



HAL
open science

Coordination Coding with Causal Encoder for Vector-Valued Witsenhausen Counterexample

Mengyuan Zhao, Maël Le Treust, Tobias Oechtering

► **To cite this version:**

Mengyuan Zhao, Maël Le Treust, Tobias Oechtering. Coordination Coding with Causal Encoder for Vector-Valued Witsenhausen Counterexample. ISIT 2024 - IEEE International Symposium on Information Theory, Jul 2024, Athens, Greece. pp.3255-3260, 10.1109/ISIT57864.2024.10619377 . hal-04679217

HAL Id: hal-04679217

<https://hal.science/hal-04679217v1>

Submitted on 27 Aug 2024

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



Distributed under a Creative Commons Attribution 4.0 International License

Coordination Coding with Causal Encoder for Vector-valued Witsenhausen Counterexample

Mengyuan Zhao*, Maël Le Treust† and Tobias J. Oechtering*

*KTH Royal Institute of Technology, EECS, Div. ISE, 100 44 Stockholm, Sweden, Email: {mzhao, oech}@kth.se

†Univ. Rennes, CNRS, Inria, IRISA UMR 6074, F-35000 Rennes, France, Email: mael.le-treust@cnrs.fr

Abstract—We investigate the Witsenhausen counterexample in a continuous vector-valued context with a causal encoder and noncausal decoder. Our main result is the optimal single-letter condition that characterizes the set of achievable Witsenhausen power costs and estimation costs, leveraging a modified weak typicality approach. In particular, we accommodate our power analysis to the causal encoder constraint, and provide an improved distortion error analysis for the challenging estimation of the interim state. Interestingly, the idea of dual role of control is explicitly captured by the two auxiliary random variables.

I. INTRODUCTION

In 1968, Witsenhausen proposed a famous counterexample [1] illustrating the challenges posed by non-classical information pattern, for which affine functions are strictly suboptimal in LQG setting. Nowadays, it still plays an important role in contemporary distributed decision making [2]–[4], and information-theoretic control fields [5]–[11].

Recent advancements in studying the Witsenhausen counterexamples have been driven by information theory and coding considering its vector-valued extensions. This enables the adoption of advanced coding techniques such as [12]–[14]. Pioneering this approach, Grover et al. explored a setting in which both decision-makers are assumed to be non-causal [15].

Empirical coordination, [16]–[19], to mention a few, has emerged as a powerful tool in characterizing the cost and capacity trade-off for state-dependent channels when the cost function jointly evaluates the source symbol and the channel input symbol. This method has shown its effectiveness in causal and non-causal encoding and decoding frameworks, such as [20, 21], and even in distributed decision making problem with power control [22]. Utilizing these insights, [23] provides the optimal characterizations for finite alphabet vector Witsenhausen problems with causal and non-causal decision-makers. Moreover, an achievability proof based on empirical coordination coding [24] for the non-causal encoding and causal decoding setup processing Gaussian states [25] is presented in [26, 27].

Diverging from the setting in [27], this paper explores the scenario where the first decision-maker encodes the state sequence causally, while the subsequent one estimates the interim state non-causally. Since some random variables are continuous, a modified version of weak typicality [28] is required in order to obtain a desired Markov Lemma-like result, as done in [29]. Moreover, to implement the Witsenhausen

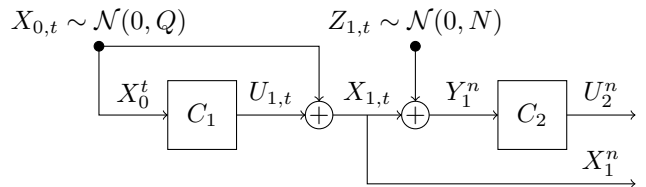


Fig. 1. The i.i.d. state and the channel noise are drawn according to Gaussian distributions $X_0^n \sim \mathcal{N}(0, Q\mathbb{I})$ and $Z_1^n \sim \mathcal{N}(0, N\mathbb{I})$.

cost constraints, we impose an average power constraint and provide a simplified approach to analyse the distortion cost. The converse proof validates the optimality of our set of the achievable pair of costs.

II. SYSTEM MODEL

Our vector-valued Witsenhausen setup consists of *causal* source states and channel noises that are drawn independently according to the i.i.d. Gaussian distributions $X_0^n \sim \mathcal{N}(0, Q\mathbb{I})$ and $Z_1^n \sim \mathcal{N}(0, N\mathbb{I})$, for some $Q, N > 0$. We denote by X_1 the memoryless interim state and Y_1 and output of the memoryless additive channel, generated by

$$X_1 = X_0 + U_1 \quad \text{with } X_0 \sim \mathcal{N}(0, Q), \quad (1)$$

$$Y_1 = X_1 + Z_1 = X_0 + U_1 + Z_1 \quad \text{with } Z_1 \sim \mathcal{N}(0, N). \quad (2)$$

We denote by $\mathcal{P}_{X_0} = \mathcal{N}(0, Q)$ the generative Gaussian probability distribution of the state, and by $\mathcal{P}_{X_1 Y_1 | X_0 U_1}$ the channel probability distribution according to (1) and (2). Work [27] considered noncausal encoder paired with causal decoder. In this work, we consider a setup with flipped causality:

Definition II.1. For $n \in \mathbb{N}$, a “control design” with causal encoder and noncausal decoder is a tuple of stochastic functions $c = (\{f_{U_{1,t}|X_0^t}^{(t)}\}_{t=1}^n, g_{U_2^n|Y_1^n})$ defined by

$$f_{U_{1,t}|X_0^t}^{(t)} : \mathcal{X}_0^t \longrightarrow \mathcal{U}_1, \quad g_{U_2^n|Y_1^n} : \mathcal{Y}_1^n \longrightarrow \mathcal{U}_2^n, \quad (3)$$

which induces a distribution over sequences of symbols:

$$\prod_{t=1}^n \mathcal{P}_{X_{0,t}} \prod_{t=1}^n f_{U_{1,t}|X_0^t}^{(t)} \prod_{t=1}^n \mathcal{P}_{X_{1,t}, Y_{1,t} | X_{0,t}, U_{1,t}} g_{U_2^n | Y_1^n}. \quad (4)$$

We denote by $\mathcal{C}_e(n)$ the set of control designs with causal encoder and non-causal decoder.

