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# A Two-Timescale Decision-Hazard-Decision Formulation for Storage Usage Values Calculation

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## Abstract

The penetration of renewable energies requires additional storages to deal with intermittency. Accordingly, there is growing interest in evaluating the usage value (opportunity cost) associated with stored energy in large storages, a cost obtained by solving a multistage stochastic optimization problem. Today, to compute usage values under uncertainties, an adequacy resource problem is solved using stochastic dynamic programming assuming a hazard-decision information structure. This modelling assumes complete knowledge of the coming week uncertainties, which is not adapted to the system operation as they occur at smaller timescale (hour). This is why, in this paper, we propose to model the decision-making process with, on the one hand, two timescales (weekly time decomposition, but uncertainties that are realized hour by hour, and hourly energy balance constraints) and, on the other hand, a decision-hazard-decision information structure considering both planning and recourse decisions. This structure is used to decompose the multistage decision-making process into a nonanticipative planning step in which the binary on/off decisions for the thermal units are made, and a recourse step in which the power modulation decisions are made once the uncertainties have been disclosed. We perform time decomposition by stochastic dynamic programming, and not by SDDP, due to the presence of binary on-off decision variables. In a numerical case, we illustrate how usage values are sensitive to how the disclosure of information is modelled.

**Keywords.** Energy system modelling, decision-hazard-decision information structure, two timescales, stochastic multistage optimization, stochastic dynamic programming

# 1 Introduction

In §1.1, we outline the context of usage values in prospective studies. In §1.2, we sketch the main features of our contribution and then, in §1.3, we make comparisons with the literature. We detail the structure of the paper in §1.4.

## 1.1 Context: usage values in prospective studies

In energy systems, prospective studies aim at identifying possible curtailment in demand or production, network congestions, or the non-satisfaction of greenhouse gas emission targets. This is done by means of simulations. The simulation of an energy system operation is carried out by solving its associated *adequacy resource* problem. Mathematically, the adequacy resource problem is a *multistage optimization* problem that aims to allocate, hour by hour, the dispatchable production means such that the demand is met while minimizing the overall production cost. The problem is formulated from the point of view of a central planner who takes all decisions in the system for the common good. In such case, the production costs are fixed and resources are allocated by merit order, i.e. the cheapest units are used before the most expensive ones. Since the electrical demand and the availability of the thermal units are uncertain, the adequacy resource problem is *stochastic*. This problem is naturally formulated using two timescales where the planning is done in coordinated manner for the week ahead but the hourly energy balance has to be met, that is, a weekly timescale and an hourly timescale. In this context, the question of when the energy in storages is going to be used arises. *Usage values* are the storages' prices — a price signal that makes it possible to choose when and how much of its energy is used — that depend on the energy system setting.

We now discuss how usage values are computed for a system composed of several storages and thermal units. Stochasticity is introduced by the residual demand of the system (difference between demand and non-dispatchable production) and the availability of the thermal units (outages). We do not focus on how the uncertainty scenarios are generated, but on *information structure*, that is, on how the available information about these scenarios is disclosed throughout the decision-making process. The dynamics in the storages introduce temporal coupling in the problem, leading to a *multistage stochastic optimization problem*. A standard way to solve it is by dynamic programming. Then, we obtain usage values by differentiating the Bellman value functions with respect to the storage levels. Once the usage values are calculated, one can carry out prospective studies by simulation. Indeed, one can compute a storage's management policy to be used in the resolution of the adequacy resource problem. However, it is important to highlight that the adequacy resource problem studied here does not intend to yield an implementable operational schedule for the system, but to give an overview of the system operation.

In practice, the French TSO Réseau de Transport d'Electricité (RTE)<sup>1</sup> is dealing with such prospective studies using the open-source tool Antares [6]. RTE tackles the resolution

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<sup>1</sup><https://www.rte-france.com/>

of the multistage stochastic optimization problem associated with the adequacy problem as follows. The timespan is one year, with a weekly timescale and an hourly timescale. Dynamic programming is performed at the weekly timescale: a Bellman value function is computed at the beginning of each week, anticipating the coming uncertainties over the week and respecting the hourly energy balance equations within the week. This structure assumes that all the decisions in a week are made with full knowledge of the uncertainties of the week. When the dispatchable units are “fast” to start (like hydropower or gas turbine), it is reasonable to assume that we can “wait-and-see” the uncertainties before making the decisions. By contrast, as “slow” dispatchable units (like nuclear or coal) need more time to start producing, start or stop decisions must be made before the uncertainties are known (especially the outages), and “wait-and-see” decisions are not adapted. This is why, our main contribution focuses on informations structures suited to handle both fast and slow dispatchable units submitted to outages, with two timescales.

## 1.2 Our contributions

Information structures are used to model the information available at each stage of the decision-making process. The current practice at RTE, as described just above, corresponds to what we call *weekly hazard-decision*. Indeed, this structure assumes that all the decisions in a week are made with full knowledge of the uncertainties of the week.

In §1.3, we present papers that deal with so-called *hazard-decision*, *decision-hazard* and *full decision-hazard*, especially in relation to the SDDP algorithm [11]. By comparison, our contributions in this paper are the following.

- We study *decision-hazard-decision* information structures *with two timescales* (already introduced in [4]), to address the usage value calculation by keeping track of hourly constraints but allowing a weekly decomposition of the yearly problem.
- We consider a model with binary variables to model the on/off decisions for the thermal (dispatchable) units, and solve the stochastic dynamic programming (SDP) equations in a decision-hazard-decision framework without relaxing these binary decisions.

Regarding this second item, the energy systems that we consider include, in addition to storage, thermal units that may be slow to go from off to on (and reverse), and which may be subject to uncertain outages. The on/off decisions are naturally modelled as binary decisions and, since we consider a problem with a small space state dimension, we choose to use SDP (with special versions of Bellman equations for nonanticipative information structures at the week timescale). The choice of using SDP is on purpose, and lies in the fact that we seek to present a framework capable of handling binary or discrete variables within the planning decisions (switch on/off), in which case SDDP is not suitable, although variants like SDDiP [15] exist. The scalability of the problem concerning the dimension of the state space will be addressed in future work by applying spatial decomposition techniques.

We now sketch what we mean by decision-hazard-decision information structures with two timescales. First, we describe the ideal case of an information structure with weekly

planning decisions and hourly recourses, as illustrated in Fig. 1. We do not consider that, when we make a (planning) decision at the beginning of the week, we know in advance the hourly random variables that will materialize during the coming week. Rather, at the end of every hour, we allow for an hourly decision variable (recourse) to handle hourly balance constraints, knowing all uncertainties up to this hour.

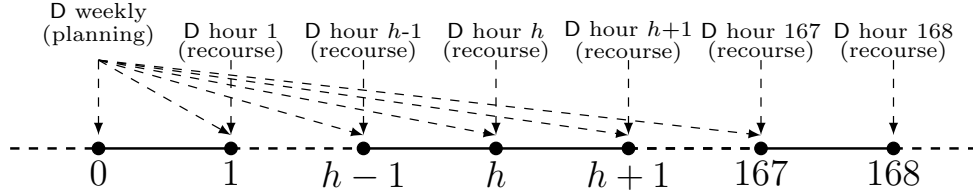


Figure 1: Sketch of information structure with weekly planning decisions and hourly recourses. An arrow maps the available information towards the decision, so that, here, the information structure is nonanticipative as all arrows go either down or from the left to the right.

We denote this information structure with weekly planning decisions and hourly recourses by  $\text{DHD}^h$  (weekly D and hourly HD), and symbolically represent it by (1):

$$\text{DHD}^h : \overbrace{\text{D} - \text{HD} - \dots - \text{HD}}^{\text{one week}} . \quad (1)$$

168 hours

The (backward) Bellman equation corresponding to  $\text{DHD}^h$  can be established from [4], and is sketched in (2) (see also §B.3):

$$\begin{aligned} \text{cost} & \\ \text{to go} & \\ \text{DHD}^h & = \quad (2) \\ \min_{\text{D weekly (planning)}} \mathbb{E} & \left[ \min_{\text{D hour 1 (recourse)}} \mathbb{E} \left[ \dots \min_{\text{D hour 167 (recourse)}} \mathbb{E} \left[ \min_{\text{D hour 168 (recourse)}} \left( \text{weekly cost} + \frac{\text{next cost to go}}{\text{DHD}^h} \right) \mid \mathbf{H}_1, \mathbf{H}_2, \dots, \mathbf{H}_{167} \right] \dots \mid \mathbf{H}_1 \right] \right] . \end{aligned}$$

The obtained Bellman functions are optimal under a (debatable) assumption of statistical independence of the random variables (weather conditions, demand, outages, etc.) between weeks. As commonly done with stochastic dual dynamic programming (SDDP), we could account for lag variables to be added aside storage, hence making an extended state. Of course, as we use SDP, we cannot extend the one dimensional state (storage level) with more than three or four entries. In the paper, we choose to keep only the storage as state.

Even without possible lag variables, the Bellman equation sketched in (2) is numerically untractable for several reasons: because of the DH part in the information structure, we cannot use SDDP without adaptation; due to the presence of binary decision variables (on-off), we cannot resort to the SDDP algorithm; as we do not suppose statistical independence of the hourly random variables inside a week, the intra-week problem is out of numerical reach.

This is why our approach consists in relaxing the problem by collecting the hourly decision variables inside the week in a single vector decision (recourse) made at the end of the week. With this, the DH phase is now followed by a hazard-decision (HD) phase, making the whole information structure *decision-hazard-decision at the weekly timescale* (DHD), as illustrated in Fig. 2.

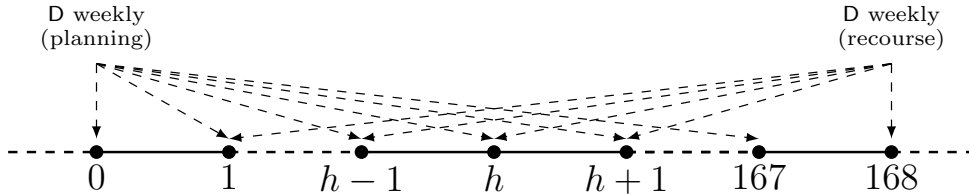


Figure 2: Decision-hazard-decision information structure at the weekly timescale. An arrow maps the available information towards the decision, so that, here, the information structure is partly anticipative as some arrows go from the right to the left.

This relaxed version condenses all the hourly recourse decisions in one vector decision, leading to a more manageable expression for the Bellman equations sketched in (3) (see also §4.3):

$$\text{cost}_{\text{DHD to go}} = \min_{\text{D weekly (planning)}} \mathbb{E} \left[ \min_{\text{D weekly (recourse)}} \left( \text{weekly cost} + \text{next cost to go DHD} \right) \right]. \quad (3)$$

The DH phase (weekly planning) is now followed by a hazard-decision (HD) phase (weekly recourse), making the whole information structure decision-hazard-decision. With this approach, we are able to handle a DHD information structure without having to extend the state by adding a control aside, contrarily to other approaches (see discussion in §1.3.1).

### 1.3 Comparison with the literature

In §1.3.1, §1.3.2 and §1.3.3, we sketch the contribution of three papers that deal with decision-hazard and hazard-decision information structures in multistage stochastic optimization problems. We present them par decreasing order of proximity with our work. Finally, in §1.3.4, we posit our contribution in comparison with the literature in a synthetic fashion.

#### 1.3.1 A. Street, D. Valladão, A. Lawson, and A. Velloso. [13] *Applied Energy*, 2020

The paper [13] is a nice one whose title *Assessing the cost of the hazard-decision simplification in multistage stochastic hydrothermal scheduling* clearly indicates the interest of the authors to discuss the impact of the hazard-decision simplification in multistage stochastic optimization problems. We share the same preoccupation as [13].

In [13, Sect. 3], the authors focus on the handling of DH information structure in multi-stage stochastic optimization problems formulated with continuous decision variables, convex costs and constraints, and linear dynamics. Their main preoccupation is how the SDDP algorithm can be adapted to the DH setting. Thus, they make a stagewise independence assumption and state Bellman equations without hint at a proof.

