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► **To cite this version:**

Antoine Rouxel, Olivier Gauthier-Lafaye, Antoine Monmayrant. Resolving ambiguities in phase correction term for optical field encoding. *Optics Letters*, 2024, 49 (16), pp.4525-4528. 10.1364/OL.533058 . hal-04676301

**HAL Id: hal-04676301**

**<https://hal.science/hal-04676301v1>**

Submitted on 23 Aug 2024

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# Resolving ambiguities in phase correction term for optical field encoding

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Compiled August 23, 2024

This article addresses ambiguities regarding the existence and definition of a phase correction term in phase and amplitude optical field encoding techniques. We present a generalized mixed Fourier-Taylor series expansion that is valid for any phase-wrapping interval. Our theoretical analysis and numerical validations confirm that maintaining consistency within a given phase-wrapping convention ensures equivalent results and reconciles previously conflicting interpretations.

Advancements in optical engineering have continually refined our ability to manipulate light at microscopic levels. Among these developments, the method introduced by Davis *et al.* [1] for encoding both amplitude and phase information onto phase-only filters stands out. This well-known approach employs a spatially-modulated blazed phase grating to tailor the diffraction efficiency in the first order. This technique has been applied for wavefront shaping tasks such as the generation of vortex beams [2–4], Bessel beams [5, 6] and mode-matching for photonic structures excitation [7]. It has permitted advances in fiber communications [8], nonlinear fluorescence microscopy [9], imaging [10] and holographic data storage [11]. It is also central for the creation and manipulation of high-dimensionality entangled states [12, 13] and more recently for optical skyrmions [14, 15]. Independent work [16] used a similar approach to achieve 1D phase and amplitude shaping using a phase-only 2D mask in the context of femtosecond laser pulse shaping using  $4f$ -lines.

Subsequently, Karimi *et al.* [2] proposed improvements to the method proposed in [1] by accounting for a phase correction term. More recently, Clark *et al.* [17] conducted a comparative study of various complex amplitude encoding approaches, concluding that the method introduced by Davis *et al.* originally yielded superior results. However, confusion persists regarding the use of the phase correction term highlighted by Karimi *et al.* [2], its application being dependent on the conventions adopted in the Fourier series expansion.

Our contribution addresses the apparent contradictions in the current literature. In particular, we demonstrate that these discrepancies are due to different phase-wrapping conventions, which are indeed equivalent provided that consistency is main-

tained within a given convention across all the calculations required to generate the blazed grating. For instance, one convention wraps the phase between  $[-\pi, +\pi]$  [17], while another wraps it between  $[0, 2\pi]$  [2]. Each convention has a corresponding correct expression for the phase correction term. Issues occur when expressions valid for one range are applied to another. We assert that any  $2\pi$  range  $[\alpha, \alpha + 2\pi]$  can be used, provided that the specific expression for that wrapping interval is correctly derived. Furthermore, we present a generalized expression that depends on  $\alpha$ , making it valid for any wrapping convention. Our work provides theoretical analysis and numerical examples to demonstrate the implications of these conventions, thereby resolving the ambiguities and enhancing the understanding of amplitude and phase encoding techniques.

## 1. CLARIFICATION ON THE FOURIER CONVENTIONS

We hypothesize that the contradictory conclusions observed in the works of Karimi *et al.* [2] and Clark *et al.* [17] stem from ambiguities concerning the valid range over which the expressions apply. Specifically, the phase correction term mentioned in [2] is applicable only when the phase-wrapping convention is  $[0, 2\pi]$ . Since many simulation tools typically wrap the phase in  $[-\pi, +\pi]$ , it is crucial to be mindful of the conventions used.

To substantiate our hypothesis, we present an analysis using a Fourier-Taylor expansion, akin to that in [1], but adapted to the phase convention in the  $[\alpha, 2\pi + \alpha]$  range, valid for any  $\alpha$ . Additionally, we provide a straightforward example to clearly demonstrate the implications of these conventions.

