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Coupled Atmosphere–Ocean Boundary Layers with the Ocean Skin Layer: Periodic Solar Forcing

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Goal

The goal of the present note is to analytically examine the impacts of the ocean skin and warm layers to the coupled dynamics of the atmosphere–ocean boundary layers. Persistent existence of a skin layer is long since observationally established by Coppin *et al.* (1991). For the purpose of a simple demonstration of the impacts, the analysis is limited only on the temperature (entropy), and no effect of the moisture and the salinity is counted for now, apart from a prescribed relative humidity for the evaporation. Full moisture contributions will be considered only later.

Furthermore, we will only consider a simplified case in which the solar forcing is purely periodic, thus the ocean experiences a solar cooling during the night. However, this idealization can easily be amended by superposing the subharmonics to the obtained solution so that the absence of solar heating during the night can be re-produced under a Fourier decomposition. Note that the following analysis is also performed under the linearization. Thus, this generalization is also straightforward.

The formulations for the skin layer and the warm layer underneath follow those by Fairall *et al.* (1996), Zeng and Beljaars (2005), and Bellenger *et al.* (2017). The atmospheric boundary layer assumes a simple convectively well-mixed layer (*cf.*, Ch. 12 of Yano 2023). Thus, the system consists of the three main equations for defining the temperatures in skin layer (ocean interface), T_{int} , the top of the warm layer (bottom of the skin layer, subskin), T_δ , and that of the atmospheric boundary layer, T_m (*cf.*, Eq. 2.8b). For solving this system, we need to specify the temperatures at the bottom of the warm layer (\bar{T}_d)

as well as a reference temperature of the atmosphere, say, as defined by the tropopause temperature (*cf.*, Eq. 2.8c).

A key question to be addressed with a simple analysis herein is whether we can reproduce a rather dramatic enhancement of the diurnal cycle in the warm layer (at the subskin) under the presence of the skin layer found by a diagnostic study by Bellenger and Duvel (2009). If this is successful, the next goal is to identify the mechanisms for such a dramatic enhancement.

2. Basic Formulation

2.1 Skin Layer

The ocean skin layer, consisting only of the first 1 mm of the ocean from the surface, is strongly controlled by the the heat flux at the ocean interface. As a result, the skin-layer temperature, or the ocean-interface temperature, T_{int} , is typically colder than the subskin temperature, T_δ , *i.e.*, of the lower warm layer by ΔT_c , thus:

$$\Delta T_c = T_\delta - T_{int} \quad (2.1a)$$

Its value is defined by a balance between the cooling rate, Q , due to the total surface flux and a downward molecular thermal diffusion of the heat:

$$\Delta T_c = Q\delta/k \quad (2.1b)$$

Here, $\delta \simeq 1$ mm is the depth of the skin layer, $k = \rho_w C_v D_H$ is the thermal conductivity defined in terms of the water density, $\rho_w = 1$ g/cm³ = 10³ kg/m³, the specific heat capacity, $C_v = 4.22 \times 10^3$ J/kg/K, and the molecular thermal diffusivity, $D_H = 1.433 \times 10^{-7}$ m²/s, of water. Note that the skin-temperature correction, ΔT_c , is the only variable that depends on the factor, $\delta/k = 1.65 \times 10^{-3}$ m²K/W, in the present problem. Thus, it plays a role of a feedback factor of the skin layer in this system, and the effect of the skin layer to the whole system is turned off simply by setting $\delta = 0$ in the following. Note that due to the smallness of this factor, δ/k , the feedback of the skin layer to the whole system is negligible, unless an extract amplifying factor involved in a full feedback coefficient.

The total cooling rate is given by a sum of the longwave radiative cooling rate, R_l , the sensible heat flux, H_s , and the evaporation flux, H_l :

$$Q = R_l + H_s + H_l \quad (2.2)$$

These fluxes are defined positive upwards, and we adopt more specific forms for them as:

$$R_l = \epsilon^* \sigma T_{int}^4 \quad (2.3a)$$

$$H_s = \frac{\rho C_p}{\tilde{\pi}} \overline{w' \theta'} \quad (2.3b)$$

$$H_l = \frac{\rho L_v}{\tilde{\pi}} \overline{w' q'} \quad (2.3c)$$

Here, $\epsilon^* = 0.97 \times 0.254 = 0.246$ is an effective emmisivity, including a contribution of the downward longwave radiation, and $\sigma = 5.6708 \times 10^{-8}$ Wm⁻²K⁻⁴ is the Stefan-Boltzmann

constant; $\rho = 1 \text{ kg/m}^3$ is the surface air density, $C_p = 1004 \text{ J/kg/K}$ is the specific heat at constant pressure of the air, $\tilde{\pi} = (p_s/p)^\kappa$ with $\kappa = 0.2854$ is the Exner function required to adjust the potential-temperature eddy flux, $\overline{w'\theta'}$, when the surface pressure, p , deviates from the reference pressure, $p_s = 10^3 \text{ hPa}$; $L_v = 2.53 \times 10^6 \text{ J/kg/K}$ is the latent heat of evaporation, and q is the water-vapor mixing ratio. In presenting the final results, we simply set $\tilde{\pi} = 1$.

The eddy fluxes are further defined by

$$\overline{w'\theta'} = C_\theta u (\theta_{int} - \theta_m) \quad (2.4a)$$

$$\overline{w'q'} = C_q u (1 - RH) q^*(T_{int}) \quad (2.4b)$$

where $C_\theta \simeq C_q \simeq 1 \times 10^{-3}$ are the bulk coefficients, u is the surface wind speed, which is also to be fixed in this study to $u = 1 \text{ m/s}$; $\theta_{int} = \tilde{\pi} T_{int}$ and θ_m are the potential temperature of the ocean interface (skin layer) and the convectively well-mixed boundary layer of the atmosphere, respectively; $q^*(T_{int})$ is the saturated water-vapor mixing ratio at the sea surface temperature, T_{int} , and $RH = 0.6$ is a fixed relative humidity.

The saturated water vapor and its derivative by temperature are evaluated by using the corresponding formulae for the saturated water-vapor pressure, e :

$$e^* = e_0 \exp\left[\frac{L_v}{R_v} \left(\frac{1}{T_0} - \frac{1}{T}\right)\right]$$

$$\frac{de^*}{dT} = \frac{L_v e^*}{R_v T^2}$$

and $q^* \simeq \tilde{\varepsilon} e^*/p$, and $\tilde{\varepsilon} = R_d/R_v = 0.622$. Here, $T_0 = 273.16 \text{ K}$, $e_0 = 611 \text{ Pa}$, and $R_v = 461.5 \text{ J/K/kg}$.

Here, the longwave radiation formulation (2.3a) follows that of Sarachik (1978), who attributes this formula to Reed (1976). Note that, as stated in the introduction, the contribution of moisture is neglected in Eq. (2.3a), i.e., a moisture contribution to the longwave radiation, R_l , found in the original formula is neglected.

