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# Role of FLR effects in magnetopause equilibrium

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The Earth magnetopause, when sufficiently plane and stationary at a local scale, can be considered as a "quasi-tangential" discontinuity, since the normal component of the magnetic field  $B_n$  is typically very small but not zero. Contrary to observations, the "Classic Theory of Discontinuities" (CTD) predicts that rotational and compressional jumps should be mutually exclusive in the general case  $B_n \neq 0$ , but allows only one exception: the tangential discontinuity provided that  $B_n$  is strictly zero. Here we show that Finite Larmor Radius (FLR) effects play an important role in the quasi-tangential case, whenever the ion Larmor radius is not fully negligible with respect to the magnetopause thickness. By including FLR effects, the results suggest that a rotational discontinuity undergoes a change comparable to the change of a Shear Alfvén into a Kinetic Alfvén wave when considering linear modes. For this new kind of discontinuity, the co-existence of rotational and compressional variations at the magnetopause does no more imply that this boundary is a strict tangential discontinuity, even in 1D-like regions far from X-lines if any. This result may lead to important consequences concerning the oldest and most basic questions of magnetospheric physics: how can the magnetopause be open, where and when? The role of FLR being established theoretically, the paper then shows that it can be proved experimentally. For that, we make use of MMS data and process them with the most recent available 4 spacecraft tools. First, we present the different processing techniques that we use to estimate spatial derivatives, such as  $grad(B)$  and  $div(P)$ , and the magnetopause normal direction. We point out why this normal direction must be determined with extremely high accuracy to make the conclusions unambiguous. Then, the results obtained by these techniques are presented in a detailed case study and on a statistical basis.

## Introduction

In space physics, there is a natural tendency of the medium to self-organize into distinct cells, separated by thin layers. This behavior can be observed at very different scales. Notable examples are planetary magnetospheres, which are bubbles in the solar wind stream and which are separated from it by bow shocks and magnetopauses (Parks 2019; Kivelson & Russell 1995; Belmont *et al.* 2014). The interaction of the solar wind with unmagnetized bodies such as comets also produces similar bubbles (Coates 1997; Bertucci 2005). The Solar System itself is a bubble in the flow of the local interstellar cloud, and it is separated from it by the heliopause and at least one shock ("termination shock") (Lallement 2001; Richardson *et al.* 2022). Similar cells and thin layers can also form spontaneously, far from any boundary condition as in the context of a turbulent medium (Frisch 1995; Chasapis *et al.* 2015).

Among all these thin layers, the terrestrial magnetopause plays a particular role. This region has been explored by a large number of spacecraft since the beginning of the space era, up to the most recent multi-spacecraft missions as Cluster (Escoubet *et al.* 1997, 2001) and MMS (Burch & Phan 2016), allowing for a detailed description of its properties. In addition, due to a very small

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43 normal component of the magnetic field with respect to the magnetopause (defined  $B_n = \mathbf{B} \cdot \mathbf{n}$   
 44 where  $\mathbf{B}$  is the magnetic field and  $\mathbf{n}$  the magnetopause's normal) it can be identified as a "quasi-  
 45 tangential" layer. This feature is a direct consequence of the frozen-in property that prevails at  
 46 large scales, on both sides of the boundary, almost preventing any penetration of magnetic flux  
 47 and matter between the solar wind and the magnetospheric media (both of them being magnetised  
 48 plasmas). By large scales here we refer to the fluid scales where an ideal Ohm's law holds,  
 49 as in the ideal magnetohydrodynamic (MHD) regime. However, small departures from a strict  
 50 separation between the two plasmas do exist, at least locally and for a given time interval, and  
 51 they are known to have important consequences for all the magnetospheric dynamics: substorms,  
 52 auroras, etc (McPherron 1979; Tsurutani *et al.* 2001).

53 Knowing when and where plasma injection occurs through the magnetopause has been  
 54 one of the hottest subjects of research since decades (Haaland *et al.* (2021) and references  
 55 therein, Lundin & Dubinin (1984); Gunell *et al.* (2012); Paschmann *et al.* (2018a)). The largest  
 56 consensus presently considers the equilibrium state of the boundary, valid on the major part  
 57 of its surface, as a tangential discontinuity, with a strictly null  $B_n$ , while plasma injection is  
 58 allowed only around a few reconnection regions, where the gradients characterizing the layer  
 59 present 2D features. For that purpose, many studies have been carried out to understand where  
 60 magnetic reconnection occurs the most (Fuselier *et al.* 2011; Trattner *et al.* 2021). Moreover,  
 61 the conditions under which the magnetopause opens due to magnetic reconnection has been  
 62 studied theoretically (Swisdak *et al.* 2003) and experimentally (Gosling *et al.* 1982; Paschmann  
 63 1984; Phan *et al.* 2000; Fuselier *et al.* 2011; Vines *et al.* 2015). The results of the present study  
 64 may allow reconsidering this paradigm by questioning the necessity of a strictly tangential  
 65 discontinuity for the basic equilibrium state.

66  
 67 In the whole paper hereafter, we will call one-dimensional all geometries in which the gradients  
 68 of all parameters are in the same direction  $\mathbf{N}$ . In this sense, a plane magnetopause with not  
 69 tangential gradient is said here to be 1D, while it would be considered 2D if considering real  
 70 space instead of  $k$  space.

## 71 1. Classic Theory of Discontinuities

72 At every layer, the downstream and upstream physical quantities are linked by the fundamental  
 73 conservation laws: mass, momentum, energy and magnetic flux (Landau & Lifshitz 1987). The  
 74 simplest case occurs whenever the number of conservation laws is equal to the number of  
 75 parameters characterizing the plasma state. When this condition is met, the possible downstream  
 76 states are uniquely determined as a function of the upstream state, regardless of the (non-ideal)  
 77 physics at play within the layer. In particular, it is possible to describe pressure variations without  
 78 any closure equation. In this case, the jumps of all quantities are determined by a single scalar  
 79 parameter (namely the "shock parameter" in neutral gas).

80 We refer hereafter to the "Classic Theory of Discontinuities" (CTD) as for the theory  
 81 corresponding to this condition, which is used both for neutral media and (magnetized) plasmas.  
 82 CTD is characterized by the following simplifying assumptions: a stationary layer, 1D variations,  
 83 and isotropic pressure on both sides. For plasmas, the additional assumption of an ideal Ohm's  
 84 law on both sides is considered (Belmont *et al.* 2019).

85 In CTD the conservation laws provide a system of jump equations between the upstream and  
 86 downstream physical quantities, namely the Rankine-Hugoniot conditions in neutral media and  
 87 generalized Rankine-Hugoniot conditions in plasmas.

88 The sets of equations used to compute the linear modes in hydrodynamics (HD) and MHD are  
 89 similar to these jump equations system. simply because the HD and MHD models rely on the  
 90 same conservation laws as Rankine-Hugoniot and generalized Rankine-Hugoniot respectively

91 A direct consequence is that many properties are shared by the solutions of the two types of  
 92 systems: linear modes and discontinuities. For a neutral medium, the linear sound wave solution  
 93 corresponds to the well-known sonic shock solution, while for a magnetized plasma, the two  
 94 magnetosonic waves correspond to the two main types of MHD shocks: fast and slow. However,  
 95 an additional discontinuity solution, the intermediate shock, has no linear counterpart. The  
 96 intermediate shock presents a reversal of the tangential magnetic field through the discontinuity,  
 97 which is not observed neither in the fast nor in the slow mode. Furthermore, a non-compressional  
 98 solution exists in both types of systems, represented by the shear Alfvén mode for linear MHD,  
 99 and by the "rotational discontinuity" solution for the generalized Rankine-Hugoniot system.

100 Focusing on magnetized plasma physics, CTD leads to distinguish compressive and rotational  
 101 discontinuities. An important feature of these solutions is that the compressional and rotational  
 102 solutions are mutually exclusive: the shock solutions are purely compressional, without any  
 103 rotation of the tangential magnetic field (this is called the "coplanarity property"), while the  
 104 rotational discontinuity does imply such a rotation but without any variation of the magnetic field  
 105 amplitude and without any compression of the particle density (Fig.1). This distinction persists  
 106 whatever the fluxes along the discontinuity normal, even when the normal components  $u_n$  and  $B_n$   
 107 of the velocity and the magnetic field are arbitrarily small. The only exception is the "tangential  
 108 discontinuity" when both normal fluxes are strictly zero. This solution would correspond, for the  
 109 magnetopause, to the case without any connection between solar wind and magnetosphere. It  
 110 appears as a singular case since the tangential discontinuity, with  $B_n = 0$ , is not the limit of any  
 111 of the general solutions with  $B_n \neq 0$ . While the limit always implies two solutions, one purely  
 112 rotational and the other purely compressional, the singular solution  $B_n = 0$  only provides one  
 113 solution where the two characters can coexist.

114 In the solar wind, discontinuities are routinely observed and several authors have performed  
 115 statistics for a long time to determine the proportion of the different kinds of discontinuities,  
 116 mainly focusing on the tangential and rotational ones. They conclude that in most cases  
 117 tangential discontinuities (*i.e.* with  $B_n$  small enough to be barely measurable) are the most  
 118 ubiquitous (see Colburn & Sonett (1966), for a pioneering work in this domain and Neugebauer  
 119 (2006); Paschmann *et al.* (2013); Liu *et al.* (2022), and references therein, for more recent  
 120 contributions). In these studies, rotational discontinuities are identified only when  $B_n$  is large  
 121 enough. However, many discontinuities present features that are typical of both rotational and  
 122 tangential discontinuities and are classified as "either" of the two. Extending these studies in the  
 123 range of small  $B_n$ , where all discontinuities are not necessarily "tangential discontinuities" in the  
 124 CTD sense, requires the study of the quasi-tangential case.

## 125 2. The Earth's magnetopause

126 Thanks to *in-situ* observations, the Earth's magnetopause has a pivotal role in testing the  
 127 discontinuity theories. Indeed, the Earth's magnetopause boundary exhibits, over its entire  
 128 surface, both a rotation of the magnetic field (Sonnerup & Ledley 1974) and a density  
 129 variation (Otto 2005) since it is the junction of two media, the magnetosheath and the  
 130 magnetosphere where the magnetic field and the density are different (Dorville *et al.* 2014).

131 As stated above, the usual paradigm is that the magnetopause is always a tangential  
 132 discontinuity and that it becomes "opens" only exceptionally at a few points where the  
 133 boundary departs from one-dimensionality due to magnetic reconnection. Does it mean that it  
 134 justifies the very radical hypothesis of a magnetopause nearly completely impermeable to mass  
 135 and magnetic flux, with strictly null  $B_n$  and  $u_n$  and quasi-independent plasmas on both sides  
 136 (apart from the normal pressure equilibrium)? From a theoretical point of view, it is clear that the  
 137 singular limit from  $B_n \simeq 0$  to  $B_n = 0$  remains to be solved. From an experimental point of view,  
 138 if the components  $B_n$  and  $u_n$  are known to be always very small, the observations can hardly

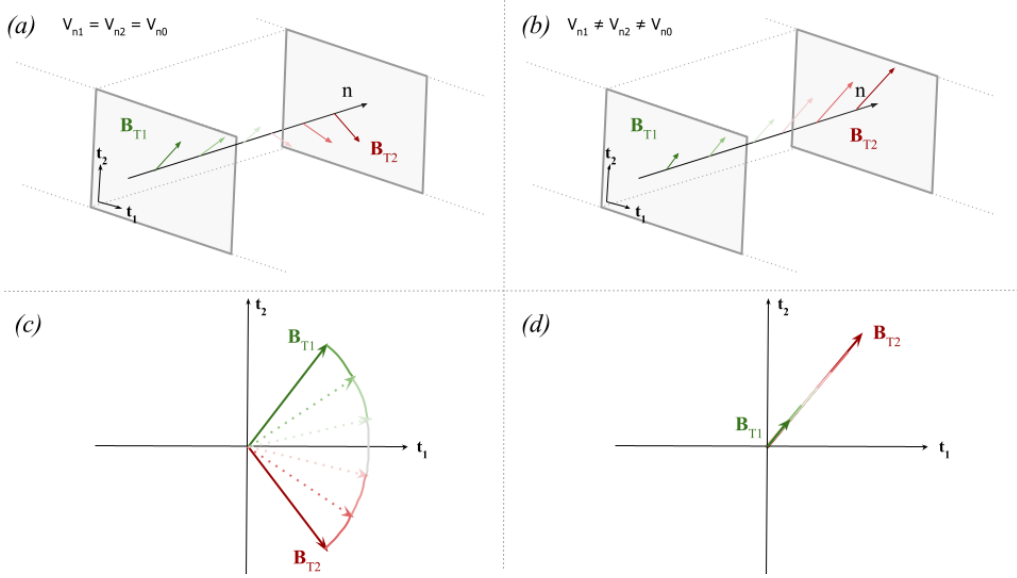


FIGURE 1. Cartoon showing the different variations of  $B$  between a rotational discontinuity (left) and a compressive one (right). The top panel shows in 3D the variation of  $B$  inside the magnetopause plane; the bottom panel shows the hodogram in this tangential plane: a circular arc for the rotational discontinuity and a radial line for shocks.

139 distinguish between  $B_n \simeq 0$  and  $B_n = 0$  because of the uncertainties, due to the fluctuations and  
 140 the limited accuracy in determining the normal direction (Rezeau *et al.* 2018; Haaland *et al.*  
 141 2004; Dorville *et al.* 2015b).

142 The results of the present paper will question the above paradigm. We will show theoretically  
 143 and experimentally that CTD fails at the magnetopause and that rotation and compression can  
 144 actually coexist with finite  $B_n$  and  $u_n$ , even in the 1D case. Such a paradigm change may be  
 145 reminiscent of a similar improvement in the theoretical modeling of the magnetotail in the 70's  
 146 studies (Coppi *et al.* (1966); Galeev (1979); Coroniti (1980) and references therein). In that case  
 147 the authors demonstrated that even a very weak component of the magnetic field across the  
 148 current layer was sufficient to completely modify the stability properties of the plasmashet, so  
 149 that the finite value of  $B_n$  had to be taken into account, contrary to the pioneer versions of the  
 150 tearing instability theories.

### 151 3. The role of pressure

152 In CTD the separation between the compressional and rotational properties of the  
 153 discontinuities comes from only two equations projected on the tangential plane. These  
 154 equations are the momentum equation and the Faraday - Ohm's law, that read:

$$155 \rho \frac{d\mathbf{u}}{dt} + \nabla \cdot \mathbf{P}_i + \nabla \cdot \mathbf{P}_e = \mathbf{J} \times \mathbf{B} \quad (3.1)$$

$$156 \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$157 \text{ where } \mathbf{E} = -\mathbf{u} \times \mathbf{B} + \frac{1}{ne} \mathbf{J} \times \mathbf{B} - \frac{1}{ne} \nabla \cdot \mathbf{P}_e \quad (3.2)$$

158 where  $\mathbf{B}$  is the magnetic field and  $\mathbf{u}$  is the flow velocity in a reference frame where the layer is steady.