### A. Derivation of the mixed series with $[\alpha, 2\pi + \alpha]$ convention

The transfer function of the SLM is written as:

$$T(u) = e^{iM(u)\phi(u)} \quad (1)$$

where  $M(u)$  represents the amplitude and  $\phi(u)$  represents the phase of the shaping as a function of the position  $u$ . Following the work of [1], we propose to rewrite this function. We perform a Taylor series expansion of  $T$  with respect to  $M(u)$  around 0:

$$T(u) = \sum_{p=0}^{+\infty} \frac{1}{p!} \frac{\partial^p T(u)}{\partial M(u)^p} \Bigg|_{M=0} M(u)^p \quad (2)$$

67 We now consider the  $p^{\text{th}}$  derivative of  $T$  with respect to  $M(u)$ ,  
68 which is written as:

$$\frac{\partial^p T(u)}{\partial M(u)^p} = [i\phi(u)]^p e^{iM(u)\phi(u)} \quad (3)$$

69 Since the function  $\phi$  is  $2\pi$ -periodic and varies between  $\alpha$  and  
70  $2\pi + \alpha$ , the  $p^{\text{th}}$  derivative of  $T$  with respect to  $M(u)$  evaluated  
71 at  $M(u) = 0$  is also  $2\pi$ -periodic. Therefore, we expand it into a  
72 Fourier series with respect to  $\phi(u)$ :

$$\frac{\partial^p T(u)}{\partial M(u)^p} \Big|_{M=0} = \sum_{n=-\infty}^{+\infty} c_{n,p} e^{in\phi(u)} \quad (4)$$

73 with  $c_{n,p}$  being Fourier coefficients defined as:

$$c_{n,p} = \frac{1}{2\pi} \int_{\alpha}^{2\pi+\alpha} \frac{\partial^p T(u)}{\partial M(u)^p} \Big|_{M=0} e^{-in\phi(u)} d\phi(u) \quad (5)$$

74 Inserting equation Eq. (4) into equation Eq. (2) yields the follow-  
75 ing expression for  $T$ :

$$T(u) = \sum_{n=-\infty}^{+\infty} \sum_{p=0}^{+\infty} B_{n,p} M(u)^p e^{in\phi(u)} \quad (6)$$

76 with  $B_{n,p}$  coefficients defined as:

$$B_{n,p}(u) = \frac{1}{2\pi p!} \int_{\alpha}^{2\pi+\alpha} \frac{\partial^p T(u)}{\partial M(u)^p} \Big|_{M=0} e^{-in\phi(u)} d\phi(u) \quad (7)$$

77 We rewrite equation Eq. (6) as:

$$T(u) = \sum_{n=-\infty}^{+\infty} T_n(u) e^{in\phi(u)} \quad \text{with} \quad T_n(u) = \sum_{p=0}^{+\infty} B_{n,p}(u) M(u)^p \quad (8)$$

78 From equation Eq. (3), we find:

$$\frac{\partial^p T(u)}{\partial M(u)^p} \Big|_{M=0} = [i\phi(u)]^p \quad (9)$$

79 Using this expression in Eq. (7) to rewrite the coefficients  $B_{n,p}$ ,  
80 yields:

$$B_{n,p}(u) = \frac{1}{2\pi p!} \int_{\alpha}^{2\pi+\alpha} [i\phi(u)]^p e^{-in\phi(u)} d\phi(u) \quad (10)$$

81 The following expression is obtained for the coefficients  $T_n$ :

$$T_n(u) = \sum_{p=0}^{+\infty} \frac{1}{2\pi p!} \int_{\alpha}^{2\pi+\alpha} [i\phi(u)M(u)]^p e^{-in\phi(u)} d\phi(u) \quad (11)$$

82 By interchanging the integral and the sum, we can evaluate the  
83 following series:

$$\sum_{p=0}^{+\infty} \frac{1}{p!} [i\phi(u)M(u)]^p = e^{iM(u)\phi(u)} \quad (12)$$