Starting from the Bellman equations [13, Equations (6)-(11)] in HD, they propose two reformulations to handle DH. Decision variables are split in two subgroups: those made under uncertainty (also called here-and-now, preventive) and those made after the observation (wait-and-see, corrective).

- The fullDH-SDDP Equations [13, Equations (12)-(17)] — although called DH formulation — can be interpreted as Bellman equations in DHD. Indeed, in the splitting of the decision variables in two subgroups, the decisions made under uncertainty (here-and-now, preventive) are the first D in the DHD information structure, hence are subject to the nonanticipativity constraints [13, Equation (16)] in a two stage stochastic optimization problem. The decisions made after the observation (wait-and-see, corrective) are the last D in the DHD information structure, hence correspond to recourse decision variables in a two stage stochastic optimization problem.
- ASDH-SDDP Equations [13, Equations (18)-(23)] and [13, Equations (24)-(31)] are Bellman equations for value functions whose argument is an extended state (nonanticipative controls are added aside the original state to make an extended state).

Our approach differs from ASDH-SDDP in that we do not extend the state. Our approach is close to fullDH-SDDP, but is not restricted to continuous decision variables, convex costs and constraints, and linear dynamics. We provide Bellman equations — whose derivation is not given in the paper, as it is lengthy, but that can be established from [4] — for general decision variables (continuous, discrete or mixed), costs and constraints (convex or not), and dynamics (linear or not). In [13], Bellman equations are stated in the SDDP setting, that is, for continuous decision variables, convex costs and constraints, and linear dynamics. Moreover, we give a formal definition of DHD, independently of SDDP, whereas [13] evokes DH and HD in the SDDP setting. Finally, we tackle the issue of DHD with two timescales, whereas [13] deals with a single timescale.

Regarding the energy application, the authors focus on operation — that is, on finding strategies to operate hydrothermal scheduling — whereas we focus on usage values in prospective studies.

This said, let us insist that [13] is a nice contribution in raising awareness about the cost of the hazard-decision simplification in multistage stochastic optimization problems in energy systems — where slow activation thermoelectric units need to be scheduled as here-and-now decisions, while fast-activating thermoelectric and hydroelectric units are considered as wait-and-see decisions.

### 1.3.2 D. Valladão, T. Silva, and M. Poggi. [14] *Annals of Operations Research*, 2019

To the difference with [13], the paper [14] is focused neither on energy systems, nor on the DH information structure as no more than two pages are devoted to this issue. However, in [14, Sect. 4], the authors consider a multistage stochastic optimization problem with risk constraints at every stage and, because of such a constraint, it would not make sense to know the uncertainty in advance, which justifies a DH information structure. As the portfolio problem under study is formulated with continuous decision variables, convex costs and constraints, and linear dynamics, the authors resort to SDDP as resolution method. Then, the authors discuss how to adapt SDDP to a DH formulation for the Markov chained stochastic dual dynamic programming (MSDDP) algorithm. They propose to solve a two-stage problem for each SDDP subproblem in the risk-constrained dynamic portfolio optimization context. A Bellman equation is given in the linear case, without hint at a proof.

Thus, one year before [13] was published, [14] raises the point that the DH information structure should be considered, a point with which we fully agree. Then, [14] proposes a way to adapt the SDDP algorithm, without state extension (hence keeping the original state as argument of Bellman functions). The approach is that of fullDH-SDDP in [13].

The comparison of [14] with our work is much more limited than with [13], since [14] is not really focused on the DH information structure. Contrarily to [14], our approach is not restricted to continuous decision variables, convex costs and constraints, and linear dynamics. We provide Bellman equations — whose derivation is not given in the paper, as it is lengthy, but that can be established from [4] — for general decision variables (continuous, discrete or mixed), costs and constraints (convex or not), and dynamics (linear or not). In [14], Bellman equations are stated in the SDDP setting, that is, for continuous decision variables, convex costs and constraints, and linear dynamics. Finally, we tackle the issue of DHD with two timescales, whereas [14] deals with a single timescale.

### 1.3.3 O. Dowson. [7] *Networks*, 2020

To the difference with [13], the paper [7] is not focused on the DH information structure as it is only tackled in slightly more than one page in [7, §2.2 (Nodes)], and then in half page in [7, §3.4 (Standart form)]. In [7, §2.2], the author adopts the formalism of stochastic optimal control in discrete time. This done, HD and DH are defined in the framework of state feedback policies: HD is when a policy depends both on current state and uncertainty, whereas DH is when a policy depends only on the current state. Our approach is more general in that the DHD structure is expressed in terms of measurability constraints, using  $\sigma$ -fields generated by the uncertainty process. At the end of [7, §2.2], the author indicates that it is possible to transform a DH node into a deterministic node followed by a HD one with an extended state space (as in ASDH-SDDP in [13]). Contrarily to [13] and [14], there are no assumption of continuous decision variables, convex costs and constraints, and linear dynamics up to [7, Sect. 4 (Proposed algorithm)]. In that, we share some proximity with [7]. For the rest, [7, Sect. 4 (Proposed algorithm)] deals with the HD information structure for



problems that resort to linear programming.

All in all, we have little in common with [7]. The author indeed evokes DH and HD — but the setting is not as formal as ours, and not as general — and does not consider DHD, which is our main object of study. We provide Bellman equations — whose derivation is not given in the paper, as it is lengthy, but that can be established from [4] — for general decision variables (continuous, discrete or mixed), costs and constraints (convex or not), and dynamics (linear or not). In [7, §3.4], a HD Bellman equation [7, Equation (3)] and a DH Bellman equation [7, Equation (4)] are stated. Finally, we tackle the issue of DHD with two timescales, whereas [7] deals with a single timescale.

### 1.3.4 Synthesis of our contributions in comparison with the literature

In Table 1, we highlight the main features of our contributions in comparison with [13, 14].

features	[13] fullDH	[13] ASDH	[14]	our paper
general costs, constraints, dynamics				✓
binary decision variables				✓
SDP				✓
SDDP	✓	✓	✓	
no state extension	✓			✓
two timescales				✓

Table 1: Synthesis of our contributions in comparison with the literature

## 1.4 Structure of the paper

The paper is organized as follows.

In Sect. 2, we present the physical modelling of the problem. First we introduce the two-timescale timeline, made of hours (because of hourly energy balance constraints) and weeks (because of weekly planning of decisions). Second, we define the physical variables needed to describe the system operation as well as the linking constraints and the economical cost functions.

In Sect. 3, once the physical model is set, we move on to the mathematical formulation of the adequacy resource problem as a multistage stochastic optimisation problem, focusing on the current practice at RTE for the information structure modelling, that is, *weekly hazard-decision*.

In Sect. 4, we present a *decision-hazard-decision* structure that considers both “here-and-now” and “wait-and-see” decisions in the context of prospective studies. The decision-hazard-decision structure is used to solve the adequacy problem with two timescales. The decision stages in the decision-making process are separated into a nonanticipative planning step and a recourse corrective step. As a consequence, the resolution of each stage in the

multistage stochastic optimisation problem becomes a two-stage problem in which the first stage decisions (*slow* decisions) are made before knowing the uncertainties, and the second stage or recourse decisions (*fast* decisions) are made once the weekly block of uncertainties is known. It can be interpreted that the slow decisions are associated with the unit commitment step and that the fast decisions are associated with the unit modulation. We obtain as a result a problem formulation that improves the information model by being less anticipative but still allows us to apply temporal decomposition methods. Once the new information structure is described, we present the corresponding mathematical formulation of the problem and the associated Bellman equations giving the usage values.

In Sect. 5, we present numerical results for a case study comparing both information structures: *hazard-decision* and *decision-hazard-decision* in the context of prospective studies. We observe that the choice of the information structure when computing usage values can modify the merit order in the system, that is, the order of the storage usage values with respect to the thermal units prices. We aim to illustrate the effect of modelling the information structure in a simple case, and it is not our goal to discuss possible algorithmic approaches to improve performance when computing usage values. We address the resolution of the two-stage problem at each stage of the multistage optimisation problem in its extensive formulation, knowing that more sophisticated techniques can be applied to scale the study. In this paper we focus in one storage (or few storages); to address the issue of scalability, we will count on decomposition methods (rather than SDDP) like in [10].

Finally, in Sect. 6, we conclude on the relevance of the choice of information structures in the computation of usage values, hence on the optimal allocation of resources.

In Appendix A, we detail the hourly composition of the storage dynamics. In Appendix B, we present the ideal *decision-hazard-decision information structure with hourly recourse* discussed in §1.2.

## 2 Physical and economical model of the energy system

In this Sect. 2, we describe the physical and economical model of the energy system we consider. In §2.1, we present the timeline with two timescales. In §2.2, we define the variables to model the system. In §2.3, we introduce the system dynamics and energy balance. Finally, in §2.4, we present the cost function modelling.

### 2.1 Timeline definition

We consider a timeline with a long timescale and a short timescale. The short and long timescales could be any two scales, as long as one is larger than the other. In this work, the long timescale is given by weeks that are represented by a finite totally ordered set  $(\mathcal{S}, \preceq)$ , where  $s^+$  is the successor of  $s \in \mathcal{S}$  and  $s^-$  its predecessor:  $\underline{s} \prec \dots \prec s^- \prec s \prec s^+ \prec \dots \prec \bar{s}$  (where  $\prec$  is the strict order associated with the order  $\preceq$ ). Then,  $\mathcal{S} = [\underline{s}, \bar{s}]$ . The short timescale, hours in this case, is represented by a finite totally ordered set  $(\mathcal{H}, \preceq)$ :  $\underline{h} \prec \dots \prec h^- \prec h \prec h^+ \prec \dots \prec \bar{h}$ . Then,  $\mathcal{H} = [\underline{h}, \bar{h}]$ .

To unify the timescale we consider the product set  $\mathcal{S} \times \mathcal{H}$  ordered as follows:

$$\begin{aligned} (\underline{s}, \underline{h}) \prec \dots \prec (s^-, \bar{h}) \prec (s, \underline{h}) \prec (s, \underline{h}^+) \prec \dots \\ \dots (s, \bar{h}^-) \prec (s, \bar{h}) \prec (s^+, \underline{h}) \prec \dots \prec (\bar{s}, \bar{h}) . \end{aligned} \quad (4)$$

We consider a period of one year, and  $(\underline{s}, \underline{h})$  is the instant corresponding to the first hour of the first week of the year, and  $(\bar{s}, \bar{h})$  is the last hour of the last week of the year. We need to define an extra time  $(\bar{s}^+, \underline{h})$  at its end to handle the resulting state of the last decision. The extended unified timeline  $\overline{\mathcal{S} \times \mathcal{H}}$  is defined as  $\mathcal{S} \times \mathcal{H} \cup \{(\bar{s}^+, \underline{h})\}$ .

We define  $\llbracket s \llbracket = ((s, \underline{h}), (s, \underline{h}^+), \dots, (s, \bar{h}))$  and  $\llbracket s \rrbracket = ((s, \underline{h}^+), \dots, (s, \bar{h}), (s^+, \underline{h}))$ . Thus, we use a simple bracket [ or ] to denote intervals of the elementary timelines  $(\mathcal{H}, \preceq)$  and  $(\mathcal{S}, \preceq)$ . By contrast, we use double brackets  $\llbracket$  or  $\rrbracket$  for the composite (product) timeline  $(\overline{\mathcal{S} \times \mathcal{H}}, \preceq)$ .

The different possibilities to index a variable (respectively a function) by time are detailed in Table 2 (respectively in Table 3).

Index	Notation	Description
$(s, h)$	$z_{(s,h)}$	Variable at $(s, h)$
$s$	$z_s$	Representative variable for the week $s$ corresponding to the variable at $(s, \underline{h})$
$\llbracket s \llbracket$	$z_{\llbracket s \llbracket$	Sequence of hourly variables given by $(z_{(s,\underline{h})}, z_{(s,\underline{h}^+)}, \dots, z_{(s,\bar{h})})$
$\rrbracket s \rrbracket$	$z_{\rrbracket s \rrbracket}$	Sequence of hourly variables given by $(z_{(s,\underline{h}^+)}, \dots, z_{(s,\bar{h})}, z_{(s^+,\underline{h})})$

Table 2: Variables notation

The characteristic aggregation in Table 3 could be a sum over  $h \in \mathcal{H}$ , a composition with respect to the state, or a combination of both (see Appendix A for further details).

