



HAL
open science

How to Use the Metropolis Algorithm for Multi-Objective Optimization?

Weijie Zheng, Mingfeng Li, Renzhong Deng, Benjamin Doerr

► **To cite this version:**

Weijie Zheng, Mingfeng Li, Renzhong Deng, Benjamin Doerr. How to Use the Metropolis Algorithm for Multi-Objective Optimization?. GECCO '24 Companion: Genetic and Evolutionary Computation Conference Companion, 2024, Melbourne, Australia. pp.71-72, 10.1145/3638530.3664078 . hal-04676964

HAL Id: hal-04676964

<https://hal.science/hal-04676964>

Submitted on 24 Aug 2024

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



Hot off the Press: How to Use the Metropolis Algorithm for Multi-Objective Optimization?

Weijie Zheng

School of Computer Science and Technology
International Research Institute for Artificial Intelligence
Harbin Institute of Technology
Shenzhen, China

Renzhong Deng

School of Computer Science and Technology
International Research Institute for Artificial Intelligence
Harbin Institute of Technology
Shenzhen, China

Mingfeng Li

School of Computer Science and Technology
International Research Institute for Artificial Intelligence
Harbin Institute of Technology
Shenzhen, China

Benjamin Doerr

Laboratoire d'Informatique (LIX)
École Polytechnique, CNRS
Institut Polytechnique de Paris
Palaiseau, France

ABSTRACT

As demonstrated by empirical and theoretical work, the Metropolis algorithm can cope with local optima by accepting inferior solutions with suitably small probability. This paper extends this research direction into multi-objective optimization.

The original Metropolis algorithm has two components, one-bit mutation and the acceptance strategy, which allows accepting inferior solutions. When adjusting the acceptance strategy to multi-objective optimization in the way that an inferior solution that is accepted replaces its parent, then the Metropolis algorithm is not very efficient on our multi-objective version of the multimodal DLB benchmark called DLTB. With one-bit mutation, this multi-objective Metropolis algorithm cannot optimize the DLTB problem, with standard bit-wise mutation it needs at least $\Omega(n^5)$ time to cover the full Pareto front. In contrast, we show that many other multi-objective optimizers, namely the GSEMO, SMS-EMOA, and NSGA-II, only need time $O(n^4)$. When keeping the parent when an inferior point is accepted, the multi-objective Metropolis algorithm both with one-bit or standard bit-wise mutation solves the DLTB problem efficiently, with one-bit mutation experimentally leading to better results than several other algorithms.

Overall, our work suggests that the general mechanism of the Metropolis algorithm can be interesting in multi-objective optimization, but that the implementation details can have a huge impact on the performance.

This paper for the Hot-off-the-Press track at GECCO 2024 summarizes the work *Weijie Zheng, Mingfeng Li, Renzhong Deng, and Benjamin Doerr: How to Use the Metropolis Algorithm for Multi-Objective Optimization? In Conference on Artificial Intelligence, AAI 2024, AAAI Press, 20883–20891. <https://doi.org/10.1609/aaai.v38i18.30078> [22].*

Permission to make digital or hard copies of part or all of this work for personal or classroom use is granted without fee provided that copies are not made or distributed for profit or commercial advantage and that copies bear this notice and the full citation on the first page. Copyrights for third-party components of this work must be honored. For all other uses, contact the owner/author(s).
GECCO '24 Companion, July 14–18, 2024, Melbourne, VIC, Australia
© 2024 Copyright held by the owner/author(s).
ACM ISBN 979-8-4007-0495-6/24/07.
<https://doi.org/10.1145/3638530.3664078>

CCS CONCEPTS

• **Theory of computation** → **Theory and algorithms for application domains**; *Theory of randomized search heuristics.*

KEYWORDS

Metropolis algorithm, runtime analysis, multi-objective optimization, multimodal, theory

ACM Reference Format:

Weijie Zheng, Mingfeng Li, Renzhong Deng, and Benjamin Doerr. 2024. Hot off the Press: How to Use the Metropolis Algorithm for Multi-Objective Optimization?. In *Genetic and Evolutionary Computation Conference (GECCO '24 Companion)*, July 14–18, 2024, Melbourne, VIC, Australia. ACM, New York, NY, USA, 2 pages. <https://doi.org/10.1145/3638530.3664078>

SUMMARY OF OUR RESULTS

Simulated annealing is widely applied in both single- and multi-objective optimization problems, see [18]. The Metropolis algorithm [14] is a special case of simulated annealing with a fixed temperature. The runtime analysis of the Metropolis algorithm for single-objective optimization started in 1988, when [16] proved that good approximations for the maximum matching problem can be computed in polynomial time. Jansen and Wegener [11] (see Doerr et al. [5] for a recent tightening of this result) proved that the Metropolis algorithm with suitable parameters solves the ONEMAX benchmark in an expected time $O(n \log n)$ fitness evaluation, a runtime many other randomized search heuristics have as well on this problem [1, 6, 8, 10, 17, 20].