159 Considering one-dimensional gradients along the normal direction  $\mathbf{n}$ , neglecting the non-ideal  
 160 terms in Ohm's law and integrating across the layer, these two equations, projected on the  
 161 tangential plane, give:

$$162 \quad \rho_2 u_{n2} \mathbf{u}_{t2} - B_{n2} \mathbf{B}_{t2} / \mu_0 = \rho_1 u_{n1} \mathbf{u}_{t1} - B_{n1} \mathbf{B}_{t1} / \mu_0 \quad (3.3)$$

$$163 \quad B_{n2} \mathbf{u}_{t2} - u_{n2} \mathbf{B}_{t2} = B_{n1} \mathbf{u}_{t1} - u_{n1} \mathbf{B}_{t1} \quad (3.4)$$

163 Due to the divergence free equation, the values  $B_{n1}$  and  $B_{n2}$  are equal and will be written as  
 164  $B_n$  without index in the following. Similarly,  $\rho_1 u_{n1}$  and  $\rho_2 u_{n2}$  are equal because of the continuity  
 165 equation and will be simply noted  $\rho u_n$  in the following. Here, the indices  $n$  and  $t$  indicate the  
 166 projection along the normal and in the tangential plane, respectively, while indices 1 and 2  
 167 indicate the two sides of the discontinuity. It is important to note that, in CTD, the pressure  
 168 divergence terms do not appear in Eq.(3.3) because of the assumption done in this theory that the  
 169 pressure is isotropic on both sides so that their integration gives terms of the form  $(p_2 - p_1)\mathbf{n}$ ,  
 170 with no component in the tangential plane.

171 We see that all terms in these two equations are proportional to  $B_n$  or  $u_n$ , so that any non-  
 172 ideal term, even small, can become dominant when these two quantities tend to zero (if these  
 173 non-ideal terms do not tend to zero at the same time). As the distinction between compressional  
 174 and rotational character fully relies on this system of equations, this evidences the necessity  
 175 of investigating the quasi-tangential case for resolving the usual singularity of the tangential  
 176 discontinuity. We note that the LHS and RHS of equation 3.4 can be put equal to zero by choosing  
 177 the "De Hoffmann-Teller" tangential reference frame where the electric field is zero (Belmont  
 178 *et al.* 2019). However, this choice, even if it can simplify some calculations, is not necessary  
 179 here. Finally, the variables  $\mathbf{u}_t$  can be eliminated from the system by a simple linear combination  
 180 of the two equations, leading to:

$$181 \quad (u_{n2} - u_{n0})\mathbf{B}_{t2} = (u_{n1} - u_{n0})\mathbf{B}_{t1} \quad (3.5)$$

181 where

$$182 \quad u_{n0} = \frac{B_n^2}{\mu_0 \rho u_n} = \text{cst} \quad (3.6)$$

182 Equation (3.5) leads to the distinction between shocks, where the tangential magnetic fields  
 183 on both sides are collinear (but with different modules), and rotational discontinuities, where  
 184 the terms inside the brackets must be equal to zero. Rotational discontinuities correspond to  
 185 a propagation velocity equal to the normal Alfvén velocity, and imply  $u_{n1} = u_{n2} = u_{n0}$ , and  
 186 therefore an absence of compression of the plasma.

187 As previously stated, the separation between the compressional and rotational characters  
 188 mainly derives, in CTD, from the assumption of isotropic pressures on both sides, which prevents  
 189 the pressure divergences to have tangential components. When the isotropic hypothesis is relaxed  
 190 (Hudson 1971), the set of conservation equations is no longer sufficient to determine a unique  
 191 downstream state for a given upstream one. As a consequence, the global result depends on the  
 192 non-ideal processes occurring within the layer. In addition to anisotropy effects, Finite Larmor  
 193 Radius (FLR) effects can be expected to break the gyrotropy of the pressure tensor around  $\mathbf{B}$   
 194 in the case of thin boundaries between different plasmas. This means that the main effect that  
 195 explains departures from CTD comes from the tangential component of the divergence of the  
 196 pressure tensor, which must be taken into account in the momentum equation. On the other hand,  
 197 the non-ideal effects related to the generalized Ohm's law are negligible, at least in the examples  
 198 shown in this paper. The possible types of discontinuities in an anisotropic plasma have been  
 199 discussed in several papers long time ago (Lynn 1967; Abraham-Shrauner 1967; Chao 1970;  
 200 Neubauer 1970), and the present paper improves the analysis in the light of the new experimental  
 201 possibilities given by the MMS measurements.

202 When the dynamics drives the conditions for the pressure tensor to become anisotropic (and  
 203 *a fortiori* in the non-gyrotropic case) the  $\nabla \cdot \mathbf{P}$  term comes into play linking upstream and  
 204 downstream quantities. Considering the "simple" anisotropic case, *i.e.* keeping the gyrotropy  
 205 around  $\mathbf{B}$ , it has been shown (Hudson 1971) that the  $\nabla \cdot \mathbf{P}$  term then just introduces a new  
 206 coefficient:

$$\alpha = 1 - \frac{P_{\parallel} - P_{\perp}}{B^2/\mu_0} \quad (3.7)$$

207 This coefficient has been interpreted as a change in the Alfvén velocity  $V_{An}^2 = \alpha V_{An}^2$ , but it  
 208 appears more basically as a change in Eq.(3.5):

$$(u_{n2} - \alpha_2 u_{n0})\mathbf{B}_{t2} = (u_{n1} - \alpha_1 u_{n0})\mathbf{B}_{t1} \quad (3.8)$$

209 This equation shows that, in this simple anisotropic case, coplanar solutions still exist ( $\mathbf{B}_{t2}$   
 210 and  $\mathbf{B}_{t1}$  are collinear), but that whenever  $\alpha_2$  is not equal to  $\alpha_1$ , the equivalent of the rotational  
 211 discontinuity now implies compression:

$$u_{n2} \neq u_{n1} \quad \text{if} \quad \alpha_2 \neq \alpha_1 \quad (3.9)$$

212 Since  $u_{n2} = \alpha_2 u_{n0}$  and  $u_{n1} = \alpha_1 u_{n0}$ . The variation of  $u_n$  explains why the modified rotational  
 213 discontinuity can be "evolutionary" (Jeffrey & Taniuti 1964), the non linear steepening being  
 214 counter-balanced at equilibrium by non-ideal effects for a thickness comparable with the  
 215 characteristic scale of these effects.

216 There is actually no additional conservation equation available that would allow the jump of  
 217 the anisotropy coefficient  $\alpha$  to be determined. Consequently, there is no universal result that gives  
 218 the downstream state as a function of the upstream one, regardless of the microscopic processes  
 219 going on within the layer. This remains valid for the full anisotropic case, with non-gyrotropy. As  
 220 soon as the ion Larmor radius  $\rho_i$  and the ion inertial length  $d_i$  are not fully negligible with respect  
 221 to the characteristic scale  $L$  of the layer, kinetic effects, and in particular FLR effects, which make  
 222 the pressure tensor non-gyrotropic, must be taken into account to describe self-consistently the  
 223 internal processes. Then, the effect of the divergence of the pressure tensor is no longer reduced  
 224 to adding a coefficient  $\alpha$  since its tangential component is no longer collinear with  $\mathbf{B}_t$ . Such  
 225 effect has been already reported and analyzed in the context of magnetic reconnection (Aunai  
 226 *et al.* 2013, 2011) and in kinetic modeling of purely tangential layers (Belmont *et al.* 2012;  
 227 Dorville *et al.* 2015a). It has been also investigated in the case of linear modes where they are  
 228 responsible for the transition from shear Alfvén into Kinetic Alfvén Wave (Hasegawa & Uberoi  
 229 1982; Belmont & Rezeau 1987; Cramer 2001). On the other hand, it has never been introduced  
 230 in the context of quasi-tangential discontinuities.

231 If a simple anisotropy preserving gyrotropy around  $\mathbf{B}$  can be straight fully taken into account  
 232 for modeling the pressure tensor and using it in fluid equations, introducing non-gyrotropy does  
 233 not lead to a general and simple modeling for the pressure tensor. It would demand *a priori*  
 234 a full kinetic description or, at least, some expansions assuming that these effects are small  
 235 enough (see Braginskii (1965) for the pioneer work in this field and Passot & Sulem (2006) and  
 236 references therein). Several papers have investigated the changes in rotational discontinuities  
 237 when such non-ideal effects are introduced (Lyu & Kan 1989; Hau & Sonnerup 1991; Hau  
 238 & Wang 2016). These theoretical papers used different analytical models based on different  
 239 simplifying assumptions. Contrary to these papers, we will not use such kind of assumptions.  
 240 Instead, we will just analyze the observed magnetic hodograms, and show that their shape is  
 241 incompatible with a gyrotropic pressure.

#### 4. The magnetopause normal

When studying the magnetopause with *in situ* measurements, the most basic geometric characteristic to be determined is the normal to its surface (which may vary during the crossing). An accurate determination of the magnetopause normal is actually a fundamental condition for determining reliable estimates of the normal components of both the magnetic and the mass fluxes. Moreover, having a good estimation of the normal direction is also necessary to determine the speed of the structure and its thickness. Quantitatively speaking, to determine the normal component of the magnetic field sufficiently well (assuming that  $B_n/|\mathbf{B}| \sim 2\%$ ), an accuracy of the normal should be of the order of  $\delta\theta < 1^\circ$ . In the literature, a good accuracy of determination of the normal is considered to be of the order of 5% (Denton *et al.* 2018).

Beyond determining the normal direction, some "reconstruction methods" can be used to provide a more global view of the large scale structure around the spacecraft. Although these methods have proven to provide remarkable results (De Keyser 2008; Hasegawa *et al.* 2005; Denton *et al.* 2020) they will not be used here (the first two studies assume the Grad-Shafranov equations to be valid, implying stationary MHD, and are therefore not appropriate to investigate the non-MHD effects such as the FLR effects).

Over the years, several methods have been developed with the purpose to precisely determine the normal direction (see e.g. (Haaland *et al.* 2004; Shi *et al.* 2019)). The most common is the minimum variance (MVA) introduced with the first measurements of the magnetic field in space (Sonnerup & Cahill 1967; Sonnerup & Scheible 1998). This method, which requires single spacecraft measurements, provides a *global* normal, *i.e.* a single normal vector for each entire time series across the boundary. The tool is based on the assumption that the boundary is a perfectly one-dimensional and stationary layer crossing the spacecraft. Other notable examples are the Generic Residue Analysis (GRA) technique (Sonnerup *et al.* 2006), which consist in a generalisation of the MVA to other parameters than  $\mathbf{B}$ , and the BV method (Dorville *et al.* 2014), which combines magnetic field and velocity data. Even though these methods can give an accurate normal determination (Dorville *et al.* 2015b), they provide, like MVA, a *global* normal and thus they cannot provide the necessary basis for investigating the variations of the magnetopause normal within the structure and test the possible departures from mono-dimensionality. Let us finally recall that waves and turbulence, which are always superimposed to the laminar magnetopause profiles, bring strong limitations in the normal direction accuracy for all methods, in particular these global ones.

In this context, multi-spacecraft missions have represented a fundamental step in increasing the accuracy of the normal determination, allowing to determinate the gradients of the measured fields. A notable example is the Minimum Directional Derivative (MDD, Shi *et al.* (2005)) method. This tool generally uses the magnetic field data, but it must be kept in mind that it is not based on specific properties of this field. The MDD technique is a so-called "gradient based method" since the calculation of the normal is based on the experimental estimation of the dyadic tensor  $\mathbf{G} = \nabla\mathbf{B}$ . This tensor gradient can be obtained from multispacecraft measurements using the reciprocal vector method (Chanteur 1998). The MDD method consists in diagonalizing the matrix  $\mathbf{L} = \mathbf{G} \cdot \mathbf{G}^T$ , finding the normal direction as the eigenvector corresponding to the maximum eigenvalue. Moreover, the gradient matrix can also be used for estimating the dimensionality of the boundary from the ratio between the eigenvalues. A way of finding a quantitative determination of this dimensionality was proposed in Rezeau *et al.* (2018).

For the vector  $\mathbf{B}$ , the MDD method makes use only of the spatial derivatives  $\partial_i\mathbf{B}$ , which are accessible at each time step thanks to the 4-point measurements today available with multi-spacecraft space missions. In this sense, it is the opposite of the MVA method, which makes use only of the temporal variances of the  $\mathbf{B}$  components. It therefore allows for an instantaneous determination of the normal at any point inside the layer, while MVA can only provide a single



291 normal for a full crossing. In addition, contrary to MVA, MDD does not make any assumption  
 292 about the geometry of the layer (1D variations or not), and about the physical properties of the  
 293 vector used. Indeed, it can be applied to the magnetic field data but also to any other vector since  
 294 the property  $\nabla \cdot \mathbf{B} = 0$  is not used.

295 However, due to waves and turbulence, the magnetopause can present locally two-dimensional  
 296 properties that are insignificant for the profiles we are looking for. For this reason, we will focus  
 297 here on intervals where the magnetopause is mainly one-dimensional, discarding the crossings in  
 298 which local 2D features are observed. The intervals considered as one-dimensional are those for  
 299 which  $\lambda_{max} \gg \lambda_{int}$ . Here  $\lambda_{max}$  and  $\lambda_{int}$  are defined as the highest and the intermediate eigenvalues  
 300 of the matrix  $\mathbf{G}$ . In this limit, the ordering between  $\lambda_{int}$  and  $\lambda_{min}$  (*i.e.* the smaller eigenvalue) is  
 301 not relevant in defining the intervals. Specifically, we use the parameter,  $D_1 = (\lambda_{max} - \lambda_{int})/\lambda_{max}$ ,  
 302 which enables us to quantify this mono-dimensionality of the magnetopause as a function of  
 303 time.

304 A more recent tool proposed to study the magnetopause is the hybrid method presented in  
 305 Denton *et al.* (2016, 2018), in which the orientation of the magnetopause is obtained through a  
 306 combination of the MDD and MVA methods, resulting in an improved accuracy of the normal  
 307 direction.