Definition II.2. We define the two long-run cost functions $c_P(u_1^n) = \frac{1}{n} \sum_{t=1}^n (u_{1,t})^2$ and $c_S(x_1^n, u_2^n) = \frac{1}{n} \sum_{t=1}^n (x_{1,t} -$

$u_{2,t})^2$. The pair of costs $(P, S) \in \mathbb{R}^2$ is achievable if for all $\varepsilon > 0$, there exists $\bar{n} \in \mathbb{N}$ such that for all $n \geq \bar{n}$, there exists a control design $c \in \mathcal{C}_e(n)$ such that

$$\mathbb{E}\left[|P - c_P(U_1^n)| + |S - c_S(X_1^n, U_2^n)|\right] \leq \varepsilon. \quad (5)$$

Remark 1. Equation (5) is a stronger requirement than

$$|P - \mathbb{E}[c_P(U_1^n)]| + |S - \mathbb{E}[c_S(X_1^n, U_2^n)]| \leq \varepsilon, \quad (6)$$

which was considered in [27]. Here, both notions lead to the same expression for the single-letter solution.

Theorem II.3 (Main result). The pair of Witsenhausen costs (P, S) is achievable if and only if there exists a joint distribution over the random variables $(X_0, W_1, W_2, U_1, X_1, Y_1, U_2)$ that decomposes according to

$$\mathcal{P}_{X_0} \mathcal{P}_{W_1} \mathcal{P}_{W_2|X_0, W_1} \mathcal{P}_{U_1|X_0, W_1} \mathcal{P}_{X_1, Y_1|X_0, U_1} \mathcal{P}_{U_2|W_1, W_2, Y_1}, \quad (7)$$

such that

$$I(W_1, W_2; Y_1) - I(W_2; X_0|W_1) \geq 0, \quad (8)$$

$$P = \mathbb{E}[U_1^2], \quad S = \mathbb{E}[(X_1 - U_2)^2], \quad (9)$$

where \mathcal{P}_{X_0} and $\mathcal{P}_{X_1, Y_1|X_0, U_1}$ are given Gaussian distributions, and W_1, W_2 are two auxiliary random variables.

Remark 2. The probability distribution (7) satisfies:

$$\begin{cases} X_0 \text{ is independent of } W_1, \\ U_1 \text{ } \textcircled{-} (X_0, W_1) \text{ } \textcircled{-} W_2, \\ (X_1, Y_1) \text{ } \textcircled{-} (X_0, U_1) \text{ } \textcircled{-} (W_1, W_2), \\ U_2 \text{ } \textcircled{-} (W_1, W_2, Y_1) \text{ } \textcircled{-} (X_0, U_1, X_1). \end{cases} \quad (10)$$

The first two properties are derived as consequences of causal encoding, where the achievability proof requires the Shannon strategy [30] for generating new codewords to the channel. The third Markov chain is due to the memoryless property of the channel and the processing order. The last Markov chain comes from the non-causal decoding and the symbol-wise reconstruction.

Remark 3. The information constraint (8) reformulates as

$$I(W_1, W_2; X_0) \leq I(W_1, W_2; Y_1).$$

The LHS corresponds to the lossy source coding involved in the quantization of X_0 using (W_1, W_2) . The RHS stands for the channel coding rate of outputting Y_1 given the input pair (W_1, W_2) . In particular, W_1 assists in enabling reliable communication, while W_2 is used to coordinate with the state.

The achievability proof of Theorem II.3 is presented in Section III and the converse proof is in Section IV.

III. ACHIEVABILITY PROOF OF THE MAIN RESULT

Preliminaries: We define the entropy and the weakly typical set for random vectors that combine discrete and continuous components as in [27]. We consider an arbitrary but fixed $\varepsilon > 0$ and assume the sequence $(X_0^n, W_1^n, W_2^n, U_1^n, X_1^n, Y_1^n, U_2^n)$ is generated i.i.d. according to a distribution that decomposes

as (7), with $P = \mathbb{E}[U_1^2]$, $S = \mathbb{E}[(X_1 - U_2)^2]$. Let $\psi^{(n)} : \mathcal{X}_0^n \times \mathcal{W}_1^n \times \mathcal{W}_2^n \times \mathcal{U}_1^n \times \mathcal{X}_1^n \times \mathcal{Y}_1^n \times \mathcal{U}_2^n \rightarrow \{0, 1\}$ denote an indicator function for sequences of length n with

$$\psi^{(n)}(x_0^n, w_1^n, w_2^n, u_1^n, x_1^n, y_1^n, u_2^n) = \begin{cases} 1 & \text{if } |c_S(x_1^n, u_2^n) - S| \geq \frac{1}{12}\varepsilon \\ & \text{or } (w_1^n, w_2^n, y_1^n) \notin \mathcal{A}_\varepsilon^n(W_1, W_2, Y_1), \\ 0 & \text{otherwise.} \end{cases}$$

Using the weak law of large numbers (LLN) and the union bound we have

$$\mathbb{E}[\psi^{(n)}(X_0^n, W_1^n, W_2^n, U_1^n, X_1^n, Y_1^n, U_2^n)] \leq \delta_n \rightarrow 0,$$

as $n \rightarrow \infty$. And similarly, we define the set

$$\mathcal{S}_\varepsilon^{(n)} = \{(x_0^n, w_1^n, w_2^n) \mid \eta^{(n)}(x_0^n, w_1^n, w_2^n) \leq \sqrt{\delta_n}\},$$

where $\eta^{(n)}(x_0^n, w_1^n, w_2^n) = \mathbb{E}[\psi^{(n)}(x_0^n, w_1^n, w_2^n, U_1^n, X_1^n, Y_1^n, U_2^n) \mid X_0^n = x_0^n, W_1^n = w_1^n, W_2^n = w_2^n]$. We take $\mathcal{B}_\varepsilon^{(n)} = \mathcal{A}_\varepsilon^{(n)} \cap \mathcal{S}_\varepsilon^{(n)}$. Then, we can easily prove the subsequent lemma:

Lemma III.1. Let the sequence (X_0^n, W_1^n, W_2^n) i.i.d. $\sim \mathcal{P}_{X_0, W_1, W_2}$, then

$$\mathbb{P}\left((X_0^n, W_1^n, W_2^n) \in \mathcal{B}_\varepsilon^{(n)}\right) \xrightarrow{n \rightarrow \infty} 1.$$

Proof of Lemma III.1. From the Markov inequality we obtain

$$\mathbb{P}\left((X_0^n, W_1^n, W_2^n) \notin \mathcal{S}_\varepsilon^{(n)}\right) \leq \frac{\delta_n}{\sqrt{\delta_n}} = \sqrt{\delta_n}. \quad \square$$

Lemma III.1 elaborates that the certainty of an outcome being jointly weakly typical AND satisfying the distortion cost constraint is guaranteed, as per the AEP, if it is generated i.i.d.

