in opposition to ... the mainstream. ... Another possibility would be to define heterodox economics positively, but the result in the current period may be an empty set.

In presence of mathematical formalisms that are congruent, that is, that are similar,

this study's formal theory infers that, feasibly new studies only advance or refine an existing formalism, that is, do not have any impact on either concepts or principles (Nowak-Posadzy 2018 discusses such a feasibility). If then a discipline is to arrive at refinements to its concepts, there is arrival at the necessity of, at the very least, contributions that embed logical arguments. Further, if the goal is a progression to concepts or principles, there is arrival at the inadequacy of logic, as such, the necessity of mathematical formalism. Clearly, there is arrival at the beneficence, as opposed to the nuisance of new mathematical formalism that are appropriate to their objectives. In presence of the foregoing, clearly a recommendation of *methodological similarities* as a platform for the integration of the *SSH* is inimical to intellectual progress. Dow (2008) concurs. If *ontology* matters, there is arrival at the requirement that mathematical formalism be robust to the parameters of the real world, clearly a desirable requirement whenever researchers claim to be engaged with *modeling the world as it should be*, as opposed to an engagement with efforts that have characterization as, *modeling the world as it feasibly could be operationalized* (see for example, the discussion in Grüne-Yanoff and Verreault-Julien 2021). Since *opposition to neoclassical theory* does not, in of itself, facilitate advancements to concepts or principles, such a premise is not a robust foundation for the integration of the *SSH*. Further, since an epistemology ought not be bounded by a philosophy, capitalism, which is not robust to general equilibrium, namely the existence of alternate ideology, a premising of the integration of the *SSH* on capitalism is not robust to the parameters of the real world. In aggregate, by elimination, it seems a foundation consisting of models of rational man that are appropriately formulated is, perhaps, the most robustly unifying common ground for the development of epistemology which embeds a common ground for either economics or all of the *SSH*. But then, the need of benchmarks for assessing the robustness of any and all engagements with mathematical formalism. This study furnishes exactly such benchmarks. In aggregate, this study postulates that the term, *heterodox theory* solely signify effort that more appropriately and robustly parameterizes man as a *rational satisficing agent*. In stated respect, in Ogrimah (2022), a rational satisficing agent searches for ends and facilitating actions which, conditional on the agents' stock of awareness' are anticipated to engender the supremum of utility, as such is an optimizing agent, not an agent who settles for just any realization of utility.

## 2.3 Some Prior Studies

Using the discussion of Rodrik (2015) in Grüne-Yanoff and Marchionni (2015), the algorithm for model choice that is advocated in Rodrik (2015) consists of three sequential steps. *First*, using the metrics, the *plausibility*, *reasonability*, and *intuitive sense* of each model, arrival at a sample of feasible models. *Second*, using the resulting sample, a juxtaposition of the critical assumptions that undergird each model for inferring the models whose critical assumptions are less robust to the specific formal theoretical endeavor. *Third*, a subjection of the remainder candidate models to empirical verification. Right off the bat, as is already highlighted in the introductory section, it is important to note that the *third step* violates an important canon that is set forth in Roll (1977), to wit, if a formal theoretical model is propositionally true, its rejection by empirical tests only can have interpretation as evidence that the empirical tests are flawed, that is, are not



adequate to the objective. The evidence that the stated canon is true is binding, namely absent the canon, each of the *Capital Asset Pricing Model (CAPM)* of Sharpe (1964), which forms the basis of the well accepted Fama and French (1993) three factor model and the Intertemporal CAPM of Merton (1973) would, on the basis of the initial empirical tests, have been rejected. In stated respect, the current consensus is the truthfulness of the CAPM, yet the inference that it only can be multifactor, as opposed to unifactor (see for example, Fama 1996; Campbell 1996; Diacogiannis and Ioannidis 2019; and Obrimah 2023c). Whereas then the concept is accepted to be propositionally true, regardless, consistent with the postulation that, feasibly empirical formalism facilitate logical inferences, it's empirical specification has been refined, improved. Whereas the *ICAPM* lacked empirical corroboration up until 2004, subsequently it has been corroborated in each of Ghysels, Santa-Clara, and Valkanov (2005) and Lanne and Saikkonen (2006). With focus on the management of a macroeconomy, Lucas (1976) concurs with Roll (1977), infers the same propositional truth, namely that epistemology cannot robustly be premised on empirical formalism.

Turning attention to the *first step*, as is noted in Grüne-Yanoff and Marchionni (2015), the metrics, *plausibility*, *reasonability*, and *intuitive sense* embed a lot of ambiguity; however, ambiguity that is mediated by the outcomes of this study's formal theory. To see this, applying this study's outcomes, this study's test for plausibility, reasonability, and intuitive sense commences with the question,

“does a study's research question have domicile in concepts or principles which do not have a native mathematical specification?”

If the answer is “*Yes*”, all mathematical formalism that are parametrically specified are excluded. If the answer is “*Nay*”, noting it is impossible for a natively mathematical concept not to have rendition, analogously as a concept or principle, there is arrival at an antecedent question, namely,

which of parametric or non-parametric formalism seem more appropriate to a workout of the formal theory?

If the objective is, at the margin, a general equilibrium, the quest is conceptual, hence, a priori, the superior robustness of non-parametric formalism;

if, otherwise, the objective is, at the margin, a partial equilibrium, the quest is parametric, hence, a priori, the seeming superior robustness of parametric formalism.

The *second step* that is recommended in Rodrik (2015), which is solely applicable to mathematical formalism that is parametrically specified, only can be tackled on a case by case basis. Importantly, with the shortcomings that are enumerated in what follows proffered in Grüne-Yanoff and Marchionni (2015), this study's outcomes address the following shortcomings of the three steps that are proposed in Rodrik (2015), namely:

- i. ambiguity as to the criteria for the inclusion of models - this study provides selection criteria.

- ii. the feasibility that critical assumptions become so entwined with study objectives, they become inseparable - in this study, only study objectives factor into the choice between models; study assumptions are excluded from consideration.
- iii. the possibility that critical assumptions remain unrealistic - realism commences with an ascertainment of the appropriateness of the candidate mathematical formalisms to the concepts or principles that the formalism seeks to refine, extend, refute, or replace.
- iv. problems inherent in efforts for model verification - as discussed, a recommendation that feasibly is plagued by *Type I* errors.