2.2 Warm Layer

A few meters of layer below the skin layer, in turn, tends to warm up due to the efficient absorption of the solar heating over this layer. We describe the evolution of the subskin temperature, T_δ , defined at the top of warm layer (bottom of the skin layer) by following Zeng and Beljaars (2005)¹⁾ as:

$$\left(\frac{\partial}{\partial t} + \frac{1}{\tau_\delta}\right) T'_\delta = \frac{-Q + f_s R_s}{C_d} - \frac{\Delta \bar{T}_\delta}{\tau_\delta} \quad (2.5)$$

Here and in the following, the prime sign, \prime , designates a deviation from the mean state, with the latter designated by the bar, $\bar{}$, thus, for example, $T_\delta = \bar{T}_\delta + T'_\delta$; we also set $\Delta \bar{T}_\delta = \bar{T}_\delta - \bar{T}_d$; Q is the total cooling rate already defined by Eq. (2.2). In the above

¹⁾ However, note that Bellenger and Duvel (2009, see especially their Sec. 3) point some problems with this model.

presentation, we have assumed that the fluctuation of the temperature is negligible at the bottom of the warm layer, as implicitly assumed in Zeng and Beljaars (2005), and set $T'_d = 0$. We also set $d = 3$ m.

The solar heating at the ocean surface, R_s , is assumed to be

$$R_s = S_0(\sin \omega t + \frac{1}{\pi}) \quad (2.6a)$$

where we set $S_0 = 1366$ W/m² to be the solar constant for simplicity. As a result, the solar forcing varies with the frequency, ω , of a day; $f_s = 0.66$ is the fraction of the solar heating reaching the bottom ($z = -d$) of the warm layer. Furthermore, a constant is added to the above so that the averaged solar heading becomes equal to the total heating during the day time, *i.e.*, $0 \leq t \leq \pi/\omega$.

For simplifying the presentation of Eq. (2.5), we have introduced an effective heat capacity, C_d , and the relaxation time scale, τ_δ as

$$C_d = d\rho_w C_v \eta / (\eta + 1) \quad (2.6b)$$

$$\tau_\delta = d\phi_l(d/L) / (\eta + 1)k_v u_{*w} \quad (2.6c)$$

where $\eta = 0.3$ is a parameter that characterizes the temperature profile in the warm layer, $\phi_l(-z/L)$ is the similarity function defined by

$$\phi_l(z/L) = 1 + 5z/L \quad (2.6d)$$

with the Obukhov length, L , defined by

$$L = \rho_w C_v u_{*w}^3 / g\alpha(-Q + f_s S_0 / \pi) \quad (2.6e)$$

where $\alpha = 2.07 \times 10^{-4}$ 1/K is the thermal expansion rate of water; g is the acceleration of gravity. Here, the factor $1/\pi$ is applied on the solar heating term with S_0 to obtain an average over a day. Furthermore, u_{*w} is the friction velocity of water; $k_v = 0.4$ is the von Karman constant.

2.3 Atmospheric Boundary Layer

Description of the atmospheric boundary layer follows a well-established formulation of convectively mixed layer as presented, for example, in Ch. 12 of Yano (2023):

$$h_m \frac{d}{dt} \Delta\theta_m = \overline{w'\theta'} - h_m Q_R \quad (2.7)$$

where h_m is the depth of the mixed layer, related to the vertically-homogeneous temperature anomaly, $\Delta\theta_m$, in the mixed layer by

$$h_m = \Delta\theta_m \left(\frac{d\bar{\theta}}{dz} \right)^{-1}, \quad (2.8a)$$

from a geometrical consideration, with the mean lapse rate, $d\bar{\theta}/dz$, and the potential temperature, θ_m of the mixed layer is

$$\theta_m = \theta_a + \Delta\theta_m \quad (2.8b)$$

and the basic profile of the atmosphere in the absence of the mixed layer is:

$$\theta = \theta_a + \left(\frac{d\bar{\theta}}{dz}\right) z \quad (2.8c)$$

Finally, Q_R is the radiative cooling rate in the well mixed layer, which is assumed to be constant ($Q_R = 1$ K/day). Note also that we also assume the lapse rate, $d\bar{\theta}/dz$, to be constant.

2.4. Surface Eddy Fluxes, $\overline{w'\theta'}$ and $\overline{w'q'}$, and the Skin-Layer Cooling rate, Q

The surface eddy fluxes, $\overline{w'\theta'}$ and $\overline{w'q'}$, have been defined by Eq. (2.4a, b). In the former, $\theta_{int} = \tilde{\pi}T_{int}$ and $T_{int} = T_\delta - \Delta T_c$ from Eq. (2.1a), and θ_m is defined by Eq. (2.8c). As already remarked immediately following Eq. (2.5), we divide all the temperatures into the mean and the perturbation, for example, $T_\delta = \bar{T}_\delta + T'_\delta$

Consequently, the surface sensible heat flux, $\overline{w'\theta'}$, is also separated into the mean and the perturbation, i.e., $\overline{w'\theta'} = \overline{w'\theta'}_0 + \overline{w'\theta'}'$:

$$\overline{w'\theta'}_0 = C_\theta u (\tilde{\pi}\bar{T}_{int} - \bar{\theta}_m) \quad (2.9a)$$

$$\overline{w'\theta'}' = C_\theta u (\tilde{\pi}T'_{int} - \Delta\theta'_m) \quad (2.9b)$$

where $\bar{T}_{int} = \bar{T}_\delta - \Delta\bar{T}_c$, $T'_{int} = T'_\delta - \Delta T'_c$, and $\bar{\theta}_m = \theta_a + \Delta\bar{\theta}_m$.

Similarly, the evaporation flux, $\overline{w'q'}$, is separate into:

$$\overline{w'q'}_0 = C_q u \bar{q}^* \quad (2.9c)$$

$$\overline{w'q'}' = C_q u q^{*'} \quad (2.9d)$$

where

$$\bar{q}^* \equiv q^*(T_{int}) = q^*(\bar{T}_d) + \left(\frac{dq^*}{dT}\right) \Delta\bar{T}_{int}$$

$$q^{*'} = \left(\frac{dq^*}{dT}\right) \Delta T'_{int}$$

representing the deviations of the saturated water vapor from the reference state, $q^*(\bar{T}_d)$, by linear approximations. Here, dq^*/dT is defined at $T = T_d$; $\Delta\bar{T}_{int} = \Delta\bar{T}_\delta - \Delta\bar{T}_c$, and $\Delta T'_{int} = T'_\delta - \Delta T'_c$.

The skin-layer cooling rate, Q , is similarly separated into:

$$\bar{Q} = \lambda_Q^{-1} [\epsilon^* \sigma \bar{T}_\delta^4 + \rho C_p C_\theta u (\bar{T}_\delta - \bar{\theta}_m / \tilde{\pi})] + \rho L_v C_q (1 - RH) u \bar{q}_\delta^* \quad (2.10a)$$

$$Q' = \lambda_Q^{-1} \{ [4\epsilon^* \sigma \bar{T}_\delta^3 + \rho C_p C_\theta u + \rho L_v C_q (1 - RH) u \frac{dq^*}{dT}] T'_\delta - \rho C_p C_\theta u \frac{\Delta\theta'}{\tilde{\pi}} \} \quad (2.10b)$$

where $\bar{q}_\delta^* = q^*(\bar{T}_\delta)$. Note that the final expressions are rather convoluted due to the feedback of Q to Q itself in its own definition through dependence on ΔT_c . Moreover, we set:

$$\lambda_Q = 1 + \frac{\delta}{k}(4\epsilon^* \sigma \bar{T}_d^3 + \rho \tilde{C}_\theta u + \rho \tilde{C}_q u \frac{dq^*}{dT}) \quad (2.11)$$

where $\tilde{C}_\theta = C_p C_\theta$ and $\tilde{C}_q = L_v C_q (1 - RH)$.

