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Replicating Mankiw's Textbook Dynamic Model: A Pedagogical Pathway to New Keynesian models

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Abstract

This paper presents a detailed replication of the dynamic model introduced in Mankiw's textbook (Chapter 14), the author introduces this model as a bridge between the simple IS-LM framework and the more advanced New Keynesian models. The accompanying Python code, developed for this replication, is made publicly available on my GitHub¹, providing a resource for students and teachers to gain a foundational understanding of key concepts such as the Phillips curve, the IS curve, and the role of monetary policy in targeting inflation and stabilizing output.

1 Introduction

The dynamic model introduced in Chapter 14 of Mankiw's Macroeconomics serves as an essential tool in the study of short-run economic fluctuations. This simplified New Keynesian framework provides an intuitive understanding of how output and inflation respond to various shocks, and the role of monetary policy in stabilizing the

*I would like to express my sincere gratitude to Professor **Greg Mankiw** from Harvard University for granting me permission to publish the replication code of the model presented in his textbook. I am deeply thankful for his support and contributions to the field of macroeconomics.

¹<https://github.com/FaicalLakhchen/MankiwReplication>

economy. The model's accessibility lies in its use of backward-looking expectations and fundamental macroeconomic relationships, making it accessible to students and those new to the study of dynamic economic models.

In the broader landscape of macroeconomic theory, Mankiw's model acts as a stepping stone towards more advanced frameworks, such as the New Keynesian models that dominate contemporary macroeconomic analysis. These models, which incorporate forward-looking expectations, micro-founded behavioral assumptions, and a richer set of frictions—including price and wage rigidities—are central to modern economic thought. For those interested in delving deeper into these models, the works of economists like Michael Woodford and Jordi Galí are particularly noteworthy.

However, jumping directly into more advanced New Keynesian models can be challenging for students. The mathematical demands are significant, involving the use of dynamic optimization, stochastic difference equations, and stochastic processes. This level of abstraction and technicity can be daunting for beginners, making it difficult to grasp the practical implications of these models. As a result, it is often beneficial for students to start with simpler frameworks that build their intuition and foundational understanding before progressing to the more intricate dynamics of New Keynesian models.

Mankiw's presents his dynamic model as a bridge between the traditional IS-LM framework and more advanced New Keynesian models. It retains the core insights of New Keynesian economics, particularly the importance of demand management, while placing them within a dynamic context that enables the analysis of economic responses to shocks over time. The model's backward-looking expectations, though a simplification, effectively illustrate the inertia observed in inflation and output adjustments, a critical feature of real-world economies. Additionally, this approach simplifies the process of solving the model, making it more accessible for educational purposes.

By engaging with Mankiw's model, students develop a foundational understanding of critical concepts such as the Phillips curve, the IS curve, and the role of monetary policy in inflation targeting and output stabilization. These concepts are crucial for progressing to more intricate New Keynesian models, where the interplay between policy, expectations, and economic outcomes is more complex.

In this paper, we will replicate the dynamic model from Mankiw's textbook, focusing on its core equations, simplifications, numerical solution and providing a Python code to replicate the model. Our goal is to illustrate how this pedagogical model can be employed to grasp fundamental economic dynamics, setting the stage for more advanced analyses within the New Keynesian framework.

2 Model Equations

In this section, we present the equations of the dynamic Mankiw textbook model. We assume that agents' expectations are backward-looking, meaning that $\mathbb{E}_t X_{t+1} = X_t$. The key equations of the model are as follows:

2.1 IS Equation

The IS equation describes the relationship between output and the real interest rate:

$$Y_t = \bar{Y} - \alpha(r_t - \rho) + \epsilon_t \quad (1)$$

where Y_t is the output, \bar{Y} is the potential output, r_t is the real interest rate, ρ is the natural rate of interest, and ϵ_t represents demand shocks.

2.2 Fisher Equation

The Fisher equation links the real interest rate to the nominal interest rate and inflation:

$$r_t = i_t - \pi_t \quad (2)$$

where i_t is the nominal interest rate and π_t is the inflation rate.

2.3 Phillips Curve

The Phillips curve illustrates the relationship between inflation and the output gap:

$$\pi_t = \pi_{t-1} + \phi(Y_t - \bar{Y}) + \nu_t \quad (3)$$

where ϕ represents the sensitivity of inflation to the output gap, and ν_t captures supply shocks.