## 2.2 Physical variables

The following is a description of the system components. We classify the variables accordingly to their type: decision (in the hand of the decision-maker), uncertainty (exogenous), state (storage) and slack (energy not supplied in the system).

### 2.2.1 Thermal units modelling

We consider a thermal fleet composed of thermal units whose variables are detailed in Table 4 for the units indexed by  $i \in \mathcal{I}$ . The ‘‘On’’ decision is associated with  $y^{\text{th},i} = 1$  and the ‘‘Off’’

Index	Notation	Description
$(s, h)$	$\phi_{(s,h)}$	Function expression at $(s, h)$
$s$	$\phi_s$	Characteristic aggregation of the hourly functions $\phi_{(s,h)}$ for the week $s$
$\llbracket s \llbracket$	$\phi_{\llbracket s \llbracket$	Sequence of hourly functions $\phi_{(s,h)}$ given by $(\phi_{(s,\underline{h})}, \phi_{(s,\underline{h}^+)}, \dots, \phi_{(s,\bar{h})})$
$\rrbracket s \rrbracket$	$\phi_{\rrbracket s \rrbracket$	Sequence of hourly functions $\phi_{(s,h)}$ given by $(\phi_{(s,\underline{h}^+)}, \dots, \phi_{(s,\bar{h})}, \phi_{(s^+,\underline{h})})$

Table 3: Functions notation

Description	Type	Notation
On/Off	Decision	$y^{\text{th},i} \in \{0, 1\}$
Modulation	Decision	$\theta^i \in \{0\} \cup [\underline{\theta}^i, \bar{\theta}^i]$
Availability	Uncertainty	$w^{\text{th},i} \in \{0, 1\}$

Table 4: Thermal units variables

decision with  $y^{\text{th},i} = 0$ . Observe that the decision  $y^{\text{th},i}$  is taken at each hour and does not represent a change in the state of the unit. The decision  $\theta^i$  denotes the power modulation once the unit is on. The availability of the thermal units is modelled with the (uncertainty) variable  $w^{\text{th},i}$ : when it is equal to 0 the unit is not available to use and, when it is equal to 1, the unit is available to use.

The collections of on/off decisions, modulation decisions and availabilities variables of all thermal units are denoted by  $y^{\text{th}} = (y^{\text{th},i})_{i \in \mathcal{I}}$ ,  $\theta = (\theta^i)_{i \in \mathcal{I}}$  and  $w^{\text{th}} = (w^{\text{th},i})_{i \in \mathcal{I}}$  respectively.

### 2.2.2 Storage modelling

In Table 5, we introduce the variables related to the storage management. We consider different variables for pumping ( $r^{\text{p}}$ ) and turbinning ( $r^{\text{t}}$ ) decisions so that we take into account the pumping efficiency in the storage. The variable  $q$  denotes the level in the storage (stock), that is, the physical state of the storage.

### 2.2.3 Residual demand modelling

The *residual demand* ( $w^{\text{d}}$ ) is the difference between demand and non-dispatchable production. This allows to group in one variable several uncertainty sources such as the wind

Description	Type	Notation
Pumping	Decision	$r^{\text{p}} \in [0, \bar{r}^{\text{p}}]$
Turbining	Decision	$r^{\text{t}} \in [0, \bar{r}^{\text{t}}]$
Storage level	Physical state	$q \in [\underline{q}, \bar{q}]$

Table 5: Storage variables

production, the solar production, the demand, etc. The variable  $w^{\text{d}}$  is classified as uncertainty.

#### 2.2.4 Slack variables

We introduce a variable  $\psi^{\text{ens}}$ , classed as *slack*, to model the *energy not supplied* in the system. This variable will appear in the forthcoming energy balance (9) and cost function (10).

### 2.3 System dynamics and energy balance

We present now the linking constraints between variables.

#### 2.3.1 Storage dynamics

The dynamics function

$$f(q, r^{\text{p}}, r^{\text{t}}) = q + \eta r^{\text{p}} - r^{\text{t}} \quad (5)$$

describes the evolution of the storage level — as a function of the current storage level  $q$ , of the pumping  $r^{\text{p}}$  and turbining  $r^{\text{t}}$  decisions — from one short time stage to the next. The parameter  $\eta \in [0, 1]$  is the pumping efficiency of the storage.

The hourly dynamics  $f$  induces a weekly temporal coupling linking the storage level at the beginning of a week  $s$  with the level at the beginning of the following week  $s^+$ . Therefore, we also consider a weekly dynamics  $f_s$  given by the hourly composition of the dynamics in (5). The composition is detailed in Appendix A. It gives, as a result, the storage level  $q_{(s^+, \underline{h})}$  at the beginning of the following week, by summing the total difference between pumping (positive taking into account its efficiency) and turbining (negative) during the week to the storage level at the beginning of the current week  $q_{(s, \underline{h})}$ .

#### 2.3.2 Thermal units' production output

The effective output of the unit is constrained by its availability and the on/off decision. More precisely, the output production  $\hat{\theta}^i$  of the  $i$ -th thermal unit not only depends on the

decision  $\theta^i$ , but also on the on/off decision  $y^{\text{th},i}$  and on the availability uncertainty  $w^{\text{th},i}$  as follows

$$\widehat{\theta}^i(y^{\text{th},i}, \theta^i, w^{\text{th},i}) = \theta^i \times \min\{y^{\text{th},i}, w^{\text{th},i}\}. \quad (6)$$

As defined, the production of the  $i$ -th unit belongs to the same set as  $\theta^i$ , that is,

$$\widehat{\theta}^i(y^{\text{th},i}, \theta^i, w^{\text{th},i}) \in \{0\} \cup [\underline{\theta}^i, \bar{\theta}^i], \quad (7)$$

taking the value zero whenever the off decision is made ( $y^{\text{th},i} = 0$ ) or the unit is not available ( $w^{\text{th},i} = 0$ ). The collection of output production of all thermal units is denoted by  $\widehat{\theta} = (\widehat{\theta}^i)_{i \in \mathcal{I}}$ .

### 2.3.3 Balance equation

Satisfying the energy balance equation is the main goal of the adequacy resource problem. That is, at every hour, the total energy production in the system should be equal to the total energy consumed in the system (which includes the pumping). Ideally, the energy balance constraint is written as the equality

$$\underbrace{r^{\text{t}} + \sum_{i \in \mathcal{I}} \widehat{\theta}^i}_{\text{total production}} + \underbrace{\psi^{\text{ens}}}_{\text{energy not supplied}} = \underbrace{r^{\text{p}} + w^{\text{d}}}_{\text{pumping demand + residual demand}}. \quad (8)$$

We rather formulate this balance equation as an inequality constraint to avoid infeasibility problems due to the thermal units' minimum power constraints: indeed, infeasibility could happen when less energy is required than the minimum power of the last (or more expensive) unit on to meet the demand. If the cause of the balance infeasibility is the lack of available production, the slack variable  $\psi^{\text{ens}}$  will take positive values, measuring how far from meeting the demand the system is. Thus, the hourly balance equation is given by

$$g(r^{\text{t}}, r^{\text{p}}, \widehat{\theta}, w^{\text{d}}, \psi^{\text{ens}}) \geq 0, \quad (9a)$$

$$\text{with } g(r^{\text{t}}, r^{\text{p}}, \widehat{\theta}, w^{\text{d}}, \psi^{\text{ens}}) = \left( r^{\text{t}} + \sum_{i \in \mathcal{I}} \widehat{\theta}^i + \psi^{\text{ens}} \right) - (r^{\text{p}} + w^{\text{d}}). \quad (9b)$$

## 2.4 Cost functions

We now introduce the hourly cost function  $\ell(y^{\text{th}}, z^{\text{th}}, \widehat{\theta}, w^{\text{th}}, \psi^{\text{ens}})$  as

$$\ell(y^{\text{th}}, z^{\text{th}}, \widehat{\theta}, w^{\text{th}}, \psi^{\text{ens}}) = \sum_{i \in \mathcal{I}} \left( \text{sc}^i \times \max\{y^{\text{th},i} - z^{\text{th},i}, 0\} + \text{vc}^i \times \widehat{\theta}^i \right) + \text{pc} \times \psi^{\text{ens}}.$$

The cost of meeting the demand is the operating cost of the thermal units. We model the thermal cost for each unit within the sum over  $i \in \mathcal{I}$  in Equation (10) with two components.

The first one is associated with decision of switching on a unit, that is, when at one hour  $y^{\text{th},i} = 0$  and in the following hour  $y^{\text{th},i} = 1$ . The second component corresponds to the variable cost, and is proportional to the power modulation  $\theta^i$  of the unit. In addition, we model the penalization on the energy not supplied as a cost. This penalization cost is much higher than the thermal units' cost to ensure that the energy demand is not provided only in cases where there is no other solution.

The parameters  $\text{sc}^i$  and  $\text{vc}^i$  correspond to the unit's start-up and modulation costs, whereas  $\text{pc}$  is the penalization parameter for the not supplied energy in the system. The variable  $z^{\text{th}}$  is introduced to take into account a temporal shift of the on/off decision  $y^{\text{th},i}$  since the start up cost is associated with a change from off to on between two consecutive hours (see Tables 6 and 7).

The weekly cost  $\ell_s$  will be defined as the sum of the hourly costs  $\ell$  within the week in Tables 6 and 7.

Up to now, we have introduced the physical and economical modelling of the problem. In Sect. 3, we will present the current practice for the adequacy problem mathematical formulation, to compute usage values, focusing on the information structure modelling.

### 3 Current information modelling in weekly hazard-decision

When modelling a stochastic multistage optimization problem, it is necessary to define the information structure, that is, a model that describes the information available at each stage of the decision-making process.

In §3.1, we introduce notations for random variables and measurability constraints. In §3.2, we present the weekly hazard-decision information structure. In §3.3, we formulate the multistage stochastic optimization problem. In §3.4, we deduce the corresponding Bellman equations in hazard-decision.

#### 3.1 Notation for random variables and measurability constraints

To formulate the multistage stochastic optimization problem, we model the uncertainties as random variables and, as a consequence, the states and controls are random variables as well. For this purpose, we consider a probability space  $(\Omega, \mathcal{F}, \mathbb{P})$ . Random variables are measurable functions from the measurable space  $(\Omega, \mathcal{F})$  towards some  $\mathbb{R}^n$ , equipped with its Borel  $\sigma$ -field  $\mathcal{B}_{\mathbb{R}^n}^o$ , and will be denoted by bold letters like  $\mathbf{Z}$ . Then, the information structures are mathematically modelled as measurability constraints. We say that  $\mathbf{Z}_1$  is measurable with respect to  $\mathbf{Z}_2$  if  $\sigma(\mathbf{Z}_1) \subset \sigma(\mathbf{Z}_2)$ , that is, the  $\sigma$ -field generated by  $\mathbf{Z}_1$  is included in (less rich than) the  $\sigma$ -field generated by  $\mathbf{Z}_2$ . Thanks to Doob Theorem [5], measurability constraints are equivalently expressed by means of functions as follows

$$\sigma(\mathbf{Z}_1) \subset \sigma(\mathbf{Z}_2) \iff \text{there exists a measurable function } \varphi \text{ such that } \mathbf{Z}_1 = \varphi(\mathbf{Z}_2). \quad (10)$$

Practically, the function  $\varphi$  is what we call policy or strategy, when  $\mathbf{Z}_2$  represents the information disclosed when making a decision.

### 3.2 Weekly hazard-decision information structure (HD)

In this Sect. 3, we consider information structures that assume a *weekly* disclosure of the information: once the uncertainty of the first hour of the week is known, the whole collection of uncertainties for the week is also known. The weekly vectors presented hereafter follow the notation given in Table 2. Let  $w_{\llbracket s \rrbracket} = (w_{\llbracket s \rrbracket}^d, w_{\llbracket s \rrbracket}^{\text{th}})$  be the vector composed of the weekly demand and weekly thermal units availabilities. In the same way, the vector composed of the collection of weekly controls is denoted by  $v_{\llbracket s \rrbracket} = (y_{\llbracket s \rrbracket}^{\text{th}}, \theta_{\llbracket s \rrbracket}, r_{\llbracket s \rrbracket}^p, r_{\llbracket s \rrbracket}^t, \psi_{\llbracket s \rrbracket}^{\text{ens}})$ .

