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A Two-Timescale Decision-Hazard-Decision Formulation for Storage Usage Values Calculation

Camila Martínez Parra^{1,2}, Michel De Lara², Jean-Philippe Chancelier²,
Pierre Carpentier³, Jean-Marc Janin¹, and Manuel Ruiz¹

¹Réseau de Transport d'Électricité, France

²CERMICS, École nationale des ponts et chaussées, Institut Polytechnique
de Paris, France

³UMA, ENSTA Paris, IP Paris, Palaiseau, France

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Abstract

The penetration of renewable energies requires additional storages to deal with intermittency. Accordingly, there is growing interest in evaluating the opportunity cost (usage value) associated with stored energy in large storages, a cost obtained by solving a multistage stochastic optimization problem. Today, to compute usage values under uncertainties, an adequacy resource problem is solved using stochastic dynamic programming assuming a *hazard-decision* information structure. This modelling assumes complete knowledge of the coming week uncertainties, which is not adapted to the system operation as the intermittency occurs at smaller timescale. We equip the two-timescale problem with a new information structure considering planning and recourse decisions: *decision-hazard-decision*. This structure is used to decompose the multistage decision-making process into a nonanticipative planning step in which the on/off decisions for the thermal units are made, and a recourse step in which the power modulation decisions are made once the uncertainties have been disclosed. In a numerical case, we illustrate how usage values are sensitive as how the disclosure of information is modelled.

Keywords. Energy system modelling, information structure, stochastic multistage optimization, dynamic programming

1 Introduction

To conduct prospective studies in energy systems under uncertainty, the operation is simulated for one year in the future considering different hypothesis of evolution of the electricity

production mix, demand and production technologies. In this article, we aim to study whether the *information structure* choice when computing *usage values* (used to design policies) has an impact on the simulation of the storage facilities operation in prospectives studies.

1.1 Context

The simulation of an energy system operation is carried out by solving its associated *adequacy resource* problem. Mathematically, the adequacy resource problem is a *multistage optimization* problem that aims to allocate, hour by hour, the production means such that the demand is met while minimizing the overall production cost. The problem is formulated from the point of view of a central planner which takes all decisions in the system. In such case, the production means costs are fixed and resources are allocated by merit order, i.e. the cheapest units are used before the most expensive ones. Since the demand and the availability of the thermal units are uncertain, the adequacy resource problem is *stochastic*. This problem is naturally formulated using two timescales where the planning is done for the week ahead but the hourly energy balance has to be met. When uncertainties are supposed probabilistic independent, it is a strong incentive to break down the annual problem week by week, and therefore to consider, in addition to the hourly (short) timescale, a weekly (long) timescale.

In this context, the question of when the energy in storages is going to be used arises. *Usage values* are the storages' prices — a price signal that makes it possible to choose when and how much of its energy is used — that depend on the energy system setting. We address the usage value computation problem for a uni-nodal system composed of several thermal units, a single storage unit and a residual demand. Stochasticity is introduced by the residual demand of the system (difference between demand and non-dispatchable production) and the availability of the thermal units that are uncertain. We do not focus on how the uncertainty scenarios are generated, but on how the available information about these scenarios is modelled throughout the decision-making process. More precisely, we define the information structure associated with a decision variable to be taken at a given instant t as the quantity of information, that is, which uncertainty variables have been revealed up to time t . The dynamics in the storage introduces a temporal coupling of the problem and, to be able to perform weekly decomposition, we turn to dynamic programming. This technique provides optimal solutions under certain hypothesis of uncertainties' independence. In this case, to perform a weekly decomposition of the yearly problem keeping the optimality, it is that weekly uncertainty sequences be statistically independent between weeks.

Usage values can be obtained from the resolution of the two-timescale stochastic multistage optimization problem (that models the adequacy resource problem), through the Bellman functions resulting from the dynamic programming at the weekly timescale. Once the usage values are calculated, one can compute a storage's management policy to be used in the resolution of the adequacy resource problem when carrying out prospectives studies (simulation).

It is important to highlight that the adequacy resource problem studied here does not

intend to yield an implementable operational schedule for the system, but to give a realistic overview of the system operation. Our aim is to develop a framework that enables to identify possible curtailment in demand or production, network congestions, or the non-satisfaction of greenhouse gas emission targets.

1.2 Structure of the paper

The paper is organized as follows.

In Section 2, we present the physical modelling of the problem. First we introduce the two-timescale timeline, made of hours (because of hourly energy balance constraints) and weeks (because of weekly planning of decisions). Second, we define the physical variables needed to describe the system operation as well as the linking constraints and the economical cost functions.

In Section 3, once the physical model is set, we move on to the mathematical formulation of the adequacy resource problem as a multistage stochastic optimisation problem, focusing on the information structure study. Information structures are used to model the information available at each stage of the decision-making process. In practice, the tools used to carry out prospective studies perform the resolution of a deterministic multistage optimization problem at each stage, anticipating the coming uncertainties. For example, in the open-source tool Antares [6], developed by the French TSO Réseau de Transport d'Electricité (RTE)¹, daily or weekly deterministic problems are solved assuming perfect information. We present the current practice for the information structure modelling when computing the usage values: *weekly hazard-decision*. This structure assumes that all the decisions in a week are made with full knowledge of the uncertainties of the week. For instance, it is the information structure considered in the SDDP algorithm in [8]. When the dispatchable units are “fast” to start, we assume that we can “wait-and-see” the uncertainties to make the decisions, in which case a hazard-decision structure is not far from reality. On the contrary, as “slow” dispatchable units (nuclear or coal) need more time to start producing, “wait-and-see” decisions are not adapted. The suitable structure for this type of unit is *decision-hazard* that considers “here-and-now” decisions as presented in [10] for the hydrothermal scheduling problem.

In Section 4, we present an innovative *decision-hazard-decision* structure that considers both “here-and-now” and “wait-and-see” decisions in the context of prospective studies. To our knowledge, such structure has never been used in prospective studies, but it has been theoretically studied in [4]. The decision-hazard-decision structure is used to solve the adequacy problem with two timescales. The decision stages in the decision-making process are separated into a nonanticipative planning step and a recourse corrective step. As a consequence, the resolution of each stage in the multistage stochastic optimisation problem becomes a two-stage problem in which the first stage decisions (*slow* decisions) are made before knowing the uncertainties, and the second stage or recourse decisions (*fast* decisions) are made once the weekly block of uncertainties is known. It can be interpreted that the

¹<https://www.rte-france.com/>

slow decisions are associated with the unit commitment step and that the fast decisions are associated with the unit modulation. We obtain as a result a problem formulation that improves the information model by being less anticipative but still allows us to apply temporal decomposition methods. Once the new information structure is described, we present the corresponding mathematical formulation of the problem and the associated Bellman equations giving the usage values.

In Section 5, we present numerical results for a case study comparing both information structures: *hazard-decision* and *decision-hazard-decision* in the context of prospective studies. We observe that the choice of the information structure when computing usage values can modify the merit order in the system, that is, the order of the storage usage values with respect to the thermal units prices.

Finally, in Section 6, we conclude on the relevance of the choice of information structures in the computation of usage values, hence on the optimal allocation of resources.

In Appendices A and B we relegate some technical results.

2 Physical and economical model of the energy system

In this Section 2, we describe the physical and economical model of the energy system we consider. In §2.1, we present the timeline with two timescales. In §2.2, we define the variables to model the system. In §2.3, we introduce the system dynamics and energy balance. Finally, in §2.4, we present the cost function modelling.

2.1 Timeline definition

We consider a timeline with a long timescale and a short timescale. The short and long timescales could be any two scales, as long as one is larger than the other. In this work, the long timescale is given by weeks that are represented by a finite totally ordered set (\mathbb{S}, \preceq) , where s^+ is the successor of $s \in \mathbb{S}$ and s^- its predecessor: $\underline{s} \prec \dots \prec s^- \prec s \prec s^+ \prec \dots \prec \bar{s}$ (where \prec is the strict order associated to the order \preceq). Then, $\mathbb{S} = [\underline{s}, \bar{s}]$. The short timescale, hours in this case, is represented by a finite totally ordered set (\mathbb{H}, \preceq) : $\underline{h} \prec \dots \prec h^- \prec h \prec h^+ \prec \dots \prec \bar{h}$. Then, $\mathbb{H} = [\underline{h}, \bar{h}]$.

To unify the timescale we consider the product set $\mathbb{S} \times \mathbb{H}$ ordered as follows:

$$\begin{aligned} (\underline{s}, \underline{h}) \prec \dots \prec (s^-, \bar{h}) \prec (s, \underline{h}) \prec (s, \underline{h}^+) \prec \dots \\ \dots \prec (s, \bar{h}^-) \prec (s, \bar{h}) \prec (s^+, \underline{h}) \prec \dots \prec (\bar{s}, \bar{h}). \end{aligned} \quad (1)$$

We consider a period of one year, and $(\underline{s}, \underline{h})$ is the instant corresponding to the first hour of the first week of the period, and (\bar{s}, \bar{h}) is the last hour of the last week of the period. We need to define an extra time $(\bar{s}^+, \underline{h})$ at its end to handle the resulting state of the last decision. The extended unified timeline $\overline{\mathbb{S}} \times \overline{\mathbb{H}}$ is defined as $\mathbb{S} \times \mathbb{H} \cup \{(\bar{s}^+, \underline{h})\}$.

We define $\llbracket s \rrbracket = ((s, \underline{h}), (s, \underline{h}^+), \dots, (s, \bar{h}))$ and $\llbracket s \rrbracket = ((s, \underline{h}^+), \dots, (s, \bar{h}), (s^+, \underline{h}))$. Thus, we use a simple bracket [or] to denote intervals of the elementary timelines (\mathbb{H}, \preceq) and

(\mathbb{S}, \preceq) . By contrast, we use double brackets \llbracket or \rrbracket for the composite (product) timeline $(\overline{\mathbb{S} \times \mathbb{H}}, \preceq)$.

The different possibilities to index a variable (respectively a function) by time are detailed in Table 1 (respectively in Table 2).

Table 1: Variables notation

| Index | Notation | Description |
|---------------------------|-------------------------------|--|
| (s, h) | $z_{(s,h)}$ | Variable at (s, h) |
| s | z_s | Representative variable for the week s corresponding to the variable at (s, \underline{h}) |
| $\llbracket s \llbracket$ | $z_{\llbracket s \llbracket$ | Sequence of hourly variables given by $(z_{(s,\underline{h})}, z_{(s,\underline{h}^+)}, \dots, z_{(s,\bar{h})})$ |
| $\rrbracket s \rrbracket$ | $z_{\rrbracket s \rrbracket}$ | Sequence of hourly variables given by $(z_{(s,\underline{h}^+)}, \dots, z_{(s,\bar{h})}, z_{(s^+,\underline{h})})$ |

Table 2: Functions notation

| Index | Notation | Description |
|---------------------------|----------------------------------|--|
| (s, h) | $\phi_{(s,h)}$ | Function expression at (s, h) |
| s | ϕ_s | Characteristic aggregation of the hourly functions $\phi_{(s,h)}$ for the week s |
| $\llbracket s \llbracket$ | $\phi_{\llbracket s \llbracket$ | Sequence of hourly functions $\phi_{(s,h)}$ given by $(\phi_{(s,\underline{h})}, \phi_{(s,\underline{h}^+)}, \dots, \phi_{(s,\bar{h})})$ |
| $\rrbracket s \rrbracket$ | $\phi_{\rrbracket s \rrbracket}$ | Sequence of hourly functions $\phi_{(s,h)}$ given by $(\phi_{(s,\underline{h}^+)}, \dots, \phi_{(s,\bar{h})}, \phi_{(s^+,\underline{h})})$ |

The characteristic aggregation in Table 2 could be a sum in $h \in \mathbb{H}$, a composition with respect of the state or a combination of both (see Appendix A for further details).