Here, we may further set:

$$\lambda_Q = 1 - \alpha_f$$

where

$$\alpha_f = -\frac{\delta}{k}(4\epsilon^* \sigma \bar{T}_d^3 + \rho \tilde{C}_\theta u + \rho \tilde{C}_q u \frac{dq^*}{dT})$$

may be considered a feedback coefficient to the heat flux by the presence of the skin layer, because with the approximations of $\lambda_Q^{-1} = 1 + \alpha_f$ and $\bar{Q}(T_{int}) \simeq \bar{Q}(T_\delta)$, we find

$$\begin{aligned} \bar{Q}(T_{int}) &= [1 + \alpha_f \bar{Q}(T_\delta)] \\ Q'(T_{int}) &= [1 + \alpha_f Q'(T_\delta)] \end{aligned}$$

Here, the arguments, T_{int} and T_δ , are added to suggest the original argument (former) and the replacement (latter). Here, $\alpha_f = -6.18 \times 10^{-3}$ and $\alpha_f = -4.15 \times 10^{-3}$ for the moist and dry (without evaporation) cases, respectively. Thus, the feedback by the skin layer to the surface heat flux is very weak.

The decomposition of the latter is given in terms of \bar{Q} and Q' as

$$\Delta \bar{T}_c = \bar{Q} \delta / k \quad (2.12a)$$

$$\Delta T'_c = Q' \delta / k \quad (2.12b)$$

using Eq. (2.1b). By further substituting Eqs. (2.10a, b), (2.11), and (2.12a, b) into Eqs. (2.9a, b):

$$\overline{w' \theta'}_0 = \frac{\tilde{\pi} C_\theta u}{\lambda_Q} \left[-\frac{\delta}{k} (\epsilon^* \sigma \bar{T}_\delta^4 + \rho \tilde{C}_q u \bar{q}_\delta^*) + \lambda_4 (\bar{T}_\delta - \frac{\bar{\theta}_m}{\tilde{\pi}}) \right] \quad (2.13a)$$

$$\overline{w' \theta'} = \frac{\tilde{\pi} C_\theta u}{\lambda_Q} (T'_\delta - \lambda_4 \frac{\theta'_m}{\tilde{\pi}}) \quad (2.13b)$$

where

$$\lambda_4 = 1 + \frac{\delta}{k}(4\epsilon^* \sigma \bar{T}_\delta^3 + \rho \tilde{C}_q u \frac{dq^*}{dT}) \simeq 1.0 \quad (2.14)$$

In the above, the standard values with $u = 1$ m/s are also quoted.

3. Solutions

The given system consisting of Eqs. (2.1b), (2.4), and (2.7) can be solved in the following manner. The solutions are presented in the order of the subsections in the last section: the skin layer, the warm layer, and the atmospheric mixed layer. Nevertheless,

the interdependence of the solutions necessitates us to refer to the later results to proceed further from time to time.

3.1 Skin Layer

The solution to the skin layer is already given formally by Eqs. (2.12a, b) in the last section. Yet, their more explicit forms become available only after obtaining the solutions for the other variables.

3.2 Warm Layer

The prognostic equation for the warm-layer temperature, T'_δ , has been given by Eq. (2.5). Introducing the decomposition, $Q = \bar{Q} + Q'$, and also with a help of the definition, Eq. (2.5) is re-written into:

$$\left(\frac{\partial}{\partial t} + \frac{1}{\tau_\delta^*}\right)T'_\delta = \frac{-\bar{Q} + \lambda_s \bar{R}_s}{C_d} + \frac{\rho C_\theta u}{\tilde{\pi}} \frac{\lambda_Q^{-1}}{C_d} \Delta\theta'_m + \frac{\lambda_s R'_s}{C_d} - \frac{\Delta\bar{T}_\delta}{\tau_\delta} \quad (3.1)$$

where $\bar{R}_s = S_0/\pi$, and

$$\frac{1}{\tau_\delta^*} = \frac{1}{\tau_\delta} + \frac{\lambda'_Q}{C_d} \quad (3.2a)$$

$$\lambda'_Q = \frac{k}{\delta} \frac{\lambda_Q - 1}{\lambda_Q} \quad (3.2b)$$

Here, $\lambda'_Q/C_d \sim 10^{-6}$, thus $\tau_\delta^* \simeq \tau_\delta$ in good approximation: Also recall that the solar heating, R_s , is defined by Eq. (2.6a).

First, we seek the mean solution assuming that the mean forcing in the right-hand side of Eq. (3.1) vanishes. Thus,

$$\frac{\bar{Q} - \lambda_s \bar{R}_s}{C_d} + \frac{\Delta\bar{T}_\delta}{\tau_\delta} - \frac{\lambda_\delta \Delta\bar{\theta}_m}{c_d} = 0 \quad (3.3a)$$

Solving the above for $\Delta\bar{T}_\delta$, we find:

$$\Delta\bar{T}_\delta = \frac{\tau_\delta}{C_d} (\lambda_\delta \Delta\bar{\theta}_m - \bar{Q} + \lambda_s \bar{R}_s) \quad (3.3b)$$

Here, $\tau_\delta = 1200$ sec, $C_d = 2.92 \times 10^6$ J/m²K, and $\tau_\delta/C_d = 4.45 \times 10^{-4}$ K/(W/m²) with $u_{*w} = 10^{-2}$, and $\lambda_\delta \simeq 1$ with $u = 1$ m/s. Note however that the value τ_δ is very sensitive to the choice of the friction velocity, u_{*w} , because the former depends on the Obukhov length, L , which increases with the cubic of u_{*w} . Note moreover that the solution (3.3b) is not yet in the closed form, because the right-hand side variables still depend $\Delta\bar{T}_\delta$. A closed solution will be derived in the next subsection.

After subtracting this mean contribution, we re-write Eq. (3.1) as

$$\frac{d}{dt} T'_\delta e^{t/\tau_\delta^*} = f_\delta e^{t/\tau_\delta^*}$$

re-setting the right-hand side of Eq. (3.1) f_δ . It can be immediately integrated into:

$$T'_\delta = T'_\delta(0)e^{-t/\tau_\delta^*} + e^{-t/\tau_\delta^*} \int_0^t f_\delta e^{t/\tau_\delta^*} dt$$

For the purpose of performing the above integral explicitly, we set:

$$\Delta\theta'_m = \Delta\theta_{m1} \sin \omega t + \Delta\theta_{m2} \cos \omega t \quad (3.5a)$$

as well as

$$T'_\delta = T_{\delta1} \sin \omega t + T_{\delta2} \cos \omega t \quad (3.5b)$$

where $\Delta\theta_{mj}$ with $j = 1, 2$ are the constants to be determined later. Consequently,

$$T_{\delta1} = \frac{1}{(\omega^2 + 1/\tau_\delta^{*2})} \left(\frac{f_1}{\tau_\delta^*} + \omega f_2 \right) \quad (3.6a)$$

$$T_{\delta2} = \frac{1}{(\omega^2 + 1/\tau_\delta^{*2})} \left(-\omega f_1 + \frac{f_2}{\tau_\delta^*} \right) \quad (3.6b)$$

where

$$f_1 = \frac{1}{C_d} (\lambda_\delta \Delta\theta_{m1} + \lambda_S S_0) \quad (3.7a)$$

$$f_2 = \frac{\lambda_\delta}{C_d} \Delta\theta_{m2} \quad (3.7b)$$

where

$$\lambda_\delta = \frac{\rho C_p C_\theta u}{\lambda_Q}$$

The basic state, $\bar{T}_\delta = \bar{T}_d + \Delta\bar{T}_\delta$, is to be determined from Eq. (3.3a), but only after the explicit forms for \bar{Q} and $\Delta\theta_{m0}$ are determined. Note that the latter variables depend on $\Delta\bar{T}_\delta$, thus the above formula is not yet closed in that sense. Similarly, the coefficients, $T_{\delta1}$ and $T_{\delta2}$, are determined from Eqs. (3.6a, b), but only after the forms for $\Delta\theta_{m1}$ and $\Delta\theta_{m2}$, are determined.