The current practice to model the information structure is a *hazard-decision* (HD) structure in the weekly timescale as illustrated in Fig. 3.

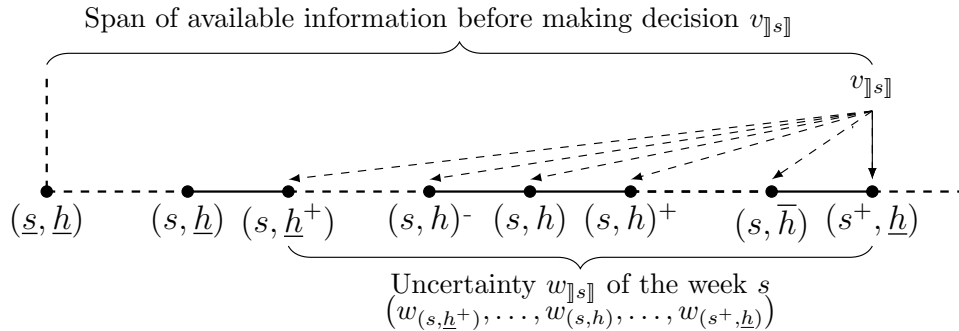


Figure 3: Weekly hazard-decision information structure. An arrow maps the available information towards the decision, so that, here, the information structure is partly anticipative as some arrows go from the right to the left.

The collection  $v_{\llbracket s \rrbracket} = (v_{(s, \underline{h}+)}, \dots, v_{(s+, \underline{h})})$  of hourly decisions for the week is made once the block  $w_{\llbracket s \rrbracket} = (w_{(s, \underline{h}+)}, \dots, w_{(s+, \underline{h})})$  of uncertainties for the week is disclosed. In other words, when making the decisions for any hour of the week, the demand and the availability of thermal units for every hour in the week are already known (in advance). We can also interpret this structure as if the hourly decisions can wait until knowing all the uncertainties of the week to be made. In this context, all the hourly decisions within the week are *one week ahead anticipative* since, when making them, the uncertainties until the end of the week are already known.

Finally, for the weekly hazard-decision structure, the information constraint is represented by the following measurability constraints

$$\sigma(\mathbf{V}_{\llbracket s \rrbracket}) \subset \sigma(\mathbf{W}_{(s, \underline{h})}, \mathbf{W}_{\llbracket s \rrbracket}, \dots, \mathbf{W}_{\llbracket s \rrbracket}) , \quad \forall s \in \mathcal{S} . \quad (11)$$

### 3.3 Multistage stochastic optimization problem formulation

In Sect. 2, we have presented the physical and economical model of the energy system. With the new notation in §3.2, we present in Table 6 the corresponding expressions and their compact mathematical versions.



Function	Notation	
	Mathematical	Physical-Economical
Storage dynamics	$\mathcal{F}_s(q_{(s,\underline{h})}, v_{\llbracket s \rrbracket})$	$f_s(q_{(s,\underline{h})}, r_{\llbracket s \rrbracket}^p, r_{\llbracket s \rrbracket}^t)$
Thermal production	$\widehat{\Theta}_{\llbracket s \rrbracket}(w_{\llbracket s \rrbracket}, v_{\llbracket s \rrbracket})$	$\left\{ \widehat{\theta}_{(s,h)^+} \left( y_{(s,h)^+}^{\text{th}}, \theta_{(s,h)^+}, w_{(s,h)^+}^{\text{th}} \right) \right\}_{h \in \mathcal{H}}$
Energy balance	$\mathcal{G}_{\llbracket s \rrbracket}(w_{\llbracket s \rrbracket}, v_{\llbracket s \rrbracket})$	$\left\{ g \left( r_{(s,h)^+}^t, r_{(s,h)^+}^p, \widehat{\theta}_{(s,h)^+} \left( y_{(s,h)^+}^{\text{th}}, \theta_{(s,h)^+}, w_{(s,h)^+}^{\text{th}} \right), w_{(s,h)^+}^d, \psi_{(s,h)^+}^{\text{ens}} \right) \right\}_{h \in \mathcal{H}}$
Weekly cost	$\mathcal{L}_s(w_{\llbracket s \rrbracket}, v_{\llbracket s \rrbracket})$	$\ell_s \left( y_{\llbracket s \rrbracket}^{\text{th}}, \widehat{\theta}_{\llbracket s \rrbracket}, w_{\llbracket s \rrbracket}^{\text{th}}, \psi_{\llbracket s \rrbracket}^{\text{ens}} \right) = \ell \left( y_{(s,\underline{h})}^{\text{th}}, 0, \widehat{\theta}_{(s,\underline{h})}, w_{(s,\underline{h})}^{\text{th}}, \psi_{(s,\underline{h})}^{\text{ens}} \right) + \sum_{h \in \mathcal{H} \setminus \{\underline{h}\}} \ell \left( y_{(s,h)^+}^{\text{th}}, y_{(s,h)^+}^{\text{th}}, \widehat{\theta}_{(s,h)^+}, w_{(s,h)^+}^{\text{th}}, \psi_{(s,h)^+}^{\text{ens}} \right)$

Table 6: Correspondence between mathematical and physical and economical notations (Sect. 2) in the weekly hazard-decision framework

The weekly cost function  $\ell_s$  is obtained as the sum of the hourly cost functions within the week. When computing the hourly cost at  $(s, h)$  with Equation (10), the variable  $z^{\text{th}}$  correspond to the on/off decision  $y_{(s,h)^-}^{\text{th}}$  at the previous hour. For simplicity, we neglect the temporal coupling of thermal units from the last hour of one week to the first hour of the next week, fixing the initial operating state of the units at the beginning of each week to off. This modelling choice is made to avoid an augmentation of the dynamic programming state's dimension for each thermal unit. This explains the presence of a zero as second entry in the first term in the expression of the weekly cost in the last line of Table 6.

Considering the definitions in Table 6, we formulate the physical adequacy problem as a stochastic multistage optimization problem using the weekly hazard-decision information structure:

$$\min_{\mathbf{Q}, \mathbf{V}} \mathbb{E} \left[ \sum_{s \in \mathcal{S}} \mathcal{L}_s(\mathbf{Q}_{(s,\underline{h})}, \mathbf{V}_{\llbracket s \rrbracket}, \mathbf{W}_{\llbracket s \rrbracket}) + K(\mathbf{Q}_{(\bar{s}^+, \underline{h})}) \right] \quad (12a)$$

such that ,  $\forall s \in \mathcal{S}$

$$\mathbf{Q}_{(\underline{s}, \underline{h})} = \mathbf{W}_{(\underline{s}, \underline{h})} , \quad (12b)$$

$$\mathbf{Q}_{(s^+, \underline{h})} = \mathcal{F}_s(\mathbf{Q}_{(s, \underline{h})}, \mathbf{V}_{\llbracket s \rrbracket}) , \quad (12c)$$

$$\mathcal{G}_{\llbracket s \rrbracket}(\mathbf{V}_{\llbracket s \rrbracket}, \mathbf{W}_{\llbracket s \rrbracket}) \geq \mathbf{0} , \quad (12d)$$

$$\sigma(\mathbf{V}_{\llbracket s \rrbracket}) \subset \sigma(\mathbf{W}_{(\underline{s}, \underline{h})}, \mathbf{W}_{\llbracket s \rrbracket}, \dots, \mathbf{W}_{\llbracket s-1 \rrbracket}, \mathbf{W}_{\llbracket s \rrbracket}) . \quad (12e)$$

The final cost  $K(\mathbf{Q}_{(\bar{s}^+, \underline{h})})$  is used to give value to the energy in the storage at the end of the yearly period.

### 3.4 Bellman equations in hazard-decision

Defining the weekly state  $x_s = q_{(s, \underline{h})}$  (storage level at the beginning of the week), we write the weekly Bellman equations [1]

$$B_{\bar{s}^+}^{\text{HD}}(x_{\bar{s}^+}) = K(x_{\bar{s}^+}) , \quad (13a)$$

$$B_s^{\text{HD}}(x_s) = \mathbb{E} \left[ \min_{v_{\llbracket s \rrbracket}} \mathcal{L}_s(x_s, v_{\llbracket s \rrbracket}, \mathbf{W}_{\llbracket s \rrbracket}) + B_{s^+}^{\text{HD}}(\mathcal{F}_s(x_s, v_{\llbracket s \rrbracket})) \right] , \quad (13b)$$

where the minimum inside Equation (13b) is computed subject to the constraint (12d).

If the sequence  $(\mathbf{W}_{\llbracket s \rrbracket}, \dots, \mathbf{W}_{\llbracket s \rrbracket}, \dots, \mathbf{W}_{\llbracket \bar{s} \rrbracket})$  of uncertainties is weekly independent, the weekly Bellman equations provide an optimal solution for Problem (12). We highlight that, to get the optimal solution, the hourly uncertainties  $\mathbf{W}_{\llbracket s \rrbracket} = (\mathbf{W}_{(s, \underline{h}^+)}, \dots, \mathbf{W}_{(s, h)^+}, \dots, \mathbf{W}_{(s^+, \underline{h})})$  within the week do not need to be assumed to be independent (from one hour to another).

It is well known that, for all  $s \in \mathcal{S}$ , the function  $B_s^{\text{HD}}$  satisfies

$$B_s^{\text{HD}}(x_s) = \min_{\mathbf{V}_{\llbracket s \rrbracket}, \dots, \mathbf{V}_{\llbracket \bar{s} \rrbracket}} \mathbb{E} \left[ \sum_{s'=s}^{\bar{s}} \mathcal{L}_{s'}(\mathbf{Q}_{(s', \underline{h})}, \mathbf{V}_{\llbracket s' \rrbracket}, \mathbf{W}_{\llbracket s' \rrbracket}) + K(\mathbf{Q}_{(\bar{s}^+, \underline{h})}) \right] \quad (14a)$$

$$\text{such that } , \quad \forall s' \in [s, \dots, \bar{s}]$$

$$\mathbf{Q}_{(s', \underline{h})} = x_s , \quad (14b)$$

$$\mathbf{Q}_{(s^+, \underline{h})} = \mathcal{F}_{s'}(\mathbf{Q}_{(s', \underline{h})}, \mathbf{V}_{\llbracket s' \rrbracket}) , \quad (14c)$$

$$\mathcal{G}_{\llbracket s' \rrbracket}(\mathbf{V}_{\llbracket s' \rrbracket}, \mathbf{W}_{\llbracket s' \rrbracket}) \geq \mathbf{0} , \quad (14d)$$

$$\sigma(\mathbf{V}_{\llbracket s' \rrbracket}) \subset \sigma(\mathbf{W}_{(\underline{s}, \underline{h})}, \mathbf{W}_{\llbracket s \rrbracket}, \dots, \mathbf{W}_{\llbracket s'-1 \rrbracket}, \mathbf{W}_{\llbracket s' \rrbracket}) , \quad (14e)$$

so that the value  $B_s^{\text{HD}}(x_s)$  of the Bellman function is interpreted as the future optimal cost when, at week  $s$ , the storage level is  $x_s$  and the weekly hazard-decision information structure is considered.

The need for another information structure arises out of the fact that the current approach is fully anticipative in the week, as illustrated in the minimum inside the expectation on the right hand side of the Bellman equation (13b). When assuming that all the uncertainties for the week are known at the moment of making a decision, we implicitly suppose that all decisions are flexible and can wait until knowing the uncertainties to be made. But it is known that certain on/off decisions cannot be made instantaneously and need to be planned in advance. For this purpose, we introduce another information structure in Sect. 4.