308 The only limitation to the MDD accuracy comes from the uncertainty of the spatial derivatives  
 309 that it uses. In particular, the local gradient matrix is calculated through the reciprocal vector  
 310 technique (Chanteur 1998), which assumes linear variations between the spacecraft. Because  
 311 small-scale waves and turbulence are always superimposed on the magnetopause profiles being  
 312 searched for, this assumption cannot be well respected without some filtering. This filtering  
 313 actually leads to introduce part of the temporal information on the variations, but it still allows  
 314 keeping local information inside the layer whenever one filters only the scales sufficiently smaller  
 315 than those associated to the full crossing duration. The quality of the filtering is therefore the  
 316 biggest challenge to complete for getting accurate results. For instance, simple gaussian filters  
 317 done independently on the four spacecraft would provide insufficient accuracy: this can be  
 318 observed by the fact that, when doing it, the relation  $\nabla \cdot \mathbf{B} = 0$  is violated in the result. In  
 319 the following section, it is shown how the MDD method can be included in a fitting procedure  
 320 of the four spacecraft simultaneously and where this relation can be imposed as a constraint. We  
 321 also show that, when no constraint is added, this procedure justifies the use of MDD with data  
 322 that are filtered independently.

## 323 5. A new tool

324 The tool we present here, namely GF2 (Gradient matrix Fitting), has been derived from the  
 325 MDD method. The digit 2 indicates that in the version of the tool that we use here the data  
 326 are fitted with a 2D model (it can be shown that fitting with a 1D model is mathematically  
 327 equivalent to the standard MDD technique used with smoothed data). Differently from the  
 328 original method, we assume that the structure under investigation can be fitted locally (*i.e.* in each  
 329 of the small sliding window used along the global crossing), by a two-dimensional model. This  
 330 does not imply that the magnetopause is assumed globally two-dimensional. As for MDD, the  
 331 instantaneous gradient matrix  $\mathbf{G}$  is obtained from the data using the reciprocal vector's technique  
 332 (Chanteur 1998). When performing the 2D fit in each sliding window, we then impose some  
 333 physical constraints, which could be checked only *a posteriori* with the classic MDD method.

334 The model  $\mathbf{G}_{fit}$  is obtained as follows:

$$\mathbf{G}_{fit} = \mathbf{e}_0 \mathbf{B}'_{e0} + \mathbf{e}_1 \mathbf{B}'_{e1} \quad (5.1)$$

335 where we define  $\mathbf{e}_0$  and  $\mathbf{e}_1$  as two unit vectors in the plane perpendicular to the direction of  
 336 invariance and  $\mathbf{B}'_{e0}$  and  $\mathbf{B}'_{e1}$  as the variation of the magnetic field along these two directions.

337 By performing the fit, we impose  $\nabla \cdot \mathbf{B} = 0$  (as used in MVA but ignored in standard MDD). In  
 338 the model, this can be written as:

$$\mathbf{e}_0 \cdot \mathbf{B}'_{e0} + \mathbf{e}_1 \cdot \mathbf{B}'_{e1} = 0 \quad (5.2)$$

339 In order to fit the experimental  $\mathbf{G}$  by the model  $\mathbf{G}_{fit}$ , the following quantity has to be minimised

$$\begin{aligned} D_2 &= \text{Tr}[(\mathbf{G}_{fit} - \mathbf{G}) \cdot (\mathbf{G}_{fit} - \mathbf{G})^T] \\ &= \mathbf{B}'_{e0}{}^2 - 2\mathbf{e}_0 \cdot \mathbf{G} \cdot \mathbf{B}'_{e0} + \mathbf{B}'_{e1}{}^2 - 2\mathbf{e}_1 \cdot \mathbf{G} \cdot \mathbf{B}'_{e1} + \text{Tr}(\mathbf{G}\mathbf{G}^T) \end{aligned} \quad (5.3)$$

340 We can disregard the last term, since it is independent of the fit parameters. To impose the  
 341 physical constraints, we use Lagrange multipliers, minimizing:

$$\begin{aligned} D_2 &= \mathbf{B}'_{e0}{}^2 - 2\mathbf{e}_0 \cdot \mathbf{G} \cdot \mathbf{B}'_{e0} + \mathbf{B}'_{e1}{}^2 - 2\mathbf{e}_1 \cdot \mathbf{G} \cdot \mathbf{B}'_{e1} + 2\lambda(\mathbf{e}_0 \cdot \mathbf{B}'_{e0} + \mathbf{e}_1 \cdot \mathbf{B}'_{e1}) \\ &= \mathbf{B}'_{e0}{}^2 - 2\mathbf{e}_0 \cdot (\mathbf{G} - \lambda\mathbf{I}) \cdot \mathbf{B}'_{e0} + \mathbf{B}'_{e1}{}^2 - 2\mathbf{e}_1 \cdot (\mathbf{G} - \lambda\mathbf{I}) \cdot \mathbf{B}'_{e1} \end{aligned} \quad (5.4)$$

342 By assuming in the first approximation that the direction of invariance  $\mathbf{e}_2$  is known, we can  
 343 choose the two vectors  $\mathbf{e}_0$  and  $\mathbf{e}_1$  as an arbitrary orthonormal basis for the plane of variance. For  
 344 performing the minimisation, we have just to impose equal to zero the derivatives with respect to  
 345  $\mathbf{B}'_{e0}$ ,  $\mathbf{B}'_{e1}$  and  $\lambda$ , obtaining Equation 5.2 and:

$$\mathbf{B}'_{e0} = \mathbf{e}_0 \cdot (\mathbf{G} - \lambda\mathbf{I}) \quad (5.5)$$

346

$$\mathbf{B}'_{e1} = \mathbf{e}_1 \cdot (\mathbf{G} - \lambda\mathbf{I}) \quad (5.6)$$

347 By introducing these two equations in Equation 5.2 we obtain:

$$\lambda = \frac{G_{00} + G_{11}}{2}, \quad (5.7)$$

348 from which we get the values of  $\mathbf{B}'_{e0}$  and  $\mathbf{B}'_{e1}$ . At this point, the matrix  $\mathbf{G}_{fit}$  is fully determined. We  
 349 can then look for its eigenvalues and eigenvectors, as in the standard MDD method, and get the  
 350 normal  $\mathbf{n}$  and the tangential directions  $\mathbf{t}_1$  (*i.e.* the one orthogonal to the direction of invariance)  
 351 from this smooth fit.

352 The choice of the direction of invariance has actually no major influence on the determination  
 353 of the normal direction, neither on the estimation of the 2D effects. For large 2D effects, one  
 354 could choose the direction of minimum variance obtained by applying directly the standard MDD  
 355 method to the data. Nevertheless, for almost 1D cases (the most common situation), the spatial  
 356 derivatives in the tangential directions are generally much smaller than the noise, so this result  
 357 is not reliable. We simply choose here the constant M direction given by MVA, which is often  
 358 considered as the direction of the X line if interpreted in the context of 2D models of magnetic  
 359 reconnection (*cf.* for instance Phan *et al.* (2013) for a typical use of this choice and Aunai *et al.*  
 360 (2016); Liu *et al.* (2018); Denton *et al.* (2018) for discussions about it).

361 Finally, another useful by-product of the method can be obtained: comparing the spatial  
 362 derivatives and the temporal ones and using a new fitting procedure, we can compute the two  
 363 components of the velocity of the structure  $V_{n0}$  and  $V_{t1}$  with respect to the spacecraft. Only the  
 364 motion along the invariant direction then remains unknown.

365

### 5.1. Normal from ions mass flux

366 This tool can be easily adapted to any other vector dataset by just changing the physical  
 367 constraint. In particular, we chose to study the structure using the ion mass flux data. In this  
 368 case we impose mass conservation  $\nabla \cdot \mathbf{\Gamma}_i = -\partial_t n_i$  (with  $\mathbf{\Gamma}_i = n_i \mathbf{u}_i$ ). Eq.5.2 now writes

$$\mathbf{e}_0 \cdot \mathbf{\Gamma}'_{e0} + \mathbf{e}_1 \cdot \mathbf{\Gamma}'_{e1} + \partial_t n_i = 0 \quad (5.8)$$

369 Therefore, when using the Lagrange multipliers, Eq.5.4 changes to:

$$\begin{aligned}
 D_2 &= \mathbf{I}'_{e0}{}^2 - 2\mathbf{e}_0 \cdot \mathbf{G} \cdot \mathbf{I}'_{e0} + \mathbf{I}'_{e1}{}^2 - 2\mathbf{e}_1 \cdot \mathbf{G} \cdot \mathbf{I}'_{e1} + 2\lambda(\mathbf{e}_0 \cdot \mathbf{I}'_{e0} + \mathbf{e}_1 \cdot \mathbf{I}'_{e1} + \partial_t n_i) \\
 &= \mathbf{I}'_{e0}{}^2 - 2\mathbf{e}_0 \cdot (\mathbf{G} - \lambda \mathbf{I}) \cdot \mathbf{I}'_{e0} + \mathbf{I}'_{e1}{}^2 - 2\mathbf{e}_1 \cdot (\mathbf{G} - \lambda \mathbf{I}) \cdot \mathbf{I}'_{e1} + 2\lambda \partial_t n_i
 \end{aligned}
 \tag{5.9}$$

370 By using the same algorithm as above, the constraint can now be written as:

$$\lambda = \frac{G_{00} + G_{11} + \partial_t n_i}{2}
 \tag{5.10}$$

371

## 5.2. Dimensionality index

372 From this procedure, we can also derive another significant result: we can obtain an indicator  
 373 of the importance of the 2D effects in the profiles, free of the parasitic noise effects. Specifically,  
 374 we can estimate the variation of the magnetic field along the normal by projecting the  $\mathbf{G}_{fit}$  matrix  
 375 along it  $\text{var}_n = |\mathbf{G}_{fit} \cdot \mathbf{n}|$ . Consequently, if we designate the variation along  $\mathbf{t}_1$  as  $\text{var}_t$ , we can  
 376 introduce a new dimensionality index:

$$\mathcal{D}_{GF2} = \frac{\text{var}_n - \text{var}_t}{\text{var}_n}
 \tag{5.11}$$

377 This index can usefully be compared with the instantaneous index  $D_1 = (\lambda_{max} - \lambda_{int})/\lambda_{max}$  of  
 378 Rezeau *et al.* (2018).

## 379 6. Expected accuracy and tests of the tool

380 In this section we test the accuracy of the GF2 tool. To accomplish this, we exploit a case  
 381 crossing, which will be investigated in detail in the following section. The crossing considered  
 382 comes from MMS data (Burch & Phan 2016), taking place at around 22:11 on 28th December  
 383 2015. For this study we use data from the FluxGate Magnetometers (FGM, Russell *et al.* (2016)),  
 384 providing the magnetic field data, the Electric Double Probe (EDP, Lindqvist *et al.* (2016); Ergun  
 385 *et al.* (2016)), for the electric field, and Dual Ions and Electrons Spectrometer instrument (DIS,  
 386 DES, Pollock *et al.* (2016)), for plasma measurements. An overview of the event is shown in Fig.  
 387 2, where both the magnetic field and ion bulk velocity are given in Geocentric Solar Ecliptic  
 388 (GSE) coordinates. For this event, the spacecraft are located in [7.6, -6.7, -0.8]  $R_E$  in GSE  
 389 coordinates (where  $R_E$  is the Earth's radius).

390 The temporal interval in which we observe the shear in the magnetic field and the crossing  
 391 in the particle structure is about 8 seconds, enough to allow for high resolution for both sets of  
 392 measurements. The crossing is chosen by also analyzing the dimensionality of the magnetic field  
 393 measurements averaged along the crossing. In particular, the dimensionality parameter defined in  
 394 Eq. 5.11, denoted as  $\mathcal{D}_{GF2}$ , and the one introduced in Rezeau *et al.* (2018), denoted as  $D_1$ , were  
 395 considered. In this interval, indeed, we have  $D_{1,mean} = 0.97 \pm 0.03$  while  $\mathcal{D}_{GF2} = 0.89 \pm 0.06$ , both  
 396 highlighting that the crossing exhibits one-dimensional features throughout the time interval. We  
 397 remind here that in burst mode, the frequency of magnetic field measurements is 132Hz while it  
 398 is 6,67 Hz for ions. To conduct the following study, it is necessary to interpolate all measurements  
 399 at the same times. We did it by testing two sampling frequencies: the magnetic field and the ion  
 400 ones. The results obtained are consistent with the two methods. All figures shown in the paper  
 401 are obtained with the sampling times of the MMS1 magnetic field. Furthermore, the crossing is  
 402 observed quasi-simultaneously for the two quantities, with a large interval where the two kinds  
 403 of results can be compared.

404 As a first test, we compare in Fig.3 the normals obtained by GF2 and those by the standard  
 405 MDD technique (using data smoothed in a 0.31s time-window), for both the magnetic field and  
 406 the ion data. For reference, we also compare the result of  $GF2_B$  with the MVA one.

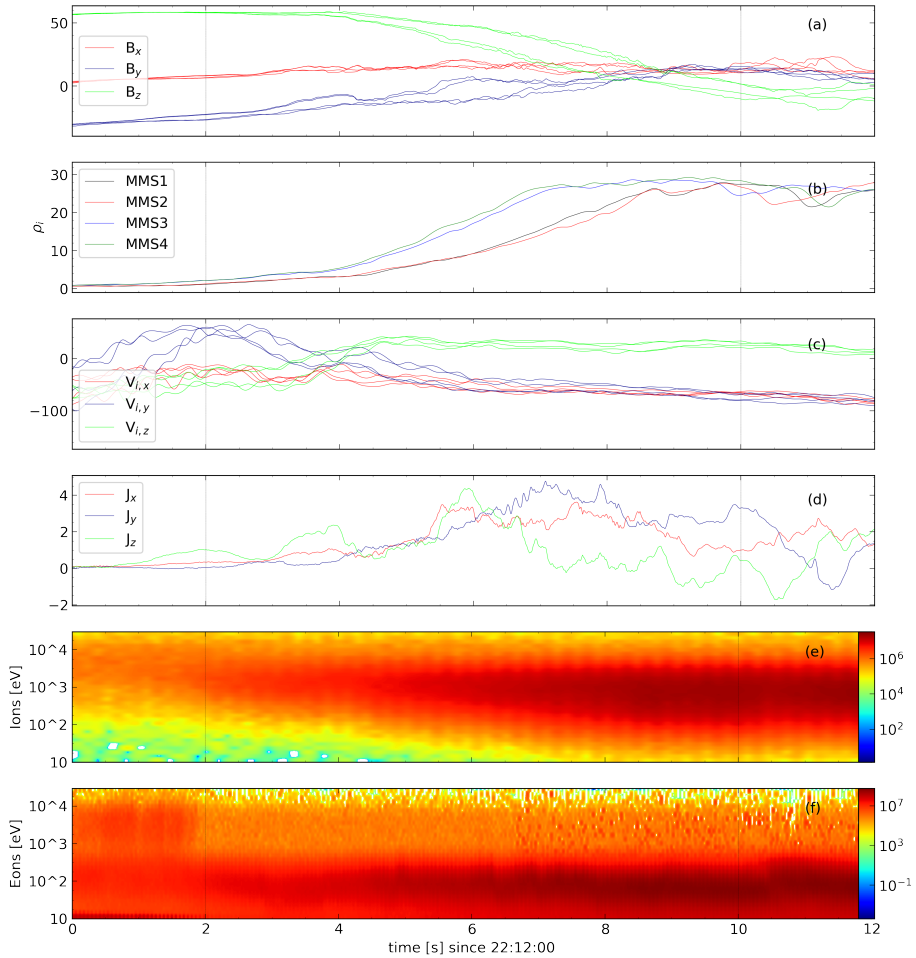


FIGURE 2. Main features of the crossing of the 28th December 2015. From top to bottom: (a) the magnetic field (in  $nT$ ), (b) the ion particle density (in  $m^{-3}$ ), (c) ion velocity (in  $km/s$ ), (d) total current (computed from the curlmeter (Dunlop *et al.* 1988), in  $nA/m^2$ ), (e) the ion and (f) electron spectrograms (energies are shown in  $eV$ ). Vertical lines indicate the time interval chosen for the case study.