2.2 Physical variables

The following is a description of the system components. We classify the variables accordingly to their type: decision (in the hand of the decision-maker), uncertainty (exogenous), state (storage) and slack (energy not supplied in the system).

2.2.1 Thermal units modelling

We consider a thermal fleet composed of thermal units whose variables are detailed in Table 3 for the units indexed by $i \in \mathbb{I}$. The ‘‘On’’ decision is associated with $y^{\text{th},i} = 1$ and the ‘‘Off’’

Table 3: Thermal units variables

| Description | Type | Notation |
|--------------|-------------|--|
| On/Off | Decision | $y^{\text{th},i} \in \{0, 1\}$ |
| Modulation | Decision | $\theta^i \in \{0\} \cup [\underline{\theta}^i, \bar{\theta}^i]$ |
| Availability | Uncertainty | $w^{\text{th},i} \in \{0, 1\}$ |

decision with $y^{\text{th},i} = 0$. Observe that the decision $y^{\text{th},i}$ is taken at each hour and does not represent a change in the state of the unit. The decision θ^i denotes the power modulation once the unit is on. The availability of the thermal units is modelled with the (uncertainty) variable $w^{\text{th},i}$: when it is equal to 0 the unit is not available to use, and when it is equal to 1 the unit is available to use.

The collections of on/off decisions, modulation decisions and availabilities variables of all thermal units are denoted by $y^{\text{th}} = (y^{\text{th},i})_{i \in \mathbb{I}}$, $\theta = (\theta^i)_{i \in \mathbb{I}}$ and $w^{\text{th}} = (w^{\text{th},i})_{i \in \mathbb{I}}$ respectively.

2.2.2 Storage modelling

In Table 4, we introduce the variables related to the storage management. We consider different variables for pumping (r^{p}) and turbinning (r^{t}) decisions so that we take into account the pumping efficiency in the storage. The variable q denotes the level of stock in the reservoir, that is, the physical state of the reservoir.

Table 4: Storage variables

| Description | Type | Notation |
|----------------|----------------|--|
| Pumping | Decision | $r^{\text{p}} \in [0, \bar{r}^{\text{p}}]$ |
| Turbinning | Decision | $r^{\text{t}} \in [0, \bar{r}^{\text{t}}]$ |
| Level of stock | Physical state | $q \in [\underline{q}, \bar{q}]$ |

2.2.3 Residual demand modelling

The *residual demand* (w^{d}) is the difference between demand and non-dispatchable production. This allows to group in one variable several uncertainty sources such as the wind production, the solar production, the demand, etc. The variable w^{d} is classified as uncertainty.

2.2.4 Slack variables

We introduce a variable ψ^{ens} , classed as *slack*, to model the *energy not supplied* in the system. This variable will appear in the forthcoming energy balance (6) and cost function (7).

2.3 System dynamics and energy balance

We present now the linking constraints between variables.

2.3.1 Storage dynamics

The dynamics function

$$f(q, r^{\text{p}}, r^{\text{t}}) = q + \eta r^{\text{p}} - r^{\text{t}} \quad (2)$$

describes the evolution of the level of stock — as a function of the current level of stock q and the pumping r^{p} and turbinning r^{t} decisions — from one short time stage to the next. The parameter $\eta \in [0, 1]$ is the pumping efficiency of the storage.

The hourly dynamics f induces a weekly temporal coupling linking the stock's level at the beginning of a week s with the stock level at the beginning of the following week s^+ . Therefore, we also consider a weekly dynamics f_s given by the hourly composition of the dynamics in (2). The composition is detailed in Appendix A. It gives, as a result, the level of stock $q_{(s^+, h)}$ at the beginning of the following week, by summing the total difference between pumping (positive taking into account its efficiency) and turbinning (negative) during the week to the stock's level at the beginning of the current week $q_{(s, h)}$.

2.3.2 Thermal units' production output

The effective output of the unit is constrained by its availability and the on/off decision. More precisely, the output production $\widehat{\theta}^i$ of the i -th thermal unit not only depends on the decision θ^i but also on the on/off decision $y^{\text{th}, i}$ and on the availability uncertainty $w^{\text{th}, i}$ as follows

$$\widehat{\theta}^i(y^{\text{th}, i}, \theta^i, w^{\text{th}, i}) = \theta^i \times \min\{y^{\text{th}, i}, w^{\text{th}, i}\}. \quad (3)$$

As defined, the production of the i -th unit belongs to the same set as θ^i , that is,

$$\widehat{\theta}^i(y^{\text{th}, i}, \theta^i, w^{\text{th}, i}) \in \{0\} \cup [\underline{\theta}^i, \bar{\theta}^i], \quad (4)$$

taking the value zero whenever the off decision is made ($y^{\text{th}, i} = 0$) or the unit is not available ($w^{\text{th}, i} = 0$). The collection of output production of all thermal units is denoted by $\widehat{\theta} = (\widehat{\theta}^i)_{i \in \mathbb{I}}$.

2.3.3 Balance equation

Satisfying the energy balance equation is the main goal of the adequacy resource problem. That is, at every hour, the total energy production in the system should be equal to the total energy consumed in the system (which includes the pumping). Ideally, the energy balance constraint is written as the equality

$$\underbrace{r^t + \sum_{i \in \mathbb{I}} \widehat{\theta}^i}_{\text{total production}} + \underbrace{\psi^{\text{ens}}}_{\text{energy not supplied}} = \underbrace{r^p + w^d}_{\text{pumping demand + residual demand}} . \quad (5)$$

We rather formulate this balance equation as an inequality constraint to avoid infeasibility problems due to the thermal units' minimum power constraints: indeed, infeasibility could happen when less energy is required than the minimum power of the last (or more expensive) unit on to meet the demand. If the cause of the balance infeasibility is the lack of available production, the slack variable ψ^{ens} will take positive values, measuring how far from meeting the demand the system is. Thus, the hourly balance equation is given by

$$g(r^t, r^p, \widehat{\theta}, w^d, \psi^{\text{ens}}) \geq 0 , \quad (6a)$$

$$\text{with } g(r^t, r^p, \widehat{\theta}, w^d, \psi^{\text{ens}}) = \left(r^t + \sum_{i \in \mathbb{I}} \widehat{\theta}^i + \psi^{\text{ens}} \right) - (r^p + w^d) . \quad (6b)$$

2.4 Cost functions

The cost of meeting the demand is the operating cost of the thermal units. In addition, we model the penalization on the energy not supplied as a cost. This penalization cost is much higher than the thermal units' cost to ensure that the energy demand is not provided only in cases where there is no other solution.

We model the thermal cost with two components for each unit. The first one is associated with decision of switching on a unit, that is, when at one hour $y^{\text{th},i} = 0$ and in the following hour $y^{\text{th},i} = 1$. The second component corresponds to the variable cost, and is proportional to the power modulation θ^i of the unit.

We now introduce the hourly cost function $\ell(y^{\text{th}}, z^{\text{th}}, \widehat{\theta}, w^{\text{th}}, \psi^{\text{ens}})$ as

$$\begin{aligned} \ell(y^{\text{th}}, z^{\text{th}}, \widehat{\theta}, w^{\text{th}}, \psi^{\text{ens}}) = & \sum_{i \in \mathbb{I}} \left(\text{sc}^i \times \max \{ y^{\text{th},i} - z^{\text{th},i}, 0 \} + \text{vc}^i \times \widehat{\theta}^i \right) \\ & + \text{pc} \times \psi^{\text{ens}} , \end{aligned} \quad (7)$$

The parameters sc^i and vc^i correspond to the unit's start-up and modulation costs and pc is the penalization parameter for the not supplied energy in the system. The variable z^{th} is introduced to take into account a temporal shift of the on/off decision $y^{\text{th},i}$ since the start up cost is associated with a change from off to on between two consecutive hours (see Tables 5 and 6).

The weekly cost ℓ_s will be defined as the sum of the hourly costs ℓ within the week in Tables 5 and 6.

Up to now, we have introduced the physical and economical modelling of the problem. In Section 3, we will present the current practice for the adequacy problem mathematical formulation to compute usage values focusing on the information structure modelling.

3 Current information modelling in weekly hazard-decision

When modelling a stochastic multistage optimization problem, it is necessary to define the information structure, that is, a model that describes the information available at each stage of the decision-making process.

In §3.1, we introduce notations for random variables and measurability constraints. In §3.2, we present the weekly hazard-decision information structure. In §3.3, we formulate the multistage stochastic optimization problem. In §3.4, we deduce the corresponding Bellman equations in hazard-decision.

3.1 Notation for random variables and measurability constraints

To formulate the multistage stochastic optimization problem, we model the uncertainties as random variables and, as a consequence, the states and controls are random variables as well. For this purpose, we consider a probability space $(\Omega, \mathcal{F}, \mathbb{P})$. Random variables are denoted by bold letters like \mathbf{Z} . The information structures are mathematically modelled as measurability constraints. We say that \mathbf{Z}_1 is measurable with respect to \mathbf{Z}_2 if $\sigma(\mathbf{Z}_1) \subset \sigma(\mathbf{Z}_2)$, that is, the σ -field generated by \mathbf{Z}_1 is included in (less rich than) the σ -field generated by \mathbf{Z}_2 . Under technical assumptions in Doob Theorem [5], measurability constraints are equivalently expressed by means of functions as follows

$$\begin{aligned} \sigma(\mathbf{Z}_1) \subset \sigma(\mathbf{Z}_2) &\iff \exists \varphi : (\mathcal{Z}_2, \mathcal{Z}_2) \rightarrow (\mathcal{Z}_1, \mathcal{Z}_1) \text{ measurable} \\ &\text{s.t. } \mathbf{Z}_1 = \varphi(\mathbf{Z}_2) . \end{aligned} \tag{8}$$

Practically, the function φ is what we call policy or strategy, when \mathbf{Z}_2 represents the information disclosed when making a decision.