## 4 Information modelling in weekly decision-hazard-decision

In this Sect. 4, we present the *weekly decision-hazard-decision* (DHD) information structure. As discussed in §1.2, this structure is a compromise between the current weekly hazard-

decision described in Sect. 3 and the ideal information structure with weekly planning decisions and hourly recourses (detailed in Appendix B). In the weekly decision-hazard-decision information structure, there are decisions that cannot be modelled as (weekly) anticipative. We classify the decisions in the system modelling between:

- *planning* or *decision-hazard* (here-and-now) decisions: denoted by  $\tilde{u}$ ,
- *recourse* or *hazard-decision* (wait-and-see) decisions: denoted by  $\tilde{v}$ .

In §4.1, we present and detail the weekly decision-hazard-decision information structure. In §4.2, we formulate the multistage stochastic optimization problem. In §4.3, we give the corresponding Bellman equations in decision-hazard-decision. In §4.4, we compare the Bellman value functions according to the underlying information structures, namely HD and DHD.

#### 4.1 Weekly decision-hazard-decision information structure (DHD)

As shown in Fig. 4, the planning decisions  $\tilde{u}_{\llbracket s \rrbracket}$  are made before knowing the uncertainties  $w_{\llbracket s \rrbracket}$  for the week (knowing only the past uncertainties); then, the weekly block of uncertainties is disclosed, and the corrective actions are made, that is, the recourse controls  $\tilde{v}_{\llbracket s \rrbracket}$ .

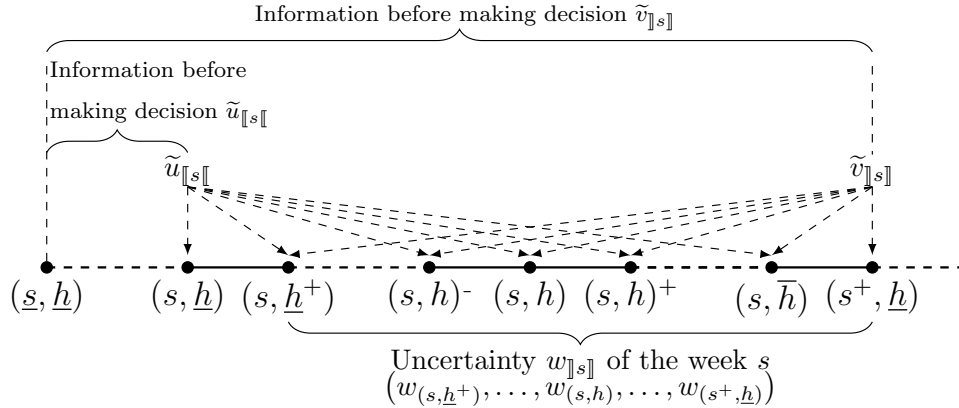


Figure 4: Weekly decision-hazard-decision information structure. An arrow maps the available information towards the decision, so that, here, the information structure is partly anticipative as some arrows go from the right to the left.

Since the recourse decisions for the beginning of the week are made knowing the uncertainties for the whole week, they are anticipative.

For the weekly decision-hazard-decision structure, the information constraints are given by the  $\sigma$ -fields inclusions

$$\sigma(\tilde{\mathbf{U}}_{\llbracket s \rrbracket}) \subset \sigma(\mathbf{W}_{(s, h)}, \mathbf{W}_{\llbracket s \rrbracket}, \dots, \mathbf{W}_{\llbracket s-1 \rrbracket}), \quad \forall s \in \mathcal{S}, \quad (15a)$$

$$\sigma(\tilde{\mathbf{V}}_{\llbracket s \rrbracket}) \subset \sigma(\mathbf{W}_{(s, h)}, \mathbf{W}_{\llbracket s \rrbracket}, \dots, \mathbf{W}_{\llbracket s-1 \rrbracket}, \mathbf{W}_{\llbracket s \rrbracket}), \quad \forall s \in \mathcal{S}. \quad (15b)$$

## 4.2 Multistage stochastic optimization problem formulation

Now we classify the physical controls described in §2.2 into planning controls  $\tilde{u}_{\llbracket s \llbracket}$  and recourse controls  $\tilde{v}_{\rrbracket s \rrbracket}$  as follows

$$\tilde{u}_{\llbracket s \llbracket} = y_{\llbracket s \llbracket}^{\text{th,slow}}, \quad (16a)$$

$$\tilde{v}_{\rrbracket s \rrbracket} = (y_{\rrbracket s \rrbracket}^{\text{th,fast}}, \theta_{\rrbracket s \rrbracket}, r_{\rrbracket s \rrbracket}^{\text{p}}, r_{\rrbracket s \rrbracket}^{\text{t}}, \psi_{\rrbracket s \rrbracket}^{\text{ens}}). \quad (16b)$$

We model as planning decisions the on and off decisions for the “slow” thermal units, and as recourse decisions all the remaining ones. We consider this classification to, in a way, model the rigidity of some thermal units, i.e. to model the fact that they cannot be switched on instantaneously. Now the collection of on/off decisions for the thermal units is composed of  $y^{\text{th}} = (y^{\text{th,slow}}, y^{\text{th,fast}})$ .

In Sect. 2, we have presented the physical and economical model of the energy system. With the new notation in §4.2, we present in Table 7 the corresponding expressions and their compact mathematical versions. As already explained in §3.3, we neglect the temporal

Function	Notation	
	Mathematical	Physical-Economical
Storage dynamics	$\tilde{\mathcal{F}}_s(q_{(s,\underline{h})}, \tilde{v}_{\rrbracket s \rrbracket})$	$f_s(q_{(s,\underline{h})}, r_{\rrbracket s \rrbracket}^{\text{p}}, r_{\rrbracket s \rrbracket}^{\text{t}})$
Thermal production	$\tilde{\Theta}_{\rrbracket s \rrbracket}(\tilde{u}_{\llbracket s \llbracket}, w_{\rrbracket s \rrbracket}, \tilde{v}_{\rrbracket s \rrbracket})$	$\left\{ \hat{\theta}_{(s,h)+} \left( \left( y_{(s,h)}^{\text{th,slow}}, y_{(s,h)+}^{\text{th,fast}} \right), \theta_{(s,h)+}, w_{(s,h)+}^{\text{th}} \right) \right\}_{h \in \mathcal{H}}$
Energy balance	$\tilde{\mathcal{G}}_{\llbracket s \llbracket}(\tilde{u}_{\llbracket s \llbracket}, w_{\rrbracket s \rrbracket}, \tilde{v}_{\rrbracket s \rrbracket})$	$\left\{ g \left( r_{(s,h)+}^{\text{t}}, r_{(s,h)+}^{\text{p}}, \hat{\theta}_{(s,h)+} \left( \left( y_{(s,h)}^{\text{th,slow}}, y_{(s,h)+}^{\text{th,fast}} \right), \theta_{(s,h)+}, w_{(s,h)+}^{\text{th}} \right), w_{(s,h)+}^{\text{d}}, \psi_{(s,h)+}^{\text{ens}} \right) \right\}_{h \in \mathcal{H}}$
Weekly cost	$\tilde{\mathcal{L}}_s(\tilde{u}_{\llbracket s \llbracket}, w_{\rrbracket s \rrbracket}, \tilde{v}_{\rrbracket s \rrbracket})$	$\ell_s \left( \left( y_{\llbracket s \llbracket}^{\text{th,slow}}, y_{\rrbracket s \rrbracket}^{\text{th,fast}} \right), \theta_{\rrbracket s \rrbracket}, w_{\rrbracket s \rrbracket}^{\text{th}}, \psi_{\rrbracket s \rrbracket}^{\text{ens}} \right) =$ $\ell \left( \left( y_{(s,\underline{h})}^{\text{th,slow}}, y_{(s,\underline{h})+}^{\text{th,fast}} \right), 0, \hat{\theta}_{(s,\underline{h})+}, w_{(s,\underline{h})+}^{\text{th}}, \psi_{(s,\underline{h})+}^{\text{ens}} \right) +$ $\sum_{h \in \mathcal{H} \setminus \{\underline{h}\}} \ell \left( \left( y_{(s,h)}^{\text{th,slow}}, y_{(s,h)+}^{\text{th,fast}} \right), \left( y_{(s,h)-}^{\text{th,slow}}, y_{(s,h)}^{\text{th,fast}} \right) + \right.$ $\left. \hat{\theta}_{(s,h)+}, w_{(s,h)+}^{\text{th}}, \psi_{(s,h)+}^{\text{ens}} \right)$

Table 7: Correspondence between mathematical and physical and economical notations (Sect. 2) in the weekly decision-hazard-decision framework

coupling of thermal units from the last hour of one week to the first hour of the next week. Considering the definitions in Table 7, we formulate the physical adequacy problem

as a stochastic multistage optimization problem using the weekly decision-hazard-decision information structure:

$$\min_{\mathbf{Q}, \tilde{\mathbf{U}}, \tilde{\mathbf{V}}} \mathbb{E} \left[ \sum_{s \in \mathcal{S}} \tilde{\mathcal{L}}_s(\mathbf{Q}_{(s,h)}, \tilde{\mathbf{U}}_{\llbracket s \rrbracket}, \mathbf{W}_{\llbracket s \rrbracket}, \tilde{\mathbf{V}}_{\llbracket s \rrbracket}) + K(\mathbf{Q}_{(\bar{s}^+, h)}) \right] \quad (17a)$$

such that ,  $\forall s \in \mathcal{S}$

$$\mathbf{Q}_{(s,h)} = \mathbf{W}_{(s,h)} , \quad (17b)$$

$$\mathbf{Q}_{(s^+, h)} = \tilde{\mathcal{F}}_s(\mathbf{Q}_{(s,h)}, \tilde{\mathbf{V}}_{\llbracket s \rrbracket}) , \quad (17c)$$

$$\tilde{\mathcal{G}}_{\llbracket s \rrbracket}(\tilde{\mathbf{U}}_{\llbracket s \rrbracket}, \mathbf{W}_{\llbracket s \rrbracket}, \tilde{\mathbf{V}}_{\llbracket s \rrbracket}) \geq \mathbf{0} , \quad (17d)$$

$$\sigma(\tilde{\mathbf{U}}_{\llbracket s \rrbracket}) \subset \sigma(\mathbf{W}_{(s,h)}, \mathbf{W}_{\llbracket s \rrbracket}, \dots, \mathbf{W}_{\llbracket s-1 \rrbracket}) , \quad (17e)$$

$$\sigma(\tilde{\mathbf{V}}_{\llbracket s \rrbracket}) \subset \sigma(\mathbf{W}_{(s,h)}, \mathbf{W}_{\llbracket s \rrbracket}, \dots, \mathbf{W}_{\llbracket s-1 \rrbracket}, \mathbf{W}_{\llbracket s \rrbracket}) . \quad (17f)$$

The final cost  $K(\mathbf{Q}_{(\bar{s}^+, h)})$  is used to give value to the energy in the storage at the end of the yearly period.

### 4.3 Bellman equations in decision-hazard-decision

Defining the weekly state  $x_s = q_{(s,h)}$ , that is, the storage level at the beginning of the week, we write the weekly Bellman equations (18) using [4, Proposition 13]

$$B_{\bar{s}^+}^{\text{DHD}}(x_{\bar{s}^+}) = K(x_{\bar{s}^+}) , \quad (18a)$$

$$B_s^{\text{DHD}}(x_s) = \min_{\tilde{u}_{\llbracket s \rrbracket}} \mathbb{E} \left[ \min_{\tilde{v}_{\llbracket s \rrbracket}} \tilde{\mathcal{L}}_s(x_s, \tilde{u}_{\llbracket s \rrbracket}, \mathbf{W}_{\llbracket s \rrbracket}, \tilde{v}_{\llbracket s \rrbracket}) + B_{s^+}^{\text{DHD}}(\tilde{\mathcal{F}}_s(x_s, \tilde{v}_{\llbracket s \rrbracket})) \right] , \quad (18b)$$

where the minimum inside the expectation term of Equation (18b) is computed subject to the constraint (17d).