407 Vertical dashed lines indicate the time interval during which all the satellites are inside the  
 408 boundary. We observe that the time required for the ions flux to complete the crossing (of about  
 409 5s) is shorter than for the magnetic field (about 8s). To perform a quantitative analysis of the  
 410 differences, we studied the angles between the different normals obtained through GF2, MDD  
 411 and MVA, as shown in Fig. 4.a.

412 The first striking result is that all these results are quite consistent. Almost all the directions are  
 413 less than ten degrees apart from each other, with an average difference of about five degrees. The  
 414 major exception concerns the comparison between MVA and  $GF2_B$  during the last second of the  
 415 interval where the two directions appear to be up to 35 degrees apart. This can be explained by  
 416 the fact that the local normals are observed (by  $GF2_B$  as well as by  $MDD_B$ ) to differ noticeably  
 417 in this part from their averaged value and that MVA is not able to detect such a change. Looking  
 418 in more detail, we can see a slight difference between the first part of the crossing (between  
 419 2s and  $\sim 6.5s$ ), where the two normals  $GF2_B$  and  $MDD_B$  differ by less than 5 degrees, and the  
 420 second part, where the angle between the normals can be up to ten degrees (probably due to a

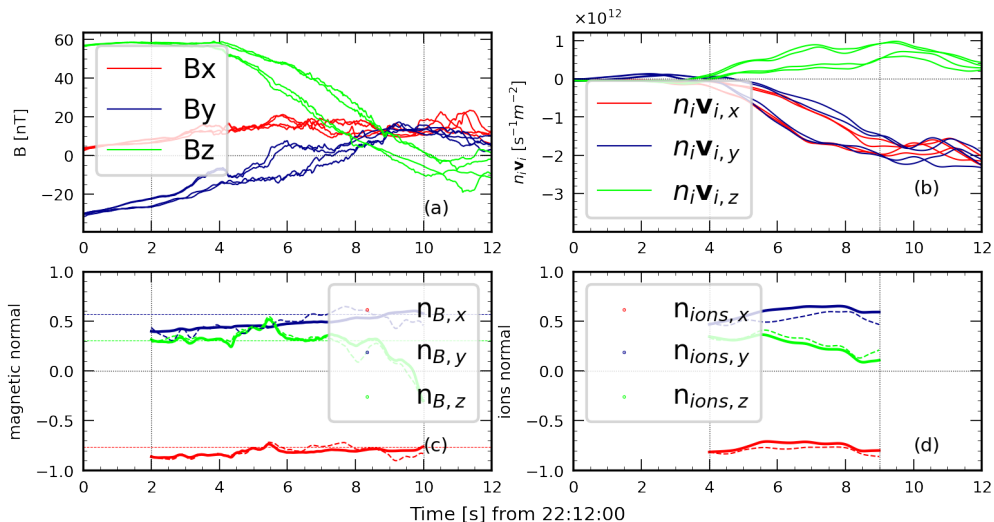


FIGURE 3. Comparison for the normals obtained with GF2 with respect to the MDD tools. (a) shows the magnetic field and (b) the ion mass flux, measured by the four MMS spacecraft. (c) and (d) shows the magnetic and the ion normal, respectively. The continuous (resp. dashed) line correspond to the components of GF2 (resp. MDD) normal. Horizontal dotted lines indicate the MVA normal obtained along the whole interval. Vertical dashed lines correspond to the time interval boundaries for the crossing, which are different for the magnetic field and the ion mass flux.

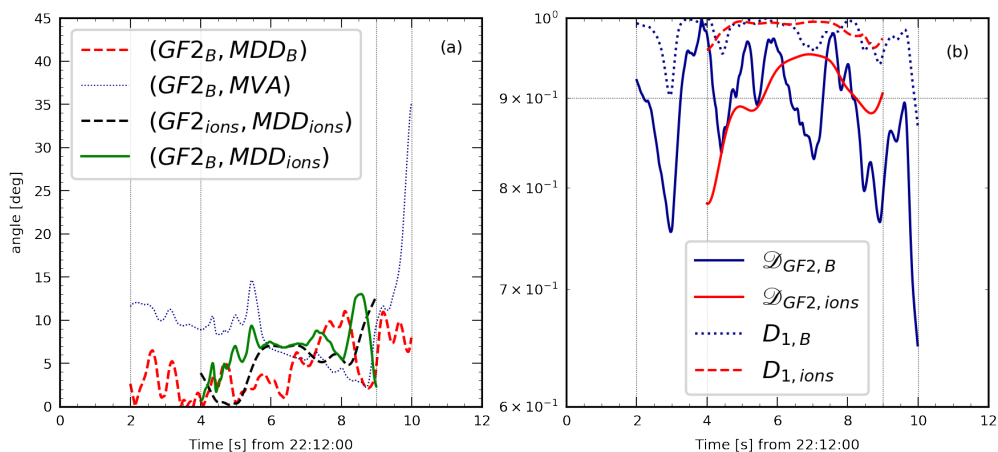


FIGURE 4. (a) Angle between the normals obtained using the state-of-the-art tools (MDD, MVA) and GF2. The subscript  $B$  and  $ions$  indicates whenever the magnetic field or the ion flux measurements are used. (b) Dimensionality of the structure as a function of time; here both the  $\mathcal{D}_{GF2}$  (continuous line) and the  $D_1$  (dashed line) indices are shown, for both the magnetic field (blue) and ions (red) data.

421 smaller ratio signal/noise for the gradient matrix  $\mathbf{G}$ ). The normals derived from ion measurements  
 422 are not much different from those derived from the magnetic field, showing that the particle and  
 423 magnetic structures are approximately identical. In Fig. (4.b), the dimensionality of the structures  
 424 is analyzed as a function of time, by using both the  $\mathcal{D}_{GF2}$  and the  $D_1$  (Rezeau *et al.* (2018))  
 425 parameters, as explained above. Even if the numerical values of the two indices are slightly  
 426 different, they both indicate structures close to one-dimensionality in the first part, with a -small

---

<b>Model</b>	<b>Normal [GSE]</b>	<b>angle with <math>\mathbf{n}_{GF2,B}</math> [deg]</b>
$\mathbf{n}_{GF2,B}$	[0.82, -0.49, -0.29]	x
$\mathbf{n}_{GF2,ions}$	[0.76, -0.59, -0.26]	7.2
MVA	[0.76, -0.57, -0.30]	6.1
MDD	[0.83, -0.49, -0.28]	0.7
Denton	[0.82, -0.49, -0.29]	0.4

---

TABLE 1. Magnetopause normal vectors obtained with the main tools presented above averaged in the time interval and their angle with respect to the normal obtained with GF2 using the magnetic field data (in degrees).

---

427 but significant- decrease in the second part. This increased departure from mono-dimensionality  
 428 can explain the slight difference between the two parts when comparing the normals from  
 429 standard MDD and GF2 techniques.

430 The present test does not allow us to state that GF2 is more accurate than standard MDD  
 431 (this will be checked in future work by comparing the two tools in a global simulation involving  
 432 realistic turbulence) but it shows at least a good agreement between the two approaches. We will  
 433 see in the following that this accuracy is anyway sufficient to prove the role of FLRs.

434 In order to smooth the small fluctuations over the time interval and to reduce the statistical  
 435 error associated with the determination of the normal, we can compare the directions averaged  
 436 along the crossing time. Mean values obtained through the tools presented above are shown in  
 437 Table 1. Here we observe that all the averaged normals differ by less than 10 degrees. Specifically,  
 438 we observe that the normals obtained with GF2, MDD and (Denton *et al.* 2018) are similar, with  
 439 a difference of less than one degree (with ours being closer to the one from Denton *et al.* (2018)).  
 440 MVA normal, instead, differs around 6 degrees from all these other normals. Finally, we also  
 441 observe that the one computed with ions flux data is the most distant. This is interpreted to be  
 442 due to the higher uncertainty of particles measurements.

## 443 7. Case study

444 In this section, we undertake a detailed analysis of the previously mentioned crossing case by  
 445 employing the normal obtained using the GF2 tool. Here, we focus on the time interval between  
 446 2 s and 9 s in Fig 2. To mitigate the potential influence of non-unidimensionality effects, we chose  
 447 to exclude the last second of the time interval studied in the preceding section for the magnetic  
 448 field (where both  $\mathcal{D}_{GF2}$  and  $D_1$  show that the structure is less one-dimensional and where we  
 449 observe that the normal is more different from the averaged one). To carry out this analysis, we  
 450 study the hodogram of the magnetic field in the tangential plane. Here the tangential results are  
 451 presented in a basis ( $\mathbf{T}_1, \mathbf{T}_2$ ) chosen as:

$$452 \quad \mathbf{T}_1 = \mathbf{n}_{mean} \times \hat{\mathbf{b}} \quad (7.1)$$

$$\quad \mathbf{T}_2 = \mathbf{n}_{mean} \times \mathbf{T}_1 \quad (7.2)$$

453 where  $\hat{\mathbf{b}} = \mathbf{B}/|\mathbf{B}|$  and  $\mathbf{n}_{mean}$  is the directions of the averaged normal in the chosen time interval.  
 454 Note that the choice of the reference frame ( $\mathbf{T}_1, \mathbf{T}_2$ ) is just a convention. The shape of the  
 455 hodogram remains unaffected by this choice except for the corresponding rotation in this  
 456 tangential plane. The direction  $\mathbf{t}_1$ , which characterizes the direction of the second dimension of  
 457 the model in GF2 and which is also in the tangential plane is generally different from  $\mathbf{T}_1$ .

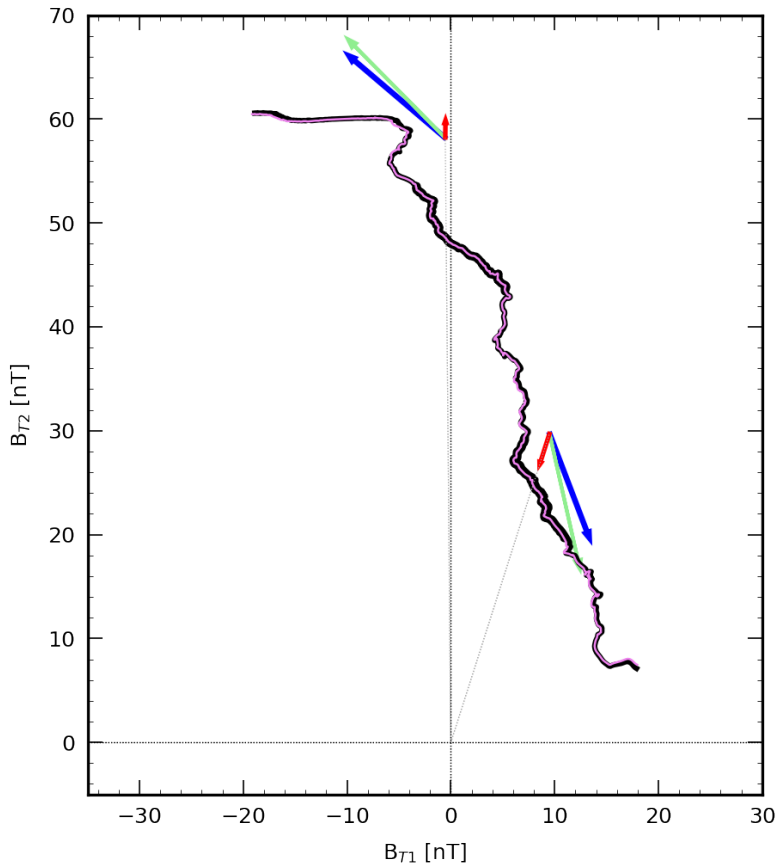


FIGURE 5. Hodogram in the tangential plane of the magnetic field for a magnetopause crossing by MMS in 28.12.2015 from 22:12:02 to 22:12:09. See text for the significance of the arrows.  $B_{T1}$  and  $B_{T2}$  are the projections of  $\mathbf{B}$  along the tangential directions computed as described in the text. The black line (resp. violet) is the hodogram when the  $\mathbf{n}_{mean}$  (resp.  $\mathbf{n}$ ) value is used to define the reference frame.

458 If CTD was valid everywhere, the hodogram of the magnetic field in the tangential plane for  
 459 a rotational discontinuity would correspond to a circular arc with constant radius while a shock  
 460 would correspond to a radial line (as shown in Fig. 1). For this reason, the hodogram is a good tool  
 461 to recognize the cases for which the CTD fails at describing the magnetopause. The hodogram  
 462 for this case is shown in Fig. 5. We observe a clear "linear" (although not radial) hodogram. This  
 463 non-radial variation of the magnetic field although not predicted by CTD, is a striking feature  
 464 of the hodogram. It cannot be explained by a departure from the 1D assumption since we have  
 465 measured that the crossing can be considered as one-dimensional to a good degree of accuracy.  
 466 It is therefore due to an intrinsic property of the layer itself. Also, in Fig. 5, we present the  
 467 hodogram derived from the local normal (un-averaged, violet line). It is clear that averaging does  
 468 not affect the shape of the hodogram.

