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Hot off the Press: Runtime Analyses of Multi-Objective Evolutionary Algorithms in the Presence of Noise

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ABSTRACT

In this work, we conduct the first mathematical runtime analysis of a simple multi-objective evolutionary algorithm (MOEA) on a classic benchmark in the presence of noise in the objective function. We prove that when bit-wise prior noise with rate $p \leq \alpha/n$, α a suitable constant, is present, the *simple evolutionary multi-objective optimizer* (SEMO) without any adjustments to cope with noise finds the Pareto front of the OneMinMax benchmark in time $O(n^2 \log n)$, just as in the case without noise. Given that the problem here is to arrive at a population consisting of $n + 1$ individuals witnessing the Pareto front, this is a surprisingly strong robustness to noise (comparably simple evolutionary algorithms cannot optimize the single-objective OneMax problem in polynomial time when $p = \omega(\log(n)/n)$). Our proofs suggest that the strong robustness of the MOEA stems from its implicit diversity mechanism designed to enable it to compute a population covering the whole Pareto front.

Interestingly this result only holds when the objective value of a solution is determined only once and the algorithm from that point on works with this, possibly noisy, objective value. We prove that when all solutions are reevaluated in each iteration, then any noise rate $p = \omega(\log(n)/n^2)$ leads to a super-polynomial runtime. This is very different from single-objective optimization, where it is generally preferred to reevaluate solutions whenever their fitness is important and where examples are known such that not reevaluating solutions can lead to catastrophic performance losses.

This paper for the Hot-off-the-Press track at GECCO 2024 summarizes the work *Matthieu Dinot, Benjamin Doerr, Ulysse Hennebelle, and Sebastian Will. 2023. Runtime analyses of multi-objective evolutionary algorithms in the presence of noise. In International Joint Conference on Artificial Intelligence, IJCAI 2023. ijcai.org, 5549–5555 [6].*

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CCS CONCEPTS

• **Theory of computation** → *Theory of randomized search heuristics.*

KEYWORDS

Runtime analysis, multi-objective optimization, robustness to noise, theory.

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SUMMARY OF OUR RESULTS

Many real-world optimization problems consist of multiple, often conflicting objectives. For these, a single optimal solution usually does not exist. Consequently, a common solution concept is to compute a set of solutions which cannot be improved in one objective without worsening in another one (Pareto optima) and then let a decision maker select one of these.

Due to their population-based nature, evolutionary algorithms (EAs) are an obvious heuristic approach to such problems, and in fact, such multi-objective evolutionary algorithms (MOEAs) have been successfully applied to many multi-objective problems [2, 16].

Evolutionary algorithms are known to be robust against different types of stochastic disturbances such as noise or dynamic changes of the problem instance [12]. Surprisingly, as regularly pointed out in the literature [10, 11, 13], only very little is known on how MOEAs cope with such stochastic optimization problems. In particular, while it is known that single-objective evolutionary algorithms without any specific adjustments can stand a certain amount of noise in the evaluation of the objective function, we are not aware of any such result in multi-objective optimization.

We approach this question via the methodology of mathematical runtime analysis [8]. This field, for more than twenty years, has significantly enlarged our understanding of the working principles of all kinds of randomized search heuristics, including both MOEAs [1] and single-objective evolutionary algorithms solving

stochastic optimization problems [14]. Despite this large amount of work, there is not a single runtime analysis discussing how a standard MOEA computes or approximates the Pareto front of a multi-objective problem in the presence of noise (and this is what we shall do in the present work). The only paper that conducts a mathematical runtime analysis of a MOEA in a noisy setting analyzes a combination of the adaptive Pareto sampling frameworks with the simple evolutionary multi-objective optimizer (SEMO) [10], so this algorithm definitely is not anymore a standard MOEA.

To start closing this gap, we conduct a runtime analysis of a simple MOEA, namely the SEMO, on the classic benchmark problem ONEMINMAX, in the presence of one-bit prior noise. We prove that when the noise rate is at most α/n , where α is a suitable constant, then the population of this MOEA, despite the noise, after an expected number of $O(n^2 \log n)$ fitness evaluations witnesses the full Pareto front. This is the same bound on the runtime that is known for the setting without noise [3, 9]. We note that in comparable single-objective settings, not much more noise can be tolerated. For example, only for $p = O(\log(\log(n))/n)$ it could be shown in [5] that the $(1+1)$ EA retains its noise-free $O(n \log n)$ runtime on the ONEMAX benchmark. Already for $p = \omega(\log(n)/n)$, the runtime is super-polynomial. Considering this and taking into account that the multi-objective ONEMINMAX problem is naturally harder (we aim at a population containing exactly one solution for each Pareto optimum), our result indicates that MOEAs cope with noise surprisingly well.

However, our work also shows one important difference to the noisy optimization of single-objective problems. Our result above assumes that each solution is evaluated only once, namely when it is generated. This possibly noisy objective value is stored with the solution unless the solution is discarded at some time. This approach is natural in that it avoids costly reevaluations, but it differs from the standards in single-objective evolutionary computation. Being afraid that a faulty fitness value can negatively influence the future optimization process, almost all works there assume that each time a solution competes with others, its fitness is reevaluated. That not reevaluating solutions can significantly disturb a noisy optimization process was rigorously shown for an ant-colony optimizer [7, 15].