Moreover, it is also necessary to quantize the output of the non-causal decoder, as done in [27], to ensure that the second cost constraint remains bounded in case an error happens. Given a joint distribution with $\mathbb{E}[(X_1 - U_2)^2] = S$, for any $\hat{\delta} > 0$ there exists a quantization $q_{U_2} : \mathcal{U}_2 \rightarrow \{\hat{u}_{2,k}\}_{k=1}^{N_{U_2}}$, as explained in [31], such that

$$\hat{S} = \mathbb{E}[(X_1 - q_{U_2}(U_2))^2] \leq (1 + \hat{\delta})S,$$

in particular $\hat{\delta}S < \frac{1}{4}\varepsilon$.

Now, we provide a coding scheme by considering a probability distribution $\mathcal{P}_{X_0, W_1, W_2, U_1, X_1, Y_1, U_2}$ that decomposes as (7) such that there exists a $\delta > 0$, and a rate $R > 0$, satisfying $I(W_2; X_0|W_1) + \delta \leq R \leq I(W_1; Y_1) + I(W_2; Y_1|W_1) - \delta$. We consider a block-Markov code $c \in \mathcal{C}_e(nB)$ defined over $B \in \mathbb{N}$ blocks of length n , similar to the scheme in [14].

Random codebook generation: We generate $|\mathcal{M}| = 2^{nR}$ sequences $W_1^n(m)$ i.i.d. $\sim \mathcal{P}_{W_1}$ with index $m \in \mathcal{M}$. For each index $m \in \mathcal{M}$, we generate the same number $|\mathcal{M}| = 2^{nR}$ sequences $W_2^n(m, \hat{m})$ with index $\hat{m} \in \mathcal{M}$ i.i.d. $\sim \mathcal{P}_{W_2|W_1}$ depending on sequence $W_1^n(m)$.

Encoding function: Due to the causal feature, the encoder performs *forward encoding*, where the encoder outputs the information associated with the *past* block. Let m_b denote

the message generated during block $b \in [1 : B]$. During the first block, without loss of generality, the encoder takes $m_1 = 1$ and returns $W_1^n(m_1)$. At the beginning of block $b \in [2 : B]$, the encoder has observed the full sequence of source symbols $X_{0,b-1}^n \in \mathcal{X}^n$ of the previous block. It recalls the index $m_{b-1} \in \mathcal{M}$ of the sequence $W_1^n(m_{b-1})$ used for block $b-1$. It finds an index m_b such that sequences $(X_{0,b-1}^n, W_1^n(m_{b-1}), W_2^n(m_{b-1}, m_b)) \in \mathcal{B}_\varepsilon^{(n)}(\mathcal{P}_{X_0, W_1, W_2})$ are jointly typical. It returns $W_1^n(m_b)$ corresponding to the current block b , and for each time $t \in [1 : n]$, it sends the symbol $U_{1,t,b}$ i.i.d. $\sim \mathcal{P}_{U_1|X_0, W_1}$ depending on $W_{1,t}(m_b)$ and $X_{0,t,b}$ causally observed in the current block $b \in [1 : B]$.

Decoding function: The decoder first returns $\tilde{m}_1 = 1$. During block $b \in [2 : B]$, the decoder recalls the past sequence $Y_{1,b-1}^n$ and the index \tilde{m}_{b-1} that corresponds to the sequence $\tilde{W}_{1,b-1}^n = W_1^n(\tilde{m}_{b-1})$. It observes the channel output $Y_{1,b}^n$ and finds the unique index \tilde{m}_b such that $(Y_{1,b}^n, W_1^n(\tilde{m}_b)) \in \mathcal{A}_\varepsilon^{(n)}(\mathcal{P}_{Y_1, W_1})$ and $(Y_{1,b-1}^n, W_1^n(\tilde{m}_{b-1}), W_2^n(\tilde{m}_{b-1}, \tilde{m}_b)) \in \mathcal{A}_\varepsilon^{(n)}(\mathcal{P}_{Y_1, W_1, W_2})$. We denote by $\tilde{W}_{1,b}^n = W_1^n(\tilde{m}_b)$ and $\tilde{W}_{2,b-1}^n = W_2^n(\tilde{m}_{b-1}, \tilde{m}_b)$ as our choice.

Forward transmission of the decoder: Upon receiving all sequences for all first $(B-1)$ blocks, the decoder non-causally generates $U_{2,b}^n \sim \mathcal{P}_{U_2|Y_1, W_1, W_2}^{\otimes n}$ depending on sequences $(Y_{1,b}^n, \tilde{W}_{1,b}^n, \tilde{W}_{2,b}^n)$ for $b \in [1 : B-1]$. Finally, the decoder transmits the quantized sequence $\hat{U}_{2,b}^n$ as the reconstruction for the interim state $X_{1,b}^n$.

Termination block B: As for the last block, the decoder simply outputs an all zero sequence, i.e., $\hat{U}_{2,B}^n = \mathbf{0}$. Usually, sequences are *not* jointly typical in the last block. Therefore we omit its error analysis in the subsequent part.

Error analysis per block: We first focus on the encoding error \mathcal{E}^e . For $b \in [2 : B]$, let $\mathcal{E}_b^e(m_{b-1})$ denote the event of a failed encoding process during block b given the knowledge of m_{b-1} , i.e., $\mathcal{E}_b^e(m_{b-1}) = \{\forall m_b \in \mathcal{M}, (X_{0,b-1}^n, W_1^n(m_{b-1}), W_2^n(m_{b-1}, m_b)) \notin \mathcal{B}_\varepsilon^{(n)}(\mathcal{P}_{X_0, W_1, W_2})\}$. Due to the independence between the codewords and Markov blocks, the probability of an encoding error in block b given no encoding errors in the previous blocks is

$$\mathbb{P}(\mathcal{E}_b^e(M_{b-1}) \mid \cap_{\beta=2}^{b-1} \bar{\mathcal{E}}_\beta^e(M_{\beta-1})) = \mathbb{P}(\mathcal{E}_b^e(M_{b-1})).$$

If $R \geq I(W_2; X_0|W_1) + \delta$, following the Covering Lemma [32] and Lemma III.1, we have $\mathbb{P}(\mathcal{E}_b^e(M_{b-1})) \rightarrow 0$ as $n \rightarrow \infty$, $\forall b \in [2 : B]$. Thus, state sequences for the first $B-1$ blocks are successfully encoded with probability $\rightarrow 1$ as $n \rightarrow \infty$.