If the sequence  $(\mathbf{W}_{\llbracket s \rrbracket}, \dots, \mathbf{W}_{\llbracket s \rrbracket}, \dots, \mathbf{W}_{\llbracket \bar{s} \rrbracket})$  of uncertainties is weekly independent, the weekly Bellman equations provide an optimal solution for Problem (17). We highlight that, to get the optimal solution, the hourly uncertainties  $\mathbf{W}_{\llbracket s \rrbracket} = (\mathbf{W}_{(s,h^+)}, \dots, \mathbf{W}_{(s,h)^+}, \dots, \mathbf{W}_{(s^+, h)})$  within the week do not need to be assumed to be independent (from one hour to another). Under this independence assumption, it is well known that, for all  $s \in \mathcal{S}$ , the function  $B_s^{\text{DHD}}$

satisfies

$$B_s^{\text{DHD}}(x_s) = \min_{\substack{\tilde{\mathbf{U}}_{\llbracket s \rrbracket}, \dots, \tilde{\mathbf{U}}_{\llbracket \bar{s} \rrbracket} \\ \tilde{\mathbf{V}}_{\llbracket s \rrbracket}, \dots, \tilde{\mathbf{V}}_{\llbracket \bar{s} \rrbracket}}} \mathbb{E} \left[ \sum_{s'=s}^{\bar{s}} \mathcal{L}_{s'}(\mathbf{Q}_{(s',h)}, \tilde{\mathbf{U}}_{\llbracket s' \rrbracket}, \mathbf{W}_{\llbracket s' \rrbracket}, \tilde{\mathbf{V}}_{\llbracket s' \rrbracket}) + K(\mathbf{Q}_{(\bar{s}^+,h)}) \right] \quad (19a)$$

such that ,  $\forall s' \in [s, \dots, \bar{s}]$

$$\mathbf{Q}_{(s',h)} = x_s , \quad (19b)$$

$$\mathbf{Q}_{(s'+,h)} = \mathcal{F}_{s'}(\mathbf{Q}_{(s',h)}, \tilde{\mathbf{V}}_{\llbracket s' \rrbracket}) , \quad (19c)$$

$$\mathcal{G}_{\llbracket s' \rrbracket}(\tilde{\mathbf{U}}_{\llbracket s' \rrbracket}, \mathbf{W}_{\llbracket s' \rrbracket}, \tilde{\mathbf{V}}_{\llbracket s' \rrbracket}) \geq \mathbf{0} , \quad (19d)$$

$$\sigma(\tilde{\mathbf{U}}_{\llbracket s' \rrbracket}) \subset \sigma(\mathbf{W}_{(s,h)}, \mathbf{W}_{\llbracket s \rrbracket}, \dots, \mathbf{W}_{\llbracket s'- \rrbracket}) , \quad (19e)$$

$$\sigma(\tilde{\mathbf{V}}_{\llbracket s' \rrbracket}) \subset \sigma(\mathbf{W}_{(s,h)}, \mathbf{W}_{\llbracket s \rrbracket}, \dots, \mathbf{W}_{\llbracket s'- \rrbracket}, \mathbf{W}_{\llbracket s' \rrbracket}) , \quad (19f)$$

so that, the value  $B_s^{\text{DHD}}(x_s)$  of Bellman function is interpreted as the future optimal cost when at week  $s$  the storage level is  $x_s$  and the weekly decision-hazard-decision information structure is considered.

#### 4.4 Theoretical comparison between Bellman functions in HD and DHD

When we compare theoretically the Bellman functions  $B_s^{\text{HD}}$  and  $B_s^{\text{DHD}}$ , given by the Bellman equations (13) and (18), we observe that the weekly hazard-decision approach is a relaxation of the weekly decision-hazard-decision approach with respect to the information constraint. In other words, at each stage of the stochastic multistage optimization problem, the decision maker has more information when making the decision in the HD case, than in the DHD case. Therefore, we have the following inequalities for all  $s \in \mathcal{S}$ :

$$B_s^{\text{HD}} \leq B_s^{\text{DHD}} . \quad (20)$$

The usage values — or prices  $p$  — are defined as the opposite of the derivative of the Bellman functions with respect to the storage level [12]:

$$p_s^{\text{HD}} = -\frac{d}{dx} B_s^{\text{HD}}(x) , \quad (21a)$$

$$p_s^{\text{DHD}} = -\frac{d}{dx} B_s^{\text{DHD}}(x) . \quad (21b)$$

Whereas we have established an inequality between Bellman functions in Equation (20), it is impossible to do so for the usage values (as they are derivatives).

In Sect. 5, we compare numerically the effect of both information structures when computing usage values for a case study of small size.

## 5 Numerical study

In this Sect. 5, we present a numerical study. In §5.1, we describe the energy system of small size considered. In §5.2, we present the method to compute the Bellman functions and usage values, either for the hazard-decision information structure in Sect. 3 or for the decision-hazard-decision information structure in Sect. 4. In §5.3, we compare the numerical results for the usage values. Finally, in §5.4, we show the effect of the information modelling choice when designing policies to carry on a simulation of the energy system dispatch.

### 5.1 Case study description

We consider a small energy system to conduct the numerical study with the following components: 3 thermal units (base unit, semi-base unit and peak unit); 1 residual demand; 1 storage unit. The thermal units differ in their prices and power capacities. Whereas the base and semi-base units are the cheapest and the less flexible ones, the peak unit is the most expensive but the most flexible one.

We model the components mentioned above as in Sect. 2, and we consider the *weekly hazard-decision* (Sect. 3) and the *weekly decision-hazard-decision* (Sect. 4) mathematical modelling of the problem. In the weekly hazard-decision case, all the decisions for the week are anticipative but, in the weekly decision-hazard-decision case, we consider that the on and off decisions for the base and semi-base units are nonanticipative (planned in advance). This classification is made taking into account the rigidity and flexibility of the units.

To model the uncertainties, we have taken a finite number of scenarios provided by RTE, with uniform probability. We divide the scenarios into two sets:

- $(w_{\lfloor s \rfloor}^n)_{s \in \mathcal{S}}$  for  $n \in \llbracket 1, N \rrbracket = \{1, \dots, N\}$  used to compute Bellman functions and their associated nonanticipative policies,
- $(w_{\lfloor s \rfloor}^c)_{s \in \mathcal{S}}$  for  $c \in \llbracket 1, C \rrbracket = \{1, \dots, C\}$  used in simulation.

For the computations, we use the programming language Julia [3], the JuMP package [9] and the Xpress solver.

### 5.2 Bellman functions computation

We compute the Bellman functions using the classical stochastic dynamic programming (SDP) algorithm [2] for the hazard-decision framework, and an adapted version for the decision-hazard-decision framework. In both cases, the problem for the entire year is decomposed in weekly problems that we solve using the backward recursions (13) and (18) (respectively) for a regular discretization grid  $G_{\mathcal{X}}$  of the state space  $\mathcal{X} = [\underline{q}, \bar{q}]$  (see Table 5). This discretization grid has been refined to achieve a satisfactory piecewise approximation of the Bellman functions.

On the one hand, we do it to be able to handle binary or discrete variables within the planning decisions, in which case SDDP is not suitable (but variants like SDDiP are). On





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**Algorithm 2:** Decision-hazard-decision stochastic dynamic programming
 

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**Data:** uncertainties scenarios  $w_{\llbracket s \rrbracket}^n$  with  $n \in \llbracket 1, N \rrbracket$  and probabilities  $\frac{1}{N}$ ,  
 space state discretization  $x \in G_{\mathcal{X}}$ ,  
 final cost  $K(x_{\bar{s}+})$   
**Result:**  $\{B_s^{\text{DHD}}(x)\}_{s \in \mathcal{S}}$   
**for**  $s = \bar{s}, \dots, \underline{s}$  **do**  
   **for**  $x \in G_{\mathcal{X}}$  **do**  
     
$$B_s^{\text{DHD}}(x) = \min_{\tilde{u}_{\llbracket s \rrbracket}} \sum_{n=1}^N \frac{1}{N} \left( \min_{\tilde{v}_{\llbracket s \rrbracket}^n} \left\{ \mathcal{L}_s(x, \tilde{u}_{\llbracket s \rrbracket}, w_{\llbracket s \rrbracket}^n, \tilde{v}_{\llbracket s \rrbracket}^n) + B_{s^+}^{\text{DHD}}(\mathcal{F}_s(x, \tilde{v}_{\llbracket s \rrbracket}^n)) \right\}, \right.$$
     such that (17d)  $\left. \right)$   
   **end**  
**end**

---

with  $\Delta x$  the discretization step of the grid  $G_{\mathcal{X}}$ .

### 5.3 Numerical results for usage values

In Fig. 5, we compare the prices of the thermal units with the usage values obtained with the Bellman functions computed using the HD and DHD structures.

We observe that, for storage levels higher than 20%, the HD usage value in blue is above the semi-base unit price (dashed yellow), whereas the DHD usage value in red is below. As a consequence, we expect different dispatches in simulation when using these usage values to design the storage policy, since the merit order of the production means changes. This change in the dispatch could lead to different conclusions when carrying out prospective studies. In §5.4 we compare the results in simulation induced by both modelling options to compute Bellman functions.

### 5.4 Numerical comparison between policies in HD and DHD

The goal of this part of the numerical study is to compare the dispatches obtained when using the policies induced by both Bellman functions  $(B_s^{\text{HD}})_{s \in \mathcal{S}}$  and  $(B_s^{\text{DHD}})_{s \in \mathcal{S}}$  computed in §5.2.

We recall that an uncertainty scenario for simulation (also called *chronicle*) is denoted by  $(w_{\llbracket s \rrbracket}^c)_{s \in \mathcal{S}}$ . We simulate the operation of the system for different chronicles  $c \in \llbracket 1, C \rrbracket$  in which the availability and demand change.

For the sake of simplicity and to be consistent with the weekly decision-hazard-decision information structure, we choose to design an “anticipative simulator”: uncertainties are disclosed at the end of the week and recourse controls are computed at the end of the week but applied within the week. The simulation algorithm is the same regardless the sequence of Bellman functions (HD or DHD) chosen to design the policies. We illustrate how

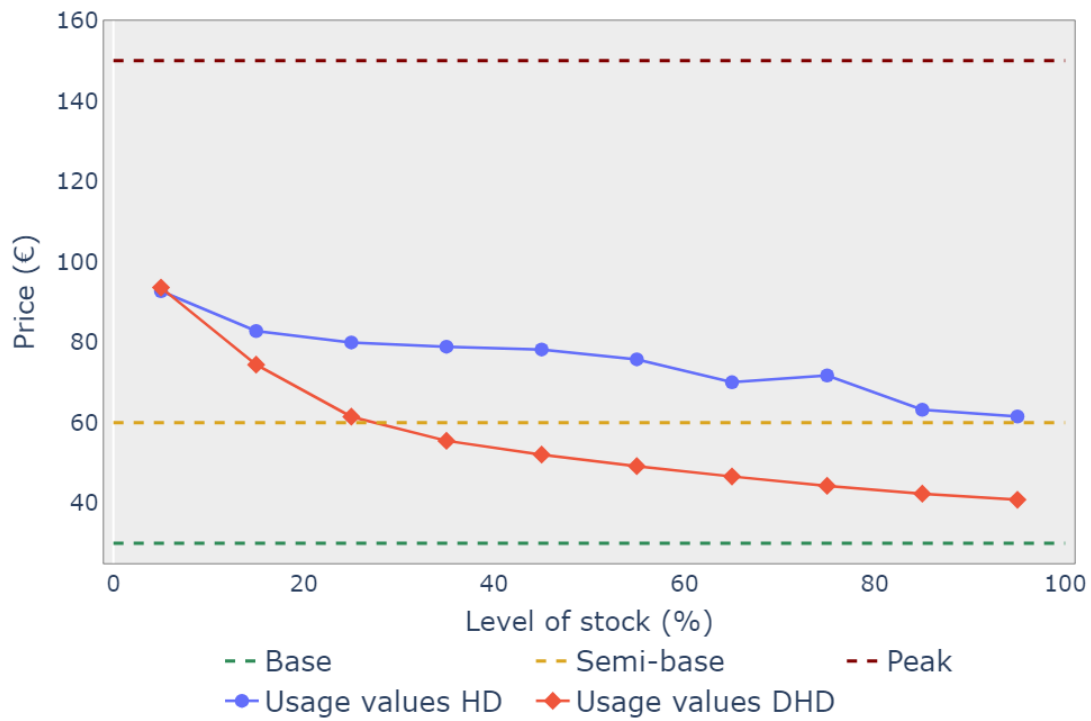


Figure 5: Prices and usage values comparison for week 20, depending on the information structure (DHD or HD) considered in the Bellman equations

the simulation is done for one uncertain chronicle in Algorithm 3. We highlight that this simulation algorithm can be used with any Bellman functions.