3.2a Mean Temperature of the Warm Layer

The mean temperature, $\Delta\bar{T}_\delta$, of the warm layer is determined from Eq. (3.3a) in the following manner. First, substituting an explicit form of \bar{Q} given by Eq. (2.10a), as well as the solution of $\Delta\bar{\theta}_m$ given by Eq. (3.13a) below to obtain:

$$\begin{aligned} & [\lambda_3 + \frac{\delta}{k} \rho \tilde{C}_\theta C_\theta u^2 \frac{d\bar{\theta}}{dz} Q_R^{-1}] (\epsilon^* \sigma \bar{T}_\delta^4 + \rho \tilde{C}_q u \bar{q}_\delta^*) + \rho \tilde{C}_\theta u (\lambda_3 - C_\theta u \frac{d\bar{\theta}}{dz} Q_R^{-1} \lambda_4) (\Delta\bar{T}_\delta - \frac{\theta'_a}{\tilde{\pi}}) \\ & + \lambda_3 \lambda_Q \frac{C_d}{\tau_\delta} \Delta\bar{T}_\delta - \lambda_3 \lambda_Q \lambda_s \bar{R}_s = 0 \end{aligned}$$

where $\Delta\bar{T}_\delta = \bar{T}_\delta - \bar{T}_d$ and $\theta'_a = \theta_a - \tilde{\pi}\bar{T}_d$. To proceed further, we assume $\bar{T}_d \gg \Delta\bar{T}_\delta$, and apply the following approximations:

$$\begin{aligned}\bar{T}_\delta^4 &\simeq \bar{T}_d^4 + 4\bar{T}_d^3\Delta\bar{T}_\delta \\ \bar{q}_\delta^* &= q^*(\bar{T}_d) + \frac{dq^*}{dT}\Delta\bar{T}_\delta\end{aligned}$$

Consequently,

$$\begin{aligned}&[\lambda_3 + \frac{\delta}{k}\rho\tilde{C}_\theta C_\theta u^2 \frac{d\bar{\theta}}{dz} Q_R^{-1}](\epsilon^* \sigma \bar{T}_d^4 + \rho\tilde{C}_q u \bar{q}^*) - \rho\tilde{C}_\theta u(\lambda_3 - C_\theta u \frac{d\bar{\theta}}{dz} Q_R^{-1} \lambda_4) \frac{\theta'_a}{\tilde{\pi}} - \lambda_3 \lambda_Q \lambda_s \bar{R}_s \\ &+ \{[\lambda_3 + \frac{\delta}{k}\rho\tilde{C}_\theta C_\theta u^2 \frac{d\bar{\theta}}{dz} Q_R^{-1}](4\epsilon^* \sigma \bar{T}_d^3 + \rho\tilde{C}_q u \frac{dq^*}{dT}) + \rho\tilde{C}_\theta u(\lambda_3 - C_\theta u \frac{d\bar{\theta}}{dz} Q_R^{-1} \lambda_4) + \lambda_3 \lambda_Q \frac{C_d}{\tau_\delta}\} \Delta\bar{T}_\delta \\ &= 0\end{aligned}$$

or

$$\begin{aligned}\Delta\bar{T}_\delta &= -\{[\lambda_3 + \frac{\delta}{k}\rho\tilde{C}_\theta C_\theta u^2 \frac{d\bar{\theta}}{dz} Q_R^{-1}](4\epsilon^* \sigma \bar{T}_d^3 + \rho\tilde{C}_q u \frac{dq^*}{dT}) + \rho\tilde{C}_\theta u(\lambda_3 - C_\theta u \frac{d\bar{\theta}}{dz} Q_R^{-1} \lambda_4) \\ &+ \lambda_3 \lambda_Q \frac{C_d}{\tau_\delta}\}^{-1} \{[\lambda_3 + \frac{\delta}{k}\rho\tilde{C}_\theta C_\theta u^2 \frac{d\bar{\theta}}{dz} Q_R^{-1}](\epsilon^* \sigma \bar{T}_d^4 + \rho\tilde{C}_q u \bar{q}^*) - \rho\tilde{C}_\theta u(\lambda_3 - C_\theta u \frac{d\bar{\theta}}{dz} Q_R^{-1} \lambda_4) \frac{\theta'_a}{\tilde{\pi}} \\ &- \lambda_3 \lambda_Q \lambda_s \bar{R}_s\}\end{aligned}\quad (3.8)$$

where

$$\lambda_3 = \frac{\delta}{k}\rho\tilde{C}_\theta u + (1 + C_\theta u \frac{d\bar{\theta}}{dz} Q_R^{-1})\lambda_4 = 1.26 \quad (3.9)$$

Note that λ_3 inverse proportionally decreases with the increasing u_{*w} due to the L -dependence on the similarity function, $\phi(z/L)$ (Eq. 2.6d).

3.2b Perturbation Temperature of the Warm Layer

The coefficients, $T_{\delta j}$ ($j = 1, 2$) for the perturbation temperature, T'_δ , defined by Eq. (3.5b) are defined from Eqs. (3.6a, b) and (3.7a, b) by substituting the solutions for $\Delta\theta_{mj}$ ($j = 1, 2$) given later by Eq. (3.13b) as:

$$\begin{aligned}&\{\omega^2 [\frac{\rho\tilde{C}_\theta C_\theta u^2}{C_d} \frac{d\bar{\theta}}{dz} Q_R^{-1}]^2 + [\frac{1}{\tau_\delta^*} \frac{\rho\tilde{C}_\theta C_\theta u^2}{C_d} \frac{d\bar{\theta}}{dz} Q_R^{-1} - \lambda_3 \lambda_Q (\omega^2 + \frac{1}{\tau_\delta^{*2}})]^2\} T_{\delta 1} \\ &+ \lambda_3^2 \lambda_Q^2 (\omega^2 + \frac{1}{\tau_\delta^{*2}}) (\frac{\rho\tilde{C}_\theta C_\theta u^2}{C_d} \frac{d\bar{\theta}}{dz} Q_R^{-1} - \frac{1}{\tau_\delta^*}) \frac{\lambda_s S_0}{C_d} = 0 \\ &\{\omega^2 [\frac{\rho\tilde{C}_\theta C_\theta u^2}{C_d} \frac{d\bar{\theta}}{dz} Q_R^{-1}]^2 + [\frac{1}{\tau_\delta^*} \frac{\rho\tilde{C}_\theta C_\theta u^2}{C_d} \frac{d\bar{\theta}}{dz} Q_R^{-1} - \lambda_3 \lambda_Q (\omega^2 + \frac{1}{\tau_\delta^{*2}})]^2\} T_{\delta 2} \\ &+ \omega \lambda_3^2 \lambda_Q^2 (\omega^2 + \frac{1}{\tau_\delta^{*2}}) \frac{\lambda_s S_0}{C_d} = 0\end{aligned}$$