Next, we analyze the decoding error \mathcal{E}^d . For $b \in [2 : B]$, let $\mathcal{E}_b^d(m_{b-1})$ denote the event of a failed decoding process in block b given the past estimated index m_{b-1} . Furthermore, let $\mathcal{E}_{b-1}^{d,t,1}(m_{b-1})$ denote the event that the sequence $Y_{1,b-1}^n$ given m_{b-1} is not jointly typical, i.e., $\mathcal{E}_{b-1}^{d,t,1}(m_{b-1}) = \{(Y_{1,b-1}^n, W_{1,b-1}^n(m_{b-1}), W_2^n(m_{b-1}, M_b)) \notin \mathcal{A}_\varepsilon^{(n)}(\mathcal{P}_{Y_1, W_1, W_2})\}$ and let $\mathcal{E}_b^{d,t,2}$ denote that the current sequence $Y_{1,b}^n$ is not jointly typical, i.e., $\mathcal{E}_b^{d,t,2} = \{(Y_{1,b}^n, W_1^n(M_b)) \notin \mathcal{A}_\varepsilon^{(n)}(\mathcal{P}_{Y_1, W_1})\}$. Note that, if a sequence

with fewer terms $(Y_{1,b}^n, W_1^n(m_b))$ is atypical, it implies that a sequence with more terms $(Y_{1,b}^n, W_1^n(m_b), W_2^n(m_b, m_{b+1}))$ is also atypical. Therefore, we have $\mathcal{E}_b^{d,t,2} \subset \mathcal{E}_b^{d,t,1}(M_b)$. Then, the decoding error probability given no past decoding error or encoding error $\mathbb{P}(\mathcal{E}_b^d(m_{b-1}) \mid \cap_{\beta=2}^{b-1} \bar{\mathcal{E}}_\beta^d(m_{\beta-1}) \cap \bar{\mathcal{E}}^e)$ can be upperbounded by

$$\begin{aligned} & \mathbb{P}(\mathcal{E}_b^d(m_{b-1}) \mid \cap_{\beta=2}^{b-1} \bar{\mathcal{E}}_\beta^d(m_{\beta-1}) \cap \bar{\mathcal{E}}^e \cap \bar{\mathcal{E}}_{b-1}^{d,t,1}(m_{b-1}) \cap \bar{\mathcal{E}}_b^{d,t,2}) \\ & + \mathbb{P}(\mathcal{E}_{b-1}^{d,t,1}(m_{b-1}) \cup \mathcal{E}_b^{d,t,2} \mid \cap_{\beta=2}^{b-1} \bar{\mathcal{E}}_\beta^d(m_{\beta-1}) \cap \bar{\mathcal{E}}^e) \end{aligned} \quad (11)$$

using the union bound. The first term of (11) can be upperbounded by

$$\begin{aligned} & \mathbb{P}(\mathcal{E}_b^d(m_{b-1}) \mid \cap_{\beta=2}^{b-1} \bar{\mathcal{E}}_\beta^d(m_{\beta-1}) \cap \bar{\mathcal{E}}^e \cap \bar{\mathcal{E}}_{b-1}^{d,t,1}(m_{b-1}) \cap \bar{\mathcal{E}}_b^{d,t,2}) \\ & = \mathbb{P}(\exists m' \neq M_b, \text{ s.t. } \{W_1^n(m') \in \mathcal{A}_\varepsilon^{(n)}(W_1|Y_{1,b}^n)\} \cap \\ & \quad \{W_2^n(M_{b-1}, m') \in \mathcal{A}_\varepsilon^{(n)}(W_2^n|Y_{1,b-1}^n, W_{1,b-1}^n)\}) \\ & \stackrel{(a)}{\leq} 2^{-n\varepsilon}, \end{aligned}$$

where (a) is due to the joint Packing Lemma [32] in order to satisfy both conditions at the same time given that $R \leq I(W_1; Y_1) + I(W_2; Y_1|W_1) - 7\varepsilon$. Moreover, as for the second term, we have

$$\begin{aligned} & \mathbb{P}(\mathcal{E}_{b-1}^{d,t,1}(m_{b-1}) \cup \mathcal{E}_b^{d,t,2} \mid \cap_{\beta=2}^{b-1} \bar{\mathcal{E}}_\beta^d(m_{\beta-1}) \cap \bar{\mathcal{E}}^e) \\ & \stackrel{(b)}{\leq} \mathbb{P}((Y_{1,b-1}^n, W_{1,b-1}^n, W_{2,b-1}^n) \notin \mathcal{A}_\varepsilon^{(n)} \mid \\ & \quad (X_{0,b-1}^n, W_{1,b-1}^n, W_{2,b-1}^n) \in \mathcal{B}_\varepsilon^{(n)}) \\ & \quad + \mathbb{P}((Y_{1,b}^n, W_{1,b}^n, W_{2,b}^n) \notin \mathcal{A}_\varepsilon^{(n)} \mid (X_{0,b}^n, W_{1,b}^n, W_{2,b}^n) \in \mathcal{B}_\varepsilon^{(n)}) \\ & \stackrel{(c)}{\leq} 2 \cdot \max_{(x_0^n, w_1^n, w_2^n) \in \mathcal{B}_\varepsilon^{(n)}} \eta^{(n)}(x_0^n, w_1^n, w_2^n) \\ & \leq 2\sqrt{\delta_n} \xrightarrow{n \rightarrow \infty} 0, \end{aligned}$$

where (b) comes from the independence of each Markov block and the fact that $\{(Y_{1,b}^n, W_{1,b}^n) \notin \mathcal{A}_\varepsilon^{(n)}\} \subset \{(Y_{1,b}^n, W_{1,b}^n, W_{2,b}^n) \notin \mathcal{A}_\varepsilon^{(n)}\}$, and (c) comes from the definition of $\mathcal{B}_\varepsilon^{(n)}$.

Therefore, following the above arguments, the encoding error and the decoding error are both asymptotically zero.

Witsenhausen costs analysis: We first analyze the power cost. To implement the power constraint, we impose a mean-square transmission power constraint as done in [14, 15]:

$$\sum_{t=1}^n \mathbb{E}[U_{1,t}^2] \leq nP, \quad (12)$$

where the expectation is taken over random sequences X_0^n and W_1^n . Therefore, the generated sequence U_1^n satisfies the condition of uniform integrability. Thus, for $b \in [1 : B]$, since at each time $U_{1,t,b}$ is generated i.i.d. with finite second moment $\mathbb{E}[U_1^2] = P$, due to the LLN, the non-negative averaged power sequence $c_P(U_{1,b}^n) = \frac{1}{n} \sum_{t=1}^n (U_{1,t,b}^2)$ converges in probability to P . Moreover, since sequence $U_{1,b}^n$ is also uniformly integrable, it holds (stronger) that

$$\mathbb{E}[|c_P(U_{1,b}^n) - P|] < \varepsilon, \quad b \in [1 : B] \quad (13)$$

for n sufficiently large, more details in [33].