For the mitigation of excessive methodological pluralism, Gräbner and Strunk (2020) propose the management of model quality and the communications that transpire between academics. As is already discussed in the foregoing, this study's outcomes impose a quality constraint on the admission of new mathematical formalism. In stated respect, evidence that a new mathematical formalism that passes this study's tests of appropriateness robustly addresses issues that are already identified in the literature, but which hitherto remained unresolved suffices as evidence of model quality. In similar vein, a focusing of communication between academics on the merits of new mathematical formalism narrows down the scope of communication, as such increases the likelihood that communication is constructive and effective. Lastly, the formal theoretical outcomes in Section 3 refine the dichotomy that is postulated, in Grüne-Yanoff and Verreault-Julien (2021), between either *how-actually formalisms (HAF)* or *how-possibly formalisms (HPF)* of economic phenomena. In stated respect, whereas mathematical formalism that are either parametrically or non-parametrically specified are appropriate to the development of the *HPF*, with the *HAF* aspiring to general equilibrium rationalizations, ideally they are implemented using mathematical formalism that are non-parametrically formulated.

### **3 Benchmarking the introduction of mathematical formalism into the development of an epistemology**

If a secular academic discipline endeavors to develop a non-vacuous epistemology, the exposition in the preceding section demonstrates the necessity, as appropriate, of some mathematical formalism. The anecdotal empirical evidence is supportive of the inference, namely non-arrival, as yet at a non-vacuous epistemology in the disciplines of Sociology or Psychology, or in *Virtue Ethics*, a sub-discipline of Philosophy. Consistent with Axiom 1, whereas the logical arguments that are proffered in the preceding section establish the enumerated inference, simultaneously they are non-robust to inferences in respect of the parameters that bound any and all efforts for the introduction of mathematical formalism into the development of an epistemology. In stated respect, since any boundaries that are inferred bound mathematical formulations, ideally they, the boundaries have specification, natively in mathematical parlance; necessarily, as such, logical arguments are non-robust to the objective. Inferences in respect of the rational benchmarks which bound the

introduction of mathematical formalism into the development of a discipline’s epistemology are the objective of the mathematical formalism that I introduce (develop) in this section. If the mathematical effort is premised on some ideology, the effort becomes the new *partial equilibrium* wall or ‘church’ which militates against any future progress (for concurrence, see Holcombe 2008 or Marglin 2023). If then the mathematical effort is to be robust, there is arrival at the demand, not for ideology, but rather for pragmatism, exactly the most pressing deficiency of neoclassical theory (for concurrence, see Dusek 2008; Gintis 2014). Evinced the pragmatism of this study’s effort, consistent with the argument that static equilibriums are inherently vacuous (see for example, Denis 2008), the formal theory is explicitly cast as a dynamically evolving equilibrium. Switching focus to model outcomes, if the outcomes of the formal theory are robust to all of the *SSH*, applying Proposition 6, the effort results in concepts and/or principles, yet study outcomes have rendition, analogously in mathematical parlance. There is arrival, as such, at an important inference, namely that the sole difference that subsists between concepts and principles that are formulated using solely language or concepts and principles that are facilitated by mathematical formalism is the absence, or otherwise of form and structure, equivalently the absence, or otherwise of rigor. The enumerated condition will be seen to be satisfied.

Let  $\Theta$  denote an academic discipline. Let  $\Gamma$  denote the concepts or principles which define  $\Theta$ . For illustration, in philosophy, *virtue ethics* is an element of  $\Gamma$ ; and in sociology, *social choice* is an element of  $\Gamma$ . Consider social choice; whereas it, social choice is, as yet, lacking in an analytical definition, it has description as the aggregate of the individual choices that are engaged with by all of the agents who make up a socioeconomy (see for example, Weber 1922; Boudon 2003; or Paternotte 2011). Regardless, at the present time there does not exist any peer reviewed published study which robustly delineates how exactly individual choice aggregates into social choice, hence the current convenience, namely the dissociation of the formation and growth of institutions from individual choice (see for example, Udehn 2003, pg. 151). Whereas then social choice is already defined, regardless the definition remains vacuous, that is, lacks rigor.

Let  $\psi$  and  $\vartheta$  denote, respectively, any *logical* or *mathematical* formalism. Also, let ‘ $\parallel$ ’ denote the presence of *commensurability* between different items. For concreteness, the concept of commensurability relates to the *properties* of things (see for example, Kuhn 1982; Nussbaum and Hursthouse 1984; Schauer 1997); in stated respect, if any two items are commensurable, the defining properties of the items are held in common.

**Theorem 1** *Let  $\Gamma$ ,  $\psi$ , and  $\vartheta$ , be as defined in the preceding. Imposing rationality on an academic discipline,  $\Theta$ ,*

$$\Gamma \parallel \psi \parallel \vartheta; \tag{1}$$

*necessarily, as such,  $\vartheta$  consists of mathematical formalism that have rendition, analogously as concepts or principles, as such that have rendition as analytical statements, equivalently*

as propositional truth. Necessarily, as such,

$$[\Gamma \cap \psi \cap \vartheta] \neq \emptyset \quad (2)$$

$$[\Gamma \cup \psi \cup \vartheta] \subseteq [\Gamma \cup \vartheta] \quad (3)$$

$$\Gamma \supset \vartheta \supset \psi \quad (4)$$

$$[\Gamma \cup \psi \cup \vartheta] \subset \Theta. \quad (5)$$

**Proof.** See the Appendix. ■

Equations (1) and (2) are commonsensically true. Equation (3) is already admitted as a logical proposition (see for example, Frege 1984, pg. 113), but is mathematically inferred in the proof of Theorem 1. It might seem, however, that equation (3) is not true.

Whereas equation (3) is mathematically established in the proof of Theorem 1, it has, logically, rationalization as follows. If all  $\vartheta$  are formulated for the advancement of  $\Gamma$ , any robustly formulated  $\Gamma$  are robust to an infinity of permutations to  $\vartheta$ ; necessarily, as such,  $\Gamma(\vartheta) \subseteq \Gamma$ . Since, however, there exist  $\Gamma$  that do not have expression as  $\vartheta$  - the existence of observational terms suffices - imposing the canon that  $\vartheta$  has constitution as ‘a specific language’,  $\Gamma(\vartheta) \equiv \vartheta$ , successively there is arrival at,  $\vartheta \subseteq \Gamma$  and the refinement,  $\vartheta \subset \Gamma$ .

Using a similar argument, to wit, a robustly formulated  $\Theta$  accommodates an infinity of improvements to each of  $\psi$  or  $\vartheta$ , allowing yet again for the existence of observational terms, equation (5) is satisfied. Importantly, with equation (4) implying,  $[\Gamma \cap \psi \cap \vartheta] = \psi$ , consistent with the discussion in the introductory section, all PhD’s or knowledge workers have characterization as Logicians, equivalently Philosophers.