84 and rewrite  $T_n$  as:

$$T_n(u) = \frac{1}{2\pi} \int_{\alpha}^{2\pi+\alpha} e^{iM(u)\phi(u)} e^{-in\phi(u)} d\phi(u) \quad (13)$$

85 We do a change of variable  $\psi = \phi - \alpha - \pi$  and obtain :

$$T_n(u) = \frac{1}{2\pi} \int_{-\pi}^{+\pi} e^{i(\psi(u)+\alpha+\pi)(M(u)-n)} d\psi(u) \quad (14)$$

86 We get the constant term out of the integral and evaluate the  
87 integral between  $-\pi$  and  $+\pi$  :

$$T_n(u) = \frac{e^{i(\pi+\alpha)(M(u)-n)}}{i2\pi(M(u)-n)} [e^{i\pi(M(u)-n)} - e^{-i\pi(M(u)-n)}] \quad (15)$$

88 We use Euler formula to simplify the exponential terms in brack-  
89 ets and obtain the corrected  $T_n(u)$  formula :

$$T_n(u) = \frac{\sin(\pi[n - M(u)])}{\pi[n - M(u)]} e^{i\Phi_c(u,\alpha,n)} \quad (16)$$

with  $\Phi_c(u,\alpha,n) = (\pi + \alpha)[M(u) - n]$

90 Examining expression Eq. (16), it becomes evident that the  
91 presence of the phase correction term is contingent on the spe-  
92 cific phase-wrapping convention used, specifically on the value  
93 of  $\alpha$ . Notably, with  $\alpha = 0$ , we obtain the correction term  
94  $\Phi_c(u,n) = \pi[M(u) - n]$  which is coherent with [2]. In con-  
95 trast, with  $\alpha = -\pi$ , we obtain the correction term  $\Phi_c(u,n) = 0$   
96 which is coherent with [17]. Our phase correction term  $\Phi_c(u,n)$   
97 is indeed a generalized phase correction term whose expression  
98 depend on the considered  $\alpha$ , thus reconciling conflicting results  
99 presented in the literature.

## 100 B. Numerical example

101 To validate the implications of employing different Fourier con-  
102 ventions, we present a numerical example comparing the exact  
103 expression  $T$  with its mixed Fourier-Taylor series expansions  $T^\alpha$   
104 for various arbitrary values of  $\alpha = -\pi, 0, \frac{3\pi}{2}$ . These expansions  
105 are defined as follows:

$$T^\alpha = \sum_{n=-N}^N e^{i\Phi_c(u,\alpha,n)} \frac{\sin(\pi(n-M))}{\pi[n-M]} e^{in\phi} \quad (17)$$

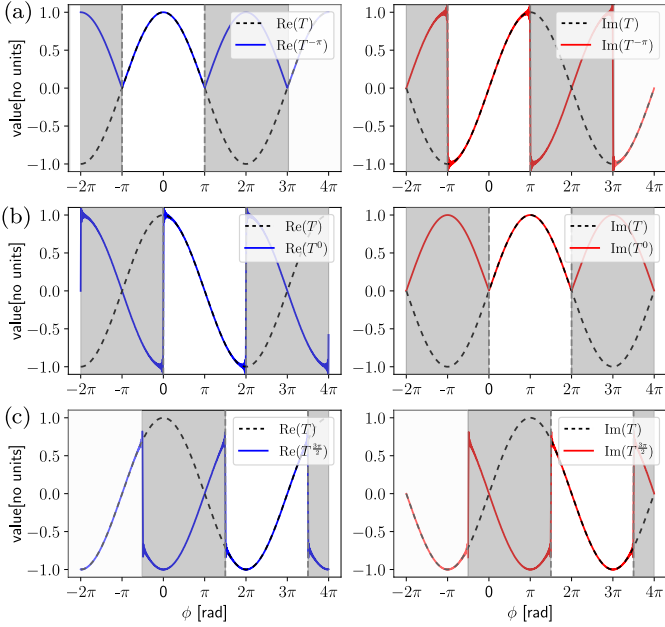
106 In these equations,  $N$  represents the number of terms in-  
107 cluded in the expansion, and  $M$  is a constant amplitude value  
108 used for the comparison. The phase term  $\phi$  is varied between  
109  $-2\pi$  and  $4\pi$  to comprehensively test the validity domains.