Next, we examine the estimation cost. Define the indicator function $\rho_b = 0$ to denote the absence of coding error in block $b \in [1 : B - 1]$. Now, take $\phi_b = (1 - \psi_b^{(n)})(1 - \rho_b)$ to indicate the occurrence of desired sequences that satisfy both the estimation cost and the typicality constraints AND no coding error event in block b . From the LLN and the error analysis, we have $\mathbb{E}[\phi_b] \rightarrow 1$ as $n \rightarrow \infty$ for $b \in [1 : B - 1]$. In particular, if $\phi_b = 1$, we have $\mathbb{E} \left[|c_S(X_{1,b}^n, \hat{U}_{2,b}^n) - S| \mid \phi_b = 1 \right] < \frac{1}{12}\varepsilon$. Therefore, from the law of total expectation, we have

$$\begin{aligned} & \mathbb{E} \left[|c_S(X_{1,b}^n, \hat{U}_{2,b}^n) - \hat{S}| \right] \\ &= \mathbb{P}(\phi_b = 1) \cdot \mathbb{E} \left[|c_S(X_{1,b}^n, \hat{U}_{2,b}^n) - \hat{S}| \mid \phi_b = 1 \right] \\ & \quad + \mathbb{P}(\phi_b = 0) \cdot \mathbb{E} \left[|c_S(X_{1,b}^n, \hat{U}_{2,b}^n) - \hat{S}| \mid \phi_b = 0 \right] \\ &\leq \frac{1}{12}\varepsilon + \mathbb{P}(\phi_b = 0) \cdot \hat{S} \\ & \quad + \mathbb{P}(\phi_b = 0) \cdot \mathbb{E} \left[|c_S(X_{1,b}^n, \hat{U}_{2,b}^n) \mid \phi_b = 0 \right]. \end{aligned}$$

For n sufficiently large, the second term $\mathbb{P}(\phi_b = 0) \cdot \hat{S} \leq \frac{1}{12}\varepsilon$, since $\hat{S} < \infty$. Next, we evaluate the last term above. In particular, we need $c_S(X_{1,b}^n, \hat{U}_{2,b}^n)$ to be stochastically bounded when an unwanted error event happens. Since

$$\begin{aligned} & \mathbb{P}(\phi_b = 0) \cdot \mathbb{E} \left[|c_S(X_{1,b}^n, \hat{U}_{2,b}^n) \mid \phi_b = 0 \right] \quad (14) \\ &= \mathbb{P}(\phi_b = 0) \cdot \mathbb{E} \left[\frac{1}{n} \sum_{t=1}^n |X_{0,t,b} + U_{1,t,b} - \hat{U}_{2,t,b}|^2 \mid \phi_b = 0 \right], \end{aligned}$$

by applying the inequality $(a + b)^2 \leq 2a^2 + 2b^2$, $\forall a, b \in \mathbb{R}$, we have

$$\begin{aligned} & \mathbb{P}(\phi_b = 0) \cdot \mathbb{E} \left[|c_S(X_{1,b}^n, \hat{U}_{2,b}^n) \mid \phi_b = 0 \right] \\ &\leq 2\mathbb{P}(\phi_b = 0) \cdot \mathbb{E} \left[|c_P(U_{1,b}^n) \mid \phi_b = 0 \right] \\ & \quad + 2\mathbb{P}(\phi_b = 0) \cdot \mathbb{E} \left[|c_S(X_{0,b}^n, \hat{U}_{2,b}^n) \mid \phi_b = 0 \right] \\ &\leq 2\mathbb{P}(\phi_b = 0) \cdot \mathbb{E} \left[|c_P(U_{1,b}^n) - P| + P \mid \phi_b = 0 \right] \\ & \quad + 2\mathbb{P}(\phi_b = 0) \cdot \mathbb{E} \left[|c_S(X_{0,b}^n, \hat{U}_{2,b}^n) \mid \phi_b = 0 \right] \\ &= 2P\mathbb{P}(\phi_b = 0) + 2\mathbb{P}(\phi_b = 0) \cdot \mathbb{E} \left[|c_P(U_{1,b}^n) - P| \mid \phi_b = 0 \right] \\ & \quad + 2\mathbb{P}(\phi_b = 0) \cdot \mathbb{E} \left[|c_S(X_{0,b}^n, \hat{U}_{2,b}^n) \mid \phi_b = 0 \right]. \quad (15) \end{aligned}$$

Here, the first term $2P\mathbb{P}(\phi_b = 0) \leq \frac{1}{36}\varepsilon$ since $P < \infty$. And the second term $2\mathbb{P}(\phi_b = 0) \cdot \mathbb{E} \left[|c_P(U_{1,b}^n) - P| \mid \phi_b = 0 \right] \leq 2\mathbb{E} \left[|c_P(U_{1,b}^n) - P| \right] \leq \frac{1}{36}\varepsilon$ due to the nonnegativity of $|c_P(U_{1,b}^n) - P|$ and the power constraint (13).

Next, we analyse the last term in (15) with Wyner's trick [31] exploiting the discretization of U_2 as follows

$$\begin{aligned} & \mathbb{E} \left[|c_S(X_{0,b}^n, \hat{U}_{2,b}^n) \mid \phi_b = 0 \right] \\ &\leq \frac{1}{n} \sum_{t=1}^n \mathbb{E} \left[D(X_{0,t,b}) \mid \phi_b = 0 \right], \end{aligned}$$

with $D(X_{0,t,b}) = \max_{\hat{u}_{2,k}} c_S(X_{0,t,b}, \hat{u}_{2,k})$. The random variables $\{D(X_{0,t,b})\}_{t=1}^n$ are i.i.d. and integrable since $c_S(\cdot, \cdot)$ is a quadratic measure and $X_{0,t,b}$ is i.i.d. Gaussian distributed. Next, let $\chi_{\{D(X_{0,t,b}) > d\}}$ denote an indicator function indicating if $D(X_{0,t,b}) > d$ for some $0 < d < \infty$. Then we have

$$\begin{aligned} & \mathbb{P}(\phi_b = 0) \cdot \mathbb{E} \left[|c_S(X_{0,b}^n, \hat{U}_{2,b}^n) \mid \phi_b = 0 \right] \quad (16) \\ &\leq d \cdot \mathbb{P}(\phi_b = 0) + \mathbb{E} \left[D(X_{0,t,b}) \chi_{\{D(X_{0,t,b}) > d\}} \right]. \end{aligned}$$

Given that $D(X_{0,t,b})$ is integrable, for any $\varepsilon_d > 0$, there must exist a d_0 such that $\mathbb{E} \left[D(X_{0,t,b}) \chi_{\{D(X_{0,t,b}) > d\}} \right] < \varepsilon_d$ for all $d > d_0$, as per the Monotone Convergence Theorem. Thus, for a sufficiently small ε_d and a sufficiently large n , term (16) can be upper bounded by $\frac{1}{72}\varepsilon$, so that (14) $\leq \frac{1}{12}\varepsilon$.