Theorem 1 implies the following. Suppose the concepts and principles,  $\Gamma$  of a discipline,  $\Theta$  tend to be solely conceptually formulated, that is, tend not to be natively mathematical. The qualifying disciplines in the *SSH* are inclusive of, *Philosophy*, *Sociology*, and *Psychology*. The demand that  $\Gamma$  be commensurable with  $\vartheta$  implies it is mostly  $\vartheta$  that have non-parametric formulations that are appropriate to the development and progression of the epistemology of the three enumerated disciplines. In presence of the insight, it is mostly the tools of *Pure Mathematics*, such as, *Set Theory*, or more broadly *Analysis*, *Topology*, etc. that are robust to the introduction of mathematical formalism into the three enumerated disciplines. Studies which provide evidence that ordinal (qualitatively specified) variables, such as ordinal utility, are measurable - evidence that they can be mathematically modeled - are inclusive of, Marschak (1950), Herstein and Milnor (1953), Alchian and Kessel (1962), and Obrimah (2022, 2023, 2024b).

Since the disciplines of Economics or Finance embed concepts or principles that have native mathematical formulations, e.g. a *Price Index*, conditional on the ethos of  $\Gamma$  - purely conceptual, but amenable to mathematical formalism or conceptual but natively mathematical - there is arrival at the appropriateness of either non-parametric formalism or parametric formalism, such as either *differential* or *integral calculus*. Whereas the physical sciences reside outside of the scope of this study, with  $\Gamma$  typically natively mathematical in the physical sciences, there is arrival at the dominance of parametric formalism in the domain of the physical sciences. Corollary 1 formalizes the foregoing.

**Corollary 1** Let  $\Theta^C \subseteq \Theta$  denote a subset of  $\Theta$  consisting of concepts or principles which reasonably cannot be deemed to be natively mathematical. Also, let  $\tilde{\Theta} \subset \Theta$  denote a subset

of  $\Theta$  consisting of concepts or principles that are natively mathematical. If the introduction of mathematical formalism into the modeling of the  $\Gamma^C \in \Theta^C$  (respectively,  $\tilde{\Gamma} \in \tilde{\Theta}$ ) is robustly implemented, all qualifying mathematical formalism are non-parametrically (respectively, parametrically or non-parametrically) specified. For concreteness, in presence of,  $\tilde{\Gamma} \in \tilde{\Theta}$ , if  $\tilde{\Gamma}$  is a general equilibrium (respectively, non-general equilibrium) concept, non-parametric (respectively, parametric) formalism are more appropriate to the parameterization of  $\tilde{\Gamma}$ . In terms, as such, of mathematical formalism, a natively mathematical, yet general equilibrium  $\tilde{\Gamma}$  is no different from a purely conceptual  $\Gamma^C \in \Theta^C$ , resulting in the inference that all concepts that are not natively mathematical have characterization as general equilibrium concepts. All of the disciplines of the SSH are parameterized by,  $\Theta^C \neq \emptyset$ , as such by,  $\tilde{\Theta} \subset \Theta$ .

**Proof.** See the Appendix. ■

Corollary 1 rationalizes neoclassical theory's fixation with partial, as opposed to general equilibrium problems, namely the non-robustness of parametric mathematics to the resolution of general equilibrium problems. For a discussion of the 'natural' restriction of neoclassical theory to the modeling of partial, as opposed to general equilibriums, see Salanti (2020).

Corollary 1 has illustration as follows. The study, Ogrimah (2024) asks, "does there exist a robust structure for the determination of discount rates?" Clearly, the question is inherently parametric, yet searches for a general equilibrium, that is, a concept. Consistent with Corollary 1, using a non-parametric structure, the study arrives at a martingale, equivalently a generic (always true) parametric equation that spans discount rates. On the other hand, Ogrimah (2023b) asks, "Does there exist a proactively formulated fiscal policy structure (concept), which applying the seminal insights in each of Lucas (1976) and Kydland and Prescott (1977) can be substituted for monetary policy as the primary tool for the management of a socioeconomy?" Whereas the study infers, via mathematical formalism, the structure of such a fiscal policy, as ought ideally be the outcome, all of the study outcomes have characterization as concepts, not implementational equations. Consider then that non-parametric formalism can be robust to either parametric or non-parametric outcomes; further, the robustness with which non-parametric formalism are implemented is decipherable from a juxtaposition of the mathematical nativeness, or otherwise of the general equilibrium concepts that are modeled and the outcomes of the formalism.

For the counterpositive evidence, particularly evidence that a violation of Corollary 1 tends to induce a violation of item (ii) of Section 2, namely a direct predication of study findings on study assumptions, consider each of Baillon and Bleichrodt (2015) or Dal Forno and Merlone (2010). With the quotes verbatim, Baillon and Bleichrodt (2015) state as follows,

To test prospect theory, we added the restriction that decision makers are less sensitive to changes in likelihood under ambiguity than under risk (Wakker 2010; Abdellaoui et al. 2011) - pg. 83.

The observed violations suggest that descriptive ambiguity models should allow for less sensitivity to changes in likelihood under ambiguity than under risk and for different ambiguity attitudes for gains and losses - pg. 78.

Clearly, the study assumes it's inference. To see this, with the  $\vartheta$  that is adopted in that study parametrically specified, but with the study parameterizing beliefs (behavior) via estimates of likelihoods (a parametric measure), the study assumes the belief, then models likelihoods; but then can a parametrically estimated likelihood deviate, in principle, from an underlying belief that, a priori, is assumed to be rational? Applying Corollary 1, the robustness of the study is enhanced if the study either models beliefs non-parametrically or abstracts away from beliefs and models likelihoods parametrically, as such does away with the endogeneity that is inherent in the implementation of the study.

Consider then Dal Forno and Merlone (2010). Under the assumption that an agent,  $x$  does not tolerate shirking from any other agent  $y$ , the study arrives at the inference that there exists some threshold effort level that, adopted by  $y$  induces  $x$  to also engage with shirking behavior. Clearly, the assumption of *intolerance* on the part of  $x$  suffices for the stated inference. Yet again, with shirking having characterization, first and foremost as a behavior, the adoption of a parametric  $\vartheta$ , which is inherently incommensurable with the modeling of a behavior, induces the necessity of an assumption - intolerance for shirking, itself also a behavior - that facilitates commensurability between the workout of the model and the study's behavioral concept. For concreteness of a snafu, consider that either shirking or non-contracting are characteristic of general equilibrium; the adverse impact of the friction is, as such, unresolved.