3.2 Weekly hazard-decision information structure (HD)

In this section, we consider information structures that assume a *weekly* disclosure of the information: once the uncertainty of the first hour of the week is known, the whole collection of uncertainties for the week is also known. The weekly vectors presented hereafter follow the notation given in Table 1. Let $w_{\llbracket s \rrbracket} = (w_{\llbracket s \rrbracket}^d, w_{\llbracket s \rrbracket}^{\text{th}})$ be the vector composed of the weekly demand and weekly availabilities. In the same way, the vector composed of the collection of weekly controls is denoted by $v_{\llbracket s \rrbracket} = (y_{\llbracket s \rrbracket}^{\text{th}}, \theta_{\llbracket s \rrbracket}, r_{\llbracket s \rrbracket}^p, r_{\llbracket s \rrbracket}^t, \psi_{\llbracket s \rrbracket}^{\text{ens}})$.

The current practice to model the information structure is a *hazard-decision* (HD) structure in the weekly timescale as illustrated in Fig. 1.

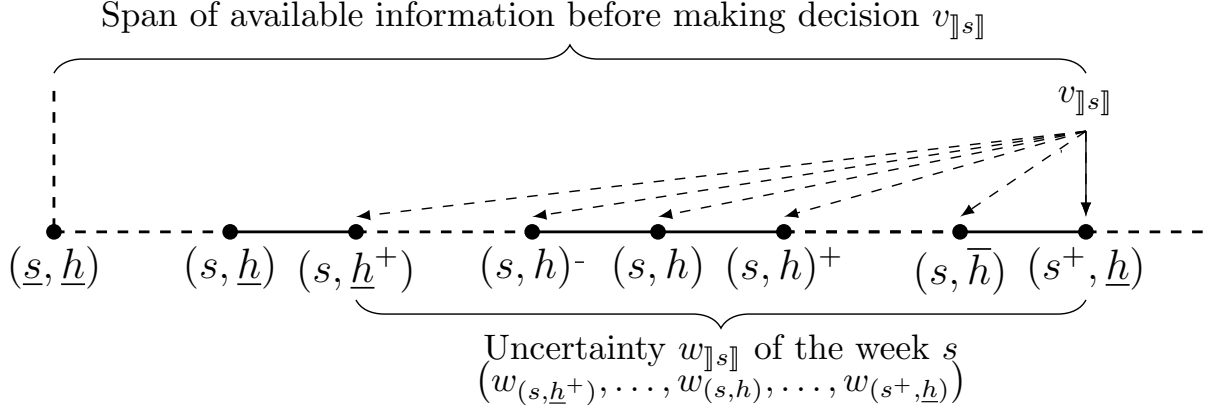


Figure 1: Weekly hazard-decision information structure

The collection $v_{[s]} = (v_{(s, \underline{h}^+)}, \dots, v_{(s^+, \underline{h})})$ of hourly decisions for the week is made once the block $w_{[s]} = (w_{(s, \underline{h}^+)}, \dots, w_{(s^+, \underline{h})})$ of uncertainties for the week is disclosed. In other words, when making the decisions for any hour of the week, the demand and the availability of thermal units for every hour in the week are already known (in advance). We can also interpret this structure as if the hourly decisions can wait until knowing all the uncertainties of the week to be made. In this context, all the hourly decisions within the week are anticipative since, when making them, the uncertainties until the end of the week are already known.

Finally, for the weekly hazard-decision structure, the information constraint is represented by the following measurability constraints

$$\sigma(\mathbf{V}_{[s]}) \subset \sigma(\mathbf{W}_{(s, h)}, \mathbf{W}_{[s]}, \dots, \mathbf{W}_{[s]}) , \quad \forall s \in \mathbb{S} . \quad (9)$$

3.3 Multistage stochastic optimization problem formulation

In Section 2, we have presented the physical and economical model of the energy system. With the new notation in §3.2, we present in Table 5 the corresponding expressions and their compact mathematical versions.

The weekly cost function ℓ_s is obtained as the sum of the hourly cost functions within the week. When computing the hourly cost at (s, h) with Equation (7), the variable z^{th} correspond to the on/off decision $y_{(s, h)^-}^{\text{th}}$ at the previous hour. For simplicity, we neglect the temporal coupling of thermal units from the last hour of one week to the first hour of the next week. This explains the presence of a zero as second entry in the first term in the expression of the weekly cost in the last line of Table 5.

Considering the definitions in Table 5, we formulate the physical adequacy problem as a stochastic multistage optimization problem using the weekly hazard-decision information

Table 5: Correspondence between mathematical and physical and economical notations in the weekly hazard-decision framework

| Function | Notation | |
|--------------------|--|--|
| | Mathematical | Physical-Economical |
| Storage dynamics | $\mathcal{F}_s(q_{(s,\underline{h})}, v_{\llbracket s \rrbracket})$ | $f_s(q_{(s,\underline{h})}, r_{\llbracket s \rrbracket}^p, r_{\llbracket s \rrbracket}^t)$ |
| Thermal production | $\widehat{\Theta}_{\llbracket s \rrbracket}(w_{\llbracket s \rrbracket}, v_{\llbracket s \rrbracket})$ | $\left\{ \widehat{\theta}_{(s,h)+} \left(y_{(s,h)+}^{\text{th}}, \theta_{(s,h)+}, w_{(s,h)+}^{\text{th}} \right) \right\}_{h \in \mathbb{H}}$ |
| Energy balance | $\mathcal{G}_{\llbracket s \rrbracket}(w_{\llbracket s \rrbracket}, v_{\llbracket s \rrbracket})$ | $\left\{ g \left(r_{(s,h)+}^t, r_{(s,h)+}^p, \widehat{\theta}_{(s,h)+} \left(y_{(s,h)+}^{\text{th}}, \theta_{(s,h)+}, w_{(s,h)+}^{\text{th}} \right), w_{(s,h)+}^d, \psi_{(s,h)+}^{\text{ens}} \right) \right\}_{h \in \mathbb{H}}$ |
| Weekly cost | $\mathcal{L}_s(w_{\llbracket s \rrbracket}, v_{\llbracket s \rrbracket})$ | $\ell_s \left(y_{\llbracket s \rrbracket}^{\text{th}}, \widehat{\theta}_{\llbracket s \rrbracket}, w_{\llbracket s \rrbracket}^{\text{th}}, \psi_{\llbracket s \rrbracket}^{\text{ens}} \right) = \ell \left(y_{(s,\underline{h})+}^{\text{th}}, 0, \widehat{\theta}_{(s,\underline{h})+}, w_{(s,\underline{h})+}^{\text{th}}, \psi_{(s,\underline{h})+}^{\text{ens}} \right) + \sum_{h \in \mathbb{H} \setminus \{\underline{h}\}} \ell \left(y_{(s,h)+}^{\text{th}}, y_{(s,h)+}^{\text{th}}, \widehat{\theta}_{(s,h)+}, w_{(s,h)+}^{\text{th}}, \psi_{(s,h)+}^{\text{ens}} \right)$ |

structure:

$$\min_{\mathbf{Q}, \mathbf{V}} \mathbb{E} \left[\sum_{s \in \mathbb{S}} \mathcal{L}_s(\mathbf{Q}_{(s,\underline{h})}, \mathbf{V}_{\llbracket s \rrbracket}, \mathbf{W}_{\llbracket s \rrbracket}) + K(\mathbf{Q}_{(\bar{s}^+, \underline{h})}) \right] \quad (10a)$$

such that , $\forall s \in \mathbb{S}$

$$\mathbf{Q}_{(\underline{s}, \underline{h})} = \mathbf{W}_{(\underline{s}, \underline{h})} , \quad (10b)$$

$$\mathbf{Q}_{(s^+, \underline{h})} = \mathcal{F}_s(\mathbf{Q}_{(s,\underline{h})}, \mathbf{V}_{\llbracket s \rrbracket}) , \quad (10c)$$

$$\mathcal{G}_{\llbracket s \rrbracket}(\mathbf{V}_{\llbracket s \rrbracket}, \mathbf{W}_{\llbracket s \rrbracket}) \geq \mathbf{0} , \quad (10d)$$

$$\sigma(\mathbf{V}_{\llbracket s \rrbracket}) \subset \sigma(\mathbf{W}_{(\underline{s}, \underline{h})}, \mathbf{W}_{\llbracket s \rrbracket}, \dots, \mathbf{W}_{\llbracket s^- \rrbracket}, \mathbf{W}_{\llbracket s \rrbracket}) . \quad (10e)$$

The final cost $K(\mathbf{Q}_{(\bar{s}^+, \underline{h})})$ is used to give value to the energy in the storage at the end of the yearly period.

3.4 Bellman equations in hazard-decision

Defining the weekly state $x_s = q_{(s,h)}$ (level of stock in the storage at the beginning of the week), we write the weekly Bellman equations (see [1] for further details)

$$B_{\bar{s}^+}^{\text{HD}}(x_{\bar{s}^+}) = K(x_{\bar{s}^+}), \quad (11a)$$

$$B_s^{\text{HD}}(x_s) = \mathbb{E} \left[\min_{v_{\llbracket s \rrbracket}} \mathcal{L}_s(x_s, v_{\llbracket s \rrbracket}, \mathbf{W}_{\llbracket s \rrbracket}) + B_{s^+}^{\text{HD}}(\mathcal{F}_s(x_s, v_{\llbracket s \rrbracket})) \right], \quad (11b)$$

where the minimum inside Equation(11b) is computed subject to the constraint (10d).

If the sequence $(\mathbf{W}_{\llbracket s \rrbracket}, \dots, \mathbf{W}_{\llbracket s \rrbracket}, \dots, \mathbf{W}_{\llbracket \bar{s} \rrbracket})$ of uncertainties is weekly independent, the weekly Bellman equations provide an optimal solution for Problem (10). We highlight that, to get the optimal solution, the hourly uncertainties $\mathbf{W}_{\llbracket s \rrbracket} = (\mathbf{W}_{(s,h^+)}, \dots, \mathbf{W}_{(s,h)^+}, \dots, \mathbf{W}_{(s^+,h)})$ within the week do not need to be assumed to be independent (from one hour to another).

It is well known that, for all $s \in \mathbb{S}$, the function B_s^{HD} satisfies

$$B_s^{\text{HD}}(x_s) = \min_{\mathbf{V}_{\llbracket s \rrbracket}, \dots, \mathbf{V}_{\llbracket \bar{s} \rrbracket}} \mathbb{E} \left[\sum_{s'=s}^{\bar{s}} \mathcal{L}_{s'}(\mathbf{Q}_{(s',h)}, \mathbf{V}_{\llbracket s' \rrbracket}, \mathbf{W}_{\llbracket s' \rrbracket}) + K(\mathbf{Q}_{(\bar{s}^+,h)}) \right] \quad (12a)$$

$$\text{such that } , \forall s' \in [s, \dots, \bar{s}]$$

$$\mathbf{Q}_{(s',h)} = x_s, \quad (12b)$$

$$\mathbf{Q}_{(s'+,h)} = \mathcal{F}_{s'}(\mathbf{Q}_{(s',h)}, \mathbf{V}_{\llbracket s' \rrbracket}), \quad (12c)$$

$$\mathcal{G}_{\llbracket s' \rrbracket}(\mathbf{V}_{\llbracket s' \rrbracket}, \mathbf{W}_{\llbracket s' \rrbracket}) \geq \mathbf{0}, \quad (12d)$$

$$\sigma(\mathbf{V}_{\llbracket s' \rrbracket}) \subset \sigma(\mathbf{W}_{(s,h)}, \mathbf{W}_{\llbracket s \rrbracket}, \dots, \mathbf{W}_{\llbracket s'-1 \rrbracket}, \mathbf{W}_{\llbracket s' \rrbracket}), \quad (12e)$$

so that the value $B_s^{\text{HD}}(x_s)$ of the Bellman function is interpreted as the future optimal cost when, at week s , the level of stock is x_s and the weekly hazard-decision information structure is considered.

