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# Prospects in P3delta Software: New Development Directions

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**Abstract:** In recent years, methodological progress has been made in the partial pole placement for infinite-dimensional systems. In particular, new control methodologies have been drawn up exploiting past data to improve the stability of such dynamical systems. A dedicated Python software called Partial Poles Placement via Delay Action (P3 $\delta$ ) has been developed<sup>\*</sup>. This brief note lists the new development directions that are planned and will be integrated into P3 $\delta$  within the framework of internships for graduate students.

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*Keywords:* Delay systems, Stability and stabilization, Partial poles placement, Exponential decay, multiplicity-induced-dominancy, coexistent-real-roots-inducing-multiplicity.

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## 1. INTRODUCTION

There are two main approaches to the stability analysis of time-delay systems; time-domain approaches and frequency-domain ones, see, for instance, Bellman and Cooke (1963); Kolmanovskii and Nosov (1986); Hale and Verduyn Lunel (1993); Gu et al. (2003); Michiels and Niculescu (2014); Breda (2022). The methods implemented in P3 $\delta$  software are affiliated with the frequency-domain approaches, which involve the characteristic roots of the corresponding characteristic function, see for example Walton and Marshall (1987). In recent years, a new *partial pole placement* method has been developed for the control of infinite-dimensional systems. It is essentially based on two properties called, respectively *multiplicity-induced-dominancy* (MID) and *coexistent-real-roots-inducing-dominancy*, see for instance Boussaada et al. (2022, 2020); Mazanti et al. (2021); Ramírez et al. (2016); Schmoderer et al. (2023, 2024). Although the MID property has been widely studied and has recently emerged as closely related to the spectral abscissa minimization problem, as pointed out in Michiels et al. (2023); Pinney (1958), the CRRID property seems more complex to characterize but offers a greater degree of robustness, see for instance Schmoderer et al. (2023).

## 2. EXISTING P3 $\delta$ FUNCTIONALITIES

Consider the scalar delay-differential equation

$$y^{(n)}(t) + \sum_{k=0}^{n-1} a_k y^{(k)}(t) + \sum_{k=0}^m b_k y^{(k)}(t - \tau) = 0 \quad (1)$$

which admits as characteristic function the quasipolynomial  $\Delta: \mathbb{C} \rightarrow \mathbb{C}$  given by

$$\Delta(s) = s^n + \sum_{k=0}^{n-1} a_k s^k + e^{-s\tau} \sum_{k=0}^m b_k s^k. \quad (2)$$

It is well known that the trivial solution of (1) is exponentially stable if and only if its *spectral abscissa* is negative, see, for instance Michiels and Niculescu (2014) and reference therein. In (1),  $y$  stands for the dependent real-valued function and the parameters  $a_0, \dots, a_{n-1}$ ,  $b_0, \dots, b_m$  are real numbers, and  $\tau$  is a positive delay. The partial pole placement exploits the affine dependency of the quasipolynomial in the parameters.

### 2.1 MID property for generic quasipolynomials

The MID property is nothing but the conditions on the equation (1) under which a multiple spectral value is dominant. Two functionalities are developed in P3 $\delta$ <sup>1</sup>, the first is called the GMID for MID property for generic quasipolynomials also called "Classic MID", the second is called the IMID for MID property in intermediate multiplicity also presented as "Control-oriented MID" property.

Classic MID: In this mode all coefficients of the quasipolynomial  $\Delta$  are considered to be free. The user inputs the values of the delay  $\tau$  and the desired spectral abscissa  $s_0$  and P3 $\delta$  computes all coefficients  $a_0, \dots, a_{n-1}$ ,  $b_0, \dots, b_m$  ensuring that the value  $s_0$  is a dominant root of  $\Delta$  of maximal multiplicity  $n+m+1$ . Notice that  $n+m+1$  corresponds to the degree of the quasipolynomial  $\Delta$ , which is nothing but the sum of the degrees of the involved polynomials plus the number of delays. The obtained settings are certified by the results of Mazanti et al. (2021); Boussaada et al. (2022).

Control-oriented MID: This mode considers the coefficients  $a_0, \dots, a_{n-1}$  corresponding to the non-delayed terms of the DDE are defined by the model and the remaining coefficients  $b_0, \dots, b_m$  of the delayed terms of the DDE are

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<sup>1</sup> <https://cutt.ly/p3delta>

left-free. The user chooses to input either the value of the delay  $\tau$  or the closed-loop exponential decay  $-s_0$  and P3 $\delta$  computes the controllers gains  $b_0, \dots, b_m$  guaranteeing the zero  $s_0$  is a root of the quasipolynomial of multiplicity  $n+1$ , which is also the spectral abscissa. Notice that this mode is certified through the results from Balogh et al. (2022). CRRID for generic quasipolynomials: The CRRID property consists in the conditions on the equation (1) under which the coexistence of real spectral values makes the rightmost root to be real. Inspired from the GMID property, but with the advantage of enhancing the robustness of the control design as emphasized in Schmoderer et al. (2023), it allows to improve the delay margin compared to the GMID property since the assigned closed-loop spectral values are simple. The implemented version of the CRRID consists in assigning equidistributed real spectra. In this mode all coefficients of the quasipolynomial  $\Delta$  are considered to be free. The user inputs the values of the delay  $\tau$  and the desired spectral abscissa  $s_1$  and P3 $\delta$  computes the distance  $d := s_1 - s_2 = \dots = s_{n+m} - s_{n+m+1}$  all coefficients  $a_0, \dots, a_{n-1}, b_0, \dots, b_m$  ensuring that the value  $s_1$  is a dominant root of  $\Delta$ , see for instance Amrane et al. (2018); Bedouhene et al. (2020); Schmoderer et al. (2024, 2023).

### 3. PLANNED EXTENTIONS

In what follow, we mention the main directions that we initialized in the framework of internship for graduate students.

- (1) In the context of the control-oriented MID property, the results from Boussaada et al. (2024), which characterize the MID property for *over-order* multiplicities, allow to further freedom on the choice of the multiplicity. Indeed, thanks to the mentioned results, one is able to exploit any multiplicity  $n+1 \leq M(s_0) \leq n+m+1$  thanks to the introduction of what is called *elimination-produced* polynomial.
- (2) In the context of the control-oriented CRRID property, the results from Schmoderer et al. (2023), suggests that assigning a number of real spectral values which is less than the quasipolynomials degree may be sufficient to ensure the realness of the rightmost root.

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