**Corollary 2** *If the introduction of mathematical formalism into  $\Theta$  is robustly implemented,  $\Theta$  has parameterization as,*

$$\left[ \Theta^C \cap \tilde{\Theta} \right] = \emptyset \quad (6)$$

$$\left[ \left( \Theta^C \cup \tilde{\Theta} \right) = \left( \Theta^C + \tilde{\Theta} \right) \right] \quad (7)$$

$$\Theta = \left[ \Theta^C \cup \tilde{\Theta} \right]. \quad (8)$$

**Proof.** Equation (6) follows directly from the incommensurability of equations (21) and (22) of the proof of Corollary 1, which combined directly connote  $\left[ \Theta^C \cap \tilde{\Theta} \right] = \emptyset$ ; combined, equation (6) and Corollary 1 directly imply equations (7) and (8). ■

Corollary 2 has illustration as follows. Suppose the assumption that a price index,  $pi$  satisfies,  $pi \in \Theta^C$ . By definition,  $pi \in [1, 100]$  or  $pi \in \mathfrak{R}^+$ ;  $pi$  is, as such, inherently mathematically specified. Having arrived at a contradiction, necessarily,  $pi \notin \Theta^C$ ; as such,  $pi \in \tilde{\Theta}$ . In similar vein, suppose the question, “*What is the price of risk (pr)?*” is deemed to satisfy,  $pr \in \Theta^C$ . By definition,  $pr$  satisfies,  $pr \in \mathfrak{R}^+$ , as such is inherently mathematically specified. Yet again, having arrived at a binding contradiction, necessarily,  $pr \in \tilde{\Theta}$ . Suppose, however, a behavioral question from the realm of Business. For concreteness, suppose the question, “*Which marketing campaign should we adopt (mc)?*”

With the question boiling down to a choice between different conceptions of a marketing effort, the question is inherently conceptual, not mathematical. Necessarily, as such,  $mc \in \Theta^C$ . For concreteness, applying Modigliani and Miller (1958), relative to the choice of the asset to purchase - the choice as to the marketing campaign to adopt - the choice as to how exactly to design and finance the winning concept is a secondary question; the design and cost question is, as such, antecedent to the following question, namely “*which sort of campaign is more likely to catch consumers’ attention?*” Naturally, there is arrival at the antecedent question, namely “*What are the different feasible designs and costs of the different feasible specifications of the winning concept (cc)?*” If cost is of paramount importance, there is arrival at a natively mathematical question, as such, at  $cc \in (\Theta^C)^c = \tilde{\Theta}$ . In presence of the foregoing,

$$[x \in \Theta^C] \rightarrow [x \notin (\Theta^C)^c = \tilde{\Theta}].$$

**Theorem 2** *Let ‘ $\succ$ ’ denote strictly preferred;  $S$ , an analytic statement consisting of a definition and an observational statement;  $\Gamma_{t-1}$ , an existing set of concepts or principles for some discipline,  $\Theta$ ; and assume that a new mathematical formalism,  $\vartheta_t^w$ , which spans concepts or principles,  $\Gamma_t^w$  is robustly formulated and implemented. Then,*

$$\vartheta_t^w(\Gamma) \succ \vartheta_{t-1}(\Gamma) \tag{9}$$

$$\longleftrightarrow \Gamma_t^w \succ \Gamma_{t-1} \tag{10}$$

$$\longleftrightarrow S(\Gamma_t^w) \succ S(\Gamma_{t-1}); \tag{11}$$

that is, there exists an ‘if and only if’ relation between the dominance relations in each of equations (9) through (11).

**Proof.** It is axiomatic that every  $\vartheta$  spans a corresponding  $\Gamma$ , and that every robustly specified  $\Gamma$  is expressible as an analytic statement,  $S$  (the citations in the introductory section). In presence of the foregoing, the relation,  $[\vartheta_t^w \succ \vartheta_{t-1}] \leftrightarrow [\Gamma_t^w \succ \Gamma_{t-1}]$  embeds a binding contradiction; further, since the rigor of a concept or principle is evident in it’s analytic statement,  $[\Gamma_t^w \succ \Gamma_{t-1}] \longleftrightarrow [S(\Gamma_t^w) \succ S(\Gamma_{t-1})]$ . Lastly, whenever  $\Gamma_t^w | \vartheta_t^w$ , necessarily,  $[\Gamma_t^w \succ \Gamma_{t-1}] \rightarrow [\vartheta_t^w \succ \vartheta_{t-1}]$ . There is arrival, as such at an *if and only if* relation between the dominance relations in each of equations (9) through (11). ■

**Corollary 3** *An analytic statement,  $S(\Gamma_t^w)$  satisfying,  $S(\Gamma_t^w) \succ S(\Gamma_{t-1})$  is a necessary and sufficient condition for inferring  $\vartheta_t^w(\Gamma) \succ \vartheta_{t-1}(\Gamma)$ .*

**Proof.** Follows directly from Theorem 2. ■

**Lemma 1** *The term, ‘progression’ does not coincide with the term, ‘refinement’.*

**Proof.** Let  $\Gamma^v$  denote a progression of  $\Gamma$  and let  $\Gamma^e$  denote a refinement of  $\Gamma$ . Let  $N$  be the aggregate number of elements in  $\Gamma$ . A refinement of  $\Gamma$  induces,  $\Gamma \subset \Gamma^e$ , but  $N(\Gamma) < N(\Gamma^e)$ , such that  $x \in \Gamma \rightarrow x \in \Gamma^e$  and for every  $y \in \Gamma^e$ ,  $\exists$  some  $\{\hat{x}, \check{x}\} \in \Gamma$

satisfying,  $y = (\hat{x} \circ \check{x})$ , but  $y \notin \Gamma$ . For illustration, let  $\mathbb{Z}$  denote the set of integers. Whereas  $\{1, 3\} \in \mathbb{Z}$  and whereas  $(1 \circ 3) = 1/3 \in \mathfrak{R}$ , regardless,  $1/3 \notin \mathbb{Z}$ .

Since a progression of  $\Gamma$  results in the introduction of new elements into  $\Gamma$ , necessarily  $\exists$  some  $x$  satisfying,  $x \in \Gamma^v$ ,  $x \notin \Gamma$ . However,  $\nexists$  any  $\{\hat{x}, \check{x}\} \in \Gamma$  satisfying,  $x = (\hat{x} \circ \check{x})$ . ■

**Theorem 3** *The ‘if and only if’ dominance relations in Theorem 2 embed the necessary conditions,*

$$\Gamma_{t-1} \not\subseteq \Gamma_t^w \quad (12)$$

$$\Gamma_t^w \not\subseteq \Gamma_{t-1} \quad (13)$$

$$\Gamma_t^w \not\parallel \Gamma_{t-1} \quad (14)$$

$$\rightarrow \Gamma_t^w \equiv \Gamma_t^w | \vartheta_t^w \quad (15)$$

$$\rightarrow \Gamma_t^w \equiv \Gamma_t^w | S_t^w; \quad (16)$$

as such imply that, relative to  $\Gamma_{t-1}$ , the new mathematical formalism,  $\Gamma_t^w$  is a disruptive (progressive) formalism; it is solely a disruptive (progressive) mathematical formalism,  $\vartheta_t^w$  that facilitates the ‘if and only if’ relations in Theorem 2. The condition,  $\Gamma_t \subset \Gamma_{t-1}$  connotes a ‘reducing refinement’ of  $\Gamma$  that is premised on either logic arguments or new non-disruptive mathematical formalism. The condition,  $\Gamma_{t-1} \subset \Gamma_t$  connotes an ‘extending refinement’ of  $\Gamma$  that is facilitated by a new non-disruptive mathematical formalism.