Given that in single-objective evolutionary computation it is more common to assume that solutions are evaluated anew when they compete with others, we also analyzed the runtime of the SEMO on ONEMINMAX under this assumption. While the reevaluations prevent one from sticking to faulty objective values for a long time, our mathematical runtime analysis shows that this approach can only tolerate much lower levels of noise. We can prove an $O(n^2 \log n)$ runtime only for $p \leq \beta/n^2$, β a suitable constant, and we prove that for $p = \omega(\log(n)/n^2)$ the algorithm needs super-polynomial time to compute the full Pareto front of the ONEMINMAX benchmark. So clearly, the reevaluation strategy recommended in single-objective optimization is less suitable in multi-objective optimization.

Overall, this first runtime analysis work of a standard MOEA in a noisy environment shows that MOEAs without specific adjustments are reasonably robust to noise and that such stochastic processes can be analyzed with mathematical means, but also that insights from the single-objective setting can be fundamentally wrong in multi-objective noisy optimization.

Parallel independent work: In [4], Dang, Opris, Salehi, and Sudholt analyze the performance of the NSGA-II on the LOTZ problem in the presence of noise. Their results in particular show that the NSGA-II can stand much stronger noise than the GSEMO.

Subsequent work: In the meantime, the authors of this works have proven a lower bound on the noise-level the SEMO without reevaluations can stand, namely that for $p = \omega(\log(n)/n)$, the runtime becomes super-polynomial.

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REFERENCES

- [1] Dimo Brockhoff. 2011. Theoretical aspects of evolutionary multiobjective optimization. In *Theory of Randomized Search Heuristics*, Anne Auger and Benjamin Doerr (Eds.). World Scientific Publishing, 101–140.
- [2] Carlos Artemio Coello Coello, Gary B. Lamont, and David A. van Veldhuizen. 2007. *Evolutionary Algorithms for Solving Multi-Objective Problems* (2nd ed.). Springer.
- [3] Edgar Covantes Osuna, Wanru Gao, Frank Neumann, and Dirk Sudholt. 2020. Design and analysis of diversity-based parent selection schemes for speeding up evolutionary multi-objective optimisation. *Theoretical Computer Science* 832 (2020), 123–142.
- [4] Duc-Cuong Dang, Andre Opris, Bahare Salehi, and Dirk Sudholt. 2023. Analysing the robustness of NSGA-II under noise. In *Genetic and Evolutionary Computation Conference, GECCO 2023*. ACM, 642–651.
- [5] Raphaël Dang-Nhu, Thibault Dardinier, Benjamin Doerr, Gautier Izacard, and Dorian Nogneng. 2018. A new analysis method for evolutionary optimization of dynamic and noisy objective functions. In *Genetic and Evolutionary Computation Conference, GECCO 2018*. ACM, 1467–1474.
- [6] Matthieu Dinot, Benjamin Doerr, Ulysse Henebelle, and Sebastian Will. 2023. Runtime analyses of multi-objective evolutionary algorithms in the presence of noise. In *International Joint Conference on Artificial Intelligence, IJCAI 2023*. ijcai.org, 5549–5557.
- [7] Benjamin Doerr, Ashish Ranjan Hota, and Timo Kötzing. 2012. Ants easily solve stochastic shortest path problems. In *Genetic and Evolutionary Computation Conference, GECCO 2012*. ACM, 17–24.
- [8] Benjamin Doerr and Frank Neumann (Eds.). 2020. *Theory of Evolutionary Computation—Recent Developments in Discrete Optimization*. Springer. Also available at http://www.lix.polytechnique.fr/Labo/Benjamin.Doerr/doerr_neumann_book.html.
- [9] Oliver Giel and Per Kristian Lehre. 2010. On the effect of populations in evolutionary multi-objective optimisation. *Evolutionary Computation* 18 (2010), 335–356.
- [10] Walter J. Gutjahr. 2012. Runtime analysis of an evolutionary algorithm for stochastic multi-objective combinatorial optimization. *Evolutionary Computation* 20 (2012), 395–421.
- [11] Walter J. Gutjahr and Alois Pichler. 2016. Stochastic multi-objective optimization: a survey on non-scalarizing methods. *Annals of Operations Research* 236 (2016), 475–499.
- [12] Yaochu Jin and Jürgen Branke. 2005. Evolutionary optimization in uncertain environments – a survey. *IEEE Transactions on Evolutionary Computation* 9 (2005), 303–317.
- [13] Arnaud Liefooghe, Matthieu Basseur, Laetitia Jourdan, and El-Ghazali Talbi. 2007. Combinatorial optimization of stochastic multi-objective problems: an application to the flow-shop scheduling problem. In *Evolutionary Multi-Criterion Optimization, EMO 2007*, Vol. 4403. Springer, 457–471.
- [14] Frank Neumann, Mojgan Pourhassan, and Vahid Roostapour. 2020. Analysis of evolutionary algorithms in dynamic and stochastic environments. In *Theory of Evolutionary Computation: Recent Developments in Discrete Optimization*, Benjamin Doerr and Frank Neumann (Eds.). Springer, 323–357. Also available at <https://arxiv.org/abs/1806.08547>.
- [15] Dirk Sudholt and Christian Thyssen. 2012. A simple ant colony optimizer for stochastic shortest path problems. *Algorithmica* 64 (2012), 643–672.
- [16] Aimin Zhou, Bo-Yang Qu, Hui Li, Shi-Zheng Zhao, Ponnuthurai Nagaratnam Suganthan, and Qingfu Zhang. 2011. Multiobjective evolutionary algorithms: A survey of the state of the art. *Swarm and Evolutionary Computation* 1 (2011), 32–49.