110 As illustrated in Figure 1, with  $N = 100$  and  $M = 0.33$ , the  
111 real and imaginary parts of  $T$  are compared with those of the  
112 mixed Fourier-Taylor series expansions  $T^\alpha$ . The value  $M =$   
113  $0.33$  is chosen arbitrarily and the number of terms  $N = 100$  is  
114 selected to ensure a high level of accuracy in the series expansion,  
115 providing a sufficient number of terms to approximate the exact  
116 transfer function closely.

117 The figures clearly show that the real and imaginary parts  
118 of the transfer function  $T$  align perfectly with their respective  
119 mixed Fourier-Taylor series expansions  $T_n^\alpha$  only within the spe-  
120 cific phase-wrapping range  $[\alpha, 2\pi + \alpha]$ , that repeats every  $2\pi$ .  
121 Outside this range, indicated by the gray areas in the figure, the  
122 expansions do not align with the exact transfer function. This  
123 highlights the necessity for consistency in applying the correct  
124 phase term across various wrapping ranges and confirms the  
125 validity of the generalized expression that depends on  $\alpha$ .

## 126 2. ENCODING OF AMPLITUDE AND PHASE IN 2D

127 To test our generalized expression Eq. (16) for 2D amplitude  
128 and phase modulation, we present simulations and experimen-  
129 tal results, based on the approach described in [1], where we  
130 generate a truncated sinusoidal pattern as described in [7]. The  
131 aim is to demonstrate the effectiveness of our generalized mixed  
132 Fourier-Taylor expansion expression for tailoring a 2D beam  
133 profile under various phase-wrapping conventions.

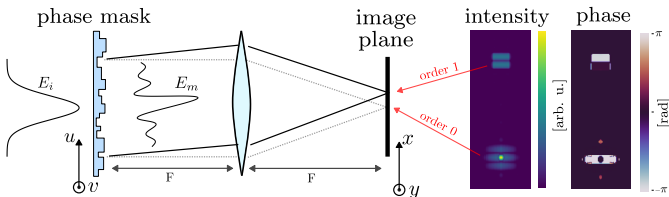


**Fig. 1.** Real and imaginary parts of the transfer function  $T$  and its corresponding mixed Fourier-Taylor series expansions  $T^\alpha$  for different phase-wrapping conventions. Each row corresponds to a different  $\alpha$  value: (a)  $-\pi$ , (b) 0, and (c)  $3\pi/2$ . The grey areas indicate ranges where the expansions are not expected to align with the exact transfer function.

### A. Experimental and Simulation Setup

The setup is a  $2f$  optical system in which a phase-only mask is placed in the Fourier plane of a lens. The optical field  $E_i$  incident on the phase mask is a 2D Gaussian with a waist  $w = 4.3$  mm which is modulated by the phase mask, thus resulting in a modulated field  $E_m$  as illustrated in figure 2. A lens with a focal length  $f = 300$  mm is used to produce the Fourier transform of the modulated beam  $E_m$  in the image plane. In practice, the phase mask is a Liquid Crystal Spatial Light Modulator (LCSLM) that accurately approximates a blazed phase grating, provided that the phase grating periodicity is large compared to the pixel size of the LCSLM.