469 To further analyze the causes of the disagreement between the hodogram of this case  
 470 crossing and what is expected from CTD, we compare the different terms of the tangential  
 471 momentum equation and Faraday/Ohm's law. As discussed above, indeed, these two equations  
 472 are responsible for the distinction between the rotational and tangential discontinuities in CTD.  
 473 This is the object of Figure 6, where we plot the different terms of the two equations projected

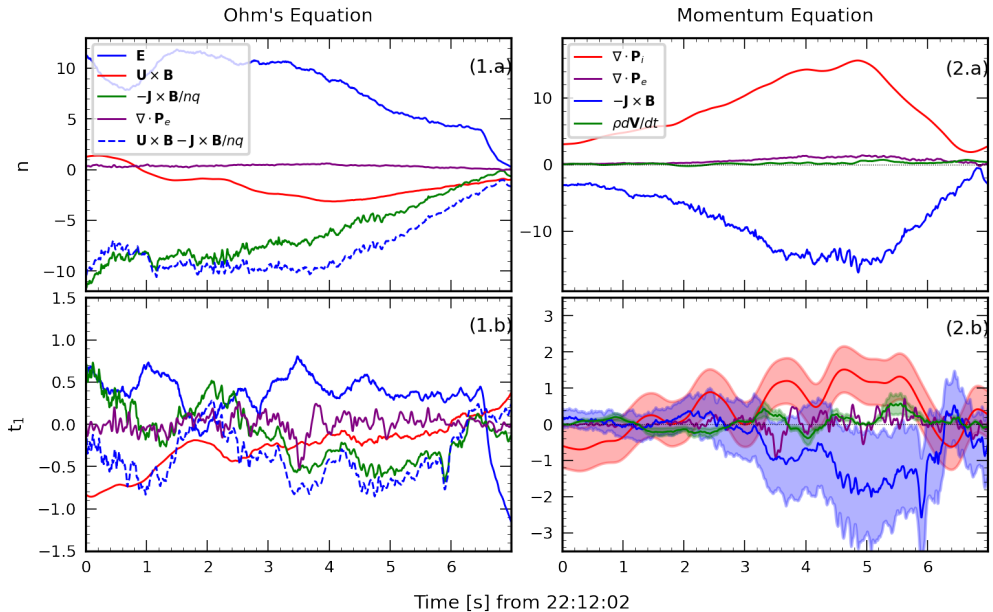


FIGURE 6. Terms of the Ohm's law (panel 1, units of  $\text{mV/m}$ ) and the momentum equation (panel 2, units of  $10^{-15} \text{kg m/s}^2$ ), projected in the normal direction  $n$  (a) and in the tangential direction  $t_1$  (b). To reduce the noise, a running average with a time window of 0.35s is applied to the electric field measurements. Shaded regions in panel 2.b represent the estimated uncertainties of the divergence of the pressure (red), the  $\mathbf{J} \times \mathbf{B}$  (blue) and the classic inertial term (green). Concerning the Ohm's law, we included the sum  $\mathbf{U} \times \mathbf{B} - \mathbf{J} \times \mathbf{B}/nq$  to facilitate the readability (blue dashed line).

*N.b.* The terms of the tangential Faraday/ Ohm's law used in the text are just the derivatives of the ones in (a) (apart from a  $\pi/2$  rotation).

474 along the  $\mathbf{n}_{mean}$  and  $\mathbf{t}_{1,mean}$  directions obtained using the GF2 tool (averaged over the whole  
 475 time interval). The influence of the averaging of the  $\mathbf{t}_{1,mean}$  direction on the results is discussed  
 476 in Appendix A. We do not show the quantities along the direction of invariance, which are  
 477 dominated by noise. The current and the gradient matrix for the pressure term are obtained via  
 478 the reciprocal vector method described in Chanteur (1998).

479 Concerning the Ohm's law (Figure 6, panel 1), we see that the electric field is well counter-  
 480 balanced by the  $\mathbf{u} \times \mathbf{B}$  and  $\mathbf{J} \times \mathbf{B}/nq$  terms (ideal and Hall terms). Outside the layer, on both sides,  
 481 the ideal Ohm's law is satisfied, as assumed in CTD (this is not visible on the figure, which is a  
 482 zoom on the inner part of the layer, and where the Hall term is important). It has been shown in  
 483 the literature that  $\nabla \cdot \mathbf{P}_e$  is not always negligible in the Ohm's law and that it can even be dominant  
 484 close to an Electron Diffusion Regions (EDR). This has been predicted theoretically (Hesse *et al.*  
 485 2011, 2014) and observed experimentally (Torbert *et al.* 2016; Genestreti *et al.* 2018), but it is not  
 486 the case for events like this one. We observe that at approximately 3.5 seconds, the  $\nabla \cdot \mathbf{P}_e$  is not  
 487 entirely negligible along the tangential direction (a similar peak can also be observed in panel 2.b  
 488 for the term associated with the electron pressure in the momentum equation). However, during  
 489 this time interval, this value is not dominant, this term being smaller than both the electric field  
 490 and the  $\mathbf{J} \times \mathbf{B}/nq$  components. Furthermore, this effect exhibits a local characteristic, as  $\nabla \cdot \mathbf{P}_e$   
 491 is only non-negligible within a small sub-interval (with respect to the magnetopause temporal  
 492 width). It is therefore not likely to be indicative of proximity to a reconnection point.

493 Concerning the momentum equation, shown in panel 2 of Figure 6, we observe that, in the  
 494 normal direction, the  $\mathbf{J} \times \mathbf{B}$  term is counter-balanced by the divergence of the ion pressure tensor,



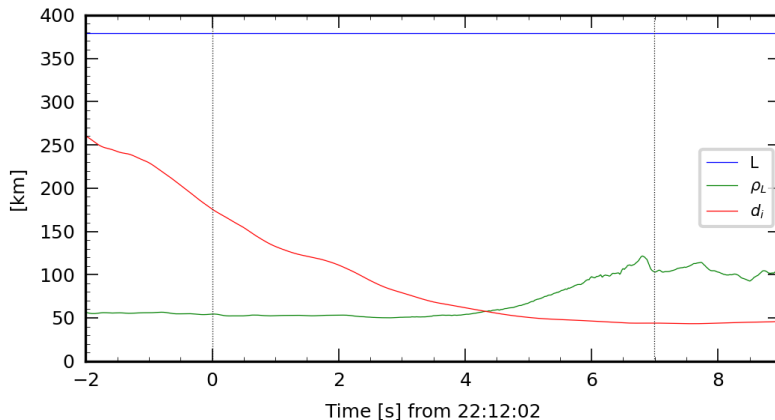


FIGURE 7. Comparison of the magnetopause width ( $L$ ) with the ion inertial length ( $d_i$ ) and the ions Larmor radius ( $\rho_L$ ). Vertical lines highlight the considered temporal interval.

495 as expected. But, if the isotropic condition assumed in CTD was valid, we would expect the  
 496 divergence of the ion pressure tensor to be zero in the tangential direction, or at least negligible  
 497 with respect to the inertial term  $\rho d\mathbf{u}/dt$ . On the contrary, we observe that the  $\mathbf{J} \times \mathbf{B}$  term along  $\mathbf{t}_1$   
 498 is of the same order of magnitude as the divergence of the ion pressure tensor, and one order of  
 499 magnitude larger than all the other terms. Panel 2.b also shows an estimation of the error on the  
 500 relevant terms:  $\mathbf{J} \times \mathbf{B}$ ,  $\nabla \cdot \mathbf{P}_i$  and the classical inertial term. It is known that measurements errors  
 501 are difficult to estimate, especially at small scales. In order to validate our results, however, we  
 502 sought to obtain an upper bound of the error associated with the quantities of interest. For that  
 503 purpose, an overestimation of the uncertainty of the measurements (acquired as the maximum  
 504 during the crossing of the errors available in FPI datasets for the pressure tensor and from the  
 505 FGM nominal error for the magnetic field) was exploited. These values are propagated as a  
 506 statistical (*i.e.* quadratic) error (by assuming that the errors on the reciprocal vectors can be  
 507 neglected with respect to that of other physical quantities).

508 From panel 2.b of Fig. 6, we see that the  $\mathbf{J} \times \mathbf{B}$  and the  $\nabla \cdot \mathbf{P}_i$  terms are pointing in opposite  
 509 directions and balancing each other. If valid in the first part of the interval, this conclusion  
 510 cannot be safely trusted due to measurement uncertainty, but we observe that in the middle part  
 511 (particularly between 3.5s and 6s) it is evident that the two quantities counterbalance each other  
 512 while the classical inertia term  $\rho d\mathbf{u}/dt$  is much lower with respect to the others. This proves that  
 513 the tangent  $\nabla \cdot \mathbf{P}_i$  term plays a fundamental role in the magnetopause equilibrium.

514 This point can be emphasized also by analyzing the hodogram. In Fig. 5, the arrows are  
 515 directed along the directions of the tangential plane that are physically relevant for the problem:  
 516 *i*) the tangent to the hodogram (green), which indicates the total variation of  $\mathbf{B}_t$ ; *ii*) the radial  
 517 direction (red), which corresponds to the plasma compression; *iii*) the  $\nabla \cdot \mathbf{P}_{it}$  direction (blue),  
 518 which is the direction of the divergence of the ion pressure tensor in the tangential plane, and  
 519 therefore corresponds to a term which is absent in CTD. The relative lengths of the arrows are  
 520 chosen proportional to the corresponding term magnitudes. These directions are averaged in two  
 521 sub-intervals (bold hodogram). The striking result is that the total variation is mainly determined  
 522 by the non-classic term  $\nabla \cdot \mathbf{P}_{it}$  and not by the radial classic one. This explains the very recurrent  
 523 (even if not reported in the literature hitherto) feature that the hodograms are almost linear but  
 524 not radial.

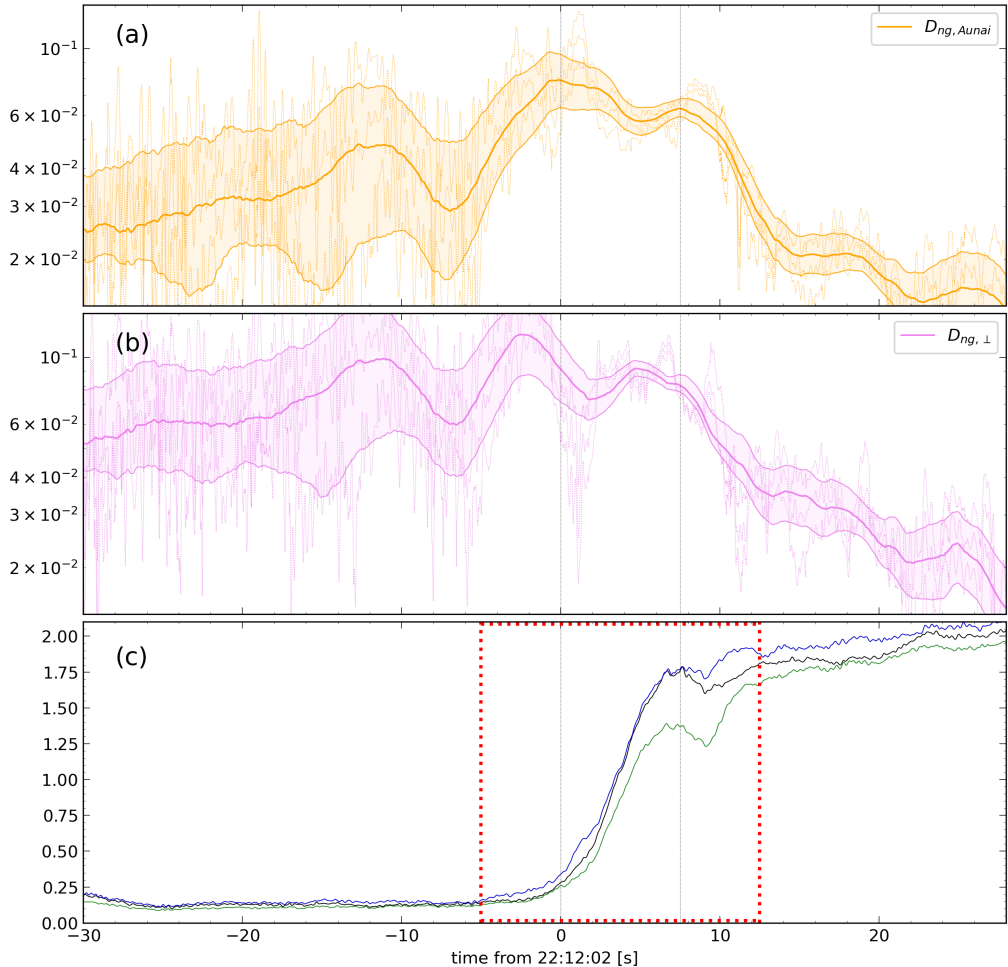


FIGURE 8. Panels *a* and *b* show the evolution of the  $D_{ng,\perp}$  and  $D_{ng,Aunai}$  (Aunai *et al.* 2013) indices, respectively, along with their estimated uncertainties. Thin lines correspond to the real-time values while thick lines to an averaged window of 1 s; (c) Evolution of the eigenvalues of the  $\mathbf{P}_i$  matrix (averaged on the four spacecraft). The dotted line indicates the magnetopause crossing. The red dotted lines in panel *c* highlight the time interval studied in Fig. 9

### 7.1. Comparison of the width of the magnetopause to relevant physical lengths

Finally, we compare the width of the magnetopause ( $L$ ) to the two main ion-related lengths: the ion Larmor radius ( $\rho_L$ ) and the ion inertial length ( $d_i$ ). The magnetopause width is estimated using the normal velocity obtained from the *GF2* tool. By averaging the velocity of the magnetopause in the normal direction, we can estimate  $L = \mathbf{V}_{n,mean}\Delta t$  (where  $\Delta t$  is the time length of the full crossing). These three scales are shown in Fig.7. We observe that this width is larger than the ion Larmor radius and the ion inertial length all across the crossing, but only two to five times larger, which appears sufficient to drive the observed kinetic effects.