The Metropolis algorithm can cope with local optima by accepting inferior solutions with suitably small probability. [11] proved that the Metropolis algorithm solves the GENTLENEGATIVESLOPE problem in polynomial time, whereas the $(1+1)$ EA needs at least exponential time; that also the move acceptance hyper-heuristic need super-polynomial time, was recently shown by [13]. [15] proved that the Metropolis algorithm solves their VALLEY function (which is different from the VALLEY problem defined in [7]) more efficiently than simple evolutionary algorithms (EAs). [19] showed that the Metropolis algorithm solves the DECEPTIVELEADINGBLOCKS (DLB) function in expected time of $O(n^2)$, while all $(1+1)$ -elitist unary unbiased black-box algorithms have an expected runtime of $\Omega(n^3)$.

Our Contribution: We note that the known runtime results for the Metropolis algorithm are all for single optimization. In this work, we take several steps towards theoretically analyzing multi-objective Metropolis algorithms. Since an advantage of the Metropolis algorithm was proven for the single-objective DLB function [19], we construct a bi-objective counterpart of the DLB problem following a general construction method from [12]. We call our new benchmark DLTB, for deceptive leading blocks for ones and deceptive trailing blocks for zeros. DLTB is the first bi-objective multimodal function for theoretical analysis where not all local optima are Pareto optimal. It is also the first bi-objective function in the theory community where the maximum size of a set of mutually non-dominating objective values is larger than the Pareto front size.

The original Metropolis algorithm uses two operators, one-bit mutation as variation operator and the selection mechanism that allows also the inferior offspring to enter the next population. Since the Metropolis algorithm differs from the randomized local search (RLS) algorithm only in the selection operator, in our discussion of multi-objective Metropolis algorithms we will build upon the simple evolutionary multi-objective optimizer (SEMO), the multi-objective analogue of RLS. Since the inferior solution in the single-objective Metropolis algorithm replaces its parent when accepted, a natural way to implement a multi-objective Metropolis algorithms would be to let also here the inferior solution replace its parent (when accepted). However, we will prove that this variant (and also the original SEMO) cannot cover the Pareto front of the DLTB. Hence the advantage of the Metropolis algorithm over RLS on the single-objective DLB problem does not extend to the bi-objective DLTB problem.

The proof of this negative result heavily exploits that as variation operator one-bit flips are used. For this reason, we also consider the multi-objective Metropolis variant where we replace one-bit mutation with the global operator of the standard bit-wise mutation (flipping each bit independently with probability $1/n$). Unfortunately, we can still prove that this variant needs an expected time of $\Omega(n^5)$ to cover the Pareto front of the DLTB problem. We note that the reason for these lower bounds is not an intrinsic difficulty of the DLTB problem. For three well-known multi-objective EAs, namely GSEMO [9], NSGA-II [4], and SMS-EMOA [2], we prove that they all cover the Pareto front of the DLTB problem in expected time $O(n^4)$. We note that our result for the SMS-EMOA, together with the parallel work [21], is only the second runtime analysis for this algorithm after the seminal work [3].

Given these unfavorable results for the Metropolis algorithm, we analyze the multi-objective Metropolis algorithm that, in case an inferior solution is accepted, only removes all individuals weakly dominated by it, but not its parent. We prove that this variant covers the Pareto front in expected time $O(n^5)$. Speculating that this runtime guarantee is not tight, we conduct a small experimental investigation and observe that this variant with standard bit-wise mutation achieves similar performance as the GSEMO, NSGA-II, and SMS-EMOA (for which we have shown a runtime guarantee of $O(n^4)$). Interestingly, the experimental results suggest an even better performance of this variant with one-bit mutation.

Overall, our work suggests that the non-elitist acceptance mechanism of the Metropolis algorithm can be interesting also in multi-objective optimization, but that its application is not straightforward. Hence more work on how to best use the key elements of the Metropolis algorithm in multi-objective optimization appears desirable.