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**Algorithm 3:** Decision-hazard-decision policy and simulation

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**Data:** uncertainties scenarios  $w_{\llbracket s \rrbracket}^n$  with  $n \in \llbracket 1, N \rrbracket$  and probabilities  $\frac{1}{N}$ ,

Bellman functions  $\{B_s\}_{s \in \mathcal{S}}$ ,

simulation chronicle  $\{w_{\llbracket s \rrbracket}^c\}_{s \in \mathcal{S}}$ ,

initial condition for the state  $x_0$

**Result:**  $\{x_s^c, u_{\llbracket s \rrbracket}^c, v_{\llbracket s \rrbracket}^c\}_{s \in \mathcal{S}}$

$x_{\underline{s}}^c = x_0$

**for**  $s = \underline{s}, \dots, \bar{s}$  **do**

**compute nonanticipative controls:**

$$u_{\llbracket s \rrbracket}^c = \arg \min_{\tilde{u}_{\llbracket s \rrbracket}} \sum_{n=1}^N \frac{1}{N} \left( \min_{\tilde{v}_{\llbracket s \rrbracket}^n} \left\{ \mathcal{L}_s(x_s^c, \tilde{u}_{\llbracket s \rrbracket}, w_{\llbracket s \rrbracket}^n, \tilde{v}_{\llbracket s \rrbracket}^n) + B_{s^+}(\mathcal{F}_s(x_s^c, \tilde{v}_{\llbracket s \rrbracket}^n)) \right\} \right)$$

**compute recourse controls:**

$$v_{\llbracket s \rrbracket}^c = \arg \min_{\tilde{v}_{\llbracket s \rrbracket}} \left\{ \mathcal{L}_s(x_s^c, u_{\llbracket s \rrbracket}^c, w_{\llbracket s \rrbracket}^c, \tilde{v}_{\llbracket s \rrbracket}) + B_{s^+}(\mathcal{F}_s(x_s^c, \tilde{v}_{\llbracket s \rrbracket})) \right\}$$

**update state:**

$$x_{s^+}^c = \mathcal{F}_s(x_s^c, \tilde{v}_{\llbracket s \rrbracket})$$

**end**

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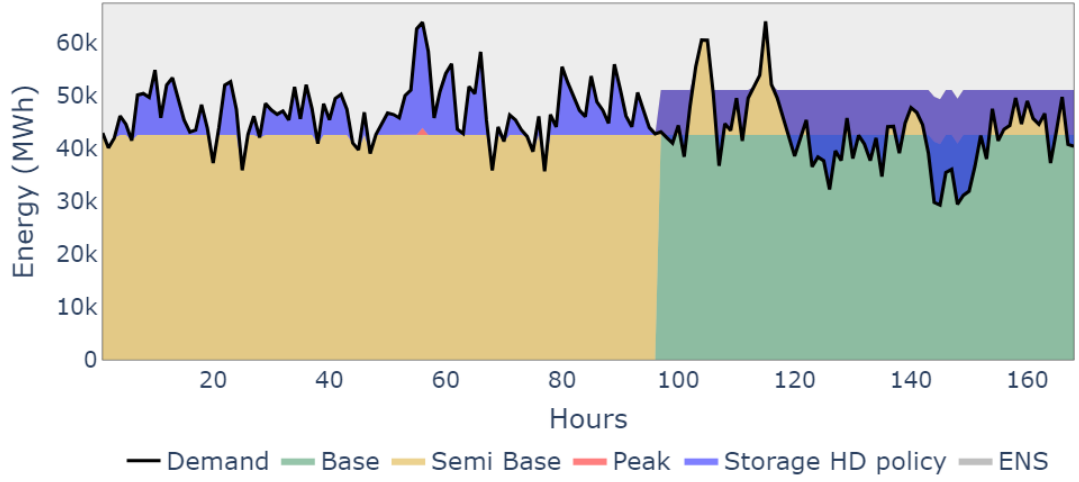
In Fig. 6, we compare the simulations of the dispatches for one chronicle of uncertainties (residual demand and availability fixed) in the week 20 of the year when using the HD and DHD Bellman functions to compute the policies. In the selected chronicle, the base unit is not available until the hour 96 of the week. The semi-base and peak units are available during the entire week.

We compare the dispatch obtained with the weekly hazard-decision policy in Fig. 6a with the one obtained with the weekly decision-hazard-decision policy in Fig. 6b. To satisfy the same demand, with the same production means available, we observe that the dispatch varies depending on the policy considered. In the case of the HD policy, the semi-base unit works at its maximum power and the marginal production is made by the storage. On the contrary, in the DHD policy dispatch, the storage operates at its maximum power and the marginal production is made by the semi-base unit.

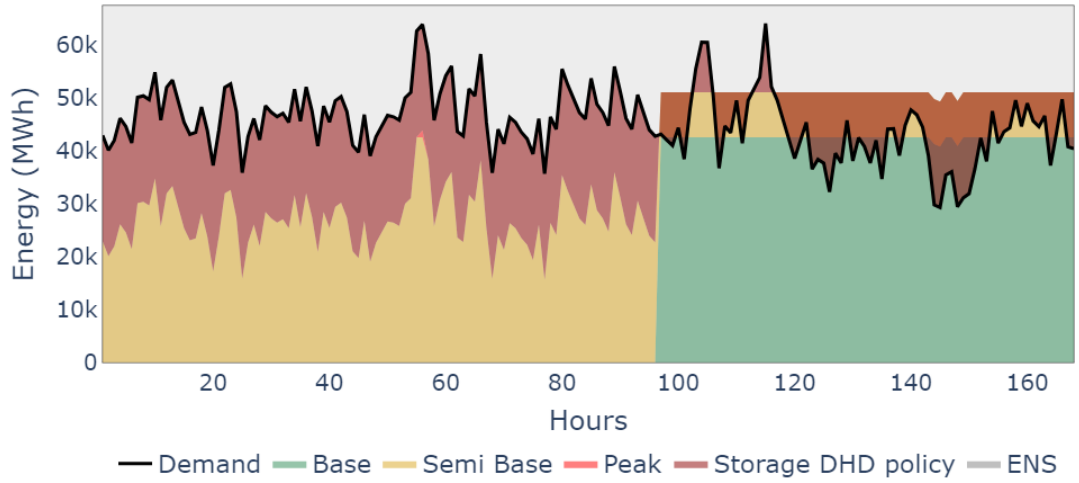
This difference comes from the fact that, as we remarked in Fig. 5, the HD usage values are above the semi-base unit price, whereas the DHD usage values are below the semi-base unit price. This changes the dispatch order.

The merit order difference in the two policies studied is observed for several weeks in the year, which leads, for all chronicles, to a higher use of the storage (more turbinning and more pumping) when using the DHD policy.

However, we observe that the HD policy tends to store more energy than the DHD policy as we can see in Fig. 7 for the same simulated yearly chronicle. This result is observed in



(a) Weekly hazard-decision policy dispatch



(b) Weekly decision-hazard-decision policy dispatch

Figure 6: Dispatch comparison using the policies induced by the Bellman functions computed either with the weekly hazard-decision (Fig. 6a) or with the weekly decision-hazard-decision information structure (Fig. 6b)

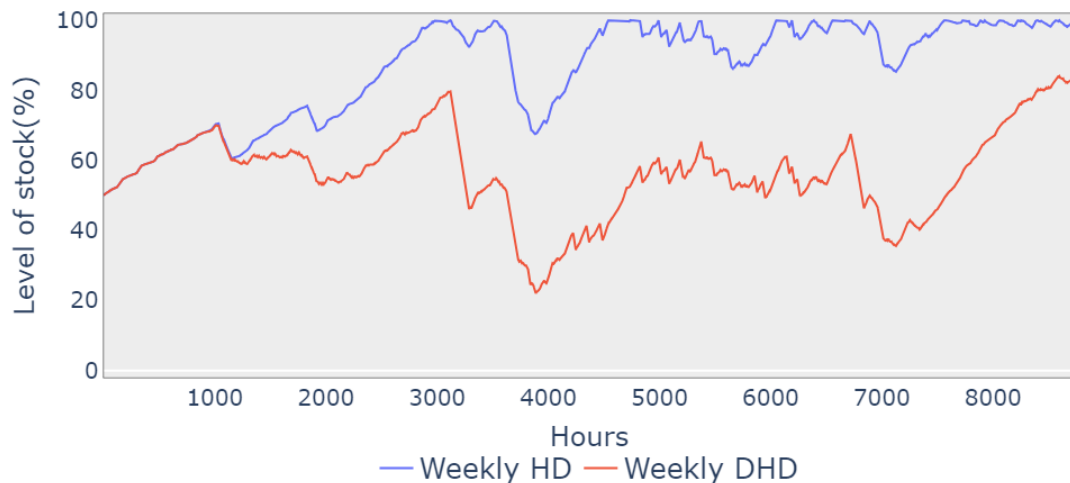


Figure 7: Comparison of the storage level obtained from the weekly hazard-decision and the weekly decision-hazard-decision policies for one given chronicle (of uncertainties)

all simulated chronicles and can be explained with the usage value difference: in the case of the DHD policy, there is less interest in storing energy since it has lower usage value.

## 6 Conclusion

In this paper, we have first formalized the weekly hazard-decision information structure in a two-timescale setting. The weekly hazard-decision information structure is the current reference framework when modelling the resource adequacy problem as a multistage stochastic optimization problem to compute usage values for prospective studies under uncertainty. We have written the corresponding weekly Bellman equations, that make it possible to compute usage values for the energy in the storage. These Bellman equations respect the hourly physical constraints.

Then, we have highlighted the need to improve this structure to account for temporal rigidities in thermal operation, that is, the fact that on/off decisions cannot be modelled as fast or last minute decisions. Therefore, we have introduced the weekly decision-hazard-decision information structure in a two-timescale setting. In this structure, the decisions for each stage in the decision-making process are separated into planning and recourse decisions, depending on the physical modelling aspects of the decisions. We have presented the mathematical formulation of the problem considering the weekly decision-hazard-decision information structure and its associated Bellman equation in the weekly timescale, that still respect the hourly physical constraints.

Afterwards, we have carried out a numerical analysis to quantify, in a case study, the consequences of the information structure modelling choice when computing Bellman

functions and their associated usage values. From this study, we have inferred that the policy induced by the weekly hazard-decision information modelling leads to an overestimation of the thermal flexibility and as a result, a lower use of the storage. Indeed, when using the policy induced by the weekly decision-hazard-decision Bellman functions, we take into account some rigidity in the thermal units (specially on/off decisions) and the storage is more used. In consequence, we concur with [13] that decision-hazard-decision information structures are of great interest to calculate usage values, taking into account that some types of thermal units are less flexible than others.

This study has been carried using a single storage, so that the well-known curse of dimensionality in SDP is not binding. In future work, we will turn to spatial decomposition techniques [10] to extend the study to multiple storage facilities.

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## A Hourly composition of the storage dynamics

We consider the hourly dynamics function given by

$$f(q, r^p, r^t) = q + \eta r^p - r^t . \quad (23a)$$

This equation represents the hourly evolution of the level in a storage. To be able to formulate the problem in a weekly framework, we define the weekly dynamics  $f_s$ . The weekly dynamics  $f_s(q_{(s,\underline{h})}, r_{\lfloor s \rfloor}^p, r_{\lfloor s \rfloor}^t)$  is obtained by composition of the hourly dynamics  $f(q_{(s,h)}, r_{(s,h)^+}^p, r_{(s,h)^+}^t)$  for  $h \in \mathcal{H}$ , but the composition being done only on storage level variable.

