



**HAL**  
open science

# Bifurcation analysis of a minimal model of a bowed string instrument

Marc Wijnand, Soizic Terrien, Frédéric Ablitzer, Sylvain Maugeais

► **To cite this version:**

Marc Wijnand, Soizic Terrien, Frédéric Ablitzer, Sylvain Maugeais. Bifurcation analysis of a minimal model of a bowed string instrument. 11th European Nonlinear Dynamics Conference 2024 (ENOC 2024), Jul 2024, Delft, Netherlands. hal-04673942

**HAL Id: hal-04673942**

**<https://hal.science/hal-04673942v1>**

Submitted on 20 Aug 2024

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Bifurcation analysis of a minimal model of a bowed string instrument

Marc Wijnand<sup>1</sup>, Soizic Terrien<sup>1</sup>, Frédéric Ablitzer<sup>1</sup>, Sylvain Maugeais<sup>1,2</sup>

<sup>1</sup>Laboratoire d'Acoustique de l'Université du Mans (LAUM), UMR 6613, Institut d'Acoustique - Graduate School (IA-GS), CNRS, Le Mans Université, Le Mans, France

<sup>2</sup>Laboratoire Manceau de Mathématiques, Le Mans Université, Le Mans, France

**Summary.** This study is concerned with a minimal model of a bowed string instrument, derived through a modal decomposition of the linear Kirchhoff-Carrier PDE with a Coulomb friction force that is subsequently regularized for the sake of numerical conditioning. A bifurcation analysis of the regularized model provides a cartography of the different sound regimes in the parameter space. These results are then compared with time-domain simulations of the unregularized Coulomb friction, to evaluate whether the regularized dynamics can converge to the unregularized dynamics.

## Introduction

Friction-induced sound and vibrations are produced by nonlinear, non-smooth dynamical systems [1], that can give rise to several self-sustained regimes depending on both design and control parameters [2]. However, the complexity of the underlying physical phenomena has not been fully understood and modeled yet.

The current study proposes a bifurcation analysis of an academic dry-friction oscillator [3, 4] used as a minimal model of a bowed string instrument. Despite its simplicity, this model has been shown to reproduce key features of the dynamics of bowed string instruments [3, 5]. A cartography of the different produced regimes as function of bowing control parameters, number of considered modes and a possible regularization of the non-smooth Coulomb friction law is produced numerically through bifurcation analysis and time-domain simulations.

It is known that certain regularized models do not allow to represent the dynamics of an unregularized model. This has been illustrated recently [5] for the particular case of the bowed string model with one mode introduced below. Therefore, it is proposed to use an alternative regularization, perform multiparametric continuation and study the influence of taking into account several modes.

## Model

### Mass-spring-damper system on conveyor belt

As a minimal model of a bowed string instrument, we consider a mass( $m$ )-spring( $k$ )-damper( $c$ ) system on a conveyor belt (Fig. 1a) with constant horizontal speed  $v_b$ , that exerts a reaction force  $F_b$  on the mass. The model is written as an ODE for the horizontal displacement  $x(t)$  of the mass from its rest position:

$$\ddot{x}(t) + 2\zeta\omega_0\dot{x}(t) + \omega_0^2x(t) = \frac{F_b}{m}\mu(\dot{x}(t) - v_b), \quad (1)$$

with natural angular frequency  $\omega_0 = \sqrt{k/m}$ , damping ratio  $\zeta = c/(2\sqrt{km})$ , and Coulomb's friction law with coefficient

$$\begin{cases} \mu(\Delta v) = -\text{sign}(\Delta v)\mu_d & \text{for } \Delta v \neq 0 \text{ (slip)} \\ \mu(\Delta v) \in [-\mu_s, \mu_s] & \text{for } \Delta v = 0 \text{ (stick)}, \end{cases} \quad (2)$$

as function of the speed difference  $\Delta v = \dot{x}(t) - v_b$  between the mass and the belt, and where  $\mu_s$  and  $\mu_d$  are the static and dynamic friction coefficients, respectively [6].