REFERENCES

- [1] Denis Antipov and Benjamin Doerr. 2021. A tight runtime analysis for the $(\mu + \lambda)$ EA. *Algorithmica* 83 (2021), 1054–1095.
- [2] Nicola Beume, Boris Naujoks, and Michael Emmerich. 2007. SMS-EMOA: Multi-objective selection based on dominated hypervolume. *European Journal of Operational Research* 181 (2007), 1653–1669.
- [3] Chao Bian, Yawen Zhou, Miqing Li, and Chao Qian. 2023. Stochastic population update can probably be helpful in multi-objective evolutionary algorithms. In *International Joint Conference on Artificial Intelligence, IJCAI 2023*. ijcai.org, 5513–5521.
- [4] Kalyanmoy Deb, Amrit Pratap, Sameer Agarwal, and T. Meyarivan. 2002. A fast and elitist multiobjective genetic algorithm: NSGA-II. *IEEE Transactions on Evolutionary Computation* 6 (2002), 182–197.
- [5] Benjamin Doerr, Taha El Ghazi El Houssaini, Amirhossein Rajabi, and Carsten Witt. 2023. How well does the Metropolis algorithm cope with local optima?. In *Genetic and Evolutionary Computation Conference, GECCO 2023*. ACM, 1000–1008.
- [6] Benjamin Doerr and Martin S. Krejca. 2020. Significance-based estimation-of-distribution algorithms. *IEEE Transactions on Evolutionary Computation* 24 (2020), 1025–1034.
- [7] Stefan Droste, Thomas Jansen, and Ingo Wegener. 2000. Dynamic parameter control in simple evolutionary algorithms. In *Foundations of Genetic Algorithms, FOGA 2000*. Morgan Kaufmann, 275–294.
- [8] Stefan Droste, Thomas Jansen, and Ingo Wegener. 2002. On the analysis of the $(1+1)$ evolutionary algorithm. *Theoretical Computer Science* 276 (2002), 51–81.
- [9] Oliver Giel. 2003. Expected runtimes of a simple multi-objective evolutionary algorithm. In *Congress on Evolutionary Computation, CEC 2003*. IEEE, 1918–1925.
- [10] Thomas Jansen, Kenneth A. De Jong, and Ingo Wegener. 2005. On the choice of the offspring population size in evolutionary algorithms. *Evolutionary Computation* 13 (2005), 413–440.
- [11] Thomas Jansen and Ingo Wegener. 2007. A comparison of simulated annealing with a simple evolutionary algorithm on pseudo-Boolean functions of unitation. *Theoretical Computer Science* 386 (2007), 73–93.
- [12] Marco Laumanns, Lothar Thiele, and Eckart Zitzler. 2004. Running time analysis of multiobjective evolutionary algorithms on pseudo-Boolean functions. *IEEE Transactions on Evolutionary Computation* 8 (2004), 170–182.
- [13] Andrei Lissovoi, Pietro S. Oliveto, and John Alasdair Warwicker. 2023. When move acceptance selection hyper-heuristics outperform Metropolis and elitist evolutionary algorithms and when not. *Artificial Intelligence* 314 (2023), 103804.
- [14] Nicholas Metropolis, Arianna W. Rosenbluth, Marshall N. Rosenbluth, Augusta H. Teller, and Edward Teller. 1953. Equation of state calculations by fast computing machines. *The Journal of Chemical Physics* 21 (1953), 1087–1092.
- [15] Pietro S. Oliveto, Tiago Paixão, Jorge Pérez Heredia, Dirk Sudholt, and Barbara Trubenová. 2018. How to escape local optima in black box optimisation: when non-elitism outperforms elitism. *Algorithmica* 80 (2018), 1604–1633.
- [16] Galen H. Sasaki and Bruce Hajek. 1988. The time complexity of maximum matching by simulated annealing. *Journal of the ACM* 35 (1988), 387–403.
- [17] Dirk Sudholt and Carsten Witt. 2019. On the choice of the update strength in estimation-of-distribution algorithms and ant colony optimization. *Algorithmica* 81 (2019), 1450–1489.
- [18] Balram Suman and Prabhat Kumar. 2006. A survey of simulated annealing as a tool for single and multiobjective optimization. *Journal of the Operational Research Society* 57 (2006), 1143–1160.
- [19] Shouda Wang, Weijie Zheng, and Benjamin Doerr. 2024. Choosing the right algorithm with hints from complexity theory. *Information and Computation* 296 (2024), 105125.
- [20] Carsten Witt. 2006. Runtime analysis of the $(\mu + 1)$ EA on simple pseudo-Boolean functions. *Evolutionary Computation* 14 (2006), 65–86.
- [21] Weijie Zheng and Benjamin Doerr. 2024. Runtime analysis of the SMS-EMOA for many-objective optimization. In *Conference on Artificial Intelligence, AAAI 2024*. AAAI Press, 20874–20882.
- [22] Weijie Zheng, Mingfeng Li, Renzhong Deng, and Benjamin Doerr. 2024. How to use the Metropolis algorithm for multi-objective optimization?. In *Conference on Artificial Intelligence, AAAI 2024*. AAAI Press, 20883–20891.