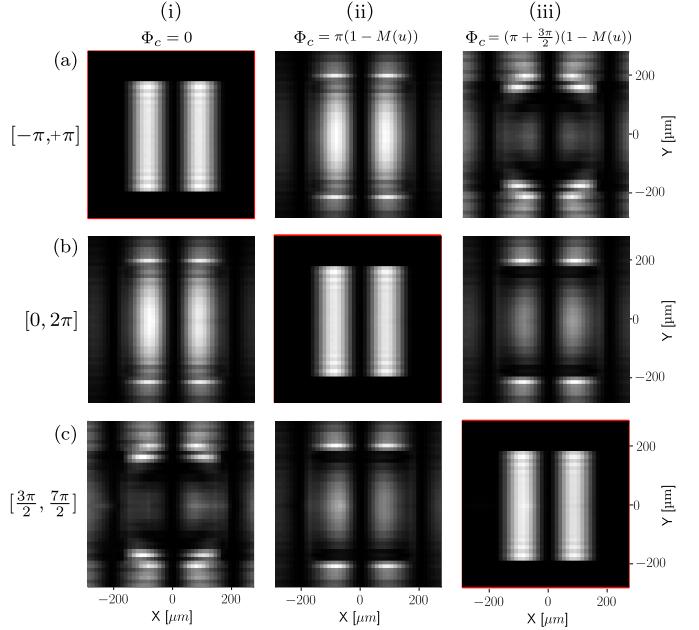
We aim at generating a truncated sinusoidal pattern with a period of  $360 \mu\text{m}$ , as illustrated on the simulations presented in Figure 3. We follow the approach described in [1] and use our mixed Fourier Taylor expansion expression Eq. (16) to generate the appropriate phase masks.



**Fig. 2.** Optical setup. The incident beam  $E_i$  phase is modulated by the mask and the Fourier transform of the modulated beam is obtained in the image plane. The targeted profile, a truncated sinusoidal pattern, is generated in the first order of diffraction as illustrated in both intensity and phase maps.

### B. Results and Discussion

We examine only the beam profile corresponding to the first order of diffraction in the image plane as it corresponds to the targeted sinusoidal profile. Figure 3 presents the resulting intensities for the truncated sinusoidal pattern under various phase-wrapping conventions and phase correction terms.



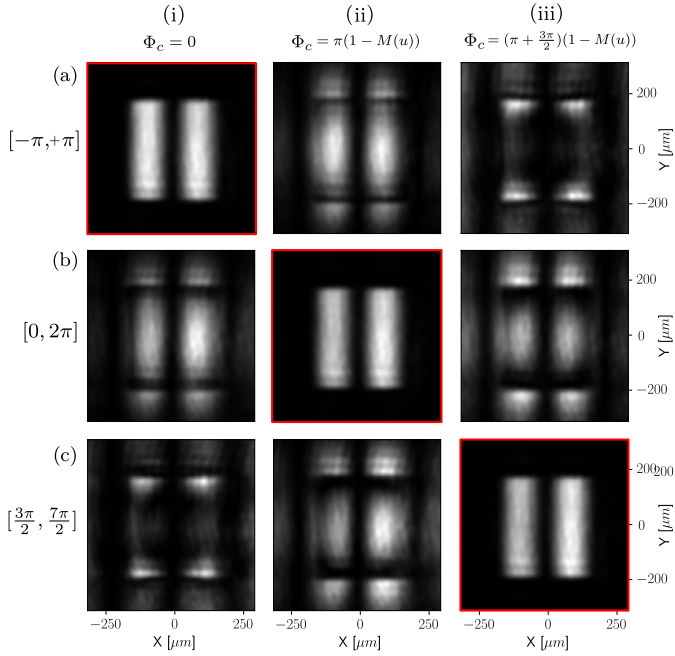
**Fig. 3.** Simulated profile intensities with different phase-wrapping conventions and phase correction terms. Each row corresponds to a different wrapping convention: (a)  $[-\pi, +\pi]$ , (b)  $[0, 2\pi]$ , and (c)  $[\frac{3\pi}{2}, \frac{7\pi}{2}]$ . Each column applies the phase correction term  $\Phi_c$  according to a different wrapping convention.