or

$$\begin{aligned} \left\{ \frac{\omega^2}{\tau_\theta^2} + \left[\frac{1}{\tau_\delta^* \tau_\theta} - \lambda_3 \lambda_Q \left(\omega^2 + \frac{1}{\tau_\delta^{*2}} \right) \right]^2 \right\} T_{\delta 1} + \lambda_3^2 \lambda_Q^2 \left(\omega^2 + \frac{1}{\tau_\delta^{*2}} \right) \left(\frac{1}{\tau_\theta} - \frac{1}{\tau_\delta^*} \right) \frac{\lambda_s S_0}{C_d} &= 0 \\ \left\{ \frac{\omega^2}{\tau_\theta^2} + \left[\frac{1}{\tau_\delta^* \tau_\theta} - \lambda_3 \lambda_Q \left(\omega^2 + \frac{1}{\tau_\delta^{*2}} \right) \right]^2 \right\} T_{\delta 2} + \omega \lambda_3^2 \lambda_Q^2 \left(\omega^2 + \frac{1}{\tau_\delta^{*2}} \right) \frac{\lambda_s S_0}{C_d} &= 0 \end{aligned}$$

where

$$\tau_\theta = \left[\frac{\rho \tilde{C}_\theta C_\theta u^2}{C_d} \frac{d\bar{\theta}}{dz} Q_R^{-1} \right]^{-1} = 1.12 \times 10^7 \text{ sec}$$

with the standard parameters. Thus,

$$\begin{aligned} T_{\delta 1} = & - \left\{ \omega^2 \left[\frac{\rho \tilde{C}_\theta C_\theta u^2}{C_d} \frac{d\bar{\theta}}{dz} Q_R^{-1} \right]^2 + \left[\frac{1}{\tau_\delta^*} \frac{\rho \tilde{C}_\theta C_\theta u^2}{C_d} \frac{d\bar{\theta}}{dz} Q_R^{-1} \right. \right. \\ & \left. \left. - \lambda_3 \lambda_Q \left(\omega^2 + \frac{1}{\tau_\delta^{*2}} \right) \right]^2 \right\}^{-1} \left\{ \lambda_3^2 \lambda_Q^2 \left(\omega^2 + \frac{1}{\tau_\delta^{*2}} \right) \left(\frac{\rho \tilde{C}_\theta C_\theta u^2}{C_d} \frac{d\bar{\theta}}{dz} Q_R^{-1} - \frac{1}{\tau_\delta^*} \right) \frac{\lambda_s S_0}{C_d} \right\} \quad (3.10a) \end{aligned}$$

$$\begin{aligned} T_{\delta 2} = & - \left\{ \omega^2 \left[\frac{\rho \tilde{C}_\theta C_\theta u^2}{C_d} \frac{d\bar{\theta}}{dz} Q_R^{-1} \right]^2 + \left[\frac{1}{\tau_\delta^*} \frac{\rho \tilde{C}_\theta C_\theta u^2}{C_d} \frac{d\bar{\theta}}{dz} Q_R^{-1} \right. \right. \\ & \left. \left. - \lambda_3 \lambda_Q \left(\omega^2 + \frac{1}{\tau_\delta^{*2}} \right) \right]^2 \right\}^{-1} \omega \lambda_3^2 \lambda_Q^2 \left(\omega^2 + \frac{1}{\tau_\delta^{*2}} \right) \frac{\lambda_s S_0}{C_d} \quad (3.10b) \end{aligned}$$

or

$$T_{\delta 1} = - \left\{ \frac{\omega^2}{\tau_\theta^2} + \left[\frac{1}{\tau_\delta^* \tau_\theta} - \lambda_3 \lambda_Q \left(\omega^2 + \frac{1}{\tau_\delta^{*2}} \right) \right]^2 \right\}^{-1} \left[\lambda_3^2 \lambda_Q^2 \left(\omega^2 + \frac{1}{\tau_\delta^{*2}} \right) \left(\frac{1}{\tau_\theta} - \frac{1}{\tau_\delta^*} \right) \frac{\lambda_s S_0}{C_d} \right] \quad (3.10a)$$

$$T_{\delta 2} = - \left\{ \frac{\omega^2}{\tau_\theta^2} + \left[\frac{1}{\tau_\delta^* \tau_\theta} - \lambda_3 \lambda_Q \left(\omega^2 + \frac{1}{\tau_\delta^{*2}} \right) \right]^2 \right\}^{-1} \left[\omega \lambda_3^2 \lambda_Q^2 \left(\omega^2 + \frac{1}{\tau_\delta^{*2}} \right) \frac{\lambda_s S_0}{C_d} \right] \quad (3.10b)$$

By further noting that $\omega \sim 1/\tau_\delta^* \gg 1/\tau_\theta$, thus all the terms containing $1/\tau_\theta$ can be dropped off from the above pair is in good approximations, and:

$$T_{\delta 1} = \left(\omega^2 + \frac{1}{\tau_\delta^{*2}} \right)^{-2} \left[\frac{1}{\tau_\delta^*} \left(\omega^2 + \frac{1}{\tau_\delta^{*2}} \right) \frac{\lambda_s S_0}{C_d} \right] \quad (3.10c)$$

$$T_{\delta 2} = \left(\omega^2 + \frac{1}{\tau_\delta^{*2}} \right)^{-2} \left[\omega \left(\omega^2 + \frac{1}{\tau_\delta^{*2}} \right) \frac{\lambda_s S_0}{C_d} \right] \quad (3.10d)$$

3.4 Atmospheric Boundary Layer

In seeking a solution of the atmospheric boundary layer, we focus on the equilibrium state of Eq. (2.7) for a practical reason that a full unsteady solution would involve sub-harmonic contributions, making the final expressions substantially more involved. This approximation is justified by the fact that a characteristic time scale for the adjustment of

the mixed layer to a given surface flux is relatively short, only about one hour, as shown in Ch. 12 of Yano (2023). The equilibrium solution for Eq. (2.7) is:

$$\Delta\theta_m = \overline{w'\theta'} \left(\frac{d\bar{\theta}}{dz} \right) Q_R^{-1} \quad (3.11)$$

Here, the eddy flux, $\overline{w'\theta'}$, is given in the two components by Eqs. (2.13a, b).

By further decomposing Eq. (3.11) into those two components:

$$\Delta\bar{\theta}_m = \overline{w'\theta'_0} \left(\frac{d\bar{\theta}}{dz} \right) Q_R^{-1} \quad (3.12a)$$

$$\Delta\theta'_m = \overline{w'\theta'} \left(\frac{d\bar{\theta}}{dz} \right) Q_R^{-1} \quad (3.12b)$$

Substitutions of Eqs. (2.13a, b) into Eq. (3.12a, b), respectively, and also taking into account of dependence on $\Delta\bar{\theta}_m$ and $\Delta\theta'_m$ on the resulting right-hand sides, we obtain:

$$\Delta\bar{\theta}_m = \frac{\tilde{\pi}C_\theta u}{\lambda_3} \frac{d\bar{\theta}}{dz} Q_R^{-1} \left[-\frac{\delta}{k} (\epsilon^* \sigma \bar{T}_\delta^4 + \rho \tilde{C}_q u \bar{q}_\delta^*) + \lambda_4 (\bar{T}_\delta - \frac{\theta_a}{\tilde{\pi}}) \right] \quad (3.13a)$$

$$\Delta\theta'_m = \frac{\tilde{\pi}C_\theta u}{\lambda_3} \frac{d\bar{\theta}}{dz} Q_R^{-1} T'_\delta \quad (3.13b)$$

Recall that λ_3 has been defined by (3.9), and $C_\theta u (d\bar{\theta}/dz) Q_R^{-1} = 0.259$. Thus, also noting that $\tilde{\pi}/\lambda_3 \simeq 1$, the amplitude of the diurnal variation of the atmospheric mixed layer becomes about a quarter of that of the ocean warm layer.

