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Computing the basins of attraction of periodic solutions in a 4D model of a musical instrument

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Summary. We investigate the multistable dynamics of a simple model of reed musical instrument written as a system of four ordinary differential equations. A bifurcation analysis is performed considering the blowing pressure – which is the main control parameter for the musician – as a bifurcation parameter. This unveils that several stable periodic solutions corresponding to different musical notes coexist on a range of the blowing pressure. We employ a machine learning technique (namely an explicit design space decomposition technique combined with an support-vector learning machine) to construct, in a well-chosen 3D subspace of the phase space, the boundary between the basins of attraction of the coexisting periodic solutions. Basins are of particular interest from a practical point of view: they relate to the *playability* of the regimes and to their sensitivity to perturbations.

Introduction

Musical instruments are dynamical systems that can produce a wide variety of sound regimes. The influence of playing (control) parameters on the final regime has been explored in the literature, both numerically and experimentally [1, 2]. These studies highlighted regions of multistability, raising the question of the practical accessibility to the different sound regimes. In practical terms, this relates to the way a musician can select a sound regime rather than an other. From a dynamical systems perspective, we investigate the basins of attraction of a simple model of saxophone.

Model

Sound production in a saxophone results from the interaction between two elements: the resonator, which is composed of the air column in the instrument, and the exciter, which is composed of a piece of reed that vibrates under the action of the musician's breath [4]. The state-of-the-art model is based on three main elements: the Bernoulli law that describes the flow entering the instrument, a harmonic oscillator equation to account for the motion of the reed, and an equation for the input acoustical impedance of the resonator. This last equation is written in the frequency domain as a sum of resonance modes [3]. For n modes, the model is written as a system of $2n$ ordinary differential equations. Because we seek here the simplest model still displaying multistability, only two resonance modes are considered. The model is then written as follows, where $P_{kr}(t)$ and $P_{ki}(t)$ are the real and imaginary parts, respectively, of the modal pressure $P_k(t)$ associated with mode k :

$$\begin{cases} \dot{P}_{kr}(t) = \text{Re}(C_k U(t) + P_k(t) S_k) \\ \dot{P}_{ki}(t) = \text{Im}(C_k U(t) + P_k(t) S_k), \quad k \in \{1, 2\} \end{cases} \quad (1)$$

where the flow $U(t)$ is written as follows:

$$U(t) = \zeta(1 - \gamma + P(t)) \sqrt{|\gamma - P(t)|} \text{sgn}(\gamma - P(t)) H(1 - \gamma + P(t)). \quad (2)$$

The pressure in the resonator is $P(t) = 2(P_{1r}(t) + P_{2r}(t))$ [2]. In eq. (1), C_k , S_k are complex parameters that depend on the geometry of the resonator [4] and their values are estimated here from the experimentally-measured input impedance of an actual saxophone [2]. The parameters γ and ζ represent respectively the dimensionless pressure in the musician's mouth and the opening of the reed at rest. In eq.(2), H is the Heaviside function according to the so-called ghost reed model [1] and the sign function accounts for a possible inversion of the flow. For numerical considerations, eq.(2) is regularised to have C^1 functions [1]. This model has been shown to reproduce accurately, at least qualitatively, key features of the dynamics of an actual saxophone [1, 2, 7]. This includes bistability between periodic regimes, as well as complex transition scenarios between static regime and periodic regimes (Figure 1-left) or quasi-periodic regimes [7].

Study of basins of attraction

Figure 1-left shows the bifurcation diagram of eq.(1-2) with respect to γ . This shows that the system is bistable on a range of γ , where two different attractors coexist, that correspond to different periodic sound regimes. Because the model is four dimensional, visualizing the boundaries between basins of attraction requires to reduce the number of dimensions. This is done here as follows: a given point of the limit cycle is considered as the origin of the subspace in which the boundaries between basins will be evaluated. The tangent vector to the cycle at this point is evaluated in the 4D phase space, and a basis of the normal subspace to this tangent vector is constructed using a Gram-Schmidt process. Because of the vicinity with the limit cycle, the