**Proof.** See the Appendix. ■

Combined, Corollary 3 and Theorems 2 and 3 establish this study’s main result. In stated respect, the main result requires that the outcomes of the introduction of mathematical formalism into the development of an epistemology have rendition as *analytical statements*, consisting of a *definition* - the outcome of the modeling - and a *descriptive (observational) statement* that links the definition to either language or to an hitherto (existing) observational statement that already links to language. Suppose then two directly competing mathematical formalism that are believed to have been properly implemented. The formalism whose analytical statements are deemed to be both more appropriate and more rigorous (less ambiguous) ought to be strictly preferred, as such, has constitution as the formalism that is more robust to the development or progression of an epistemology. Consider then that this study’s outcomes do not relate to the robustness of the implementation of a model, rather relate to the robustness of *model choice* and *model outcomes*. Since evidence that either model choice or model outcomes are non-robust facilitates abstraction away from the robustness with which a model is implemented - an exercise which requires more effort - study outcomes decrease the costs of model verification. Importantly, consistent with the logical propositions in Section 2, the main results establish that a concept or principle that is solely formulated in language cannot be the source of a refinement, extension, or progression to an epistemology. Whereas logical arguments are able to refine or extend an existing epistemology, it is solely disruptive mathematical formalism that progress an epistemology. If a new mathematical formalism truly progresses an epistemology, there exist elements of the hitherto epistemology that are jettisoned and replaced by new elements. Simultaneously, the new formalism introduces



elements that did not previously exist into the epistemology. Applied to a juxtaposition of value maximization vis-a-vis the parameterization of rationality in Obrimah (2022), study outcomes recommend the parameterization in Obrimah (2022), as follows:

- Obrimah (2022):
  - replaces *value maximization* with *rational satisficing behavior*: more appropriately, substitutes a behavior for an activity
  - replaces *effort for value maximization* with *effort for the unraveling of unawareness*': more appropriately, substitutes a general equilibrium activity (an activity that is integral to man even in informal situations) for a 'formal' natively mathematical activity
  - replaces 'absence of an *how to*' with an '*how to*': any unawareness' that are unraveled are rational only whenever they emerge out of efforts for the refinement of an existing rational awareness; contrarily, the '*subject to*' boundaries on value maximization are solely mathematical, not behavioral; clearly Obrimah (2022) is more appropriate; to see this, whereas per Obrimah (2022), 'subject to' is, qualitatively, exactly the same for each and every agent at any realization of wealth, namely '*subject to existing rational awareness*', there is not any such uniformity in the context of value maximization
  - whereas neoclassical theory confers utility on the items that are subjected to value maximization, in Obrimah (2022), utility is the outcome of improvements to agents' awareness'; utility is, as such, a parameter of man, not a parameter of the items that are consumed or the returns from investments; whereas then value maximization is unable to associate *learning from mistakes* with utility, with improvements to awareness as the outcomes of learning from mistakes, the Obrimah (2022) parameterization of rationality confers positive utility on learning that is the outcome of mistakes; clearly Obrimah (2022) is more appropriate
  - whereas value maximization is arrived at in the context of a feasible mathematical translation of, 'non-satiation with respect to wealth', Obrimah (2022) introduces new mathematical formalism which facilitates a formal theoretical parameterization of rationality, as such, has constitution as the sole rigorous effort.

Whereas the illustration is from the physical sciences, it seems to me that it's familiarity and centrality to the progression of each of welfare and the *social order* on earth renders it a fitting illustration (in part, the six permanent members of the *United Nations Security Council* all owe their permanent status' to *Isaac Newton, Albert Einstein, Marie Curie, etc.*). Consider then *Isaac Newton's Principia*. In order to actualize the *Laws of Motion*, Newton needed to be able to model motion. The mathematical canon at the time consisted, principally, however, in the tools of *Geometry*, which primarily map spaces, not motion. Let the concept,  $\Gamma_t$  to be modeled then be motion, and let the concept previously modeled, namely the *Organization of Space* have designation as,  $\Gamma_{t-1}$ . We have that

$[\Gamma_{t-1} \cap \Gamma_t] = \emptyset$ , resulting in the necessity, to wit, Newton had to develop a new tool, *Calculus*, as such, the satisfaction, simultaneously of equations (12) through (14). Consistent with the highlighted inference, at the timing of his penning of *Special Relativity* in 1905, there did not exist any tool that Albert Einstein could apply towards a reconciliation of special relativity with Newton’s work on the laws of motion. The reconciliation, which resulted in the publication of Einstein’s work on *General Relativity* in 1915 had to wait until the development of *Riemann Integrals*, a tool that, simultaneously is robust to the modeling of each of gravity and relativity.

Alternately, consider the transition between Georgescu-Roegen (1936) and Samuelson (1938). In Georgescu-Roegen (1936), utility is premised on ordered preferences, but supports a parametric specification. As then is the case in Corollary 1, utility,  $U$  is deemed to be inherently parametric, as such robust to the imposition of mathematical formalism that is parametrically specified. But then Samuelson (1938) establishes the non-necessity of all of the parametric specifications that are imposed on  $U$  in Georgescu-Roegen (1936), as such establishes that  $U$  is primarily ordinal and only feasibly parametric (Houthakker 1950 reconfirms the refinement). Let  $\hat{\varkappa}_{t-1} \in \Gamma_{t-1}$  and  $\check{\varkappa}_{t-1} \in \Gamma_{t-1}$  be, respectively the parametric and ordinal properties that are either deciphered or assumed in Georgescu-Roegen (1936). Let ‘ $\succ$ ’ denote ‘*strictly preferred to*’. Using a similar mathematical formalism, Samuelson (1938) arrives at,  $\check{\varkappa}_t$  satisfying,  $\check{\varkappa}_t \equiv \check{\varkappa}_{t-1} \in \Gamma_t \subset \Gamma_{t-1}$ ;  $\hat{\varkappa}_{t-1} \notin \Gamma_t$ ; and  $\Gamma_t \succ \Gamma_{t-1}$ , a refinement that now is termed, the ‘*revealed preference structure*’.

Switching attention to Sociology, let  $\Theta$  be *social choice* theory. If, as in Weber (1922), Boudon (2003), or Paternotte (2011),  $\Gamma(\Theta)$  has specification as, “*social choice is the aggregate of the individual choices of socioeconomic agents*”, as is argued in Gintis (2014) efforts for the modeling of social choice commence in decision theory then extend to game theoretic interactions. Suppose then two sets of mathematical formulations, namely formulations,  $\hat{\vartheta}$  that are decision theoretic, and formulations,  $\check{\vartheta}$  that are game theoretic. Since the  $\hat{\vartheta}$  formulate individual decision making, but the  $\check{\vartheta}$  model the ‘*give and take*’ that facilitate social equilibriums, necessarily if a property  $\varkappa$  satisfies,  $\varkappa \in \hat{\vartheta}$ , it does not embed any ‘*give and take*’; as such, rational expectations demands,  $(\varkappa \in \hat{\vartheta}) \rightarrow (\varkappa \notin \check{\vartheta})$ . If then decision theoretic and game theoretic efforts are composable, they solely are independently directly composable with  $\Gamma$ . Let  $\varsigma$  denote a property satisfying,  $\varsigma \in \check{\vartheta}$ . Applying equation (4), which is proven to be true in Theorem 1,  $\Gamma \supset [\hat{\vartheta} \cup \check{\vartheta}]$ , such that  $\check{\vartheta}$  and  $\hat{\vartheta}$  are complements, not substitutes.