## 8. Ion pressure tensor analysis

To further investigate the question of the ion non-gyrotropy with respect to the magnetic field and quantify this effect, let us now examine the properties of the ion pressure tensor and introduce

536 a new non-gyrotropy index. For that purpose, we define the matrices  $\mathbf{P}_{\parallel} = p_{\parallel} \mathbf{b}\mathbf{b}$ , where  $\mathbf{b} = \mathbf{B}/|\mathbf{B}|$   
 537 and  $p_{\parallel} = \mathbf{b} \cdot \mathbf{P}_i \cdot \mathbf{b}$ , and  $\mathbf{P}_{\perp} = \mathbf{P}_i - \mathbf{P}_{\parallel}$ . By defining  $p_1$  and  $p_2$  the maximum and intermediate  
 538 eigenvalues of the  $\mathbf{P}_{\perp}$  matrix, we define:

$$D_{ng,\perp} = \frac{p_1 - p_2}{p_1 + p_2} \quad (8.1)$$

539 In Fig. 8.a, this parameter is compared to the non-gyrotropy index presented in Aunai *et al.*  
 540 (2013). The two indices define nongyrotropy differently, (Aunai *et al.* 2013)'s index defining  
 541 nongyrotropy as the ratio of the nongyrotropic to the gyrotropic part of the tensor (instantaneous),  
 542 while ours makes use of the 2D modeling of the data used in GF2 (averages on sliding windows).  
 543 We note how both indices are significantly different from zero, approximately of the order of 0.1  
 544 within the boundary, corresponding to clearly present, although not predominant, non-gyrotropic  
 545 effects. We note a decrease in both indices outside the magnetopause, as expected, but it is worth  
 546 noting also that, despite a continuous decrease, these indices remain relatively high in the time  
 547 interval just preceding the crossing, in a region where magnetic field, density and pressure tensor  
 548 are almost constant. This can be understood by noting that an ion velocity gradient is observed in  
 549 this interval, suggesting that the non-diagonal terms of the pressure tensor could be due there to a  
 550 kind of gyroviscous effect, the non-diagonal terms of the pressure tensor (Braginskii 1965) being  
 551 due to FLRs (Stasiewicz 1993). One must take care that, in this interval, the pressure tensor has  
 552 low values characterized by larger relative errors, which could partially influence this result. To  
 553 further analyze this question, we have estimated the uncertainties on both non-gyrotropy indices.  
 554 This estimation is derived from the nominal uncertainties of the FPI dataset. The diagonal terms  
 555 have higher values and lower relative errors. Concerning the time interval before the crossing  
 556 that we discuss here, the diagonal terms have errors of approximately 5%, whereas off-diagonal  
 557 terms have an average relative error about 50%. We observe on Fig. 6 that this way of estimating  
 558 the uncertainty well encompasses the variance of the results. It confirms that, within the crossing  
 559 interval, all relative errors are smaller than 10 %, as considered in the Ohm's law study (Fig. 6).  
 560 In addition, a preliminary study appears to confirm the validity of the gyroviscous interpretation.  
 561 Using the theoretical expressions given in Stasiewicz (1989), we can compare the variations of  
 562 the non-diagonal terms of the pressure tensor with the spatial derivatives of the flow velocity, and  
 563 evidence a fairly good correlation (see Appendix B).

564 Fig. 8 (panel *c*) also shows the evolution of the eigenvalues of the  $\mathbf{P}_i$  tensor, averaged on the  
 565 four spacecraft. This figure shows how outside of the magnetopause the three eigenvalues tend to  
 566 converge towards each other meaning that these media are close to isotropy. However, inside the  
 567 magnetopause, we note a transition in the behavior of the intermediate eigenvalue, shifting from  
 568 a value close to the minor one to being closer to the major eigenvalue. The minor eigenvalue  
 569 exhibits a significant deviation from the other two towards the last part of the crossing.

570 Focusing on the temporal interval marked by the red square in Fig. 8, this transition is further  
 571 investigated in Fig. 9 where we show the ions' distribution functions in the tangential plane  
 572 (with respect to the magnetopause) for four different intervals during the crossing, highlighting  
 573 the non-gyrotropy of the ions' distribution function over time. VDFs (printed using a linear 2D  
 574 interpolation on a cartesian grid in the chosen plane using the Pyspedas library) are here averaged  
 575 in the corresponding time intervals framed with the same color as in the bottom plot where the  
 576 eigenvalues of the ion pressure tensor are plotted again (the time length decreases as the density  
 577 increases).

578 Finally, we analyzed the non-gyrotropy with respect to a generic direction, *i.e.* without  
 579 imposing that this direction is the magnetic field direction. Specifically, we have looked at a

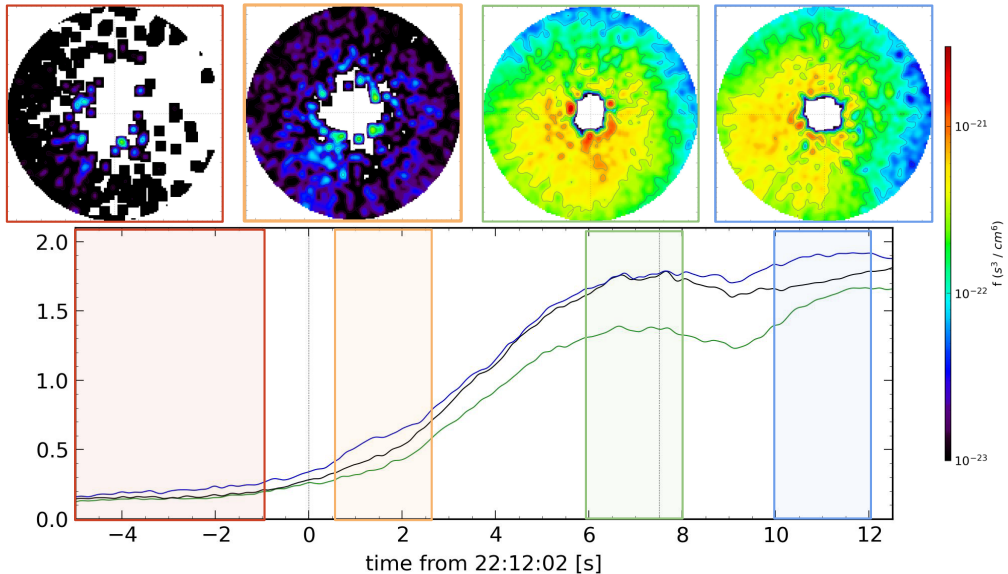


FIGURE 9. Top: Ions' velocity distribution functions in the tangential plane (the  $\mathbf{T}_1$ -  $\mathbf{T}_2$  plane) averaged in four different time periods. Velocities axes are between  $-220$  km/s and  $220$  km/s. Bottom: Eigenvalues of the pressure tensor (same interval as in the red dashed square of Fig. 8.c). The four colored boxes are used to distinguish the four time intervals.

580 direction, denoted as  $\mathbf{g}$ , around which the rotated matrix could be rewritten as follows:

$$\begin{pmatrix} P_2 & 0 & 0 \\ 0 & P_1 & 0 \\ 0 & 0 & P_1 \end{pmatrix} \quad (8.2)$$

581 To achieve this, we employ a minimization algorithm to derive the rotation matrix  $\mathbf{M}$  that allows  
 582 us to put the pressure tensor data under a form as close as possible to this one. Results from this  
 583 study are shown in Fig. 10 (here shown for MMS2). Panel *a* displays the variation of  $P_1$  and  $P_2$   
 584 along the crossing, where  $P_2$  consistently exceeds  $P_1$ . In addition, we imposed an upper limit on  
 585 the temporal variation of the gyrotropic direction  $\mathbf{g}$ , excluding points with significant temporal  
 586 variations (indicated by the thin line). Consequently, the remaining points reflect instances where  
 587 the direction of  $\mathbf{g}$  can be considered as stable and reliable. The vector  $\mathbf{g}$  itself is represented in  
 588 panels *b* and *c*, where it is clear that the direction of gyrotropy is not close to the magnetic  
 589 field direction: it is close to  $\mathbf{n}_{mean} \times \mathbf{B}$ , the component along  $\mathbf{B}$  being smaller and varying. This  
 590 result reminds us that at boundaries such as the magnetopause, the strong gradients can break  
 591 the isotropy as much, and even more here, than the magnetic field, so that gyrotropy can be  
 592 around another vector than  $\mathbf{B}$ . A similar remark had already been made in Belmont *et al.* (2012)  
 593 concerning the modeling of a tangential discontinuity.

## 594 9. Dataset selection

595 In order to expand the results on a statistical basis, we selected a dataset of 146 crossings,  
 596 chosen from the largest one reported in Nguyen *et al.* (2022) and Michotte De Welle *et al.* (2022).  
 597 From this database, the following conditions were required in order to carry out an accurate  
 598 study:

599 (i) MMS data are in burst mode.

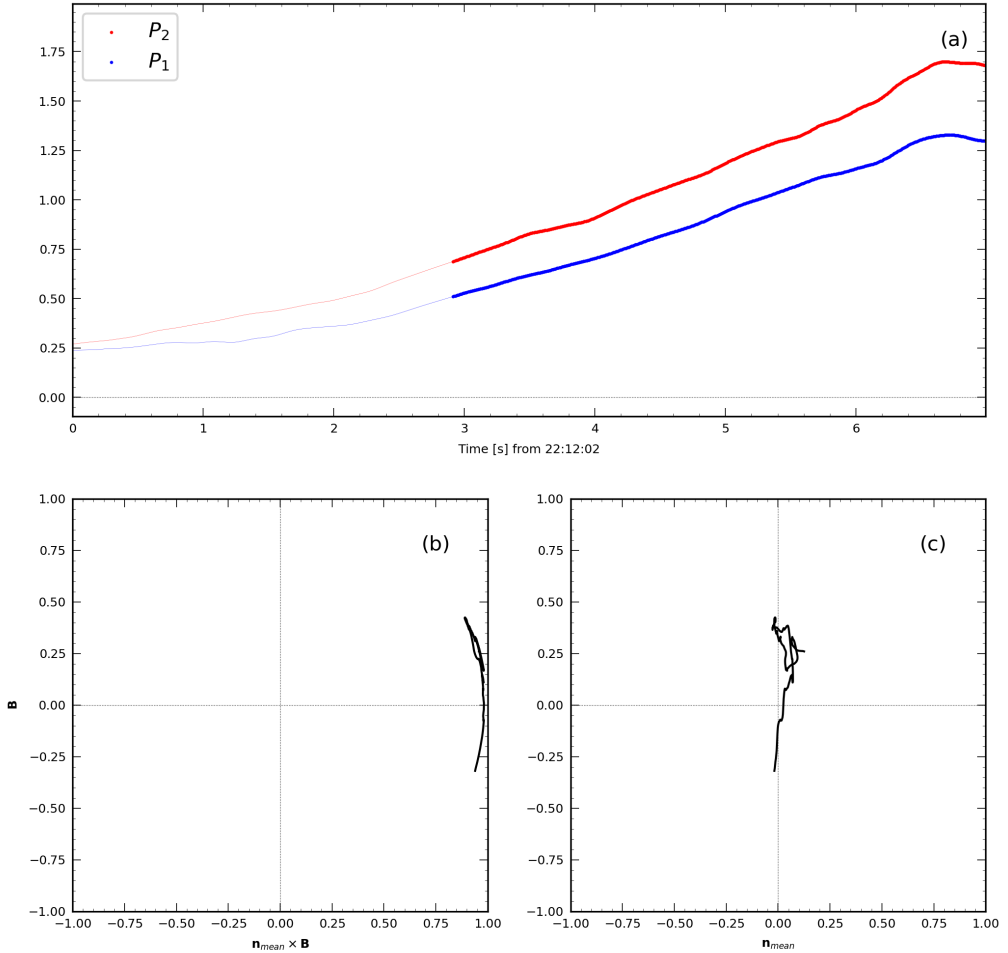


FIGURE 10. (a): Evolution of parameters  $P_1$  and  $P_2$ . (b) and (c): projections of the gyrotropy direction in two planes. The ordinate is the direction of  $\mathbf{B}$ , the abscissa is the direction of  $\mathbf{n}_{mean} \times \mathbf{B}$  for panel (b) and  $\mathbf{n}_{mean}$  for panel (c).

- 600 (ii) The crossing duration is between 3 and 15 seconds. Too short crossings do not have a sufficient  
 601 number of points within the structure (ion measurements are every 0.15 s). Too long crossings  
 602 may imply non-stationary structures.  
 603 (iii) Partial crossings are discarded. For that, we impose a density threshold less than  $4 \text{ cm}^{-3}$   
 604 in the magnetosphere and larger than  $15 \text{ cm}^{-3}$  in the magnetosheath.  
 605 (iv) Only cases presenting simultaneous crossing features in particles and magnetic field are  
 606 considered, in order to compare normals computed at the same time.

607 In addition to these basic requirements, we also excluded some of the selected crossings for  
 608 criteria that demand a more detailed analysis of the internal structure of the boundary. First,  
 609 we excluded two-dimensional features. The quantitative determination of the dimensionality  
 610 was done with the parameters presented in Rezeau *et al.* (2018) and the dimensionality index  
 611 presented in Section 5.2, which are functions of the ratio between the eigenvalues of the gradient  
 612 matrix. Namely, we considered only crossings with  $D_1 > 0.9$  and  $\mathcal{D}_{GF2} > 0.8$ , these two  
 613 parameters being averaged on the crossing interval. These parameters are calculated at each time

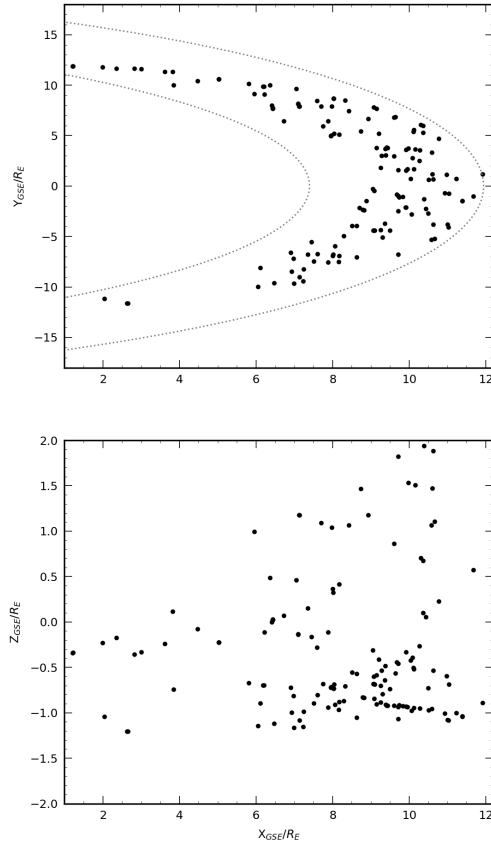


FIGURE 11. Spatial distribution of the selected database of crossings on the  $x, y$  (top) and  $x, z$  planes (bottom). The dashed grey lines represent the magnetopause location (Shue *et al.* 1997).

614 step but, due to waves and turbulence, attention must be paid that some of these two-dimensional  
 615 features can be only local and insignificant for the profiles we are looking for. It is the reason  
 616 why we use only the averaged values. The 146 selected crossings span from September 2015 to  
 617 December 2017 (included). We can observe in Figure 11 that the crossings are evenly distributed  
 618 in the  $x, y$  plane. Regarding the  $z$  component, there is a prevalence of cases at negative  $z$ .

619 The list of crossings can be found in Supplementary Materials. For each crossing, the  
 620 classification and the physical quantities relevant for the study (normals, dimensionality index,  
 621 non-gyrotropy index, and the main characteristic lengths discussed above for the case crossing)  
 622 are included.

## 623 10. Statistical study of the magnetic hodograms

624 The previous results about the role of the FLR effects at the magnetopause are now carried out  
 625 statistically. This study aims to generalize the results obtained from the case crossing studied in  
 626 the previous section and to estimate the role played by FLRs at the magnetopause.