4. Analysis

We begin by re-iterating some of the basic parameter dependence remarked in the previous two sections, then proceed to more in-depth discussions on the feedback effects.

4.1 Feedback of the Skin Layer to the Surface Processes

The feedback to the skin layer to the total surface heat flux, Q , is presented as

$$\bar{Q}(T_{int}) = [1 + \alpha_f \bar{Q}(T_\delta)] \quad (4.1a)$$

$$Q'(T_{int}) = [1 + \alpha_f \bar{Q}'(T_\delta)] \quad (4.1b)$$

for the mean and the perturbation, respectively. Here, the feedback factor, α_f , is defined by

$$\alpha_f = -\frac{\delta}{k} (4\epsilon^* \sigma \bar{T}_d^3 + \rho \tilde{C}_\theta u + \rho \tilde{C}_q u \frac{dq^*}{dT}) \quad (4.2)$$

with the standard values are $\alpha_f = -6.18 \times 10^{-3}$ and $\alpha_f = -4.15 \times 10^{-3}$ for the moist and dry (without evaporation) cases, respectively. This feedback factor is related to the factor, λ_Q , defined by Eq. (2.10a), by $\lambda_Q = 1 - \alpha_f$. Here, this feedback factor is negligibly small, and we can safely set $\lambda_Q \simeq 1$: the skin layer does not have any significant control on the surface fluxes.

More generally, the interface (surface) temperature, T_{int} , is modified from the subskin temperature, T_δ , but the surface-flux cooling, Q , in the skin layer by

$$T_{int} = T_\delta - \frac{\delta}{k}Q$$

Thus, any tendency, $\mathcal{F}(T_{int})$, by a surface process, as a function of T_{int} is modified by the skin-layer feedback process into:

$$\mathcal{F}(T_{int}) = (1 + \alpha_{\mathcal{F}})\mathcal{F}(T_\delta) \quad (4.3a)$$

where $\alpha_{\mathcal{F}}$ is the general feedback rate defined by

$$\alpha_{\mathcal{F}} = -\frac{\delta}{k}\mathcal{F}^{-1}(T_\delta)\left[\frac{\partial\mathcal{F}}{\partial T}(T_\delta)\right]Q \quad (4.3b)$$

By applying this formula, the feedback rates for the sensible and latent heat fluxes are, respectively, given by

$$\alpha_{H_s} = -\frac{\delta}{k}\frac{Q}{\tilde{\pi}T_\delta - \bar{\theta}_m} \quad (4.4a)$$

$$\alpha_{H_l} = -\frac{\delta}{k}\frac{L_v Q}{T_\delta^2} \quad (4.4b)$$

Here, in Eq. (4.4b), we have used the relation $(dq^*/dT)/q^* = L_v/T^2$. An important implication of this relation is that although the saturated moisture increases exponentially with the temperature, its fractional rate of the increase with the temperature decreases with the increasing temperature. This relatively weak dependence on the temperature makes the feedback of the skin-layer effect to the evaporation rate makes weaker than intuitively expected.

Those estimated linear feedback rates are plotted in Fig. 1: A strong negative feedback rate, α_{H_s} , to the sensible heat less than -1 is found only with a very small temperature difference between the ocean and the atmosphere at the interface (Fig. 1(a)). It appears that the linear feedback is a reasonably good approximation when the temperature difference is larger than 0.5 K. Yet, this feedback is clearly significant.

However, a major caveat to keep in mind is that a stronger feedback with a smaller temperature difference, $T_\delta - \bar{\theta}_m/\tilde{\pi}$, is only in a relative sense, and the absolute change by feedback remains the same, because the sensible heat flux itself decreases linearly with the decreasing $T_\delta - \bar{\theta}_m/\tilde{\pi}$, perfectly compensating the increasing tendency of the feedback rate. Along this line, it can also be shown that the feedback of the skin layer to the mixed layer potential temperature anomaly, $\Delta\theta_m$, defined by Eq. (3.11) is also rather weak.

In contrast, the feedback rate, α_{H_l} , to the evaporation (Fig. 1(b)) is less than -1 over the full range considered, demonstrated that the feedback of the skin layer to the evaporation rate is so strong that it cannot be appropriately represented by the linear feedback rate defined here (Eq. 4.4b). This strong nonlinear feedback could also potentially

affect the other parts of the results. However, as it turns out, the strong feedback of the skin layer to the evaporation rate does not play a key role in any other parts.

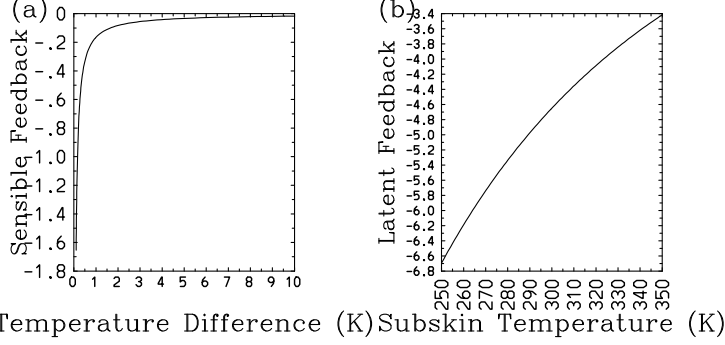


Fig. 1: Feedback rates, α , for (a) the sensible heat flux (Eq. 4.4a) and (b) the latent heat flux (Eq. 4.4b), respectively, as functions of the temperature difference, $\tilde{\pi}T_\delta - \bar{\theta}_m$, and the subskin temperature, T_δ . Here, we fix the total heat flux to be $Q = 100 \text{ W/m}^2$ for simplicity

4.2 Friction-Velocity Dependence

The subskin temperature anomaly is defined by Eq. (3.3b), in which $\tau_\delta/C_d = 4.45 \times 10^{-4} \text{ K/(W/m}^2\text{)}$ with $u_{*w} = 10^{-2}$, and $\lambda_\delta \simeq 1$ with $u = 1 \text{ m/s}$. Note however that the value τ_δ is very sensitive to the choice of the friction velocity, u_{*w} , because the former depends on the Obukhov length, L , which increases with the cubic of u_{*w} . Thus, we investigate this issue further now.

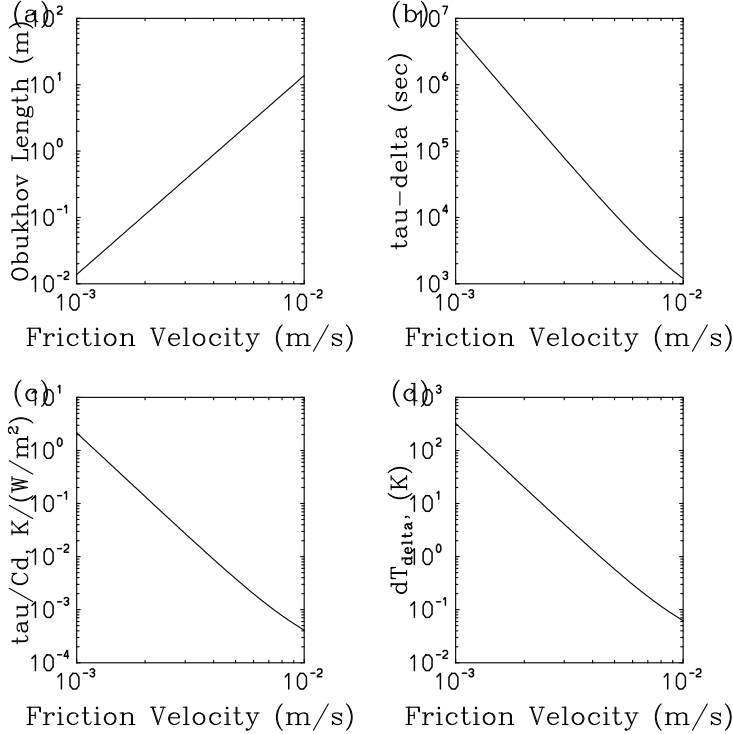


Fig. 2: As functions of the friction velocity, u_{*w} : (a) Obukhov length, L (m: Eq. 2.6e), (b) characteristic time, τ_δ (sec: Eq. 2.6c), (c) Respond rate of the subskin temperature, T_δ , to the surface, τ_δ/C_d (K/(W/m²): Eq. 3.3b),

and 9d) the subskin temperature anomaly, $\Delta\bar{T}_\delta$, estimated by Eq. (4.7). assuming $\theta_a = 299$ K and $\bar{T}_d = 300$ K.