The need for a new information structure arises out of the fact that the current approach is fully anticipative in the week, as illustrated in the minimum inside the expectation on the right hand side of the Bellman equation (11b). When assuming that all the uncertainties for the week are known at the moment of making a decision, we implicitly suppose that all decisions are flexible and can wait until knowing the uncertainties to be made. But it is known that certain on/off decisions cannot be made instantaneously and need to be planned in advance. For this purpose, we introduce a new information structure in Section 4.

4 New information modelling in weekly decision-hazard-decision

In the new weekly *decision-hazard-decision* (DHD) information structure, there are decisions that cannot be modelled as (weekly) anticipative.

We classify the decisions in the system modelling between:

- *planning or decision-hazard* (here-and-now) decisions: denoted by \tilde{u} ,
- *recourse or hazard-decision* (wait-and-see) decisions: denoted by \tilde{v}

Ideally, we should consider an information structure in which at the beginning of the week the planning for the nonanticipative decisions $\tilde{u}_{\llbracket s \rrbracket}$ is made knowing only the past uncertainties, that is, the uncertainties up to (s, \underline{h}) . Then, when the uncertainties $w_{(s,h)}$ begin to be disclosed hour by hour, there are so-called hourly recourse decisions $\tilde{v}_{(s,h)}$ that are also made hour by hour knowing the uncertainties up to (s, h) . Such information structure is illustrated in Fig. 2.

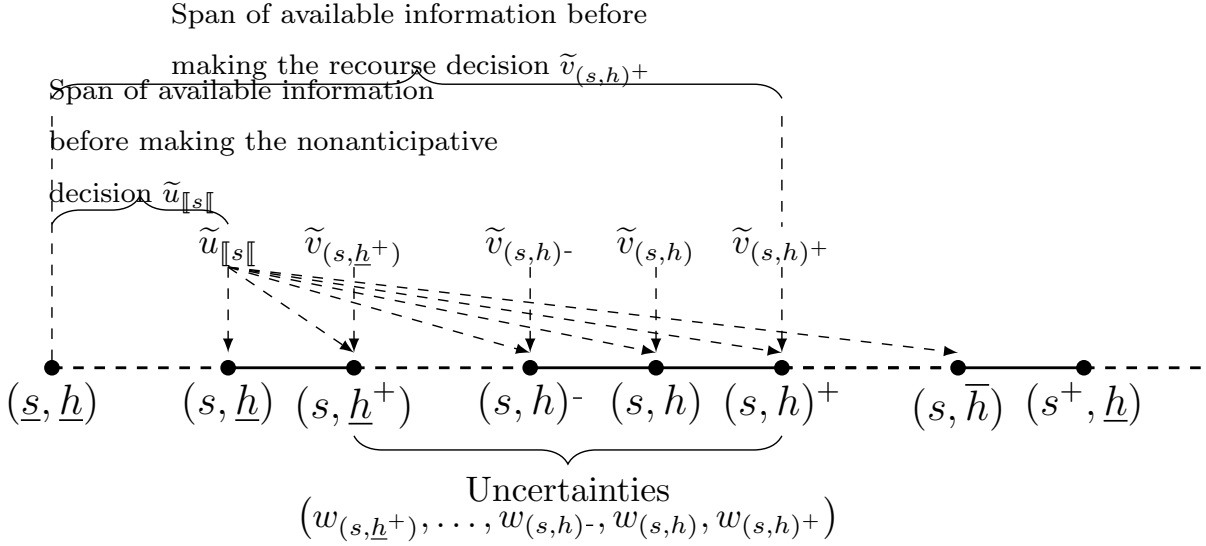


Figure 2: Decision-hazard-decision information structure with hourly recourse

This structure leads to Bellman equations (see Appendix B) with 168 nested mathematical expectations and minimizations, hence out of reach to compute. This is why we opt for a compromise solution, detailed in this section.

Remark 1 *In real operation it is possible to adjust the production “in real time” to meet demand, that is, there is some anticipativity at the fast timescale for the hourly recourse decision $\tilde{v}_{(s,h)}$. Therefore, it is not unrealistic to consider the ideal model described above. Note that, it is not possible to consider a fully nonanticipative (or decision-hazard) structure, in which the hourly recourse decisions $\tilde{v}_{(s,h)}$ are made knowing the uncertainties up to $(s, h)^-$ (instead of (s, h)), because it would lead to an excessive usage of the slack variable ψ^{ens} in the energy balance equation (6).*

In §4.1, we present the weekly decision-hazard-decision information structure. In §4.2, we formulate the multistage stochastic optimization problem. In §4.3, we give the corresponding Bellman equations in decision-hazard-decision. In §4.4, we compare Bellman function according to the underlying information structures, namely HD and DHD.

4.1 Weekly decision-hazard-decision information structure (DHD)

As shown in Fig. 3, the planning decisions $\tilde{u}_{\llbracket s \rrbracket}$ are made before knowing the uncertainties $w_{\llbracket s \rrbracket}$ for the week (knowing only the past uncertainties); then, the weekly block of uncertainties is disclosed, and the corrective actions are made, that is, the recourse controls $\tilde{v}_{\llbracket s \rrbracket}$.

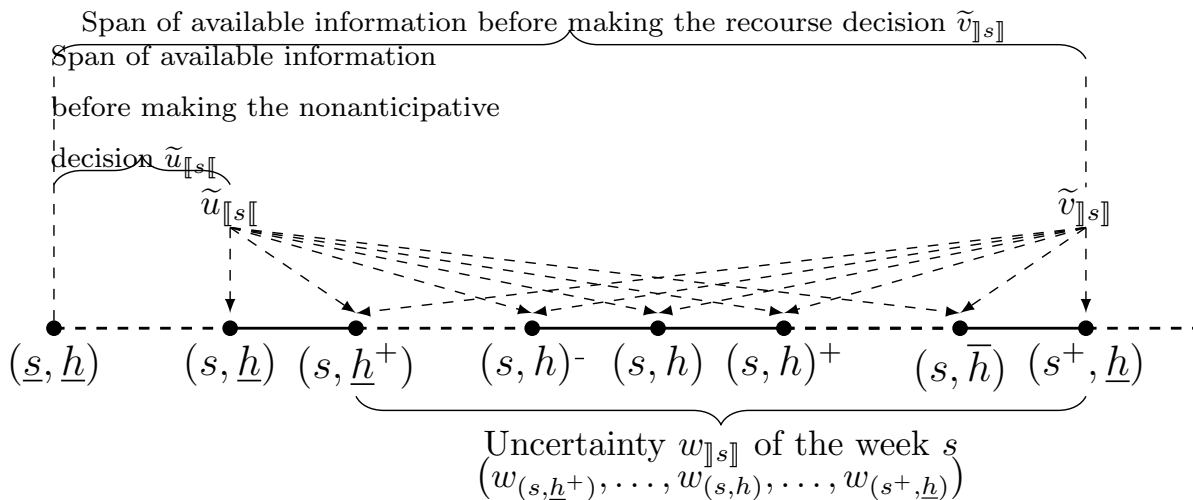


Figure 3: Weekly decision-hazard-decision information structure

Since the recourse decisions for the beginning of the week are made knowing the uncertainties for the whole week, they are anticipative.

For the weekly decision-hazard-decision structure, the information constraints are given by the σ -fields inclusions

$$\sigma(\tilde{\mathbf{U}}_{\llbracket s \rrbracket}) \subset \sigma(\mathbf{W}_{(s, \underline{h})}, \mathbf{W}_{\llbracket s \rrbracket}, \dots, \mathbf{W}_{\llbracket s-1 \rrbracket}), \quad \forall s \in \mathbb{S}, \quad (13a)$$

$$\sigma(\tilde{\mathbf{V}}_{\llbracket s \rrbracket}) \subset \sigma(\mathbf{W}_{(s, \underline{h})}, \mathbf{W}_{\llbracket s \rrbracket}, \dots, \mathbf{W}_{\llbracket s-1 \rrbracket}, \mathbf{W}_{\llbracket s \rrbracket}), \quad \forall s \in \mathbb{S}. \quad (13b)$$

4.2 Multistage stochastic optimization problem formulation

Now we classify the physical controls described in §2.2 into planning controls $\tilde{u}_{\llbracket s \rrbracket}$ and recourse controls $\tilde{v}_{\llbracket s \rrbracket}$ as follows

$$\tilde{u}_{\llbracket s \rrbracket} = y_{\llbracket s \rrbracket}^{\text{th,slow}} \quad (14a)$$

$$\tilde{v}_{\llbracket s \rrbracket} = (y_{\llbracket s \rrbracket}^{\text{th,fast}}, \theta_{\llbracket s \rrbracket}, r_{\llbracket s \rrbracket}^{\text{p}}, r_{\llbracket s \rrbracket}^{\text{t}}, \psi_{\llbracket s \rrbracket}^{\text{ens}}). \quad (14b)$$

We model as planning decisions the on and off decisions for the “slow” thermal units, and as recourse decisions all the remaining ones. We consider this classification to, in a way, model the rigidity of some thermal units, i.e. to model the fact that they cannot be switched on instantaneously. Now the collection of on/off decisions for the thermals is composed of $y^{\text{th}} = (y^{\text{th,slow}}, y^{\text{th,fast}})$.