627 The database described above has first been used to perform a statistical study on the hodogram  
 628 shapes, to determine how often linear hodograms are observed in magnetopause crossing. Having  
 629 an estimation of the percentage of crossings that do not conform to CTD allows us to gauge how  
 630 frequently the assumptions made by this theory do not accurately represent the magnetopause.

631 For this purpose, we separate the crossings in different classes, this classification being based on  
 632 CTD distinctions and on the preceding findings:

- 633 (i) *Linear* crossings, *i.e.* straight lines not passing through the origin as in the above case study.
- 634 (ii) *Radial* crossings, including all linear crossings whose best fit line passes through the origin  
 635 (considering uncertainty). These crossings correspond to CTD compressional discontinuities.
- 636 (iii) *Circular* crossings, when the distance from the origin is constant. These cases correspond to  
 637 CTD rotational discontinuities.
- 638 (iv) *Other* crossings, whose features are not included in the previous classes. This class includes  
 639 crossings with various features, *e.g.* circular hodograms not centered on the origin, crossing  
 640 characterised by two different hodograms in two sub-intervals, etc., and crossings that do not  
 641 have an obvious distinction between the previous classes, due to noise.

642 To classify each crossing, we only focus on its central time interval, where the gradients  
 643 are maximum. By considering larger time intervals, the hodograms' shape becomes more  
 644 complex because the variations out of this interval are generally unrelated to the main boundary  
 645 jumps. Selecting only the middle part of the crossing provides simpler and more conformal  
 646 hodograms. Even if the boundary jumps are not fully completed in this part, this will not prevent  
 647 comparing the experimental results with CTD predictions since this theory, when valid, is  
 648 based on conservation laws for any sub-interval of the discontinuity. When this theory fails to  
 649 reproduce the observed properties, we can interpret those new features as coming from kinetic  
 650 effects, therefore confirming the limitation of CTD to describe the magnetopause boundary. To  
 651 that purpose, for each dataset we selected the crossing temporal interval following the algorithm  
 652 used in Haaland *et al.* (2004, 2014) and Paschmann *et al.* (2018b) to estimate the spatial scale of  
 653 the magnetopause (intervals are identified as 75% of the magnetic field  $\mathbf{B}_L$  component variation).

654 The classification performed here differs from previous attempts to classify magnetopause  
 655 hodograms, as seen in studies such as Sonnerup & Ledley (1974); Berchem & Russell (1982)  
 656 and Panov *et al.* (2011). In these previous works, hodograms were categorized as C-shaped  
 657 or S-shaped based on their form in the tangential plane. However, unlike those studies, we  
 658 considered the central part of the crossing, rather than considering the entire temporal interval.  
 659 Our classification of hodograms involves a two-step process:

- 660 (1) Visual Inspection: Initially, all hodograms are visually inspected to identify the cases that  
 661 are clearly not linear or circular, which are classified separately as 'Others'. Additionally, a  
 662 preliminary distinction is made between crossings with circular and linear features.
- 663 (2.a) Analysis of hodograms with possible circular features: For these crossings, we analyze  
 664 the variation of the modulus of the magnetic field in the plane, allowing for a maximum  
 665 possible variation of 20%. This accounts for factors such as turbulence and waves propagating  
 666 alongside the magnetopause. Any crossings exceeding this 20% threshold are categorized as  
 667 'Others.'
- 668 (2.b) Analysis of hodograms with possible linear or radial features: These crossings undergo an  
 669 initial assessment to confirm their linearity. This involves examining the width-to-length ratio  
 670 of the crossing, with any ratio exceeding 20% classified as 'Other.' Finally, the remaining  
 671 crossings are classified as either radial or linear based on whether their projection passes  
 672 through the origin.

673 From this database, we found the following distribution:

- 674 - 36.3% (53/146) of the crossings present linear features.
- 675 - 2.7% (4/146) of the crossings present circular features (rotational discontinuity).
- 676 - 15.8% (23/146) of the crossings present radial features (compressional discontinuity).

677 - 45.2% (66/146) of the crossings could not be interpreted definitely as either of the three before  
678 (presenting more than one feature at the same time).

679 It follows that more than a third of the selected crossings show linear features, emphasizing  
680 that the fundamental role FLR effects have on magnetopause structure is found in a significant  
681 number of crossings.

682 It could be interesting to compare the above results with the several classifications that were  
683 previously published (see Liu *et al.* (2022) and references therein). These previous classifications  
684 were not based on the analysis of the rotational and compressional properties as done here, but  
685 on the normal component of the magnetic field and its magnitude (background and variation)  
686 (Smith 1973; Burlaga *et al.* 1977; Tsurutani & Smith 1979; Neugebauer *et al.* 2010). For such a  
687 comparison, however, one should take care that there are important differences in the definitions:  
688 in these previous classifications in particular, any discontinuity is named "tangential", whatever  
689 its other properties, as soon as the measured  $B_n$  is sufficiently smaller than  $B$ , the threshold for  
690 this ratio being for instance of the order of 0.3 (Liu *et al.* 2022; Smith 1973; Burlaga *et al.* 1977;  
691 Tsurutani & Smith 1979; Neugebauer *et al.* 2010). This is of course a very different approach  
692 from the one we use here since, even when  $B_n$  is small (and even if barely measurable), we  
693 consider that different kinds of discontinuities exist, with different properties.

694 As done for the case study above, it was possible to study on a statistical basis *i*) the ratio  
695 between the width of the magnetopause and the ion Larmor radius and *ii*) the non-gyrotropy  
696 index. For both parameters, the case study appears rather typical. On average, the magnetopause  
697 was found to be approximately 6.5 times the ion Larmor radius, only slightly smaller (6.1) for  
698 linear hodograms. Similarly, the non-gyrotropy index  $D_{ng,\perp}$ , has an average value of 0.07, only  
699 slightly higher (0.08) for linear hodograms. The  $D_{ng,Aumai}$  index has even comparable averages for  
700 the four different classes. It therefore seems that, although non gyrotropy has been demonstrated  
701 above to play an important role, the non-gyrotropy index alone is not decisive for predicting  
702 unequivocally the shape of the hodograms. This question should be the subject of future works.

## 703 11. A comparison between the magnetic and the particles normals

704 For each crossing, both the magnetic and the particles normals were computed with the  
705 GF2 tool. Thanks to the high resolution of the MMS measurements, we can measure the local  
706 fluctuations of the normals inside the magnetopause around their mean values. However, in order  
707 to compare the magnetic and ion geometries, a single average normal was used for each case.  
708 The mean normal is obtained inside the same time interval as in the previous study.

709 To study the differences between the two normals, we compared them via their departure from  
710 the Shue model's normal (where the magnetopause is assumed to be a paraboloid, (Shue *et al.*  
711 1997)). This normal was obtained using the solar wind and IMF properties from the OMNI data  
712 set (King & Papitashvili 2005). The time delay between the crossing time and the measurement  
713 time of the solar wind relevant parameters is estimated by using the propagation method used  
714 in Michotte De Welle *et al.* (2022) (which was adapted from Šafránková *et al.* (2002)). The  
715 procedure for acquiring these parameters is as follows: *i*) the distance from the bow shock's nose  
716 (where OMNI data are defined) to the crossing location, projected along the Earth-Sun axis,  
717 is estimated; *ii*) we estimate the solar wind's propagation time ( $t_{est}$ ) between these two points,  
718 assuming an average solar wind velocity of 400 km/s; *iii*) the solar wind velocity  $V_{sw}$  is then  
719 determined from the OMNI dataset, averaging over a 2-minute interval centered on the crossing  
720 time adjusted by the time delay  $t_{est}$ ; and (IV) ultimately, a final time delay is computed based on  
721  $V_{sw}$ , which is subsequently utilized to obtain final values of solar wind and IMF parameters. The  
722 crossings for which OMNI data computed with this procedure are missing (10 out of 146) were  
723 left out of this analysis.



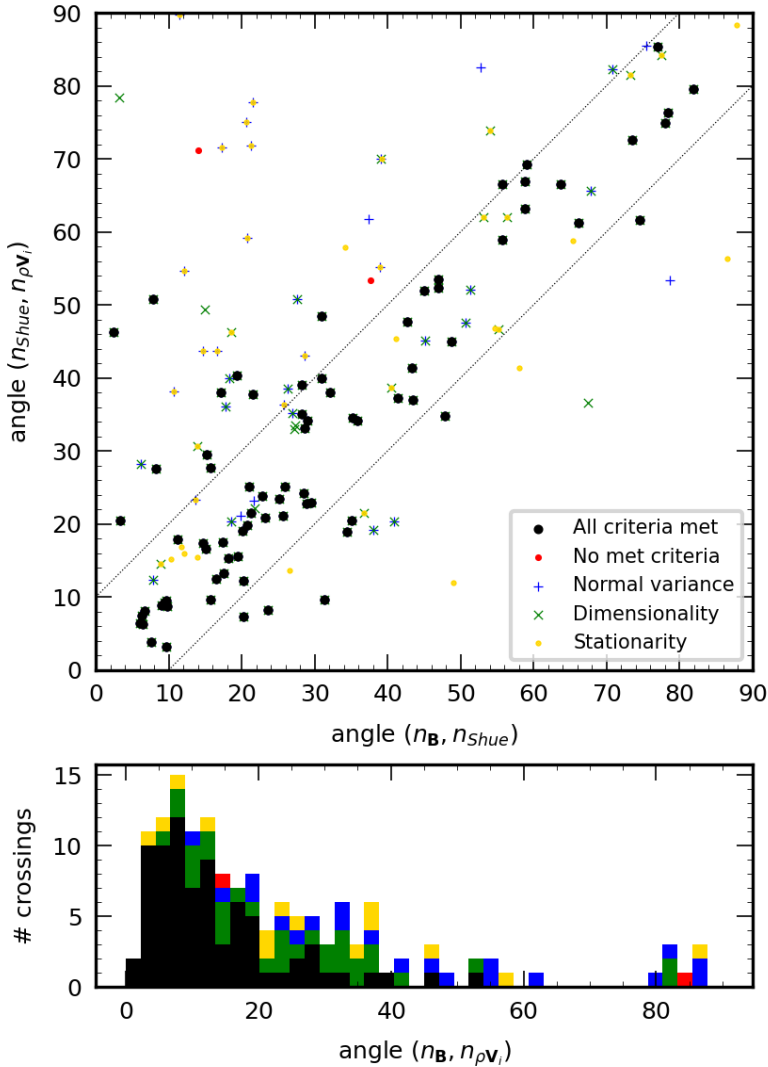


FIGURE 12. (a) Comparison between the angle between the theoretical normal (Shue *et al.* 1997) and the magnetic and ion normals, (b) Distribution of the angle between the magnetic and ion normals. Here the markers for each point is chosen depending on whether each crossing respects the criteria on dimensionality, stationarity, and normal variance on the ions flux measurements (see Appendix C for further details). Colors in the histograms are used accordingly. Blue, green and yellow points indicate the crossing with small variance on the ions normal direction within the crossing, good one-dimensionality and good stationarity. Black points indicate the crossings respecting all criteria, red points not any criteria.

724 In Figure 12.a, we plot the angle between the nominal normal and the magnetic and particles  
 725 normals respectively. In this figure we observe that most of the crossings are along the diagonal,  
 726 corresponding to cases where the two normals, ionic and magnetic, are similar (82 points out of  
 727 146 are between the two thin lines, which indicate differences of  $\pm 10^\circ$ ).

728 The cases are distributed throughout the plane, with many cases above  $40^\circ$ , although we  
 729 observe a cluster at lower angles, between zero and  $30^\circ$ . The largest angles correspond to a  
 730 magnetopause very far from the paraboloid shape assumed in Shue's model, which relies on the

731 assumption of a magnetopause at (or near) equilibrium. The departures are likely to be related to  
 732 surface waves on the boundary itself.

733 Finally, the distribution of the angles between the two normals is shown in Fig. 12.*b*. Here we  
 734 evidence again that most of the cases studied (82 out of 146) are below  $20^\circ$ , with the maximum  
 735 of the distribution at  $10^\circ$ . However, we also observe again that several cases have much larger  
 736 angles, up to  $90^\circ$ . The strongest departures are problematic and deserve further investigation.  
 737 This appears to be due to the more complex ion structure with respect to the magnetic one. As the  
 738 criteria used for the dataset selection were built from magnetic data, they are not as relevant when  
 739 considering ion normals. This is evidenced in Figure 12, where the colors indicate how several  
 740 ion criteria are satisfied. These criteria concern respectively the dimensionality, the stationarity,  
 741 and the variance of the normal direction. All details are given in Appendix C. Focusing on points  
 742 respecting all the criteria for the ions flux (black markers and hodogram), we observe that only a  
 743 few crossings are outside the diagonal. Only two of these crossings have angles above  $40^\circ$ .

## 744 12. Conclusions

745 The study of the properties of the magnetopause is a very important issue for understanding  
 746 the penetration of the solar wind plasma into the magnetosphere. In the theoretical part, we  
 747 show that the notion of "quasi-tangential" discontinuity has to be introduced to complete the  
 748 theory of discontinuities and understand the limit when the crossing fluxes tend to zero as in  
 749 the magnetopause case. We emphasize that, in presence of anisotropy, the physical processes  
 750 occurring inside the layer play a fundamental role because they are responsible for the conditions  
 751 linking the downstream and upstream quantities. In particular, for thin current layers, the FLR  
 752 corrections corresponding to the non-gyrotropic pressure tensor components must be taken into  
 753 account.

754 The tool GF2 presented in the paper and used for determining the normal direction to the  
 755 boundary derives from the MDD method. It includes in addition a fitting procedure, which allows  
 756 introducing a part of the temporal information via a 4-point filtering of the data and adding  
 757 constraints such as  $\nabla \cdot \mathbf{B} = 0$ . It is shown here to provide results quite compatible with the  
 758 original method (when used with smoothed data), which is enough for drawing reliable physical  
 759 conclusions on the magnetopause equilibrium. We expect that this approach could bring more  
 760 precise information concerning the magnetopause gradients. Unfortunately, investigating this  
 761 point in more detail cannot be done using MMS data but requires testing the tool in fully 3D  
 762 kinetic simulations with realistic turbulence. This point is the subject of future work. Here, we  
 763 have applied this tool on a particular crossing case and compared with other state-of-the-art  
 764 normals. We have shown that the local normal (at each time step during the crossing) differs by  
 765 less than ten degrees from the one calculated by all the other models. When averaging over the  
 766 whole crossing, the normal obtained with the GF2 is even less than one degree apart from the  
 767 normals from Shi *et al.* (2005); Denton *et al.* (2018).