2.4 Monetary Policy Rule

The central bank sets the nominal interest rate according to a policy rule:

$$i_t = \pi_t + \rho + \theta_\pi(\pi_t - \pi_t^*) + \theta_Y(Y_t - \bar{Y}) \quad (4)$$

where θ_π and θ_Y are the policy response coefficients to the inflation gap and the output gap, respectively, and π_t^* is the target inflation rate.

3 Simplification and Solution of the Model

To solve the model, we first simplify it by reducing the number of equations and then express it as a gap model.

3.1 Simplifying the Model

We start by substituting the expression for the nominal interest rate i_t from the Fisher equation into the monetary policy rule. This gives us:

$$r_t + \pi_t = \pi_t + \rho + \theta_\pi(\pi_t - \pi_t^*) + \theta_Y(Y_t - \bar{Y})$$

By canceling out π_t on both sides and rearranging terms, we obtain:

$$r_t - \rho = \theta_\pi(\pi_t - \pi_t^*) + \theta_Y(Y_t - \bar{Y})$$

Notice that the nominal interest rate i_t is eliminated from the model. However, this will not cause any issues as we can recover its dynamics later.

3.2 Gap Model Formulation

Next, we express the model in terms of gaps by introducing the following new variables:

$$Y_t^{Gap} = Y_t - \bar{Y}, \quad \pi_t^{Gap} = \pi_t - \pi_t^*, \quad r_t^{Gap} = r_t - \rho, \quad i_t^{Gap} = i_t - \pi_t^* - \rho$$

Substituting these into the model equations results in the following gap model:

$$Y_t^{Gap} = -\alpha r_t^{Gap} + \epsilon_t \tag{5}$$

$$\pi_t^{Gap} = \pi_{t-1}^{Gap} + \phi Y_t^{Gap} + \nu_t \tag{6}$$

$$r_t^{Gap} = \theta_\pi \pi_t^{Gap} + \theta_Y Y_t^{Gap} \tag{7}$$

3.3 Matrix Representation

We can represent the gap model in matrix form as:

$$Ax_t = Bx_{t-1} + Cz_t \tag{8}$$

where x_t is a (3×1) vector of endogenous variables, A is a (3×3) matrix associated with the endogenous variables, B is a (3×3) matrix associated with the lagged endogenous variables, and z_t is a (3×1) vector of exogenous variables.

The matrices A , B , and C are given by:

$$\underbrace{\begin{bmatrix} 1 & 0 & \alpha \\ -\phi & 1 & 0 \\ -\theta_Y & -\theta_\pi & 1 \end{bmatrix}}_A \underbrace{\begin{bmatrix} Y_t^{Gap} \\ \pi_t^{Gap} \\ r_t^{Gap} \end{bmatrix}} = \underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}}_B \underbrace{\begin{bmatrix} Y_{t-1}^{Gap} \\ \pi_{t-1}^{Gap} \\ r_{t-1}^{Gap} \end{bmatrix}} + \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}}_C \underbrace{\begin{bmatrix} \epsilon_t \\ \nu_t \\ 0 \end{bmatrix}}$$

3.4 Solving the Model

We look for a solution to the equation 8 of the form:

$$x_t = \Psi_1 x_{t-1} + \Psi_2 z_t \quad (9)$$

To find the matrices Ψ_1 and Ψ_2 , we multiply both sides of equation 8 by the inverse of matrix A , denoted as A^{-1} :

$$\begin{aligned} Ax_t &= Bx_{t-1} + Cz_t \\ A^{-1}Ax_t &= A^{-1}Bx_{t-1} + A^{-1}Cz_t \\ x_t &= A^{-1}Bx_{t-1} + A^{-1}Cz_t \end{aligned}$$

Thus, the solution is given by:

$$x_t = \Psi_1 x_{t-1} + \Psi_2 z_t \quad (10)$$

where $\Psi_1 = A^{-1}B$ and $\Psi_2 = A^{-1}C$.

3.5 Numerical Solution

To solve the model numerically, we need to assign values to the parameters. We use the calibration from Mankiw's textbook, as summarized in Table 1.