In Section 2, we have presented the physical and economical model of the energy system. With the new notation in §4.2, we present in Table 6 the corresponding expressions and their compact mathematical versions. As already explained in §3.3, we neglect the temporal

Table 6: Correspondence between mathematical and physical and economical notations in the weekly decision-hazard-decision framework

| Function | Notation | |
|--------------------|--|---|
| | Mathematical | Physical-Economical |
| Storage dynamics | $\tilde{\mathcal{F}}_s(q_{(s,\underline{h})}, \tilde{v}_{\llbracket s \rrbracket})$ | $f_s(q_{(s,\underline{h})}, r_{\llbracket s \rrbracket}^p, r_{\llbracket s \rrbracket}^t)$ |
| Thermal production | $\tilde{\Theta}_{\llbracket s \rrbracket}(\tilde{u}_{\llbracket s \rrbracket}, w_{\llbracket s \rrbracket}, \tilde{v}_{\llbracket s \rrbracket})$ | $\left\{ \hat{\theta}_{(s,h)+} \left(\left(y_{(s,h)}^{\text{th,slow}}, y_{(s,h)+}^{\text{th,fast}} \right), \theta_{(s,h)+}, w_{(s,h)+}^{\text{th}} \right) \right\}_{h \in \mathbb{H}}$ |
| Energy balance | $\tilde{\mathcal{G}}_{\llbracket s \rrbracket}(\tilde{u}_{\llbracket s \rrbracket}, w_{\llbracket s \rrbracket}, \tilde{v}_{\llbracket s \rrbracket})$ | $\left\{ g \left(r_{(s,h)+}^t, r_{(s,h)+}^p, \hat{\theta}_{(s,h)+} \left(\left(y_{(s,h)}^{\text{th,slow}}, y_{(s,h)+}^{\text{th,fast}} \right), \theta_{(s,h)+}, w_{(s,h)+}^{\text{th}} \right), w_{(s,h)+}^d, \psi_{(s,h)+}^{\text{ens}} \right) \right\}_{h \in \mathbb{H}}$ |
| Weekly cost | $\tilde{\mathcal{L}}_s(\tilde{u}_{\llbracket s \rrbracket}, w_{\llbracket s \rrbracket}, \tilde{v}_{\llbracket s \rrbracket})$ | $\ell_s \left(\left(y_{\llbracket s \rrbracket}^{\text{th,slow}}, y_{\llbracket s \rrbracket}^{\text{th,fast}} \right), \theta_{\llbracket s \rrbracket}, w_{\llbracket s \rrbracket}^{\text{th}}, \psi_{\llbracket s \rrbracket}^{\text{ens}} \right) =$ $\ell \left(\left(y_{(s,\underline{h})}^{\text{th,slow}}, y_{(s,\underline{h})+}^{\text{th,fast}} \right), 0, \hat{\theta}_{(s,\underline{h})+}, w_{(s,\underline{h})+}^{\text{th}}, \psi_{(s,\underline{h})+}^{\text{ens}} \right) +$ $\sum_{h \in \mathbb{H} \setminus \{\underline{h}\}} \ell \left(\left(y_{(s,h)}^{\text{th,slow}}, y_{(s,h)+}^{\text{th,fast}} \right), \left(y_{(s,h)-}^{\text{th,slow}}, y_{(s,h)}^{\text{th,fast}} \right) + \right.$ $\left. \hat{\theta}_{(s,h)+}, w_{(s,h)+}^{\text{th}}, \psi_{(s,h)+}^{\text{ens}} \right)$ |

coupling of thermal units from the last hour of one week to the first hour of the next week. Considering the definitions in Table 6, we formulate the physical adequacy problem as a stochastic multistage optimization problem using the weekly decision-hazard-decision information structure:

$$\min_{\mathbf{Q}, \tilde{\mathbf{U}}, \tilde{\mathbf{V}}} \mathbb{E} \left[\sum_{s \in \mathbb{S}} \tilde{\mathcal{L}}_s(\mathbf{Q}_{(s,\underline{h})}, \tilde{\mathbf{U}}_{\llbracket s \rrbracket}, \mathbf{W}_{\llbracket s \rrbracket}, \tilde{\mathbf{V}}_{\llbracket s \rrbracket}) + K(\mathbf{Q}_{(\bar{s}^+, \underline{h})}) \right] \quad (15a)$$

such that , $\forall s \in \mathbb{S}$

$$\mathbf{Q}_{(s,\underline{h})} = \mathbf{W}_{(s,\underline{h})}, \quad (15b)$$

$$\mathbf{Q}_{(s^+, \underline{h})} = \tilde{\mathcal{F}}_s(\mathbf{Q}_{(s,\underline{h})}, \tilde{\mathbf{V}}_{\llbracket s \rrbracket}), \quad (15c)$$

$$\tilde{\mathcal{G}}_{\llbracket s \rrbracket}(\tilde{\mathbf{U}}_{\llbracket s \rrbracket}, \mathbf{W}_{\llbracket s \rrbracket}, \tilde{\mathbf{V}}_{\llbracket s \rrbracket}) \geq \mathbf{0}, \quad (15d)$$

$$\sigma(\tilde{\mathbf{U}}_{\llbracket s \rrbracket}) \subset \sigma(\mathbf{W}_{(s,\underline{h})}, \mathbf{W}_{\llbracket s \rrbracket}, \dots, \mathbf{W}_{\llbracket s \rrbracket}), \quad (15e)$$

$$\sigma(\tilde{\mathbf{V}}_{\llbracket s \rrbracket}) \subset \sigma(\mathbf{W}_{(s,\underline{h})}, \mathbf{W}_{\llbracket s \rrbracket}, \dots, \mathbf{W}_{\llbracket s \rrbracket}, \mathbf{W}_{\llbracket s \rrbracket}). \quad (15f)$$

The final cost $K(\mathbf{Q}_{(\bar{s}^+, \underline{h})})$ is used to give value to the energy in the storage at the end of the yearly period.

4.3 Bellman equations in decision-hazard-decision

Defining the weekly state $x_s = q_{(s, \underline{h})}$, that is, the level of stock in the storage at the beginning of the week, we write the weekly Bellman equations (16) using [4, Proposition 13]

$$B_{\bar{s}^+}^{\text{DHD}}(x_{\bar{s}^+}) = K(x_{\bar{s}^+}) , \quad (16a)$$

$$B_s^{\text{DHD}}(x_s) = \min_{\tilde{u}_{\llbracket s \rrbracket}} \mathbb{E} \left[\min_{\tilde{v}_{\llbracket s \rrbracket}} \tilde{\mathcal{L}}_s(x_s, \tilde{u}_{\llbracket s \rrbracket}, \mathbf{W}_{\llbracket s \rrbracket}, \tilde{v}_{\llbracket s \rrbracket}) + B_{s^+}^{\text{DHD}}(\tilde{\mathcal{F}}_s(x_s, \tilde{v}_{\llbracket s \rrbracket})) \right] , \quad (16b)$$

where the minimum inside the expectation term of Equation (16b) is computed subject to the constraint (15d).

If the sequence $(\mathbf{W}_{\llbracket s \rrbracket}, \dots, \mathbf{W}_{\llbracket s \rrbracket}, \dots, \mathbf{W}_{\llbracket \bar{s} \rrbracket})$ of uncertainties is weekly independent, the weekly Bellman equations provide an optimal solution for Problem (15). We highlight that, to get the optimal solution, the hourly uncertainties $\mathbf{W}_{\llbracket s \rrbracket} = (\mathbf{W}_{(s, \underline{h}^+)}, \dots, \mathbf{W}_{(s, h^+)}, \dots, \mathbf{W}_{(s^+, \underline{h})})$ within the week do not need to be assumed to be independent (from one hour to another).

It is well known that, for all $s \in \mathbb{S}$, the function B_s^{DHD} satisfies

$$B_s^{\text{DHD}}(x_s) = \min_{\substack{\tilde{\mathbf{U}}_{\llbracket s \rrbracket}, \dots, \tilde{\mathbf{U}}_{\llbracket \bar{s} \rrbracket} \\ \tilde{\mathbf{V}}_{\llbracket s \rrbracket}, \dots, \tilde{\mathbf{V}}_{\llbracket \bar{s} \rrbracket}}} \mathbb{E} \left[\sum_{s'=s}^{\bar{s}} \mathcal{L}_{s'}(\mathbf{Q}_{(s', \underline{h})}, \tilde{\mathbf{U}}_{\llbracket s' \rrbracket}, \mathbf{W}_{\llbracket s' \rrbracket}, \tilde{\mathbf{V}}_{\llbracket s' \rrbracket}) + K(\mathbf{Q}_{(\bar{s}^+, \underline{h})}) \right] \quad (17a)$$

such that , $\forall s' \in [s, \dots, \bar{s}]$

$$\mathbf{Q}_{(s', \underline{h})} = x_s , \quad (17b)$$

$$\mathbf{Q}_{(s'+, \underline{h})} = \mathcal{F}_{s'}(\mathbf{Q}_{(s', \underline{h})}, \tilde{\mathbf{V}}_{\llbracket s' \rrbracket}) , \quad (17c)$$

$$\mathcal{G}_{\llbracket s' \rrbracket}(\tilde{\mathbf{U}}_{\llbracket s' \rrbracket}, \mathbf{W}_{\llbracket s' \rrbracket}, \tilde{\mathbf{V}}_{\llbracket s' \rrbracket}) \geq \mathbf{0} , \quad (17d)$$

$$\sigma(\tilde{\mathbf{U}}_{\llbracket s' \rrbracket}) \subset \sigma(\mathbf{W}_{(\underline{s}, \underline{h})}, \mathbf{W}_{\llbracket s \rrbracket}, \dots, \mathbf{W}_{\llbracket s'- \rrbracket}) , \quad (17e)$$

$$\sigma(\tilde{\mathbf{V}}_{\llbracket s' \rrbracket}) \subset \sigma(\mathbf{W}_{(\underline{s}, \underline{h})}, \mathbf{W}_{\llbracket s \rrbracket}, \dots, \mathbf{W}_{\llbracket s'- \rrbracket}, \mathbf{W}_{\llbracket s' \rrbracket}) , \quad (17f)$$

so that, the value $B_s^{\text{DHD}}(x_s)$ of Bellman function is interpreted as the future optimal cost when at week s the level of stock is x_s and the weekly decision-hazard-decision information structure is considered.

4.4 Theoretical comparison between Bellman functions in HD and DHD

When we compare theoretically the Bellman functions B_s^{HD} and B_s^{DHD} , given by the Bellman equations (11) and (16), we observe that the weekly hazard-decision approach is a relaxation

of the weekly decision-hazard-decision approach with respect to the information constraint, as sketched below

$$\begin{array}{ccc}
 \begin{array}{c} \text{weekly} \\ \text{hazard-decision} \\ \overbrace{\mathbb{E} \min} \\ v_{[s]} \\ \underbrace{\hspace{1.5cm}} \\ \text{slow} \\ \text{fast} \end{array} & \text{versus} & \begin{array}{c} \text{weekly decision-} \\ \text{hazard-decision} \\ \overbrace{\min \mathbb{E} \min} \\ \underbrace{\tilde{u}_{[s]}}_{\text{slow}} \quad \underbrace{\tilde{v}_{[s]}}_{\text{fast}} \end{array}
 \end{array} \tag{18}$$

In other words, at each stage of the stochastic multistage optimization problem, the decision maker has more information when making the decision in the HD case. Therefore, we have the following relation for all $s \in \mathbb{S}$:

$$B_s^{\text{HD}} \leq B_s^{\text{DHD}} . \tag{19}$$

The usage values — or prices p — are defined as the opposite of the derivative of the Bellman functions with respect to the level of stock [9]:

$$p_s^{\text{HD}} = -\frac{d}{dx} B_s^{\text{HD}}(x) , \tag{20a}$$

$$p_s^{\text{DHD}} = -\frac{d}{dx} B_s^{\text{DHD}}(x) . \tag{20b}$$

Whereas we have established an inequality between Bellman functions in Equation (19), it is impossible to do so for the usage values (as they are derivatives).