768 Although we cannot claim to have achieved the ideal accuracy of about one degree, the reached  
 769 accuracy is sufficient to evidence the correct physics at play, resumed as FLR effects. We have  
 770 presented the results for a crossing observed by the four MMS spacecraft. For this crossing,  
 771 the "linear" hodogram in the tangential plane shows that the boundary properties differ from  
 772 those predicted by CTD. This discrepancy is explained by looking at the tangential components  
 773 of the momentum equation, which highlights the role of the pressure tensor symmetries in the  
 774 magnetopause equilibrium. This result agrees with the theoretical results of the first part and it is  
 775 likely to hold more generally for all quasi-tangential discontinuities. The ion pressure tensor has  
 776 been analyzed for this purpose. We have used two indices of non-gyrotropy, which both confirm  
 777 the presence of a significant, even if small, non-gyrotropic part in this tensor. Furthermore,  
 778 we have shown that the non-gyrotropy direction differs from the magnetic field one, aligning

779 approximately with the  $\mathbf{n}_{mean} \times \mathbf{B}$  direction. Finally, the analysis of the VDFs directly confirms  
780 the presence of non-gyrotropic distributions.

781 To show that our methodology applies to cases that CTD cannot handle, we have selected a  
782 substantial number of magnetopause crossings with one-dimensional characteristics to have a  
783 proper statistical basis for our findings. For all these crossings, we have plotted the hodogram  
784 of the magnetic field in the tangential plane and classified them depending on their geometry.  
785 Our results show that 36.3% of the crossings evidence clear linear features, incompatible with  
786 the CTD description, while only 18.5% of the crossings show either circular or radial hodograms  
787 as predicted by CTD. In other words, a significant number of cases escapes the classic theory,  
788 proving that the relevance, even if not a predominance, of FLR effects at the magnetopause can  
789 be generalized and that the case crossing presented in the first section is rather typical. It is well-  
790 known that the linear version of the rotational discontinuity is the MHD shear Alfvén wave. Here  
791 it appears that the magnetopause-like "quasi-tangential" discontinuities correspond in the same  
792 way to the quasi-perpendicular "Kinetic Alfvén Waves" (Hasegawa & Uberoi 1982; Belmont &  
793 Rezeau 1987; Cramer 2001).

794 Several papers have investigated the changes in rotational discontinuities when various non-  
795 ideal effects are introduced. These theoretical papers have addressed the problem as a Riemann  
796 problem using the methodology of a "piston" to study the formation of different discontinuities.  
797 Some introduced FLRs and gyroviscosity in the layer while assuming isotropy on both sides  
798 (Lyu & Kan 1989; Hau & Sonnerup 1991), and others introduced anisotropy everywhere while  
799 assuming gyrotropy in the layer (Hau & Wang 2016). These different papers lead to different  
800 conclusions; in particular concerning the role of electron inertia in the layer equilibrium.

801 It is worth noticing that the hodograms of  $\mathbf{B}$  obtained with these theoretical studies were never  
802 far from circular ones, contrary to the almost linear shapes shown in the present paper. Our  
803 methodology has been different here: without assuming pre-defined forms for the non-ideal  
804 terms, we look experimentally to the hodograms and the form of the  $\mathbf{P}$  tensor and explain  
805 theoretically how the second can explain the first ones.

806 Finally, we have used the same database of crossings to compare the geometric properties  
807 of the magnetic and ion structures. We have compared the normal obtained from the magnetic  
808 field and the ion flux measurements to the one expected from Shue *et al.* (1997) model. Many  
809 crossings differ by more than 40 degrees from the nominal equilibrium condition, underlining a  
810 very dynamical environment, but it is worth noticing that the two kinds of determination are most  
811 often in agreement with each other, and therefore confirm the result. Furthermore, an accurate  
812 study of the ion flux measurements have shown that crossings showing bigger discrepancies  
813 between the magnetic field and ion flux normals are generally due to non-stationarities, non-one-  
814 dimensionality, or variations in the ion flux normals. When excluding these cases from the study,  
815 the ion and magnetic flux normals are compatible with only two crossings (over 77) showing  
816 angles larger than forty degrees.

## 817 Data availability

818 Magnetospheric Multiscale satellite data were accessed through the MMS Science  
819 Data Center, <https://lasp.colorado.edu/mms/sdc/public/>. Furthermore, all the  
820 softwares employed, from interpolation of the data to the analysis itself, can be found at  
821 [https://github.com/GiulioBallerini/Notebooks\\_FLR.git](https://github.com/GiulioBallerini/Notebooks_FLR.git).

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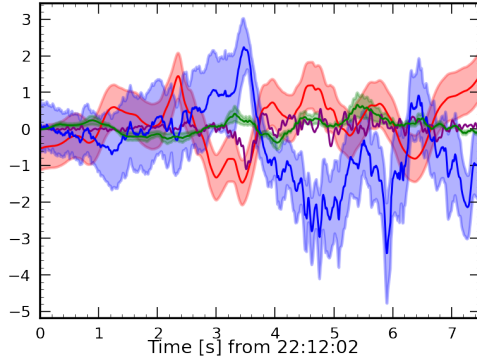


FIGURE 13. Terms of the momentum equation (units of  $10^{-15} \text{kg m/s}^2$ ), projected on the local tangential direction ( $\mathbf{t}_1$ ). Shaded regions are estimated uncertainties of the divergence of the pressure (red), the  $\mathbf{J} \times \mathbf{B}$  (blue) and the classic inertial term (green).

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### 827 **Appendix A. Influence of averaging the $\mathbf{t}_1$ direction in the momentum equation** 828 **balance**

829 In this section, we investigate the impact of using an averaged tangential direction along the  
830 crossing on the outcomes concerning the role of the pressure tensor in the momentum equation.  
831 In Fig. 13 we show the projection of the terms of the momentum equation along the local  $\mathbf{t}_1$   
832 direction (*i.e.* without averaging). We observe here some reversals of the sign of the dominant  
833 terms, that were not observed in the averaged case. Nonetheless, it is still evident that the pressure  
834 tensor counterbalances the  $\mathbf{J} \times \mathbf{B}$  term, hereby confirming our earlier findings.

### 835 **Appendix B. Analysis of the gyroviscous effects**

836 In this section, we use the magnetopause crossing analysed in detail above to study the validity  
837 of the gyroviscous interpretation. In particular, we employ the Braginskii gyroviscosity term  
838 (Braginskii 1965) as applied by Stasiewicz (1989) to the magnetopause, to analyze the pressure  
839 tensor. In this case, the pressure tensor is considered as the sum of an isotropic component,  $\mathbf{P}_{iso}$   
840 and a viscosity term,  $\sigma$ :

$$\mathbf{P}_i = \mathbf{P}_{iso} - \sigma \quad (\text{B } 1)$$

841 To investigate the viscosity term, we use the reference system where the normal direction is  
842 aligned with the z-axis (the x and y directions are chosen accordingly to form an orthogonal  
843 triad). By exploiting the definition of  $\sigma$ , we focus here on its projection along the normal yielding  
844 the following relation:

$$-\sigma \cdot \mathbf{n} = \begin{pmatrix} P_{nx} \\ P_{ny} \\ P_{nz} \end{pmatrix} = \rho \nu \begin{pmatrix} 0 & b_n & b_y \\ -b_n & 0 & -b_x \\ b_y & -b_x & 0 \end{pmatrix} \cdot \begin{pmatrix} u'_x \\ u'_y \\ u'_n \end{pmatrix} \quad (\text{B } 2)$$

845 Here  $\nu$  is the gyroviscosity coefficient,  $\hat{\mathbf{b}} = (b_x, b_y, b_z)$  the normalized magnetic field, and  $\mathbf{u}' =$   
846  $(u'_x, u'_y, u'_z)$  is the vector of the spatial derivatives of the velocity components along the normal.  
847 We now consider the first two components of this equation, yielding the following expressions

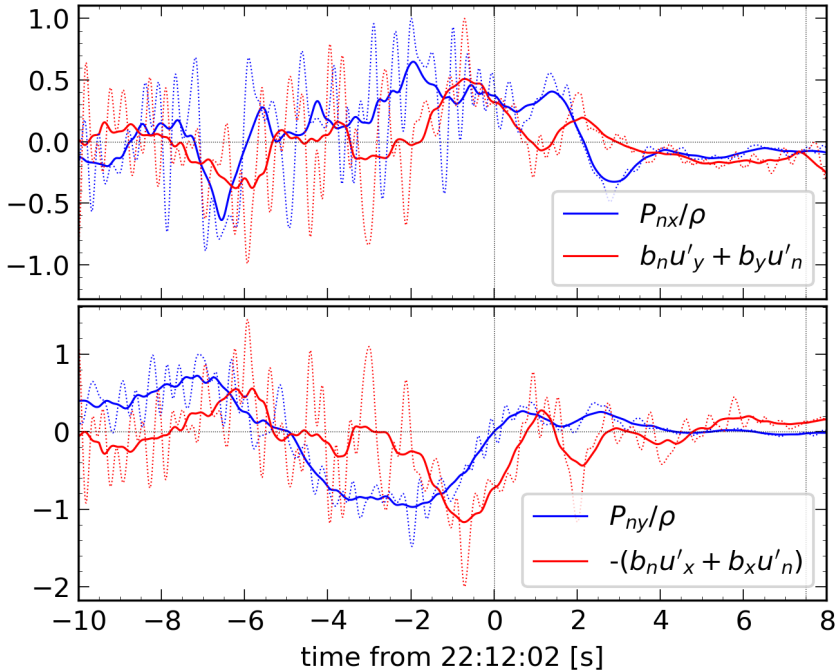


FIGURE 14. Left (blue) and right (red) hand sides for Equations B 3 (top) and B 4 (bottom). Thin-dotted lines correspond to the real-time values while thick lines to an averaged window of 1 s. All terms are normalized.

848 that allow us to compare the non-diagonal terms with the velocity changes:

$$\frac{P_{nx}}{\rho} = v(b_n u'_y + b_y u'_n) \quad (\text{B } 3)$$

849

$$\frac{P_{ny}}{\rho} = -v(b_n u'_x + b_x u'_n) \quad (\text{B } 4)$$

850 The terms of these equations are shown (normalized) in Figure 14. Here we observe a fairly good  
 851 correlation between the non-diagonal terms of the pressure tensor and the spatial derivatives of  
 852 the flow velocity.

### 853 Appendix C. Quality indices for the ion normals

854 In the absence of additional caution, Figure 12 shows that the angle between the normal  
 855 obtained with the magnetic field and the one with the ion flux reaches very high values, up to 90  
 856 degrees. This result requires a more accurate study, as the criteria used for the dataset selection  
 857 are based on the magnetic field (except for the threshold imposed on the density values).

858 To interpret the results accurately, the following parameters were considered:

- 859 (i) Dimensionality of ion flux. For this purpose, we exploit the dimensionality index defined in  
 860 Equation 5.11, computed from the ion flux measurements.
- 861 (ii) Stationarity of the ion flux measurements. To evaluate stationarity, we exploit the GF2 tool.  
 862 Specifically, we consider the quality of the fit of the gradient matrix as an index of stationarity.

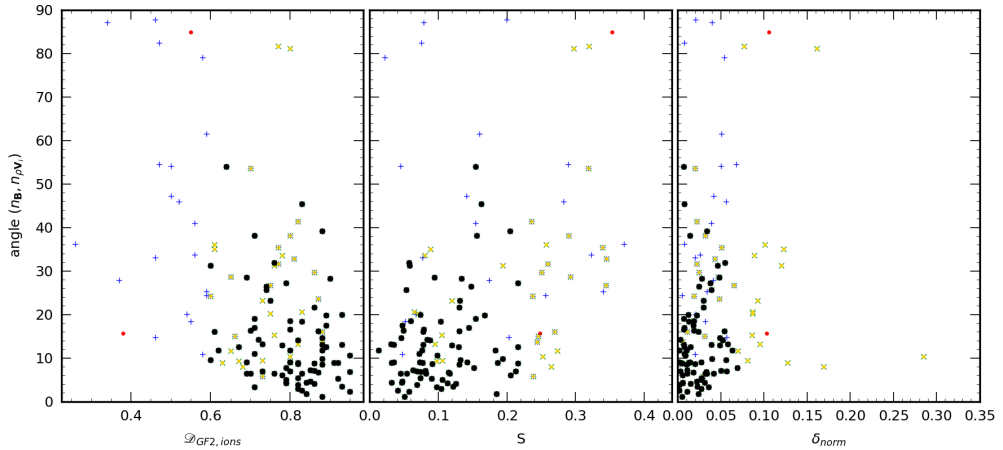


FIGURE 15. Dimensionality (left), stationarity (center), and normal variance (right) averaged for each crossing as a function of the angle between the magnetic field normal and the ion flux one. Green, blue, and yellow indicate crossings respecting the  $\mathcal{D}_{GF2,ions} > 0.6$ ,  $\delta_{norm} > 0.07$ ,  $S > 0.22$  criteria individually. Black dots indicate the crossings for which all the criteria are met, and red dots (two cases) when no condition is met.

863 By defining  $\mathbf{D} = \mathbf{G}_{fit} - \mathbf{G}$  we can introduce the stationarity index:

$$S = \frac{\text{Tr}(\mathbf{D}\mathbf{D}^T)}{\text{Tr}(\mathbf{G}\mathbf{G}^T)} \quad (\text{C } 1)$$

864 Since for a truly stationary magnetopause,  $S$  should be equal to zero, deviations from zero  
865 suggest potential non-stationarity.

866 (iii) Variance of the normal. In some crossings of the database, the normal associated with ion  
867 flux exhibits local differences with respect to the mean value, such as fluctuations or rotations  
868 within a plane, with one component varying within the crossing. In these cases, the ion flux is  
869 therefore characterized by more complex structures and the mean normal is not meaningful.  
870 To exclude such cases, we examined the variation of the normal around the mean value,  
871 defined as follows:

$$\delta_{norm} = \langle |\mathbf{n}_i - \mathbf{n}_{mean,i}|^2 \rangle \quad (\text{C } 2)$$

872 Small values of  $\delta_{norm}$  indicate almost constant normals.

873 The average values of these three parameters for each crossing are shown in Figure 15 as a  
874 function of the angles between the normal of the magnetic field and the ion flux. We observe  
875 here that crossings showing the largest angles occur when at least one of these conditions fails.  
876 To select the cases for which the ions are characterized by a stationary and one-dimensional  
877 structure, for which the normal has no variations around the mean value, we applied the following  
878 thresholds:  $\mathcal{D}_{GF2,ions} > 0.6$ ,  $\delta_{norm} > 0.07$ ,  $S > 0.22$ . Specifically, crossings individually meeting  
879 one of these criteria are shown in green, blue, and yellow, respectively. When all criteria are met,  
880 crossings are indicated by black dots. This Figure underlines a correlation between the difference  
881 between the two normals and the values of these three parameters, showing how cases with higher  
882  $\mathcal{D}_{GF2,ions}$  and smaller  $\delta_{norm}$  and  $S$  are the ones with smaller differences between the two normals.

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