Lastly, consider either Merton (1973) or Black and Scholes (1973). In either study, whereas the research question possesses a behavioral component, the questions are natively mathematical. For concreteness, Black and Scholes (1973) ask the question, “*How should a Call Option that is written on an Asset be priced?*” In Merton (1973), the question is, “*Intertemporally, how should investors go about the formulation of the risk premiums that they demand from the assets in which they seek to invest?*” Clearly, either question is natively mathematical. Whereas feasibly a non-parametric  $\vartheta$  is robust to a resolution of either study’s research question, because each searches for a general equilibrium, not a partial equilibrium, feasibly a parametrically specified  $\vartheta$  is also robust to the same objective. Famously, both studies apply parametric mathematical formulations towards

their respective research questions, resulting in the award of the 1997 Nobel Prize in Economics to each of Robert Merton and Myron Scholes. Consistent with Theorem 3, regardless of the brilliance of the Black and Scholes (1973) model, the consensus is, the decision to model a partial equilibrium which feasibly is the source of the general equilibrium that actually materializes induces the Black and Scholes option pricing model to be lacking in some pragmatism (see for example, Rubinstein 1994; Bakshi et al. 1997).

## 4 Conclusions

The evolution of mathematical formalism in the *social sciences and humanities* (*SSH*), particularly in Economics or Finance has, feasibly description as follows, namely iteratively the putting up of walls around formalisms that are deemed to be appropriate to the modeling of the phenomena that are of interest (see for example, Dow 2008; Holcombe 2008; Lee 2011; Dobusch and Kapeller 2012). Regardless, with the insight emerging out of economics and finance, there is agreement that economic development, equivalently sustainable growth that improves agents' welfare is premised on *technical change*, that is, on either improvements to existing *concepts or principles* (COPR) or the emergence of new COPR and the proficiencies that span the new COPR. Clearly, there is arrival at a binding contradiction, namely an ubiquity of resistance to newly emergent COPR and the proficiencies that span the new COPR, yet the admission that it is advancements to each of COPR and the proficiencies which span the new COPR that sustainably progress man's welfare.

This study provides formal theoretical evidence that it is not mathematical formalisms or methodologies that connect literature either across studies or across time. Rather contrarily, it is the evolution of the analytical formulations of a discipline's concepts or principles, combined a discipline's epistemology that connect literature both across studies and across time. Whereas it is mathematical formalism that solely facilitate progression to the analytical formulations of a discipline's epistemology, the impact of a new mathematical formalism on a discipline's epistemology is shown to increase with the extent to which it is incommensurable with the existing mathematical formalisms. Since then the sizes of the feasible improvements to the welfare of either a socioeconomy or an academic discipline increase with the severity of the incommensurability that subsists between the existing and the new mathematical formalisms, there is arrival at the irrationality of intransigence to the emergence of new mathematical formalisms that are both appropriately applied and rigorously formulated. For illustration, the mathematical formalism in Merton (1973) is highly incommensurable with that in Sharpe (1964). The rationale is straightforward, namely whereas the market portfolio is not a free parameter in Sharpe (1964), it is the sole free parameter in Merton (1973). Similarly, Black and Scholes (1973) is incommensurable with the efforts for the development of option pricing models that preceded it, such as, Boness (1964), Samuelson (1965), or Samuelson and Merton (1969). For concreteness, whereas the prior studies assume representative agents who are risk averse, Black and Scholes (1973) develop a mathematical formalism in which agents' risk preferences are immaterial to the determination of the pricing equilibrium. Responding honestly to the incommensurability and the superior robustness of the latter

study, Robert Merton would abandon the structure in Samuelson and Merton (1969) and in Merton (1973, 1976) build on the new structure in Black and Scholes (1973).

In addition to the foregoing, if either concepts or principles are inherently and solely rooted in language, that is, are not natively mathematical, the formal theory shows only mathematical formulations that are non-parametrically specified are robust to a progression of the relevant epistemology; otherwise, mathematical formulations which are either parametric or non-parametric are appropriate. Applying the highlighted inference, outside, perhaps of the development of measures, it is mathematical formalism that are non-parametrically specified that are mostly robust to the introduction of formalism into the development of the epistemology of any of *Philosophy, Sociology, Psychology, or Anthropology*; the modeling of utility functions or more generally, functions, is, as such largely excluded. In stated respect, with the four enumerated disciplines inherently *ecological*, the tools of *Analysis*, such as *Set Theory*, which serve to delineate and interact ecosystems (see for example, Buzaglo 2002) are appropriate to the development of the disciplines' epistemology. In similar vein, whereas the question, “*what is the price of risk*” is both *conceptual* and *natively mathematical*, as such amenable to the adoption of parametric formalism, contrarily, the question, “*what is rationality*” is not natively mathematical; as such, ideally is solely amenable to mathematical formalism that is non-parametrically specified. In aggregate, there is arrival at two sources of non-robustness in the efforts for the development of an epistemology, namely either the absence of mathematical formalism, or the introduction of mathematical formalism that is incommensurable with the properties of the concepts or principles that are modelled. On basis of the evidence, economics' epistemology consists of a mixture of epistemology that are robustly or non-robustly formulated. In stated respect, in presence of the evidence, in this study, that neoclassical economics' epistemology is premised on an epistemology of rationality that is non-robustly formulated, there is arrival at the relevance of heterodox economics, or better yet, the need to abandon neoclassical formalism.