Thus, compared to the original cost constraint S , for block $b \in [1 : B - 1]$ we have

$$\begin{aligned} & \mathbb{E} \left[|c_S(X_{1,b}^n, \hat{U}_{2,b}^n) - S| \right] \\ &\leq |\hat{S} - S| + \mathbb{E} \left[|c_S(X_{1,b}^n, \hat{U}_{2,b}^n) - \hat{S}| \right] \\ &\leq \frac{1}{4}\varepsilon + \mathbb{E} \left[|c_S(X_{1,b}^n, \hat{U}_{2,b}^n) - \hat{S}| \right] \leq \frac{1}{2}\varepsilon. \quad (17) \end{aligned}$$

Lastly, we estimate the cost of the last block where the output $\hat{U}_{2,B}$ is set to be an all-zero sequence. In this case

$$\begin{aligned} & \mathbb{E} \left[|c_S(X_{1,B}^n, \hat{U}_{2,B}^n) - \hat{S}| \right] \\ &\leq \mathbb{E} \left[|2c_P(X_{0,B}^n) + 2c_P(U_{1,B}^n) - \hat{S}| \right] \\ &\leq 2 \cdot \mathbb{E} \left[|c_P(X_{0,B}^n)| \right] + 2 \cdot \mathbb{E} \left[|c_P(U_{1,B}^n)| \right] + \hat{S} \\ &\stackrel{(a)}{\leq} 2 \cdot Q + 2 \cdot P + \hat{S} < \infty, \end{aligned}$$

where (a) comes from that $X_{0,B}^n$ is generated i.i.d. $\sim \mathcal{N}(0, Q\mathbb{I})$. Therefore, similar to (17), we have

$$\mathbb{E} \left[|c_S(X_{1,B}^n, \hat{U}_{2,B}^n) - S| \right] \leq \frac{1}{4}\varepsilon + 2 \cdot Q + 2 \cdot P + \hat{S} \triangleq \tilde{S}.$$

Synthesizing power costs for all B blocks, we have

$$\mathbb{E} \left[|c_P(U_1^{Bn}) - P| \right] \leq \frac{1}{B} \sum_{b=1}^B \mathbb{E} \left[|c_P(U_{1,b}^n) - P| \right] \leq \varepsilon, \quad (18)$$

for n sufficiently large.

For the average estimation cost, the impact of the last pathological block can be effectively diminished, since

$$\begin{aligned} & \mathbb{E} \left[|c_S(X_1^{Bn}, \hat{U}_2^{Bn}) - S| \right] \\ &\leq \frac{1}{B} \sum_{b=1}^B \mathbb{E} \left[|c_S(X_{1,b}^n, \hat{U}_{2,b}^n) - S| \right] \\ &\leq \frac{1}{B} \tilde{S} + \frac{B-1}{B} \cdot \frac{1}{2}\varepsilon \leq \varepsilon, \quad (19) \end{aligned}$$

for n and B sufficiently large.

Existence of the desired code: With the aid of random coding arguments and applying the Selection Lemma [34] to

both (18) (19), we can obtain that there exists a control design $c \in \mathcal{C}_e(n)$ that simultaneously satisfies

$$\mathbb{E} [|c_P(U_1^{Bn}) - P|] < \varepsilon, \quad \mathbb{E} [|c_S(X_1^{Bn}, \hat{U}_2^{Bn}) - S|] < \varepsilon.$$

Closedness: So far we have only shown under strict inequality of rate constraint (8) the cost constraints holds. As for equality, we consider a sequence of probability distribution $\mathcal{P}^k \xrightarrow{k \rightarrow \infty} \mathcal{P}^*$, where each \mathcal{P}^k satisfies the rate constraint with strict inequality, and \mathcal{P}^* satisfies it with equality. Therefore,

$$\begin{aligned} & \mathbb{E}_{\mathcal{P}^*} [|c_S(X_1^n, U_2^n) - S|] \\ & \leq \mathbb{E}_{\mathcal{P}^k} [|c_S(X_1^n, U_2^n) - S|] + \mathbb{E}_{\mathcal{P}^* - \mathcal{P}^k} [|c_S(X_1^n, U_2^n) - S|] \\ & \leq 1/2 \cdot \varepsilon + 1/2 \cdot \varepsilon \leq \varepsilon \end{aligned}$$

where the first $1/2 \cdot \varepsilon$ comes from the achievability proof with strict inequality as n large enough, and the second $1/2 \cdot \varepsilon$ could similarly be derived repeating the estimation cost analysis steps above since the probability measure $\rightarrow 0$ as $k \rightarrow 0$. Moreover, the power constraint for \mathcal{P}^* also holds due to (12).

IV. CONVERSE PROOF OF THE MAIN RESULT

We consider a control design $c \in \mathcal{C}_e(n)$ of block length $n \in \mathbb{N}$ that achieves a pair of costs (P, S) . According to Csiszár sum identity, we have

$$\begin{aligned} 0 &= \sum_{t=1}^n I(X_0^{t-1}; Y_{1,t} | Y_{1,t+1}^n) - \sum_{t=1}^n I(Y_{1,t+1}^n; X_{0,t} | X_0^{t-1}) \\ &\leq \sum_{t=1}^n I(X_0^{t-1}, Y_{1,t+1}^n; Y_{1,t}) - \sum_{t=1}^n I(Y_{1,t+1}^n; X_{0,t} | X_0^{t-1}) \\ &\stackrel{(a)}{=} \sum_{t=1}^n I(W_{1,t}, W_{2,t}; Y_{1,t}) - \sum_{t=1}^n I(W_{2,t}; X_{0,t} | W_{1,t}) \\ &\stackrel{(b)}{=} n \cdot (I(W_{1,T}, W_{2,T}; Y_{1,T} | T) - I(W_{2,T}; X_{0,T} | W_{1,T}, T)) \\ &\leq n \cdot (I(W_{1,T}, T, W_{2,T}; Y_{1,T}) - I(W_{2,T}; X_{0,T} | W_{1,T}, T)) \\ &\stackrel{(c)}{=} n \cdot (I(W_1, W_2; Y_1) - I(W_2; X_0 | W_1)), \end{aligned}$$

where (a) is from the identification of the auxiliary random variables $W_{1,t} = X_0^{t-1}$, $W_{2,t} = Y_{1,t+1}^n$, (b) is from introducing the uniform random variable $T \in \{1, \dots, n\}$ and the corresponding mean auxiliary random variables $X_{0,T}, W_{1,T}, W_{2,T}, Y_{1,T}$, where $Y_{1,T}$ is distributed according to

$$\mathbb{P}(Y_{1,T} = y_1) = \frac{1}{n} \sum_{t=1}^n \mathbb{P}(Y_{1,t} = y_1), \quad \forall y_1 \in \mathcal{Y}_1$$

and (c) comes from the introduction of $X_0 \triangleq X_{0,T}, W_1 \triangleq (W_{1,T}, T), W_2 \triangleq W_{2,T}, Y_1 \triangleq Y_{1,T}$.