Each row represents a different phase-wrapping convention used in the simulations: row (a) uses the wrapping convention  $[-\pi, +\pi]$ , similar to the convention used in [1], row (b) uses the wrapping convention  $[0, 2\pi]$ , corresponding to what is proposed in [2], and row (c) uses the arbitrary wrapping convention  $[\frac{3\pi}{2}, \frac{7\pi}{2}]$ . Each column shows the application of the phase correction term  $\Phi_c$  from Eq. (16) according to a specific wrapping convention: column (i) applies the phase correction term for  $\alpha = -\pi$ , column (ii) for  $\alpha = 0$ , and column (iii) for  $\alpha = \frac{3\pi}{2}$ .

The results illustrate that when using the  $[-\pi, +\pi]$  convention, i.e.  $\alpha = -\pi$ , no phase correction term is needed, as the phase naturally aligns within this range. For other conventions, as illustrated by the red frames, applying the appropriate phase correction term according to the selected wrapping range ensures accurate phase shaping and avoids discrepancies. Specifically, the intensity profiles show that the target beam profile is correctly formed only when the phase correction term matches the phase-wrapping convention used in the beam shaping simulation.

This validation further confirms the robustness of our generalized mixed Fourier-Taylor expansion expression for phase correction across different phase-wrapping conventions. It enables precise amplitude and phase shaping in optical beam shaping applications, ensuring that the desired 2D beam profile is accurately achieved regardless of the phase-wrapping convention employed.

Note the crucial difference between the wrapping convention



**Fig. 4.** Experimental profile intensities with different phase-wrapping conventions and phase correction terms. Each row corresponds to a different wrapping convention : (a)  $[-\pi, +\pi]$ , (b)  $[0, 2\pi]$ , and (c)  $[\frac{3\pi}{2}, \frac{7\pi}{2}]$ . Each column applies the phase correction term  $\Phi_c$  according to a different wrapping convention.

used to calculate the phase mask that defines the blazed grating and the effective phase modulation range of experimental systems used to actually implement this blazed grating. The wrapping convention impacts the position of the grating grooves relative to the amplitude modulation envelope, thus affecting the generated profile. However, once this profile is generated, the actual phase modulation range  $[\gamma, \gamma + 2\pi]$  of the device used experimentally does not matter.

To illustrate, we present experimental results obtained with the setup described at the beginning of section A. Our LCSLM used for the phase mask induces phase delays over a range  $[\gamma, \gamma + 2\pi]$  with  $\gamma \gg 1$ . However, the generated profile for the blazed grating is the same but for a global phase constant applied uniformly to the diffracted shaped wavefront. The quality of the generated profiles, highlighted in red in Figure 4, provides experimental proof that the effective phase modulation range does not need to correspond to the phase-wrapping convention used to calculate the mask.

### 3. CONCLUSION

This study addresses and resolves ambiguities in the phase correction term for optical field encoding by analyzing different Fourier conventions. We have shown that discrepancies in the literature arise from varying phase-wrapping conventions, which, when consistently applied, yield equivalent results. Our theoretical analysis and numerical validations demonstrate the importance of selecting and adhering to appropriate Fourier conventions.

We presented a generalized expression for the phase correction term that is valid for any phase-wrapping convention. This expression allows for consistent and accurate phase encoding, crucial for the effective implementation of phase-only filters

across various optical systems. Our approach was validated through simulations and experiments of 2D beam shaping, illustrating the robustness of our method in achieving precise amplitude and phase modulation regardless of the phase-wrapping convention used.

We hope this clarification work will help enhance the understanding of amplitude and phase encoding techniques, providing a clearer theoretical framework and helping to resolve existing ambiguities. We expect it will pave the way for more reliable wavefront shaping and advance the capabilities of optical engineering at microscopic levels.

### Funding.

This work was supported by the Agence Nationale de la Recherche under grants ASTRID RESON ANR-19-ASTR-0019 and MinixQuanta ANR-20-CE47-0008-03.

**Acknowledgments.** This work was supported by French Defense Innovation Agency (AID) under grant ASTRID RESON ANR-19-ASTR-0019.

**Disclosures.** The authors declare no conflicts of interest.

**Data availability.** Data underlying the results presented in this paper are not publicly available at this time but may be obtained from the authors upon reasonable request.

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