## Appendix

**Proof of Theorem 1.** Suppose  $\Gamma \nparallel \psi$ ,  $\Gamma \nparallel \vartheta$ , or  $\psi \nparallel \vartheta$ . Then  $\Gamma$ ,  $\psi$ , and  $\vartheta$  do not share any defining property,  $\varkappa$  in common, yet combined, define  $\Theta$ , implying  $\Theta = [\Gamma \cup \psi \cup \vartheta]$  satisfying,

$$[\Gamma \cap \psi \cap \vartheta] = \emptyset,$$

which implies,  $\Theta$  is segmented, such that,

$$\begin{aligned} \Theta(\Gamma) \cup \Theta(\psi) \cup \Theta(\vartheta) &= \Theta \\ \Theta(\Gamma) \cap \Theta(\psi) \cap \Theta(\vartheta) &= \emptyset; \end{aligned}$$

that is, is segmented into a sub-discipline,  $\Theta(\Gamma)$  consisting solely of concepts, but lacking any logic; a second sub-discipline,  $\Theta(\psi)$  consisting in logic, but lacking concepts or principles; and a third sub-discipline,  $\Theta(\vartheta)$  consisting solely of mathematical formalism that lacks logic and concepts, clearly a binding contradiction. In presence of the

contradiction, necessarily, either,

$$\Theta(\Gamma) \cap \Theta(\psi) \neq \emptyset \quad (17)$$

$$\Theta(\Gamma) \cap \Theta(\vartheta) \neq \emptyset \quad (18)$$

$$\Theta(\psi) \cap \Theta(\vartheta) \neq \emptyset \quad (19)$$

$$\Theta(\Gamma) \cap \Theta(\psi) \cap \Theta(\vartheta) \neq \emptyset. \quad (20)$$

If solely equation (17) is true,

$$[\Theta(\Gamma) \cap \Theta(\psi)] \cap \Theta(\vartheta) = \emptyset,$$

implying  $\vartheta$  generates mathematical formalism that does not translate into either logic or concepts, yet attempts to formalize a concept,  $\Theta$ , clearly a binding contradiction. Applying similar arguments to each of equations (18) and (19), there is arrival at the acceptance, solely of equation (20), which asserts the existence of some property,  $\varkappa$  satisfying,  $\varkappa \in [\Theta(\Gamma) \cap \Theta(\psi) \cap \Theta(\vartheta)]$ , as such the satisfaction of equation (2). Equation (2) directly connotes equation (1). For additional concreteness, given  $\Gamma$  consists in concepts, as such consists in language, and given logic refines man's understanding of language (see for example, the discussion of Wittgenstein's work in Hacker 2001, pgs. 89-90),  $\varkappa$  exists *if and only if*  $\vartheta$  has expression, analogously in non-mathematical language. If  $\vartheta$  has expression in non-mathematical language it models concepts, which require language to express; since logic refines language, the output of  $\vartheta$  is required to be robust to the imposition of logic.

Suppose equation (3) is not true. Necessarily,  $\psi \subseteq \vartheta$  and  $\psi \supseteq \vartheta$ . As is already discussed in the introductory section, *Wittgenstein* induced *Russell* to admit the impossibility of,  $\psi \supseteq \vartheta$ , the exclusion of which directly implies,  $\psi \subset \vartheta$ . For additional concreteness, consider the logical statement, '*A is true*'. Necessarily, '*Not A is also true*', such that in the domains of logic, *A* (the color *blue*, say) and *Not A* (the color *red*, say) are irreconcilable for arrival at a harmonious logic. Consider, however, Mathematics. Suppose  $\alpha > 2$  is true. If  $\beta \leq 2$ ,  $\alpha$  is not  $\beta$ , and  $\beta$  is not  $\alpha$ , yet with  $\{\alpha, \beta\} \in \mathfrak{R}$ ,  $\alpha$  and  $\beta$  remain composable, that is, can be composed into some new insight,  $c = f(\alpha, \beta)$ , say, which is neither  $\alpha$  nor  $\beta$ . For illustration, *red* ( $\alpha$ ) composed with *blue* ( $\beta$ ) produces magenta ( $c$ ), which is *neither blue* nor *red*. Equation (3), as such, is true.

Suppose equation (4) is not true. Then every  $\Gamma$  is mathematically specified, clearly a non-robust assertion, because all *observational terms* belong in  $\Gamma$  and all are non-mathematical. Contrarily, else the mathematical formalism is non-communicable in language, every  $\vartheta$  has, necessarily rationalization as either a concept or principle,  $\Gamma(\vartheta)$ . Given  $\Gamma$  and  $\vartheta$  are commensurable, the inference,  $\Gamma \subsetneq \vartheta$ , implies, necessarily,  $\Gamma \supset \vartheta$ . Since  $\Gamma \supset \vartheta$  and  $\vartheta \supset \psi$ , there is arrival at,  $\Gamma \supset \vartheta \supset \psi$ , as such, yet again, evidence that equations (1) and (2) are always true.

Suppose equation (5) is not true, that is,  $[\Gamma \cup \psi \cup \vartheta] \supseteq \Theta$ ; clearly there is arrival at the statement of an irrationality. Necessarily, as such,  $[\Gamma \cup \psi \cup \vartheta] \not\supseteq \Theta$ , which directly implies,  $[\Gamma \cup \psi \cup \vartheta] \subset \Theta$ , as such the possibility of new  $\Gamma$ , new  $\psi$ , or new  $\vartheta$  that further parameterize  $\Theta$ , clearly an assertion of rationality. ■

**Proof of Corollary 1.** Suppose  $\Theta^C$ .

Applying the philosophical canon (see the discussion in the introductory section), all  $\Gamma^C \in \Theta^C$  have rendition as the combination of definitional statements,  $\zeta^d$  and descriptive (observational) statements,  $\zeta^o$ . By assumption, the  $\Gamma^C$  are not natively mathematical, yet are amenable to mathematical formalism, because they are analytic terms, not observational terms.

Let  $\zeta^{d,m}$  be the outcome of a mathematical formalism,  $\vartheta^C$ ; imposing Theorem 1,  $(\zeta^d \cup \zeta^o) \parallel \zeta^{d,m}$ ; imposing rationality,  $\zeta^{d,m}$  has some mathematical support - the formal theoretical workout,  $\zeta^{d,m,s}$  that is the source of  $\zeta^{d,m}$ , else  $\zeta^{d,m}$  is non-analytically specified (*LP3*). Imposing yet again rationality, that is, Theorem 1, and setting  $\zeta^d$  equivalent to  $\Gamma$ , there is arrival at:

$$\begin{aligned} (\zeta^d \cup \zeta^o) \parallel \zeta^{d,m} \parallel \zeta^{d,m,s} \\ \rightarrow (\zeta^d \cup \zeta^o) \supset \zeta^{d,m} \supset \zeta^{d,m,s}, \end{aligned} \quad (21)$$

with the outcome, the latency of  $\zeta^{d,m,s}$  is rational. If  $\zeta^{d,m}$  is parametric, necessarily,  $\zeta^{d,m,s}$  is parametric, clearly a contradiction to  $\Gamma^C \in \Theta^C$ . If  $\zeta^{d,m,s}$  is non-parametric,  $\zeta^{d,m}$ , it's outcome cannot be parametric, because by assumption,  $\Gamma^C \in \Theta^C$ ; necessarily, as such,  $\zeta^{d,m}$  is non-parametric and robust to  $\Gamma^C \in \Theta^C$ ; there is arrival, as such, at the rationality (non-vacuousness) and robustness of equation (21).

Suppose then,  $\tilde{\Gamma} \in \tilde{\Theta}$ .