In Section 5, we compare numerically the effect of both information structures when computing usage values for a case study of small size.

5 Numerical study

In this section, we present a numerical study. In §5.1, we describe the electrical system of small size considered. In §5.2, we present the method to compute the Bellman functions and usage values, either for the hazard-decision information structure in Section 3 or for the decision-hazard-decision information structure in Section 4. In §5.3, we compare the numerical results for the usage values. Finally, in §5.4, we show the effect of the information modelling choice when designing policies to carry on a simulation of the energy system dispatch.

5.1 Case study description

We consider a small electrical system to conduct the numerical study with the components listed hereafter.

- 3 thermal units: base unit, semi-base unit and peak unit.
- 1 residual demand.

- 1 storage unit.

The thermal units differ in their prices and power capacities. Whereas the base and semi-base units are the cheapest and the less flexible ones, the peak unit is the most expensive but the most flexible one.

We model the components mentioned above as in Section 2, and we consider the weekly *hazard-decision* (Section 3) and the weekly *decision-hazard-decision* (Section 4) mathematical modelling of the problem. In the weekly hazard-decision case, all the decisions for the week are anticipative but, in the weekly decision-hazard-decision case, we consider that the on and off decisions for the base and semi-base units are nonanticipative (planned in advance). This classification is made taking into account the rigidity and flexibility of the units.

To model the uncertainties, we have taken a finite number of scenarios provided by RTE, with uniform probability. We divide the scenarios into two sets:

- $(w_{\lfloor s \rfloor}^n)_{s \in \mathbb{S}}$ for $n \in \llbracket 1, N \rrbracket = \{1, \dots, N\}$ used to compute Bellman functions and their associated nonanticipative policies,
- $(w_{\lfloor s \rfloor}^c)_{s \in \mathbb{S}}$ for $c \in \llbracket 1, C \rrbracket = \{1, \dots, C\}$ used in simulation.

We use for the computations the programming language Julia [3], the JuMP package [7] and the Xpress solver.

5.2 Bellman functions computation

We compute the Bellman functions using the classical stochastic dynamic programming algorithm [2] for the hazard-decision framework, and an adapted version for the decision-hazard-decision framework. In both cases, the problem for the entire year is decomposed in weekly problems that we solve using the backward recursions (11) and (16) (respectively) for a discretization of the state space \mathbb{X} .

The expectations in the Bellman equations are computed as finite sums $\sum_{n \in \llbracket 1, N \rrbracket} \frac{1}{N} \Phi(w_{\lfloor s \rfloor}^n)$, where $w_{\lfloor s \rfloor}^n$ is a weekly scenario and $\frac{1}{N}$ its corresponding probability.

The objective is to obtain two sequences $(B_s^{\text{HD}})_{s \in \mathbb{S}}$ and $(B_s^{\text{DHD}})_{s \in \mathbb{S}}$ of Bellman functions corresponding to the weekly hazard-decision and weekly decision-hazard-decision frameworks. They are represented by real numbers $(B_s^{\text{HD}}(x))_{s \in \mathbb{S}}$ and $(B_s^{\text{DHD}}(x))_{s \in \mathbb{S}}$ with x varying in a grid ($\subset \mathbb{X}$). In this case study, we consider that the cost-to-go $B_{s^+}^{\text{HD}}$ and $B_{s^+}^{\text{DHD}}$ in (11) and (16) respectively are piecewise linear functions. Therefore, for both structures, we solve MILP problems for each week. The algorithm sketches are presented in Alg. 1 and Alg. 2.

Whereas in the HD Algorithm 1, at each week and each point of the discretization of the state space, N deterministic mixed integer linear problems are solved, in the DHD Algorithm 2, a stochastic two-stage mixed integer linear problem is solved.

When comparing the Bellman functions for all the weeks in the year we obtain that $B_s^{\text{HD}}(x) < B_s^{\text{DHD}}(x)$ for all x , which numerically confirms the theoretical result in (19).

Algorithm 1: Hazard-decision stochastic dynamic programming

Data: uncertainties scenarios $w_{\lfloor s \rfloor}^n$ with $n \in \llbracket 1, N \rrbracket$ and probabilities $\frac{1}{N}$,
 space state discretization $x \in \mathbb{X}$,
 final cost $K(x_{\bar{s}+})$
Result: $\{B_s^{\text{HD}}(x)\}_{s \in \mathbb{S}}$
for $s = \bar{s}, \dots, \underline{s}$ **do**
 for $x \in \mathbb{X}$ **do**
 $B_s^{\text{HD},0}(x) = 0;$
 for $n \in \llbracket 1, N \rrbracket$ **do**
 $B_s^{\text{HD},n}(x) = B_s^{\text{HD},n-1}(x) + \frac{1}{N} \min_{v_{\lfloor s \rfloor}^n} \left\{ \mathcal{L}_s(x, w_{\lfloor s \rfloor}^n, v_{\lfloor s \rfloor}^n) + B_{s^+}^{\text{HD}}(\mathcal{F}_s(x, v_{\lfloor s \rfloor}^n)) \right\},$
 s.t. (10d)
 end
 end
end

The usage values or prices, p , are calculated as the opposite of the derivative of the Bellman functions with respect to the level of stock as in Equations (20). As the Bellman functions are computed on a discrete grid, and as usage values in (20) have been defined as derivatives, usage values are approximated by increments between middle points of the state space grid by the formula

$$\frac{d}{dx} B_s(x) \approx \frac{B_s(x + \frac{\Delta x}{2}) - B_s(x - \frac{\Delta x}{2})}{\Delta x}, \quad (21)$$

with Δx the discretization step.

5.3 Numerical results for usage values

In Fig. 4, we compare the prices of the thermal units with the usage values obtained with the Bellman functions computed using the HD and DHD structures.

We observe that, for levels of stock higher than 20%, the HD usage value in blue is above the semi-base unit price (dashed yellow), whereas the DHD usage value in red is below. As a consequence, we expect different dispatches in simulation when using these usage values to design the storage policy, since the merit order of the production means changes. This change in the dispatch could lead to different conclusions when carrying out prospective studies. In §5.4 we compare the results in simulation induced by both modelling options to compute Bellman functions.

Algorithm 2: Decision-hazard-decision stochastic dynamic programming

Data: uncertainties scenarios $w_{\llbracket s \rrbracket}^n$ with $n \in \llbracket 1, N \rrbracket$ and probabilities $\frac{1}{N}$,
 space state discretization $x \in \mathbb{X}$,
 final cost $K(x_{\bar{s}+})$
Result: $\{B_s^{\text{DHD}}(x)\}_{s \in \mathbb{S}}$
for $s = \bar{s}, \dots, \underline{s}$ **do**
 for $x \in \mathbb{X}$ **do**

$$B_s^{\text{DHD}}(x) = \min_{\tilde{u}_{\llbracket s \rrbracket}} \sum_{n=1}^N \frac{1}{N} \left(\min_{\tilde{v}_{\llbracket s \rrbracket}} \left\{ \mathcal{L}_s(x, \tilde{u}_{\llbracket s \rrbracket}, w_{\llbracket s \rrbracket}^n, \tilde{v}_{\llbracket s \rrbracket}^n) + B_{s+}^{\text{DHD}}(\mathcal{F}_s(x, \tilde{v}_{\llbracket s \rrbracket}^n)) \right\}, \right.$$
 s.t. (15d) $\left. \right)$
 end
end

5.4 Numerical comparison between policies in HD and DHD

The goal of this part of the numerical study is to compare the dispatches obtained when using the policies induced by both Bellman functions $(B_s^{\text{HD}})_{s \in \mathbb{S}}$ and $(B_s^{\text{DHD}})_{s \in \mathbb{S}}$ computed in §5.2 .

We recall that an uncertainty scenario for simulation (also called *chronicle*) is denoted by $(w_{\llbracket s \rrbracket}^c)_{s \in \mathbb{S}}$. We simulate the operation of the system for different chronicles $c \in \llbracket 1, C \rrbracket$ in which the availability and demand change.

For the sake of simplicity and to be consistent with the weekly decision-hazard-decision information structure, we choose to design an “anticipative simulator”: uncertainties are disclosed at the end of the week and recourse controls are computed at the end of the week but applied within the week. The simulation algorithm is the same regardless the sequence of Bellman functions (HD or DHD) chosen to design the policies.

We illustrate how the simulation is done for one uncertain chronicle in Algorithm 3. We highlight that this simulation algorithm can be used with any Bellman function.

In Fig. 5, we compare the simulations of the dispatches for one chronicle of uncertainties (residual demand and availability fixed) in the week 20 of the year when using the HD and DHD Bellman functions to compute the policies. In the selected chronicle, the base unit is not available until the hour 96 of the week. The semi-base and peak unit are available during the entire week.

We compare the dispatch obtained with the weekly hazard-decision policy in Fig. 5a with the one obtained with the weekly decision-hazard-decision policy in Fig. 5b. To satisfy the same demand, with the same production means available, we observe that the dispatch varies depending on the policy considered. In the case of the HD policy, the semi-base unit works at its maximum power and the marginal production is made by the storage. On the contrary, in the DHD policy dispatch, the storage operates at its maximum power and the marginal production is made by the semi-base unit.

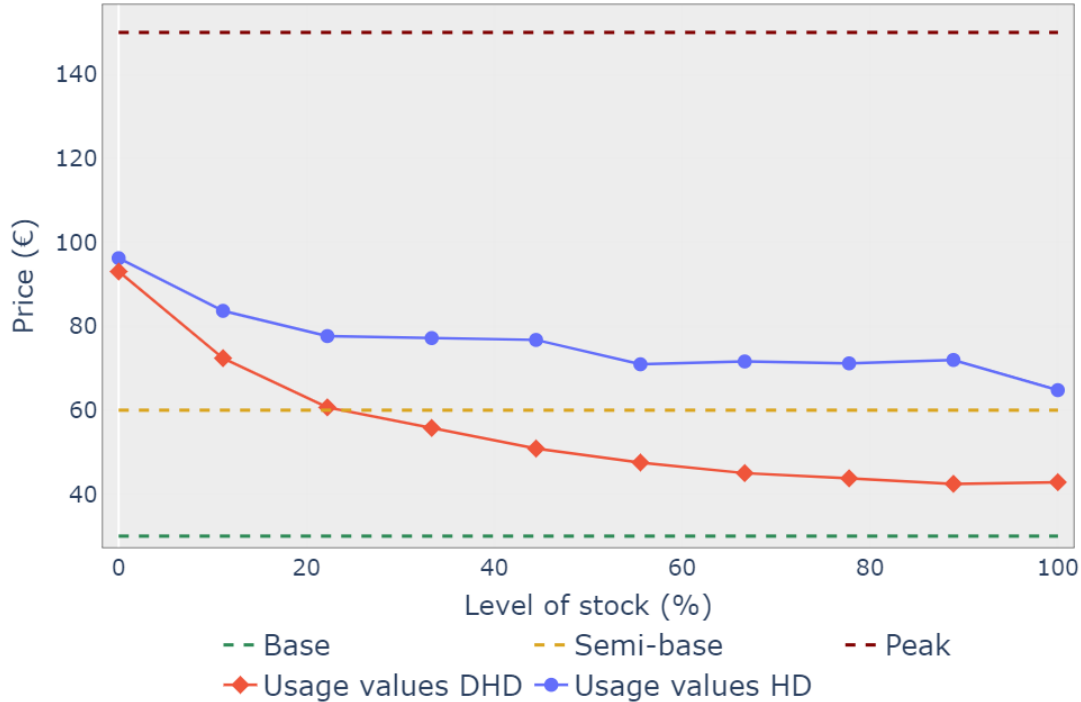


Figure 4: Prices and usage values comparison for week 20

This difference comes from the fact that, as we remarked in Fig. 4, the HD usage values are above the semi-base unit price, whereas the DHD usage values are below the semi-base unit price. This changes the dispatch order.