Next, we show that the introduced auxiliary random variables $W_{1,t} = X_0^{t-1}$, and $W_{2,t} = Y_{1,t+1}^n$ in equation (a) satisfy the Markov chain properties (10).

For $t \in \{1, \dots, n\}$:

- $X_{0,t}$ and $W_{1,t} = X_0^{t-1}$ are independent because of the i.i.d. property of the source.
- $U_{1,t} \ominus (X_{0,t}, X_0^{t-1}) \ominus Y_{1,t+1}^n$, is given by the causal encoding, that the current encoder's output $U_{1,t}$ depends

on the future symbols $Y_{1,t+1}^n$ only through the past and current input states $(X_{0,t}, X_0^{t-1})$.

- $(X_{1,t}, Y_{1,t}) \ominus (X_{0,t}, U_{1,t}) \ominus (X_0^{t-1}, Y_{1,t+1}^n)$ comes from the memoryless property of the channel.
- $U_{2,t} \ominus (X_0^{t-1}, Y_{1,t+1}^n, Y_{1,t}) \ominus (X_{0,t}, U_{1,t}, X_{1,t})$ is stated in Subsection IV-A below.

This implies that the auxiliary random variables $X_0 = X_{0,T}, W_1 = (W_{1,T}, T), W_2 = W_{2,T}, U_1 = U_{1,T}, X_1 = X_{1,T}, Y_1 = Y_{1,T}, U_2 = U_{2,T}$ satisfy the Markov chains (10). Therefore, the distribution of auxiliary random variables decomposes as in (7).

Now, we reformulate n -stage costs with a control design $c \in \mathcal{C}_e(n)$. By defining distributions $\mathbb{P}(U_{1,T} = u_1) = \frac{1}{n} \sum_{t=1}^n \mathbb{P}(U_{1,t} = u_1)$, $\forall u_1 \in \mathcal{U}_1$ and $\mathbb{P}(X_{1,T} = x_{1,T}, U_{2,T} = u_{2,T}) = \frac{1}{n} \sum_{t=1}^n \mathbb{P}(X_{1,t} = x_{1,t}, U_{2,t} = u_{2,t})$, $\forall (x_{1,t}, u_{2,t}) \in \mathcal{X}_1 \times \mathcal{U}_2$ and using auxiliary random variables $U_1 = U_{1,T}, X_1 = X_{1,T}, U_2 = U_{2,T}$, we have

$$\begin{aligned} \mathbb{E}[c_P(U_1^n)] &= \mathbb{E} \left[\frac{1}{n} \sum_{t=1}^n U_{1,t}^2 \right] = \mathbb{E} [U_{1,T}^2] = \mathbb{E} [U_1^2], \\ \mathbb{E}[c_S(X_1^n, U_2^n)] &= \mathbb{E} \left[\frac{1}{n} \sum_{t=1}^n (X_{1,t} - U_{2,t})^2 \right] \\ &= \mathbb{E} [(X_{1,T} - U_{2,T})^2] = \mathbb{E} [(X_1 - U_2)^2]. \end{aligned}$$

In conclusion, if a pair of costs $(P, S) \in \mathbb{R}^2$ is achievable, then for all $\varepsilon > 0$, there exists $\bar{n} \in \mathbb{N}$ such that for all $n \geq \bar{n}$, there exists a control design $c \in \mathcal{C}_e(n)$ such that (7) and (8) are satisfied. Moreover, from (5), we also have

$$\begin{aligned} & \mathbb{E} \left[|P - c_P(U_1^n)| + |S - c_S(X_1^n, U_2^n)| \right] \leq \varepsilon \\ & \implies |P - \mathbb{E}[c_P(U_1^n)]| + |S - \mathbb{E}[c_S(X_1^n, U_2^n)]| \leq \varepsilon \\ & \implies |P - \mathbb{E}[U_1^2]| + |S - \mathbb{E}[(X_1 - U_2)^2]| \leq \varepsilon \quad (20) \end{aligned}$$

The equation (20) is valid for all $\varepsilon > 0$. This shows that $(X_0, W_1, W_2, U_1, X_1, Y_1, U_2)$ satisfy (7), (8), (9).

A. Sketch Proof of the Markov Chain

$$\begin{aligned} & \mathcal{P}(U_{2,t} | Y_{1,t}, W_{1,t}, W_{2,t}, X_{0,t}, U_{1,t}, X_{1,t}) \\ &= \sum_{U_1^{t-1}, X_1^{t-1}, Y_1^{t-1}} \mathcal{P}(U_1^{t-1} | Y_{1,t}, X_0^{t-1}, Y_{1,t+1}^n) \\ & \quad \times \mathcal{P}(X_1^{t-1} | Y_{1,t}, X_0^{t-1}, Y_{1,t+1}^n, U_1^{t-1}) \\ & \quad \times \mathcal{P}(Y_1^{t-1} | Y_{1,t}, X_0^{t-1}, Y_{1,t+1}^n, U_1^{t-1}, X_1^{t-1}) \\ & \quad \times \mathcal{P}(U_{2,t} | Y_{1,t}, X_0^{t-1}, Y_{1,t+1}^n, U_1^{t-1}, X_1^{t-1}, Y_1^{t-1}), \\ &= \sum_{U_1^{t-1}, X_1^{t-1}, Y_1^{t-1}} \mathcal{P}(U_1^{t-1}, X_1^{t-1}, Y_1^{t-1} U_{2,t} | Y_{1,t}, X_0^{t-1}, Y_{1,t+1}^n) \\ &= \mathcal{P}(U_{2,t} | Y_{1,t}, X_0^{t-1}, Y_{1,t+1}^n) \\ &= \mathcal{P}(U_{2,t} | Y_{1,t}, W_{1,t}, W_{2,t}). \end{aligned}$$

REFERENCES

- [1] H. S. Witsenhausen, "A counterexample in stochastic optimum control," *SIAM Journal on Control*, vol. 6, no. 1, pp. 131-147, 1968.