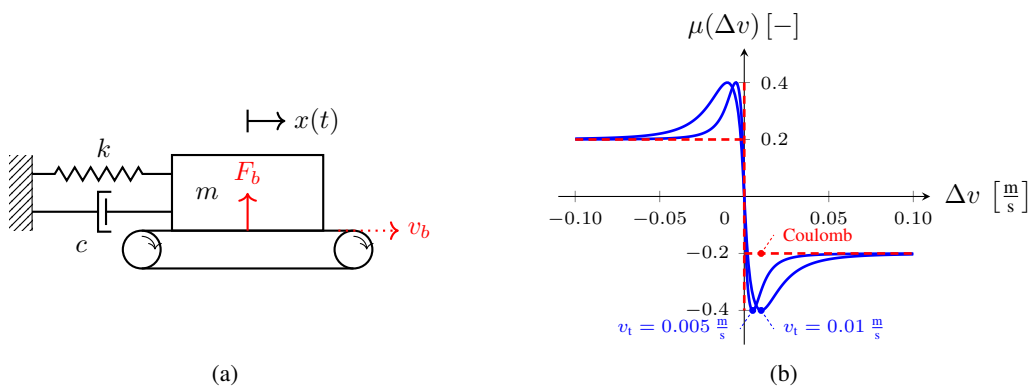


Figure 1: (a) Schematic representation of the mass-spring-damper system on a conveyor belt. (b) Coulomb friction law (2) with  $\mu_s = 0.4$  and  $\mu_d = 0.2$  (---) and its regularized counterpart (3) with different values of  $v_t$  (—).

## Friction regularization

For sake of numerical conditioning, the piecewise smooth Coulomb friction law (2) is replaced by the following regularized counterpart:

$$\mu(\Delta v) = -\mu_d \tanh\left(\frac{4\Delta v}{v_t}\right) - \frac{(\mu_s - \mu_d) \frac{\Delta v}{v_t}}{\left(\frac{1}{4}\left(\frac{\Delta v}{v_t}\right)^2 + \frac{3}{4}\right)^2}, \quad (3)$$

where the regularization parameter  $v_t$  [ $\frac{m}{s}$ ] determines the abscissa of the extrema of the curve (cf. Fig. 1b): for  $v_t \rightarrow 0$ , Coulomb's friction law is approached. This regularization using a hyperbolic tangent smoothing function is an alternative to the regularizations mentioned in [5, Eq. 15-16 and refs. 24-26 therein], that preserves extremal ( $\mu_s$ ) and asymptotic ( $\mu_d$ ) values.

## Bifurcation analysis

A numerical bifurcation analysis of the regularized model (1)-(3) is performed using MATLAB toolbox COCO (Computational Continuation Core, [7]). A cartography of the different oscillation regimes as function of the control parameters ( $v_b, F_b$ ), number of modes  $N$  and regularization parameter  $v_t$  is obtained. Fig. 2 shows an example of such results in the ( $v_b, F_b$ ) plane: a curve of Hopf bifurcations, at which a periodic solution, whether stable or unstable, emerges, and a curve of saddle-node bifurcations of periodic orbits where a stable and an unstable periodic solution collide and disappear. Overall, these curves bound the region of the parameter space where stick-slip oscillations corresponding to friction-induced sound production exist. Moreover, the comparison between the three panels of Figure 2 shows quantitative changes in the bifurcation diagram depending on the regularization parameter  $v_t$ . This motivates a comparison with time-domain simulation of the unregularized model (1)-(2) using a dedicated solver [8].