After performing the computations, we obtain the following solutions:

$$\begin{bmatrix} Y_t^{Gap} \\ \pi_t^{Gap} \\ r_t^{Gap} \end{bmatrix} = \begin{bmatrix} 0 & -0.3077 & 0 \\ 0 & 0.9231 & 0 \\ 0 & 0.3077 & 0 \end{bmatrix} \begin{bmatrix} Y_{t-1}^{Gap} \\ \pi_{t-1}^{Gap} \\ r_{t-1}^{Gap} \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \epsilon_t \\ \nu_t \end{bmatrix} \quad (11)$$

Parameter	Value
α	1
ϕ	0.25
θ_Y	0.5
θ_π	0.5
π^*	2
\bar{Y}	100
ρ	2

Table 1: Calibration

The solution in equation 10 describes the dynamic behavior of the model. We find that the IS curve drives output fluctuations, while the Phillips curve determines the evolution of inflation. The feedback between these two equations is key to understanding the propagation of shocks in the economy. The monetary policy rule introduces a stabilization mechanism, with the central bank responding to deviations in output and inflation from their respective targets.

To analyze the dynamics, one can simulate various scenarios by altering the shock variables ϵ_t and ν_t and observing the response of output, inflation, and the real interest rate over time. These simulations offer insights into how well the central bank can manage shocks and maintain economic stability.

4 Simulation of a Demand Shock

A demand shock typically refers to an increase or decrease in the aggregate demand for goods and services. As in Mankiw's textbook let's break down the effects of a positive demand shock—such as an increase in consumer confidence or government spending—on the key macroeconomic variables of the model (figure 1):

Effect on the Output Gap:

A positive demand shock leads to an increase in aggregate demand. This shifts the IS curve to the right, resulting in a higher level of output. Since output is now above its natural level (potential output), the output gap (the difference between actual output and potential output) becomes positive. The positive output gap reflects the economy operating above its potential, which is unsustainable in the long run. This increase in demand and output pressures firms to produce more, often by increasing labor input or utilizing resources more intensively.

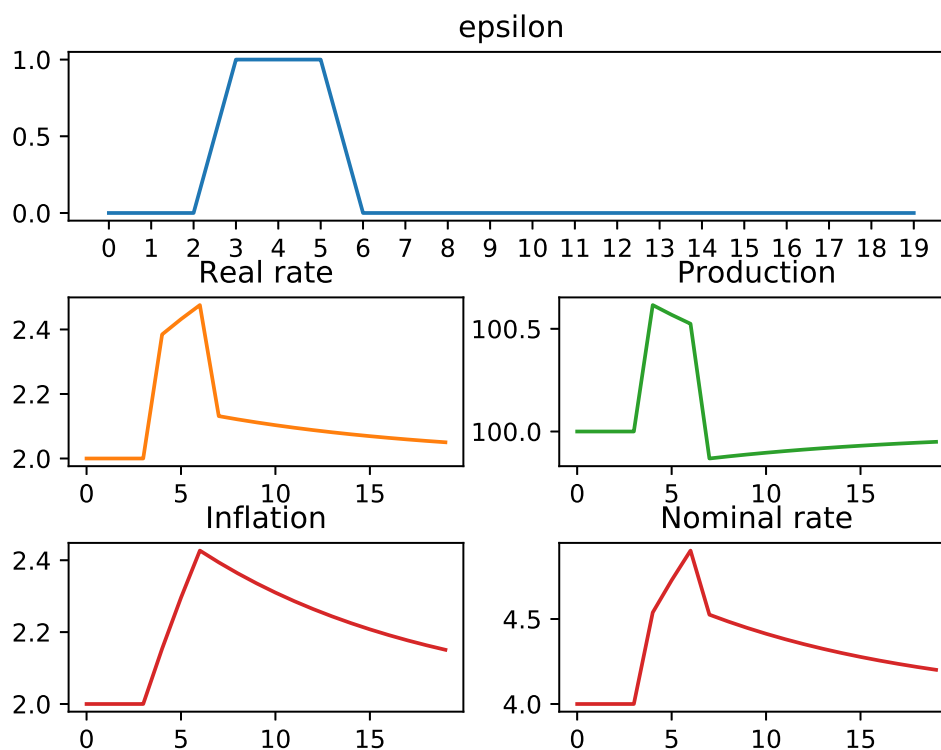


Figure 1: Simulation of a Demand Shock

Effect on Inflation:

The positive output gap exerts upward pressure on prices, leading to higher inflation. In the New Keynesian model, this relationship is captured by the Phillips curve, which suggests that inflation is driven by both expected inflation and the output gap. As the output gap increases, firms raise prices due to higher production costs and increased demand, leading to a rise in inflation.

