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Introduction of the magnetic scalar potential ϕ in the **T-A** and **J-A** formulations for efficient electromagnetic simulations of High Temperature Superconductors

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Abstract—Superconducting devices are likely to play a crucial role in the energy transition addressing power generation, transmission, distribution systems, transportation and even quantum computing. Therefore, developing models that enable fast and highly accurate simulations is important for advancing the technology. This work introduces the **T-A- ϕ** and **J-A- ϕ** formulations in 2D, offering the potential to reduce the number of degrees of freedom by modeling the non-conductive parts of a device using magnetic scalar potential. The coupling between the **A** and ϕ formulations is key for this approach and is detailed in the text. The case studies are a single 2G HTS tape and a CORC[®] cable. The formulations are compared in terms of AC losses, the distributions of magnetic flux densities, and current densities. The validation is carried out using experimental data of AC losses. The results are in good agreement. The key parameter for achieving high accuracy is the boundary mesh between the **A** and ϕ domains.

Keywords—Finite Element Method, **J-A- ϕ** formulation, **T-A- ϕ** formulation.

I. INTRODUCTION

Superconducting devices hold significant potential for contributing to the global energy transition. However, their effective implementation requires studies relying on complex simulations. These models, subsequently referred to as Finite

Element Models or FEM require considerable computational effort, especially in 3D.

Over the years, various formulations of the Maxwell equations have been proposed to reduce the computation burden related to HTS modelling. These formulations include **H**, **T-A** [1], **H-A**, **H- ϕ** , and more recently, the **J-A** formulation [2]. However, even with these advancements, simulating the problem while considering a non-conductive surrounding medium still adds numerous degrees of freedom (DOF).

The present paper introduces two formulations: the **T-A- ϕ** and **J-A- ϕ** formulations. These formulations aim to decrease the DOF and consequently to reduce the computational time. A comparative analysis of the magnetic field, current density, and AC losses in superconducting tapes and stack is conducted hereinafter.

II. THE **T-A- ϕ** AND **J-A- ϕ** FORMULATIONS

The **T-A- ϕ** and **J-A- ϕ** formulations consist of two couplings: the coupling of **T** or **J** with **A**, and the coupling of **A** with ϕ . Here, **T** represents the current vector potential, **J** denotes the current density, **A** stands for the magnetic vector potential, and ϕ represents the scalar magnetic potential. The concept behind these formulations is to reduce the DOF by utilizing the scalar magnetic potential in non-conductive regions. The **T-A** and **J-A** formulations are expressed in

equations (1) and (2), respectively.

$$\begin{cases} \nabla \times \frac{1}{\mu} \nabla \times \mathbf{A} = \mathbf{J} \\ \nabla \times \rho \nabla \times \mathbf{T} = -\frac{\partial \mathbf{B}}{\partial t} \end{cases} \quad (1)$$

$$\begin{cases} \nabla \times \frac{1}{\mu} \nabla \times \mathbf{A} = \mathbf{J} \\ \rho \mathbf{J} = -\frac{\partial \mathbf{A}}{\partial t} \end{cases} \quad (2)$$

In the surrounding medium where there is no current, Ampère's law can be reformulated as $\nabla \times \mathbf{H} = \mathbf{0}$. Since the curl of the magnetic field is zero, it is customary to define the magnetic field as the gradient of a scalar magnetic potential ϕ . Applying Gauss's law for magnetic fields, an equation in ϕ is derived [3], as given by (3).

$$\nabla \cdot \nabla \phi = 0 \quad (3)$$

A similar approach as presented in [3] is used to couple the ϕ and \mathbf{A} formulations. The magnetic field computed in the \mathbf{A} formulation is enforced in the ϕ formulation using a Neumann boundary condition, see (4). Conversely, the magnetic field computed in the ϕ formulation is imposed in the \mathbf{A} formulation through a Dirichlet boundary condition, see (5). This process effectively couples the $\mathbf{T-A}$ and $\mathbf{J-A}$ formulations with ϕ , resulting in the $\mathbf{T-A-\phi}$ and $\mathbf{J-A-\phi}$ formulations.

$$\mathbf{n} \cdot \mathbf{B} = \mathbf{n} \cdot (\nabla \times \mathbf{A}) \quad (4)$$

$$\mathbf{n} \times \mathbf{H} = \mathbf{n} \times (-\nabla \phi) \quad (5)$$

In the thin strip approximation of 2G HTS tapes, the current over the superconducting layer is imposed as described in [1] and [2].

III. CASE STUDIES: SINGLE TAPE AND CORC[®] CABLE

For the single-tape case study, a tape with a width of 12 mm is simulated using the thin strip approximation, with the surrounding medium represented by a circle with a radius of 1000 mm. To model the resistivity of the superconductor, the critical electrical field (E_c), critical current (I_c), and transition index (n) are set to $1 \mu\text{V}/\text{cm}$, 150 A, and 25, respectively. A sine wave with a peak current of 100 A and a frequency of 50 Hz is applied as a current source. In the second case, a CORC[®] cable is modeled in the 2D xy plane. In this case, the I_c is set to 235 A and n to 33 keeping the same value of critical field E_c . A sine wave with a peak current equal to 80% of I_c and a frequency of 50 Hz is considered. The critical current density is dependent on the magnetic field [4].

IV. RESULTS

Figure 1 illustrates the losses computed by the $\mathbf{T-A}$, $\mathbf{J-A}$, $\mathbf{T-A-\phi}$, and $\mathbf{J-A-\phi}$ formulations. It is apparent that all formulations exhibit strong agreement, supported by a coefficient of determination (R^2) equal to 99%. For this simple problem the DOF of $\mathbf{T-A-\phi}$ and $\mathbf{J-A-\phi}$ are 4970 and 4937, respectively, while the $\mathbf{T-A}$ and $\mathbf{J-A}$ have 7893 and 7892, respectively, consequently the computation time of the computation time

of the proposed formulations are faster 30% when compared with the $\mathbf{T-A}$ and $\mathbf{J-A}$ formulation.

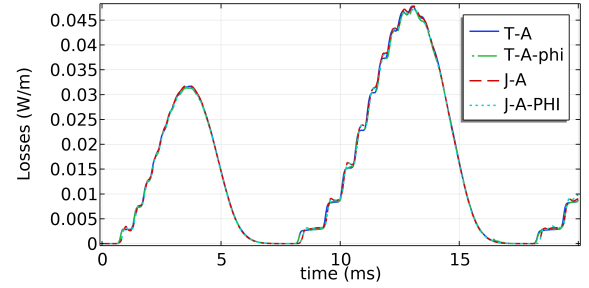


Fig. 1. Losses of single tape for the computed formulations

In the case of the CORC[®] cable, the losses, magnetic field, and current density agree well with a R^2 exceeding 98%.

V. CONCLUSION

This study introduces two innovative formulations that capitalize on the benefits of implementing a magnetic scalar potential in non-conductive regions reducing the overall DOF. This reduction leads to a diminution of the computation time. A further gain in addition to the fast $\mathbf{J-A}$ formulation. The results obtained through the $\mathbf{T-A-\phi}$ and $\mathbf{J-A-\phi}$ formulations are validated against those obtained with $\mathbf{T-A}$ and $\mathbf{J-A}$, resulting in an R^2 exceeding 98% for all analyzed variables. This indicates a high level of agreement between the proposed formulations and the existing ones. It is the first step to move to 3D problems for which the reduction of the DOF becomes particularly substantial dealing with the magnetic vector potential \mathbf{A} . However, in two model the approximation still is useful as the a linear shape function can be used in the ϕ -formulation.

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