To describe the dynamics composition we introduce the following extra notation

$$f_{(s,h)}^{r^p, r^t}(q_{(s,h)}) = f(q_{(s,h)}, r_{(s,h)^+}^p, r_{(s,h)^+}^t) , \quad (23b)$$

so that we get

$$f_s(q_{(s,\underline{h})}, r_{\lfloor s \rfloor}^p, r_{\lfloor s \rfloor}^t) = \left( f_{(s,\bar{h})}^{r^p, r^t} \circ f_{(s,\bar{h}^-)}^{r^p, r^t} \circ \dots \circ f_{(s,\underline{h})}^{r^p, r^t} \right) (q_{(s,\underline{h})}) . \quad (23c)$$

From the hourly dynamics  $f$  expression (23a), the weekly dynamics  $f_s$  is given by:

$$f_s(q_{(s,\underline{h})}, r_{\lfloor s \rfloor}^p, r_{\lfloor s \rfloor}^t) = q_{(s,\underline{h})} + \sum_{h \in \mathcal{H}} (\eta r_{(s,h)^+}^p - r_{(s,h)^+}^t) . \quad (23d)$$

## B Ideal information structure in decision-hazard-decision

As discussed in §1.2, we should ideally consider an information structure in which, at the beginning of the week  $s$ , the planning for the nonanticipative decisions  $\tilde{u}_{\llbracket s \rrbracket}$  is made knowing only the past uncertainties, that is, the uncertainties up to  $(s, \underline{h})$ . Then, when the uncertainties  $w_{(s,h)}$  begin to be disclosed hour by hour, there are so-called hourly recourse decisions  $\tilde{v}_{(s,h)}$  that are also made hour by hour knowing the uncertainties up to  $(s, h)$ . Such information structure, illustrated in Fig. 8, is called *decision-hazard-decision information structure with hourly recourse*.

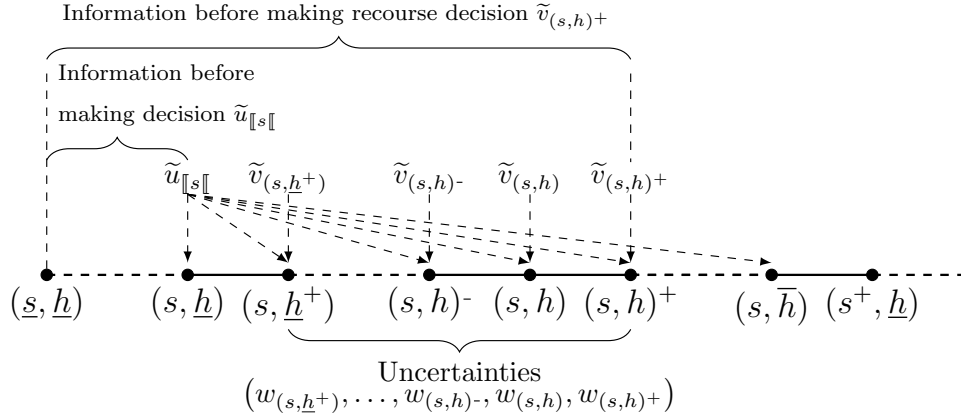


Figure 8: Decision-hazard-decision information structure with hourly recourse. An arrow maps the available information towards the decision, so that, here, the information structure is nonanticipative as all arrows go either down or from the left to the right.

This structure leads to Bellman equations (see §B.3) with nested mathematical expectations and minimizations, hence out of reach to compute. This is why we opted in Sect. 4 for a compromise solution, namely, the weekly decision-hazard information structure.

**Remark 1** *In real operation it is possible to adjust the production “in real time” to meet demand, that is, there is some anticipativity at the fast timescale for the hourly recourse decision  $\tilde{v}_{(s,h)}$ . Therefore, it is not unrealistic to consider the ideal model described above. Note that, it is not possible to consider a fully nonanticipative (or decision-hazard) structure, in which the hourly recourse decisions  $\tilde{v}_{(s,h)}$  are made knowing the uncertainties up to  $(s, h)^-$  (instead of  $(s, h)$ ), because it would lead to an excessive usage of the slack variable  $\psi^{\text{ens}}$  in the energy balance equation (9).*

### B.1 Weekly decision-hazard-decision information structure with hourly recourse

We consider an information structure in which the nonanticipative control  $\tilde{u}_{\llbracket s \rrbracket}$  is made knowing the uncertainties up to the beginning of week  $s$ , and the hourly recourse control  $\tilde{v}_{(s,h)+}$  is



made knowing the uncertainties up to the hour  $(s, h)^+$ . As illustrated in Fig. 8, at the beginning of the week  $s$  the nonanticipative decision  $\tilde{u}_{\llbracket s \rrbracket}$  is made (for all  $h \in \mathcal{H}$ ) and then, once the uncertainties are disclosed *hour by hour*, the recourse decisions  $v_{(s,h)^+}$  are sequentially made.

For the weekly decision-hazard-decision structure with hourly recourse the information constraints are given by the  $\sigma$ -fields inclusions

$$\forall s \in \mathcal{S}, \quad \sigma(\tilde{\mathbf{U}}_{\llbracket s \rrbracket}) \subset \sigma(\mathbf{W}_{(s,h)}, \mathbf{W}_{\llbracket s \rrbracket}, \dots, \mathbf{W}_{\llbracket s-1 \rrbracket}), \quad (24a)$$

$$\forall (s, h) \in \mathcal{S} \times \mathcal{H}, \quad \sigma(\tilde{\mathbf{V}}_{(s,h)^+}) \subset \sigma(\mathbf{W}_{(s,h)}, \mathbf{W}_{\llbracket s \rrbracket}, \dots, \mathbf{W}_{\llbracket s-1 \rrbracket}, \mathbf{W}_{(s,h^+)}, \dots, \mathbf{W}_{(s,h)^+}). \quad (24b)$$

## B.2 Multistage stochastic optimization problem formulation

Considering the definitions in Table 7, we formulate the physical-economical adequacy problem as a stochastic multistage optimization problem using the weekly decision-hazard-decision information structure with hourly recourse:

$$\min_{\tilde{\mathbf{U}}, \tilde{\mathbf{V}}} \mathbb{E} \left[ \sum_{s \in \mathcal{S}} \tilde{\mathcal{L}}_s(\mathbf{Q}_{(s,h)}, \tilde{\mathbf{U}}_{\llbracket s \rrbracket}, \mathbf{W}_{\llbracket s \rrbracket}, \tilde{\mathbf{V}}_{\llbracket s \rrbracket}) + K(\mathbf{Q}_{(\bar{s}^+, h)}) \right] \quad (25a)$$

such that ,  $\forall (s, h) \in \mathcal{S} \times \mathcal{H}$

$$\mathbf{Q}_{(s,h)} = \mathbf{W}_{(s,h)}, \quad (25b)$$

$$\mathbf{Q}_{(s^+, h)} = \tilde{\mathcal{F}}_s(\mathbf{Q}_{(s,h)}, \tilde{\mathbf{V}}_{\llbracket s \rrbracket}), \quad (25c)$$

$$\tilde{\mathcal{G}}_{\llbracket s \rrbracket}(\tilde{\mathbf{U}}_{\llbracket s \rrbracket}, \mathbf{W}_{\llbracket s \rrbracket}, \tilde{\mathbf{V}}_{\llbracket s \rrbracket}) \geq \mathbf{0}, \quad (25d)$$

$$\sigma(\tilde{\mathbf{U}}_{\llbracket s \rrbracket}) \subset \sigma(\mathbf{W}_{(s,h)}, \mathbf{W}_{\llbracket s \rrbracket}, \dots, \mathbf{W}_{\llbracket s-1 \rrbracket}), \quad (25e)$$

$$\sigma(\tilde{\mathbf{V}}_{(s,h)^+}) \subset \sigma(\mathbf{W}_{(s,h)}, \mathbf{W}_{\llbracket s \rrbracket}, \dots, \mathbf{W}_{\llbracket s-1 \rrbracket}, \mathbf{W}_{(s,h^+)}, \dots, \mathbf{W}_{(s,h)^+}). \quad (25f)$$

The final cost  $K(\mathbf{Q}_{(\bar{s}^+, h)})$  is used to give value to the energy in the storage at the end of the yearly period.

## B.3 Bellman equations in decision-hazard-decision information structure with hourly recourse

Here, we provide Bellman equations corresponding to the stochastic multistage optimization problem (25). Their derivation is not given, as it is lengthy, but can be established from [4]. Defining the weekly state  $x_s = q_{(s,h)}$ , that is, the storage level at the beginning of the week,



assumption, it is well known that, for all  $s \in \mathcal{S}$ , the function  $B_s^{\text{DHD}^h}$  satisfies

$$B_s^{\text{DHD}^h}(x_s) = \min_{\substack{\tilde{\mathbf{U}}_{\llbracket s \rrbracket, \dots, \llbracket \bar{s} \rrbracket} \\ \tilde{\mathbf{V}}_{\llbracket s \rrbracket, \dots, \llbracket \bar{s} \rrbracket}}} \mathbb{E} \left[ \sum_{s'=s}^{\bar{s}} \mathcal{L}_{s'}(\mathbf{Q}_{(s', \underline{h})}, \tilde{\mathbf{U}}_{\llbracket s' \rrbracket}, \mathbf{W}_{\llbracket s' \rrbracket}, \tilde{\mathbf{V}}_{\llbracket s' \rrbracket}) + K(\mathbf{Q}_{(\bar{s}^+, \underline{h})}) \right] \quad (27a)$$

such that ,  $\forall s' \in \llbracket s, \dots, \bar{s} \rrbracket$

$$\mathbf{Q}_{(s', \underline{h})} = x_s , \quad (27b)$$

$$\mathbf{Q}_{(s'+, \underline{h})} = \mathcal{F}_{s'}(\mathbf{Q}_{(s', \underline{h})}, \tilde{\mathbf{V}}_{\llbracket s' \rrbracket}) , \quad (27c)$$

$$\mathcal{G}_{\llbracket s' \rrbracket}(\tilde{\mathbf{U}}_{\llbracket s' \rrbracket}, \mathbf{W}_{\llbracket s' \rrbracket}, \tilde{\mathbf{V}}_{\llbracket s' \rrbracket}) \geq \mathbf{0} , \quad (27d)$$

$$\sigma(\tilde{\mathbf{U}}_{\llbracket s' \rrbracket}) \subset \sigma(\mathbf{W}_{(\underline{s}, \underline{h})}, \mathbf{W}_{\llbracket \underline{s} \rrbracket}, \dots, \mathbf{W}_{\llbracket s'- \rrbracket}) , \quad (27e)$$

$$\sigma(\tilde{\mathbf{V}}_{(s', h)^+}) \subset \sigma(\mathbf{W}_{(\underline{s}, \underline{h})}, \mathbf{W}_{\llbracket \underline{s} \rrbracket}, \dots, \mathbf{W}_{\llbracket s'- \rrbracket}, \mathbf{W}_{(s', h)^+}, \dots, \mathbf{W}_{(s', h)^+}) . \quad (27f)$$

so that the value  $B_s^{\text{DHD}^h}(x_s)$  of Bellman function is interpreted as the future optimal cost when at week  $s$  the storage level is  $x_s$  and the weekly decision-hazard-decision information structure with hourly recourse is considered. More details can be found in [4, Proposition 10].

## B.4 Theoretical comparison between Bellman functions in HD, DHD and DHD<sup>h</sup>

When we compare theoretically the Bellman functions  $B_s^{\text{HD}}$ ,  $B_s^{\text{DHD}}$  and  $B_s^{\text{DHD}^h}$ , given by the Bellman equations (13), (18) and (26), we observe that the weekly hazard-decision approach is a relaxation of the weekly decision-hazard-decision approach, and this last one is a relaxation of the weekly decision-hazard-decision with hourly recourse approach with respect to the information constraint.

In other words, at each stage of the stochastic multistage optimization problem, the decision maker has less information when changing from the HD to the DHD approach. The same happens when changing from the DHD to the DHD<sup>h</sup> approach. Therefore, we have the following inequalities for all  $s \in \mathcal{S}$ :

$$B_s^{\text{HD}} \leq B_s^{\text{DHD}} \leq B_s^{\text{DHD}^h} . \quad (28)$$