As we find, the strongest sensitivity of the studied system is found with the ocean friction velocity, u_{*w} , which is expected to span over the range of 10^{-3} – 10^{-2} m/s. Over this range, the Obukhov length, L , defined by Eq. (2.6e) varies by the three order of magnitude (Fig. 2(a)). As a result, the characteristic response time, τ_δ , of the subskin temperature to the surface processes, as defined by Eq. (2.6c), decrease by the same order of magnitude (Fig. 2(b)): the response time, τ_δ , is much longer with the smaller frictional velocity, u_{*w} . With the smallest u_{*w} , it is much longer than 10 days, and with the largest u_{*w} , it is as short as 10^3 sec. This variation further reflects upon the projection factor, τ_δ/C_d , of the surface processes. As seen in Eq. (3.3b), the response of the subskin temperature, T_δ , to the surface processes is enhanced by this factor (Fig. 2(c)): the smaller the friction velocity (i.e., the weaker surface wind speed), the subskin temperature more sensitively responds to the surface processes.

4.3 Feedback of the Skin Layer to the Subskin Temperature

The feedback of the skin layer to the subskin temperature can be seen by examining Eq. (3.8). This equation can be much simplified by introducing various approximations. First recall $\lambda_Q \simeq \lambda_4 \simeq 1$. However, keep in mind that $\lambda_3 = 1.26$ significantly deviating from the unity. We further note that typically $C_\theta u(d\bar{\theta}/dz)Q_R^{-1} = 0.259$, thus we can set $(\delta/k)C_\theta u(d\bar{\theta}/dz)Q_R^{-1} \simeq 0$. We also note that $\lambda_3 - C_\theta u(d\bar{\theta}/dz)Q_R^{-1}\lambda_4 \simeq 1$. As a result, Eq. (3.8) is simplified into:

$$\Delta\bar{T}_\delta \simeq \frac{\tau_\delta}{C_d}(\Delta Q_s - \frac{\delta}{k}Q_{R+l}) \quad (4.5)$$

where

$$\Delta Q_s = \lambda_s \bar{R}_s - Q_{R+l} \quad (4.6a)$$

$$Q_{R+l} = \epsilon^* \sigma \bar{T}_d^4 + \rho \tilde{C}_q u \bar{q}^* \quad (4.6b)$$

Thus, so long as $\Delta Q_s \sim Q_{R+l}$, the feedback of the skin layer to the mean subskin temperature, $\Delta\bar{T}_\delta$, is negligible. Here, under the standard parameters, $\lambda_s \bar{R}_s = 287$ W/m², $Q_{R+l} = 136$ W/m², $\Delta Q_s = 151$ W/m².

Thus, it is at the regime just described, and the solar heating dominates to heat the warm layer, leading to the approximation:

$$\Delta\bar{T}_\delta \simeq \frac{\tau_\delta}{C_d} \Delta Q_s \quad (4.7)$$

However, with the standard value of $u_{*w} = 10^{-2}$ m/s, it only leads to a weak response with $\Delta\bar{T}_\delta = 6.21 \times 10^{-2}$ K. As already pointed out in discussing Fig. 2(a)–(c), the response of the warm layer to the solar heating is extremely sensitive to the friction velocity, u_{*w} , and only by halving the above value, the subskin temperature anomaly, $\Delta\bar{T}_\delta$ increase by factor 10. With a relatively calm state with $u_{*w} = 3 \times 10^{-3}$ m/s, we obtain $\Delta\bar{T}_\delta = 4.1$ K,

corresponding to typically quoted temperature at the top of the warm layer (e.g., Fig. 1(a) of Webster *et al.* 1996). Yet, it should be emphasized that at the normal state, as estimated above, the warm layer anomaly is extremely weak, as also known in the literature (e.g., Fig. 1(b) of Webster *et al.* 1996). Dependence of $\Delta\bar{T}_\delta$ on u_{*w} over the full range is shown in Fig. 2(d).

Nevertheless, it is possible to envision a particular situation in which the skin layer controls the subskin temperature: under a cloudy (overcast) situation with a smaller factor, λ_2 , when the solar heating well balances with the compensating upward heat flux, *i.e.*, $\Delta Q_s \simeq 0$. In this case, the second term may dominates in Eq. (4.5). However, the resulting heating tendency is relatively weak and $(\delta/k)Q_{R+l} \sim 10^{-1}$ W/m² with $\delta/k \sim 10^{-3}$ m²K/W and $Q_{R+l} \sim 10^2$ W/m².

4.4 Diurnal Cycle

The diurnal cycle of the warm layer is presented by Eq. (3.5b) with the coefficients, $T_{\delta j}$ ($j = 1, 2$), determined by Eqs. (3.10c, d) in good approximations, is evaluated, and the results are plotted in Fig. 3. Here, for the further simplification, we set $\tau_\delta^* \simeq \tau_\delta$, although the approximation gradually breaks down for smaller frictional velocities. The response amplitude, $(T_{\delta 1}^2 + T_{\delta 2}^2)^{1/2}$ (Fig. 3(a): short dash), asymptotically approaches to 4.2 K, and the lag of the warm-response (Fig. 3(b)) asymptotically approaches to a perfect off phase as $u_{*w} \rightarrow 0$. A typically observed response amplitude about 3 K is obtained with $u_{*w} \simeq 4 \times 10^{-3}$ m/s with the phase lag of about 4 hours.²⁾

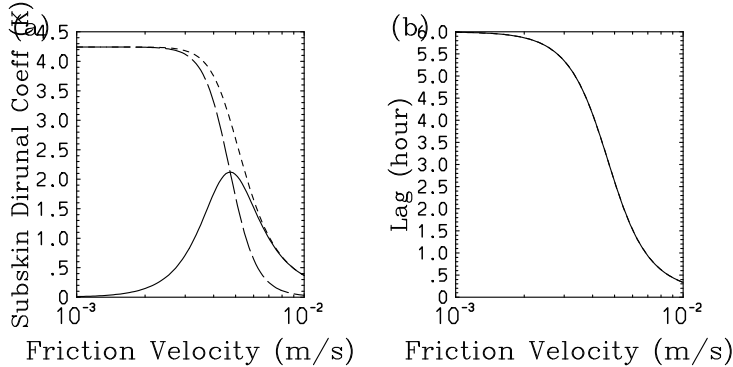


Fig. 3: The diurnal cycle of the subskin temperature, as described by Eq. (3.5b): (a) the coefficients in phase and off phase with the solar cycle, $T_{\delta 1}$ (solid) and $T_{\delta 2}$ (long dash). Also plotted is the amplitude, $(T_{\delta 1}^2 + T_{\delta 2}^2)^{1/2}$ of the cycle. (b) The phase lag of the subskin temperature response in hours.