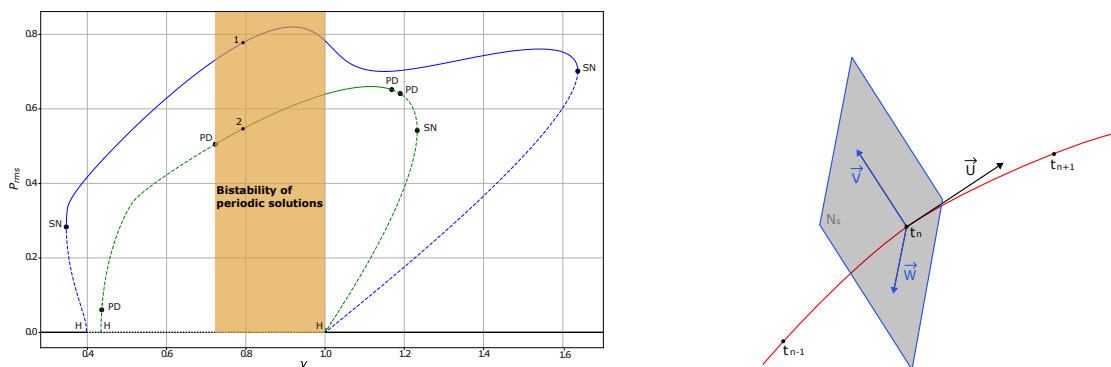


Figure 1: Left: Bifurcation diagram of eq.(1-2), showing a branch of equilibrium solution (black line) and two branches of periodic solutions (blue and green) with different frequencies. Solid and dashed lines represent stable and unstable solutions, respectively. H , SN and PD indicate Hopf bifurcations, saddle-node bifurcations of periodic orbits and period doubling bifurcations respectively. Attractors at points 1 and 2 are shown in Figure 2-right. Branches of period-doubled solutions emerging from period-doubling bifurcations are unstable and not represented. Right: Schematic representation of the moving frame method for the visualisation of the basins in 3D. Black vector is tangent to the cycle in red and blue vectors are orthogonal vectors obtained through Gram-Schmidt process and spanning the normal bundle (N_s).

system stays on the considered regime when perturbed along the tangent vector direction, and this direction is discarded to evaluate the basins. A 3D example is shown in Figure 1-right: the 2D subspace N_s spanned by the vectors \vec{V} and \vec{W} is orthogonal to the tangent vector \vec{U} . This process applied to a 4D space gives a 3D subspace N_s with coordinates X, Y and Z (Figure 2-left and center). System of eq.(1-2) is integrated in this subspace using a Runge-Kutta method, and the frequency of the solution is analysed with Yin algorithm [5]. In practice, the boundary between basins, as shown in Figure 2-left and center, is approached using a machine learning technique, namely and Explicit Design Space Decomposition method and a Support Vector Machine [6]. Because the subspace properties are local only, the process is repeated for different points along the cycle (t_{n-1} and t_{n+1} in Figure 1-right) via a moving frame. The time is used as the fourth dimension, and the basins boundary is finally visualised as a movie in a 3D subspace. In Figure 2-left and center, the thinner shape of one basin close to the origin makes the system sensitive to disturbances, which could lead to a regime shift. This increased sensitivity is attributed to details of the basin's shape. A 3D phase portrait is also provided in order to visualize the attractors of the two periodic regimes for the value of $\gamma = 0.79$ (Figure 2-right).

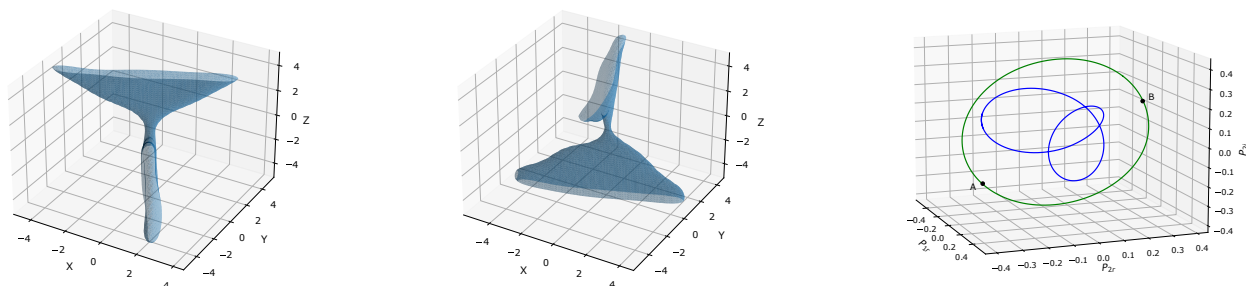


Figure 2: Left and center: Separatrix between the basins of attraction of both periodic solutions, represented in the 3D normal subspace, for the points A and B indicated in the left panel. Right: Phase portrait of eq.(1-2) projected in 3D, for $\gamma = 0.79$, showing the two coexisting periodic solutions. The self-intersections of orbits are due to the projection.

Conclusion

The shape of basins provides valuable insight on the likeliness of transition between different stable regimes. In particular, their structure relates to the sensitivity of these regimes to external perturbations and give a visual representation of the stability landscape. Regimes with more extended basins may indicate greater stability and resilience, as larger perturbations are required to induce a regime change. Conversely, regimes with narrower basins might be more sensitive to disturbances, making them prone to abrupt and uncontrolled transitions. In the context of musical instruments, we argue that basins shape relate to the practical *playability* of the different musical registers.

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