Effect on the Nominal Interest Rate:

The central bank, following its monetary policy rule (Taylor rule), will increase the nominal interest rate in response to rising inflation and a positive output gap. The size of the rate hike depends on the central bank's parameters in the policy rule—specifically, its responsiveness to deviations in inflation (inflation targeting) and the output gap.

The rise in the nominal interest rate serves to cool down the economy, reducing

aggregate demand, and hence reducing the output gap and inflation over time. This restores equilibrium in the economy, with output returning to its potential level and inflation stabilizing around the central bank's target.

Python Code

```
1 import pandas as pd
2 import matplotlib.pyplot as plt # Simplified matplotlib import
3 import numpy as np # Imported numpy
4 from matplotlib import pyplot as plt # Imported pyplot again for
  plotting
5
6 # Time period
7 T = 20
8
9 # Initialize arrays to store values
10 Time = np.zeros(T)
11 GapY = np.zeros(T)
12 GapPi = np.zeros(T)
13 GapI = np.zeros(T)
14 GapR = np.zeros(T)
15 epsilon = np.zeros(T)
16
17 # Parameters
18 alpha = 1
19 phi = 0.25
20 thetaPi = 0.5
21 thetaY = 0.5
22
23 # Steady state & Targets
24 BarY = 100 # Steady-state output
25 StarPi = 2 # Target inflation
26 rho = 2 # Steady-state real interest rate
27
28 # System matrices
29 A = np.array([
30     [1, 0, alpha],
31     [-phi, 1, 0],
32     [-thetaY, -thetaPi, 1]
33 ])
34
35 B = np.array([
36     [0, 0, 0],
37     [0, 1, 0],
```

```
38     [0, 0, 0]
39 ] )
40
41 C = np.array([
42     [1, 0, 0],
43     [0, 1, 0],
44     [0, 0, 0]
45 ])
46
47 # Calculate the inverse of matrix A
48 invA = np.linalg.inv(A)
49
50 # Compute the psi matrices
51 psi1 = invA.dot(B)
52 psi2 = invA.dot(C)
53
54 # Print the matrices for verification
55 print("The matrix A=")
56 print(A)
57 print("\nThe psi1 matrix is")
58 print(psi1)
59
60 # Loop over time to simulate the system's response to shocks
61 for x in range(T - 1):
62     Time[x + 1] = x + 1
63     if 3 <= x <= 5: # Demand shock occurs between periods 3 and 5
64         epsilon[x] = 1
65     else:
66         epsilon[x] = 0
67
68     # Update the gaps using the psi matrices
69     GapPi[x + 1] = psi1[1, 1] * GapPi[x] + psi2[1, 0] * epsilon[x]
70     GapY[x + 1] = psi1[0, 1] * GapPi[x] + psi2[0, 0] * epsilon[x]
71     GapR[x + 1] = psi1[2, 1] * GapPi[x] + psi2[2, 0] * epsilon[x]
72
73 # Calculate actual output, inflation, real interest rate, and
74     nominal interest rate
75 Y = GapY + BarY # Output
76 Pi = GapPi + StarPi # Inflation
77 r = GapR + rho # Real interest rate
78 i = r + Pi # Nominal interest rate
79
80 # Plot the results
81 fig1 = plt.figure(1)
82 # Plot epsilon (shock)
```

```
83 ax1 = plt.subplot2grid((3, 2), (0, 0), colspan=2)
84 ax1.plot(Time, epsilon)
85 ax1.set_title('Epsilon (Shock)')
86 ax1.set_xticks(Time)
87
88 # Plot real interest rate
89 ax2 = plt.subplot2grid((3, 2), (1, 0))
90 ax2.plot(Time, r, 'tab:orange')
91 ax2.set_title('Real Rate')
92
93 # Plot output
94 ax3 = plt.subplot2grid((3, 2), (1, 1))
95 ax3.plot(Time, Y, 'tab:green')
96 ax3.set_title('Output')
97
98 # Plot inflation
99 ax4 = plt.subplot2grid((3, 2), (2, 0))
100 ax4.plot(Time, Pi, 'tab:red')
101 ax4.set_title('Inflation')
102
103 # Plot nominal interest rate
104 ax5 = plt.subplot2grid((3, 2), (2, 1))
105 ax5.plot(Time, i, 'tab:red')
106 ax5.set_title('Nominal Rate')
107
108 # Show the plots
109 plt.tight_layout() # Adjust layout to prevent overlapping
110 plt.show()
```