- [2] R. Bansal and T. Başar, "Stochastic teams with nonclassical information revisited: When is an affine law optimal?," in *1986 American Control Conference*, pp. 45–50, IEEE, 1986.
- [3] E. I. Silva, G. C. Goodwin, and D. E. Quevedo, "Control system design subject to SNR constraints," *Automatica*, vol. 46, no. 2, pp. 428–436, 2010.
- [4] S. Yüksel and T. Başar, *Stochastic networked control systems: Stabilization and optimization under information constraints*. Springer Science & Business Media, 2013.
- [5] N. C. Martins and M. A. Dahleh, "Fundamental limitations of performance in the presence of finite capacity feedback," in *Proceedings of the 2005, American Control Conference, 2005.*, pp. 79–86, IEEE, 2005.
- [6] J. S. Freudenberg and R. H. Middleton, "Feedback control performance over a noisy communication channel," in *2008 IEEE Information Theory Workshop*, pp. 232–236, IEEE, 2008.
- [7] M. S. Derpich and J. Ostergaard, "Improved upper bounds to the causal quadratic rate-distortion function for gaussian stationary sources," *IEEE Transactions on Information Theory*, vol. 58, no. 5, pp. 3131–3152, 2012.
- [8] E. Akyol, C. Langbort, and T. Başar, "Information-theoretic approach to strategic communication as a hierarchical game," *Proceedings of the IEEE*, vol. 105, no. 2, pp. 205–218, 2017.
- [9] C. D. Charalambous, C. Kourtellis, and I. Tzortzis, "Hierarchical optimality of linear controllers-encoders-decoders operating at control-coding capacity of lqg control systems," in *2017 IEEE 56th Annual Conference on Decision and Control (CDC)*, pp. 3682–3687, IEEE, 2017.
- [10] M. Wiese, T. J. Oechtering, K. H. Johansson, P. Papadimitratos, H. Sandberg, and M. Skoglund, "Secure estimation and zero-error secrecy capacity," *IEEE Transactions on Automatic Control*, vol. 64, no. 3, pp. 1047–1062, 2018.
- [11] P. A. Stavrou, M. Skoglund, and T. Tanaka, "Sequential source coding for stochastic systems subject to finite rate constraints," *IEEE Transactions on Automatic Control*, vol. 67, no. 8, pp. 3822–3835, 2022.
- [12] Y.-H. Kim, A. Sutivong, and T. M. Cover, "State amplification," *IEEE Transactions on Information Theory*, vol. 54, no. 5, pp. 1850–1859, 2008.
- [13] O. Sumszyk and Y. Steinberg, "Information embedding with reversible stegotext," in *2009 IEEE International Symposium on Information Theory*, pp. 2728–2732, IEEE, 2009.
- [14] C. Choudhuri, Y.-H. Kim, and U. Mitra, "Causal state communication," *IEEE Transactions on Information Theory*, vol. 59, no. 6, pp. 3709–3719, 2013.
- [15] P. Grover and A. Sahai, "Witsenhausen's counterexample as assisted interference suppression," *International Journal of Systems, Control and Communications*, vol. 2, pp. 197–237, 2010.
- [16] P. W. Cuff, H. H. Permuter, and T. M. Cover, "Coordination capacity," *IEEE Transactions on Information Theory*, vol. 56, no. 9, pp. 4181–4206, 2010.
- [17] P. Cuff and C. Schieler, "Hybrid codes needed for coordination over the point-to-point channel," in *2011 49th Annual Allerton Conference on Communication, Control, and Computing (Allerton)*, pp. 235–239, IEEE, 2011.
- [18] P. Cuff and L. Zhao, "Coordination using implicit communication," in *2011 IEEE Information Theory Workshop*, pp. 467–471, IEEE, 2011.
- [19] G. Kramer and S. A. Savari, "Communicating probability distributions," *IEEE Transactions on Information Theory*, vol. 53, no. 2, pp. 518–525, 2007.
- [20] B. Larrousse, S. Lasaulce, and M. Wigger, "Coordination in state-dependent distributed networks: The two-agent case," in *2015 IEEE International Symposium on Information Theory (ISIT)*, pp. 979–983, IEEE, 2015.
- [21] B. Larrousse, S. Lasaulce, and M. Wigger, "Coordinating partially-informed agents over state-dependent networks," in *2015 IEEE Information Theory Workshop (ITW)*, pp. 1–5, IEEE, 2015.
- [22] B. Larrousse, S. Lasaulce, and M. R. Bloch, "Coordination in distributed networks via coded actions with application to power control," *IEEE Transactions on Information Theory*, vol. 64, no. 5, pp. 3633–3654, 2018.
- [23] M. Le Treust and T. J. Oechtering, "Optimal control designs for vector-valued Witsenhausen counterexample setups," in *2018 56th Annual Allerton Conference on Communication, Control, and Computing (Allerton)*, pp. 532–537, 2018.
- [24] M. Le Treust, "Joint empirical coordination of source and channel," *IEEE Transactions on Information Theory*, vol. 63, no. 8, pp. 5087–5114, 2017.
- [25] A. Sutivong, M. Chiang, T. Cover, and Y.-H. Kim, "Channel capacity and state estimation for state-dependent gaussian channels," *IEEE Transactions on Information Theory*, vol. 51, no. 4, pp. 1486–1495, 2005.
- [26] T. J. Oechtering and M. Le Treust, "Coordination coding with causal decoder for vector-valued Witsenhausen counterexample setups," in *2019 IEEE Information Theory Workshop (ITW)*, pp. 1–5, 2019.
- [27] M. Le Treust and T. J. Oechtering, "Power-estimation trade-off of vector-valued Witsenhausen counterexample with causal decoder," *IEEE Transactions on Information Theory*, vol. 70, no. 3, pp. 1588–1609, 2024.
- [28] T. M. Cover and J. A. Thomas, *Elements of information theory*. John Wiley & Sons, 1999.
- [29] M. T. Vu, T. J. Oechtering, and M. Skoglund, "Hierarchical identification with pre-processing," *IEEE Transactions on Information Theory*, vol. 66, no. 1, pp. 82–113, 2020.
- [30] C. E. Shannon, "Channels with side information at the transmitter," *IBM Journal of Research and Development*, vol. 2, no. 4, pp. 289–293, 1958.
- [31] A. D. Wyner, "The rate-distortion function for source coding with side information at the decoder," *Information and Control*, vol. 38, no. 1, pp. 60–80, 1978.
- [32] A. El Gamal and Y. Kim, *Network Information Theory*. Cambridge University Press, 2011.
- [33] V. I. Bogachev, "The spaces L_p and spaces of measures," *Measure Theory*, pp. 249–328, 2007.
- [34] M. R. Bloch and J. Barros, *Physical-Layer Security: From Information Theory to Security Engineering*. Cambridge University Press, 2011.