The merit order difference in the two policies studied is observed for several weeks in the year, which leads, for all chronicles, to a higher use of the storage (more turbinning and more pumping) when using the DHD policy.

However, we observe that the HD policy tends to store more energy than the DHD policy as we can see in Fig. 6 for the same simulated yearly chronicle. This result is observed in all simulated chronicles and can be explained with the usage value difference: in the case of the DHD policy there is less interest in storing energy since it has lower usage value.

6 Conclusion

During this work, we first formalized the weekly hazard-decision information structure in a two-timescale setting. This is the most common information framework nowadays when modelling the resource adequacy problem as a multistage stochastic optimization problem to compute usage values for prospective studies under uncertainty. We wrote the weekly Bellman equations in this framework, that make it possible to compute usage values for the

Algorithm 3: Decision-hazard-decision policy and simulation

Data: uncertainties scenarios $w_{\llbracket s \rrbracket}^n$ with $n \in \llbracket 1, N \rrbracket$ and probabilities $\frac{1}{N}$,
 space state discretization $x \in \mathbb{X}$,
 Bellman functions $\{B_s\}_{s \in \mathbb{S}}$,
 simulation chronicle $\{w_{\llbracket s \rrbracket}^c\}_{s \in \mathbb{S}}$,
 initial condition for the state x_0
Result: $\{x_s^c, u_{\llbracket s \rrbracket}^c, v_{\llbracket s \rrbracket}^c\}_{s \in \mathbb{S}}$
 $x_s^c = x_0$
for $s = \underline{s}, \dots, \bar{s}$ **do**
 compute nonanticipative controls:

$$u_{\llbracket s \rrbracket}^c = \arg \min_{\tilde{u}_{\llbracket s \rrbracket}} \sum_{n=1}^N \frac{1}{N} \left(\min_{\tilde{v}_{\llbracket s \rrbracket}^n} \left\{ \mathcal{L}_s(x_s^c, \tilde{u}_{\llbracket s \rrbracket}, w_{\llbracket s \rrbracket}^n, \tilde{v}_{\llbracket s \rrbracket}^n) + B_{s^+}(\mathcal{F}_s(x_s^c, \tilde{v}_{\llbracket s \rrbracket}^n)) \right\} \right)$$

 compute recourse controls:

$$v_{\llbracket s \rrbracket}^c = \arg \min_{\tilde{v}_{\llbracket s \rrbracket}} \left\{ \mathcal{L}_s(x_s^c, u_{\llbracket s \rrbracket}^c, w_{\llbracket s \rrbracket}^c, \tilde{v}_{\llbracket s \rrbracket}) + B_{s^+}(\mathcal{F}_s(x_s^c, \tilde{v}_{\llbracket s \rrbracket})) \right\}$$

 update state:
 $x_{s^+}^c = \mathcal{F}_s(x_s^c, \tilde{v}_{\llbracket s \rrbracket})$
end

energy in the storage. These Bellman equations respect the hourly physical constraints.

We highlighted the need to improve this structure to account for temporal rigidities in thermal operation, that is, the fact that on/off decisions cannot be modelled as fast or last minute decisions. Therefore, we introduced the weekly decision-hazard-decision information structure. In this structure, the decisions for each stage in the decision-making process are separated into planning and recourse decisions depending on the physical modelling aspects of the decisions. We presented the mathematical formulation of the problem considering the weekly decision-hazard-decision information structure and its associated Bellman equations in the weekly timescale, that still respect the hourly physical constraints.

Afterwards, we carried out a numerical analysis to quantify, in a case study, the consequences of the information structure modelling choice when computing Bellman functions and their associated usage values. From this study, we inferred that the policy induced by the weekly hazard-decision information modelling leads to an overestimation of the thermal flexibility and as a result, a lower use of the storage. When using the policy induced by the weekly decision-hazard-decision Bellman functions, we take into account some rigidity in the thermal units (specially on/off decisions) and the storage is more used. In consequence, we think that decision-hazard-decision information structures are of great interest to calculate usage values taking into account that some types of thermal units are less flexible.

References

- [1] R. E. Bellman. *Dynamic Programming*. Princeton University Press, Princeton, N.J., 1957.
- [2] D. P. Bertsekas. *Dynamic Programming and Optimal Control*. Athena Scientific, Belmont, Massachusetts, second edition, 2000. Volumes 1 and 2.
- [3] J. Bezanson, A. Edelman, S. Karpinski, and V. B. Shah. Julia: A fresh approach to numerical computing. *SIAM review*, 59(1):65–98, 2017.
- [4] P. Carpentier, J.-P. Chancelier, M. D. Lara, T. Martin, and T. Rigaut. Time blocks decomposition of multistage stochastic optimization problems. *Journal of Convex Analysis*, 30(2):627–658, 2023.
- [5] C. Dellacherie and P. A. Meyer. *Probabilités et potentiel*. Hermann, Paris, 1975.
- [6] M. Doquet, R. Gonzalez, S. Lepy, E. Momot, and F. Verrier. A new tool for adequacy reporting of electric systems: Antares. CIGRE, 2008. 42nd International Conference on Large High Voltage Electric Systems.
- [7] M. Lubin, O. Dowson, J. Dias Garcia, J. Huchette, B. Legat, and J. P. Vielma. JuMP 1.0: Recent improvements to a modeling language for mathematical optimization. *Mathematical Programming Computation*, 2023.
- [8] M. V. F. Pereira and L. M. V. G. Pinto. Multi-stage stochastic optimization applied to energy planning. *Math. Program.*, 52:359–375, Oct. 1991.
- [9] G. Steeger, L. Barroso, and S. Rebennack. Optimal bidding strategies for hydro-electric producers: A literature survey. *Power Systems, IEEE Transactions on*, 29:1758–1766, 07 2014.
- [10] A. Street, D. Valladão, A. Lawson, and A. Velloso. Assessing the cost of the hazard-decision simplification in multistage stochastic hydrothermal scheduling. *Applied Energy*, 280:115939, 2020.

A Hourly composition of the storage dynamics

We consider the hourly dynamics function given by

$$f(q, r^p, r^t) = q + \eta r^p - r^t. \quad (22a)$$

This equation represents the hourly evolution in the level of stock in a storage. To be able to formulate the problem in a weekly framework, we define the weekly dynamics f_s .

The weekly dynamics $f_s(q_{(s,\underline{h})}, r_{\llbracket s \rrbracket}^p, r_{\llbracket s \rrbracket}^t)$ is obtained by composition of the hourly dynamics $f(q_{(s,h)}, r_{(s,h)^+}^p, r_{(s,h)^+}^t)$ for $h \in \mathbb{H}$, but the composition being done only on level of stock variable.

To describe the dynamics composition we introduce the following extra notation,

$$f_{(s,h)}^{r^p, r^t}(q) = f(q_{(s,h)}, r_{(s,h)^+}^p, r_{(s,h)^+}^t) . \quad (22b)$$

Then,

$$f_s(q_{(s,\underline{h})}, r_{\llbracket s \rrbracket}^p, r_{\llbracket s \rrbracket}^t) = \left(f_{(s,\bar{h})}^{r^p, r^t} \circ f_{(s,\bar{h}^-)}^{r^p, r^t} \circ \dots \circ f_{(s,\underline{h})}^{r^p, r^t} \right) (q_{(s,\underline{h})}) . \quad (22c)$$

From the hourly dynamics f expression (22a), the weekly dynamics f_s is given by:

$$f_s(q_{(s,\underline{h})}, r_{\llbracket s \rrbracket}^p, r_{\llbracket s \rrbracket}^t) = q_{(s,\underline{h})} + \sum_{h \in \mathbb{H}} (\eta r_{(s,h)^+}^p - r_{(s,h)^+}^t) . \quad (22d)$$

B Ideal information structure in decision-hazard-decision

B.1 Weekly decision-hazard-decision information structure with hourly recourse

In addition to the weekly decision-hazard-decision structure presented in Section 4, we consider an information structure in which the nonanticipative control $\tilde{u}_{\llbracket s \rrbracket}$ is made knowing the uncertainties up to the beginning of week s , and the hourly recourse control $\tilde{v}_{(s,h)^+}$ is made knowing the uncertainties up to the hour $(s, h)^+$. As illustrated in Fig. 2, at the beginning of the week s the nonanticipative decision $\tilde{u}_{\llbracket s \rrbracket}$ is made (for all $h \in \mathbb{H}$) and then, once the uncertainties are disclosed *hour by hour*, the recourse decisions $v_{(s,h)^+}$ are sequentially made.