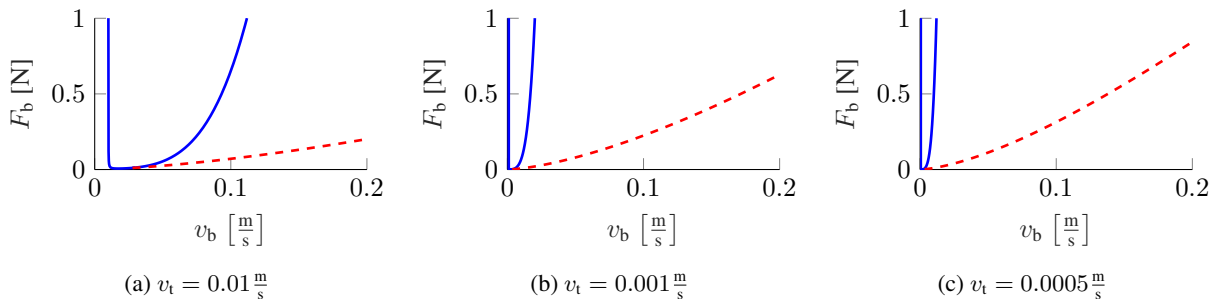


Figure 2: Evolution of the two Hopf bifurcations (—) and the saddle-node bifurcation (---) for the regularized MSD system (1)-(3) in the ( $v_b, F_b$ ) parameter plane for decreasing regularization  $v_t$ . (Model parameters:  $m = 4.6 \cdot 10^{-3}$  kg,  $\zeta = 0.005$ ,  $\omega_0 = 2\pi \cdot 196.1$  rad/s,  $\mu_s = 0.4$ ,  $\mu_d = 0.2$ .)

## Conclusion

A numerical bifurcation analysis of a minimal model of a bowed string instrument as function of the bowing control parameters and friction law regularization parameter allows to obtain a cartography of the different oscillation regimes in the space of physically relevant parameters. A comparison with the unregularized case aims to better understand and model the physics of sound production in dry-friction oscillators, including in particular bowed string instruments [9]. Moreover, a similar bifurcation analysis is performed for a multimodal model derived from the linear Kirchhoff-Carrier PDE. It shows that the bifurcation diagram in the ( $v_b, F_b$ ) plane becomes very complex as soon as two modes are considered. As a future work, an experimental testbench will be developed for model-free bifurcation analysis of a self-oscillating system.

## Acknowledgments

The authors acknowledge the support of ANR project FRICTIONAL (ANR 22 CE51 0010).

## References

- [1] A. Akay. Acoustics of friction. *The Journal of the Acoustical Society of America*, 111(4):1525–1548, 2002.
- [2] M. Di Bernardo, C.J. Budd, A.R. Champneys, P. Kowalczyk, A.B. Nordmark, G.O. Tost, and P.T. Piironen. Bifurcations in nonsmooth dynamical systems. *SIAM review*, 50(4):629–701, 2008.
- [3] K. Popp and P. Stelzer. Stick-slip vibrations and chaos. *Philosophical Transactions: Physical Sciences and Engineering*, pages 89–105, 1990.
- [4] J.H. Smith and J. Woodhouse. The tribology of rosin. *Journal of the Mechanics and Physics of Solids*, 48(8):1633–1681, 2000.
- [5] P. Vigué, C. Vergez, S. Karkar, and B. Cochelin. Regularized friction and continuation: Comparison with Coulomb's law. *Journal of Sound and Vibration*, 389:350–363, 2017.
- [6] E. Pennestrì, V. Rossi, P. Salvini, and P.P. Valentini. Review and comparison of dry friction force models. *Nonlinear Dyn.*, 83:1785–1801, 2016.
- [7] H. Dankowicz and F. Schilder. *Recipes for continuation*. SIAM, 2013.
- [8] M. Demoucron. *On the control of virtual violins - Physical modelling and control of bowed string instruments*. PhD thesis, Université Pierre et Marie Curie - Paris VI & Royal Institute of Technology, Stockholm, 2008.
- [9] J. Woodhouse and P.M. Galluzzo. The bowed string as we know it today. *ACTA Acustica united with Acustica*, 90(4):579–589, 2004.