As already remarked earlier in Sec. 3.4, the response of the atmospheric mixed layer is only about a quarter of the warm-layer response, substantially less significant.

5. Summary

5.1 Skin Layer

A cold anomaly of the ocean skin temperature can significantly influences negatively both the sensible and latent heat fluxes, as naively expected and also argued in the literature (Ref!), when the effects are measured by the fractional feedback rates. That is

²⁾ Hugo: est-ce-que tu pense que le resultat ici est coherent avec l'observations?

especially the case with the latent heat flux, when this rate is evaluated with an assumption of the fixed relative humidity, as herein. However, in reality, this tendency is expected to be much compensated by an active response of the atmosphere diminishing the relative humidity as the latent heat flux suppressed.

Moreover, the evolution of the mixed layer is in joint consort of the evolution of the potential temperature and the moisture, maintaining the identical mixed layer depth. Thus, a very strong negative feedback found for the latent heat flux with a simple method would be reduced to the same level as that of the sensible heat flux by various adjustment processes.

The feedback rate to the sensible heat flux is also significant, but only to the limit of the vanishing temperature difference between the ocean and the atmosphere at the interface, and only in a relative sense. Towards this limit, the sensible heat flux also tends to vanish, thus the absolute magnitude of the skin-layer feedback becomes vanishingly small. Thus, against the native anticipations, the skin-layer temperature does not provide significant impacts to the atmospheric mixed layer.

Overall, in the atmosphere-ocean coupled boundary layer system considered herein, the skin layer is not found to play any significant role. The main reason is attributed to the thinness of the skin layer, only about $\delta \sim 1$ mm thick, and a relative effectiveness of the diffusive transport rate, as measured by the thermal conductivity, k . The impact of the skin layer to the other components of the system is measured by the ratio of those two quantities, which is $\delta/k \sim 10^{-3}$ m²K/W, *i.e.*, an increase of the heat flux by 1 W/m² only leads to a change of the temperature by 10^{-3} K. Throughout our investigation, we have simply failed to identify any process that can enhance this factor dramatically.

5.2 Warm Layer

A similar conclusion also follows with the warm layer, immediately below the skin layer. Under a normal state of the ocean surface layer with an expected value of a typical ocean friction velocity, $u_{*w} \simeq 10^{-2}$ m/s, the warm layer does not develop in any significant manner.

However, the main difference of the warm layer from the skin layer is that the characteristic evolution of the latter is extremely sensitive to the value of the friction velocity, u_{*w} . This sensitivity arises from the fact that the mixing efficiency estimated by the Monin-Obukhov similarity theory is scaled by the Obukhov length. The latter increases with the cubic of u_{*w} . A smaller Obukhov length suggests a longer mixing time scale, as measured by τ_δ defined by Eq. (2.6c).

Thus, under a clam situation with a smaller friction velocity, a substantial heat anomaly can be developed in the warm layer due to the ineffectiveness of the vertical mixing. Importantly, only a slightly smaller friction velocity is required to see a significant effect due to its strong sensitivity on the mixing efficiency, as demonstrated by Fig. 2.

5.3 Diurnal Cycle

The characteristics of the diurnal cycle in the warm layer also sensitively depends on the adjustment time scale, τ_δ . More precisely, the response of the warm layer to the diurnal solar forcing is defined by a competition between the two time scales: the response time scale, τ_δ , and the diurnal-cycle scale, as seen from Eqs. (3.10c, d). When the friction velocity, u_{*w} , is reasonably large, the response time, τ_δ , is short enough that the warm

layer immediately responds to the solar forcing only with a small lag (*cf.*, Fig. 3(b)), but with a weak response (*cf.*, Fig. 3(a)). As u_{*w} decreases, the response time, τ_δ , increases, and the diurnal response of the warm layer lags behind longer, and to the asymptotic limit of $u_{*w} \rightarrow 0$, the response approaches totally off phase, but with the response amplitude approaches to a constant value, 4.3 K. The result with an intermediate value of $u_{*w} = 5 \times 10^{-3}$ m/s appears to provide a consistent result with observations of the response amplitude 2.0 K with a lag of 2 hours.

5.4 Remaining Issues and the Perspectives

The present note has examined the behavior of the skin and warm layers and their impacts on the atmospheric mixed layer over the tropical oceans with a simple atmosphere–ocean coupled boundary–layer system. Although the model is quite simple, an effort is made to include all the essential basic physics. Yet, some of the physical parameters have been fixed for simplicity, including the thickness of the skin layer, which is fixed to 1 mm in this study.

The main external parameters to be specified in this system is the reference atmospheric potential temperature, θ_a , as introduced in Eq. (2.8b), and the reference temperature, \bar{T}_d , of the deep ocean, defined at the bottom of the warm layer. Since those two key parameters cannot be determined by the given model, no attempt has been made in the present analysis, for example, to define a reference eddy heat flux,

$$\overline{\theta'w'} = \tilde{\pi} C_\theta u (\bar{T}_\delta - \frac{\bar{\theta}_m}{\tilde{\pi}}).$$

Being linearly depending on the unspecified difference, $\bar{T}_\delta - \bar{\theta}_m/\tilde{\pi}$, the reference flux can take any value by simply adjusting the latter. This is clearly a major limitation of the present model formulation; to obtain the reference atmospheric value, θ_a , we need to specify the radiation at the top of the atmosphere. To do this part more consistently, a full radiation calculation would be required. Defining the deep–ocean temperature is even less obvious.

The present study based on a simple model, but containing all the essential physics, has essentially excluded a possibility that the skin layer has a significant impact on the atmospheric state. More precisely, the estimated relative feedback rate to the moisture eddy flux is rather large. Yet, we simply discounted this result based on the fact that this strong feedback must be curbed down by a requirement of a consistency of a simple mixed layer model with the weak feedback expected for the dry component. This constraint would be removed, when clouds are formed at the top of the well–mixed layer, and we may find a strong skin–layer feedback to the moisture field significantly affects the evolution of the boundary layer top clouds. Inclusion of the top cloud would be an important next step to consider.

Appendix: Some Useful Formulas

$$\begin{aligned}
\int_0^t e^{t/\tau_\delta^*} dt &= \tau_\delta^* (e^{t/\tau_\delta^*} - 1) \\
\int_0^t \sin \omega t e^{t/\tau_\delta^*} dt &= \frac{1}{\omega^2 + 1/\tau_\delta^{*2}} [(-\omega \cos \omega t + \frac{1}{\tau_\delta^*} \sin \omega t) e^{t/\tau_\delta^*} + \omega] \\
\int_0^t \cos \omega t e^{t/\tau_\delta^*} dt &= \frac{1}{\omega^2 + 1/\tau_\delta^{*2}} [(\omega \sin \omega t + \frac{1}{\tau_\delta^*} \cos \omega t) e^{t/\tau_\delta^*} - \frac{1}{\tau_\delta^*}]
\end{aligned}$$

Friction Velocity:

According to Price (1979), the surface wind stress is typically, $\tau \sim 1 \text{ dyn/cm}^2 \sim 10^{-1} \text{ N/m}^2$. Thus, a typical friction velocity for the atmosphere is $u_{*a} = (\tau/\rho)^{1/2} \sim 0.3 \text{ m/s}$, and for the ocean is $u_{*w} = (\tau/\rho_w)^{1/2} \sim 10^{-2} \text{ m/s}$.

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