For the weekly decision-hazard-decision structure with hourly recourse the information constraints are given by the σ -fields inclusions

$$\forall s \in \mathbb{S}, \quad \sigma(\tilde{\mathbf{U}}_{\llbracket s \rrbracket}) \subset \sigma(\mathbf{W}_{(s,h)}, \mathbf{W}_{\llbracket s \rrbracket}, \dots, \mathbf{W}_{\llbracket s-1 \rrbracket}), \quad (23a)$$

$$\forall (s, h) \in \mathbb{S} \times \mathbb{H}, \quad \sigma(\tilde{\mathbf{V}}_{(s,h)^+}) \subset \sigma(\mathbf{W}_{(s,h)}, \mathbf{W}_{\llbracket s \rrbracket}, \dots, \mathbf{W}_{\llbracket s-1 \rrbracket}, \mathbf{W}_{(s,h^+)}, \dots, \mathbf{W}_{(s,h)^+}). \quad (23b)$$

B.2 Multistage stochastic optimization problem formulation

Considering the definitions in Table 6, we formulate the physical-economical adequacy problem as a stochastic multistage optimization problem using the weekly decision-hazard-decision information structure with hourly recourse:

$$\min_{\tilde{\mathbf{U}}, \tilde{\mathbf{V}}} \mathbb{E} \left[\sum_{s \in \mathbb{S}} \tilde{\mathcal{L}}_s(\mathbf{Q}_{(s,h)}, \tilde{\mathbf{U}}_{\llbracket s \rrbracket}, \mathbf{W}_{\llbracket s \rrbracket}, \tilde{\mathbf{V}}_{\llbracket s \rrbracket}) + K(\mathbf{Q}_{(\bar{s}^+, h)}) \right] \quad (24a)$$

such that , $\forall (s, h) \in \mathbb{S} \times \mathbb{H}$

$$\mathbf{Q}_{(s,h)} = \mathbf{W}_{(s,h)}, \quad (24b)$$

$$\mathbf{Q}_{(s^+, h)} = \tilde{\mathcal{F}}_s(\mathbf{Q}_{(s,h)}, \tilde{\mathbf{V}}_{\llbracket s \rrbracket}), \quad (24c)$$

$$\tilde{\mathcal{G}}_{\llbracket s \rrbracket}(\tilde{\mathbf{U}}_{\llbracket s \rrbracket}, \mathbf{W}_{\llbracket s \rrbracket}, \tilde{\mathbf{V}}_{\llbracket s \rrbracket}) \geq \mathbf{0}, \quad (24d)$$

$$\sigma(\tilde{\mathbf{U}}_{\llbracket s \rrbracket}) \subset \sigma(\mathbf{W}_{(s,h)}, \mathbf{W}_{\llbracket s \rrbracket}, \dots, \mathbf{W}_{\llbracket s-1 \rrbracket}), \quad (24e)$$

$$\sigma(\tilde{\mathbf{V}}_{(s,h)^+}) \subset \sigma(\mathbf{W}_{(s,h)}, \mathbf{W}_{\llbracket s \rrbracket}, \dots, \mathbf{W}_{\llbracket s-1 \rrbracket}, \mathbf{W}_{(s,h^+)}, \dots, \mathbf{W}_{(s,h)^+}). \quad (24f)$$

The final cost $K(\mathbf{Q}_{(\bar{s}^+, h)})$ is used to give value to the energy in the storage at the end of the yearly period.

It is well known that, for all $s \in \mathbb{S}$, the function B_s^{DHD} satisfies

$$B_s^{\text{DHD}^h}(x_s) = \min_{\substack{\tilde{\mathbf{U}}_{\llbracket s \rrbracket, \dots, \tilde{\mathbf{U}}_{\llbracket \bar{s} \rrbracket} \\ \tilde{\mathbf{V}}_{\llbracket s \rrbracket, \dots, \tilde{\mathbf{V}}_{\llbracket \bar{s} \rrbracket}}} \mathbb{E} \left[\sum_{s'=s}^{\bar{s}} \mathcal{L}_{s'}(\mathbf{Q}_{(s',h)}, \tilde{\mathbf{U}}_{\llbracket s' \rrbracket}, \mathbf{W}_{\llbracket s' \rrbracket}, \tilde{\mathbf{V}}_{\llbracket s' \rrbracket}) + K(\mathbf{Q}_{(\bar{s}^+,h)}) \right] \quad (26a)$$

such that , $\forall s' \in \llbracket s, \dots, \bar{s} \rrbracket$

$$\mathbf{Q}_{(s',h)} = x_s , \quad (26b)$$

$$\mathbf{Q}_{(s'+,h)} = \mathcal{F}_{s'}(\mathbf{Q}_{(s',h)}, \tilde{\mathbf{V}}_{\llbracket s' \rrbracket}) , \quad (26c)$$

$$\mathcal{G}_{\llbracket s' \rrbracket}(\tilde{\mathbf{U}}_{\llbracket s' \rrbracket}, \mathbf{W}_{\llbracket s' \rrbracket}, \tilde{\mathbf{V}}_{\llbracket s' \rrbracket}) \geq \mathbf{0} , \quad (26d)$$

$$\sigma(\tilde{\mathbf{U}}_{\llbracket s' \rrbracket}) \subset \sigma(\mathbf{W}_{(s,h)}, \mathbf{W}_{\llbracket s \rrbracket}, \dots, \mathbf{W}_{\llbracket s'- \rrbracket}) , \quad (26e)$$

$$\sigma(\tilde{\mathbf{V}}_{(s',h)^+}) \subset \sigma(\mathbf{W}_{(s,h)}, \mathbf{W}_{\llbracket s \rrbracket}, \dots, \mathbf{W}_{\llbracket s'- \rrbracket}, \mathbf{W}_{(s',h)^+}, \dots, \mathbf{W}_{(s',h)^+}) . \quad (26f)$$

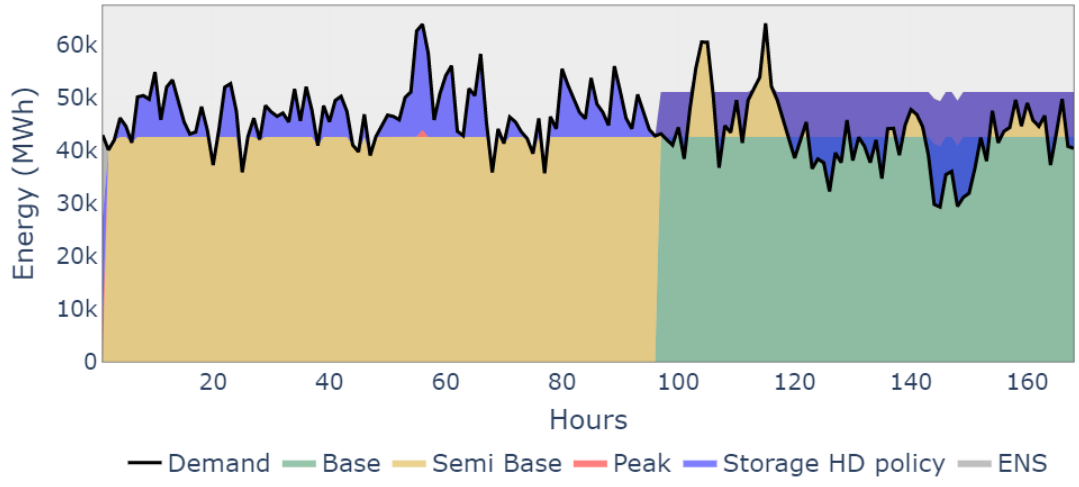
so that the value $B_s^{\text{DHD}^h}(x_s)$ of Bellman function is interpreted as the future optimal cost when at week s the level of stock is x_s and the weekly decision-hazard-decision information structure with hourly recourse is considered. More details can be found in Proposition 10 in [4].

B.4 Theoretical comparison between Bellman functions in HD, DHD and DHD^h

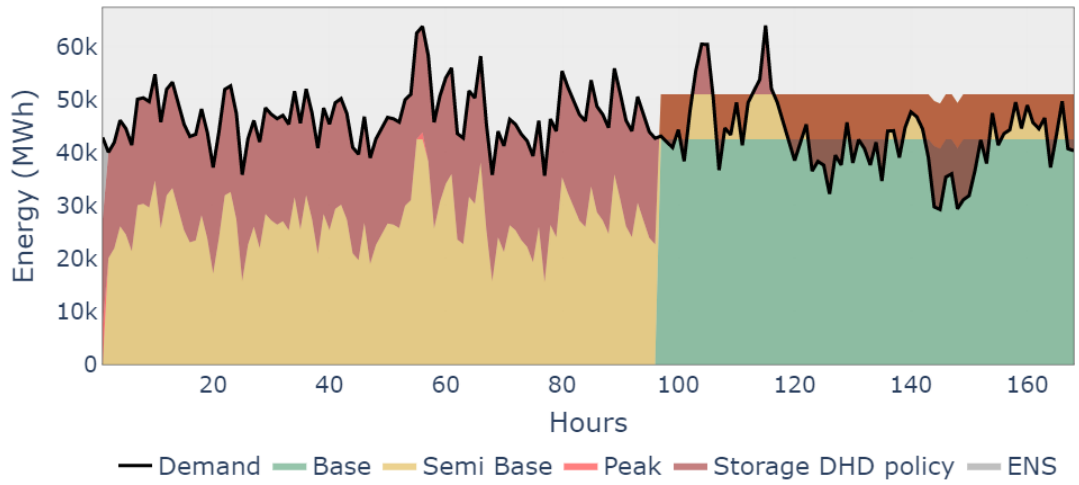
When we compare theoretically the Bellman functions B_s^{HD} , B_s^{DHD} and $B_s^{\text{DHD}^h}$, given by the Bellman equations (11), (16) and (25), we observe that the weekly hazard-decision approach is a relaxation of the weekly decision-hazard-decision approach, and this last one is a relaxation of the weekly decision-hazard-decision with hourly recourse approach with respect to the information constraint.

In other words, at each stage of the stochastic multistage optimization problem, the decision maker has less information when changing from the HD to the DHD approach. The same happens when changing from the DHD to the DHD^h approach. Therefore, we have the following relation for all $s \in \mathbb{S}$:

$$B_s^{\text{HD}} \leq B_s^{\text{DHD}} \leq B_s^{\text{DHD}^h} . \quad (27)$$



(a) Weekly hazard-decision policy dispatch



(b) Weekly decision-hazard-decision policy dispatch

Figure 5: Dispatch comparison using the policies induced by the weekly hazard-decision and weekly decision-hazard-decision Bellman functions

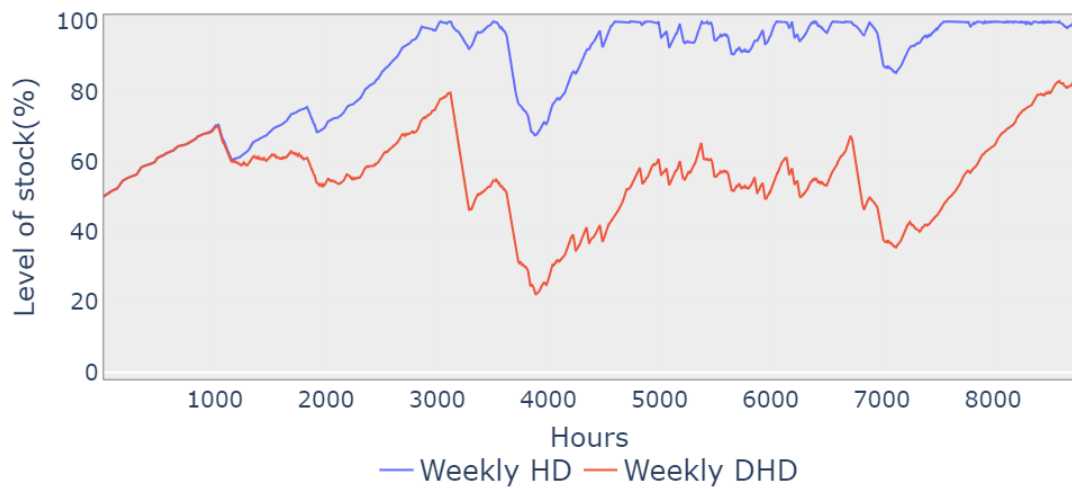


Figure 6: Comparison of the level of stock obtained from the weekly hazard-decision and the weekly decision-hazard-decision policies for one given chronicle (of uncertainties)