Let  $\tilde{\zeta}^{d,m}$  be the outcome of a mathematical formalism,  $\tilde{\vartheta}$  that is applied towards the formalization of a natively mathematical concept,  $\tilde{\zeta}^d$ ; imposing Theorem 1,  $\tilde{\zeta}^d \parallel \tilde{\zeta}^{d,m}$ ; necessarily, as such,  $\tilde{\zeta}^{d,m}$  is natively mathematical; imposing rationality (Theorem 1), regardless,  $\tilde{\zeta}^{d,m}$  has some conceptual interpretation,  $\tilde{\zeta}^{d,m,s}$ , else there is not any basis for inferring that it conveys a propositional truth. Imposing yet again rationality, that is, Theorem 1, and setting  $\tilde{\zeta}^d$  equivalent to the  $\Gamma$  in Theorem 1, there is arrival at:

$$\begin{aligned} \tilde{\zeta}^d \parallel \tilde{\zeta}^{d,m} \parallel \tilde{\zeta}^{d,m,s} \\ \rightarrow \tilde{\zeta}^d \supset \tilde{\zeta}^{d,m,s} \supset \tilde{\zeta}^{d,m}, \end{aligned} \quad (22)$$

such that,  $\zeta^{d,m,s}$  is an interpretation that connects  $\tilde{\zeta}^d$  and  $\tilde{\zeta}^{d,m}$ , both of which are natively mathematical. Suppose,  $\zeta^{d,m,s}$  is inherently parametrically specified. Since it is the source of  $\tilde{\zeta}^d$ , necessarily  $\tilde{\zeta}^d$  is parametrically specified. Since a non-parametric formalism,  $\tilde{\zeta}^{d,m}$  is unable to span a parametric,  $\zeta^{d,m,s}$ , necessarily  $\tilde{\zeta}^{d,m}$  is parametric. Suppose, contrarily that  $\zeta^{d,m,s}$  is non-parametric; since it is the source of,  $\tilde{\zeta}^d$ , it is the non-parametric version of  $\tilde{\zeta}^d$  that is modeled; further, since only a non-parametric  $\tilde{\zeta}^{d,m}$  spans a non-parametric  $\tilde{\zeta}^{d,m,s}$ , the workout,  $\tilde{\zeta}^{d,m}$  is non-parametric. But in the latter scenario, there is arrival at,

$$\begin{aligned} \zeta^d \parallel \zeta^{d,m} \parallel \zeta^{d,m,s} \\ \rightarrow \zeta^d \supset \zeta^{d,m} \supset \zeta^{d,m,s}, \end{aligned}$$

as such, arrival at a scenario that is not any different from the modeling of a  $\Gamma$  that is not natively mathematical.

Suppose  $\Theta^C(SSH) = \emptyset$ . We have that all  $\Gamma^C(SSH)$  have statement, inherently as

mathematical propositions, clearly a non-tenable proposition, because all study man and man is a concept that is not natively mathematical. Having arrived at a contradiction, necessarily,  $\Theta^C(SSH) \neq \emptyset \rightarrow \tilde{\Theta} \subset \Theta$ . ■

**Proof of Theorem 6.** Suppose, contrarily that,  $\Gamma_{t-1} \subseteq \Gamma_t$ . Necessarily,  $\Gamma_t \parallel \Gamma_{t-1}$ , implying using Theorem 2 that,  $\vartheta_t \parallel \vartheta_{t-1}$ , with the outcome  $\vartheta_{t-1}$  is a sufficient statistic for  $\vartheta_t$ ; necessarily, as such,  $\vartheta_t \neq \vartheta_t^w$ .

Suppose, contrarily that,  $\Gamma_{t-1} \subset \Gamma_t$ . Necessarily,  $\Gamma_t \parallel \Gamma_{t-1}$ , and  $\vartheta_t \parallel \vartheta_{t-1}$ , but applying Lemma 1,  $N(\Gamma_{t-1}) < N(\Gamma_t)$ ; whereas then  $\vartheta_t$  facilitates some new elements,  $x \in \Gamma(\Theta)$ , it remains commensurable with  $\Gamma_{t-1}$ , as such only extends  $\Gamma_{t-1}$ . We have then that  $\vartheta_t \neq \vartheta_t^w$ .

Suppose  $\Gamma_t \subseteq \Gamma_{t-1}$ ; necessarily,  $\Gamma_{t-1} \not\subseteq \Gamma_t$ , else  $\Gamma_t = \Gamma_{t-1}$ . There is arrival, as such, at,  $\Gamma_t \subset \Gamma_{t-1}$  and  $\Gamma_t$  serves solely to restrict the elements,  $x \in \Gamma(\Theta)$  to a smaller domain. Necessarily, as such,  $\vartheta_t \parallel \vartheta_{t-1}$  and  $\vartheta_t \neq \vartheta_t^w$ . Having excluded each of,  $\Gamma_t \subseteq \Gamma_{t-1}$  and  $\Gamma_t \subset \Gamma_{t-1}$ , necessarily,  $\Gamma_t \not\subseteq \Gamma_{t-1}$ .

Applying the foregoing, namely,  $\Gamma_{t-1} \not\subseteq \Gamma_t$  and  $\Gamma_t \not\subseteq \Gamma_{t-1}$ , if  $\Gamma_t^w$ ,  $\exists$  some  $\hat{x}$  such that,  $\hat{x} \in \Gamma_{t-1} \rightarrow \hat{x} \notin \Gamma_t^w$  and some  $\check{x}$  such that,  $\check{x} \in \Gamma_t^w \rightarrow \check{x} \notin \Gamma_{t-1}$  with the outcome,  $\Gamma_t^w \not\parallel \Gamma_{t-1}$ .

Suppose the dominance relations in equations (9) through (11) can be rendered as,

$$\psi_t^w(\Gamma) \succ \psi_{t-1}(\Gamma) \quad (23)$$

$$\longleftrightarrow \psi_t^w \succ \psi_{t-1} \quad (24)$$

$$\longleftrightarrow S(\Gamma_t^w) \succ S(\Gamma_{t-1}); \quad (25)$$

equations (23) through (25) assert that logic can be a source of propositional truth, a claim that is already well accepted to be false. Suppose also that the dominance relations in equations (9) through (11) can be rendered as:

$$\Gamma_t^w(\Gamma) \succ \Gamma_{t-1}(\Gamma) \quad (26)$$

$$\longleftrightarrow \Gamma_t^w \succ \Gamma_{t-1} \quad (27)$$

$$\longleftrightarrow S(\Gamma_t^w) \succ S(\Gamma_{t-1}); \quad (28)$$

straightforwardly there is arrival at the vacuousness of equations (26) through (28), a vacuousness especially evident in the identity that subsists between equations (26) and (27). There is arrival, as such, at equation (15). Applying Theorem 2, there is arrival at equation (16). ■

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