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# Nonlinear Dynamics of a Rotary Drill-String Immersed in a 3D Geometry Well

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**Abstract.** Oil or geothermic rotary drilling is composed with a very slenderness drill-string which is subjected in particular to the tool-bit excitations. Therefore a great number of vibratory phenomena concerned with the axial, lateral and torsional behavior are exhibited: whirling, bit bouncing, stick slip to cite just a few. In order to predict the rotordynamics of such a structure, the model proposed is based on Timoshenko beam elements immersed in a 3D geometry well. A constant rotation speed is imposed at the top of drill-string. A fluid-structure interaction model that takes into account the drilling mud is used. The effect of drilling mud in drill-string vibration is studied by varying the well trajectories.

**Keywords:** Fluid-structure interaction · Drill-string dynamics

## 1 Introduction

Rotary drilling requires a very long drill-string made of screwed pipes, which brings the rotation and the drilling fluid (mud) to the drill-bit. The mud is injected downward from the pump into the drill-pipes, and then ejected at the drill-bit by the drill-nozzle. It goes upward to the surface by the drill-string - wellbore annular space. The mud carries the rock-cutting, lubricates and cools the drill-string and drill-bit. The presence of the mud inside provides the hydrostatic pressure acting on the drill-string. Consequently, the slenderness structure which runs in a 60 to 120 rpm range, is subject to various and variable excitations caused mainly by the pulsating flow of the mud, the distributed unbalance masses, the pulsating axial torque and the force of the drill-bit. Therefore this structure exhibits very intensive unexpected static and dynamic phenomena, which cause mechanical failures and reduce the MTBF: torsional and axial buckling, axial drill-bit bouncing due to the axial vibration, torsional stick-slip, lateral vibrations, forward and backward whirls are studied in [4–7]. In these studies, the effect of drilling fluid is neglected. In order to understand and to control as

good as possible the dynamics of such a complicated mechanical system, a reliable model is being established. It couples the axial, lateral and torsional deflections, also taking into account the fluid-structure interaction and multi-contacts between the string and the well. Gravity, 3D-geometric well, soil-hardness, variable viscosity and density of the mud, complicate the deal.

Chen et al. [8], studied the vibration of a rod in Newtonian viscous, incompressible fluid enclosed by a rigid, concentric and cylindrical shell. The reaction fluid force acting on the rod is interpreted as an added mass and a damping coefficient. An approximation of added mass coefficient for different types of drilling fluid based on Chen et al. [8]’s research is proposed by Shyu [9]. Heisig [11] provided the analytic formulas to compute the eigen frequencies of drill-string in horizontal well-bore. As the fluid is considered with a null viscosity, the added damping coefficient can not be calculated. The drilling fluid is taken into account only by the added mass coefficient proposed by Sinyavskii [12]. Axisa and Antunes [13,14] studied the influences of hydrodynamic force acting on a rotor immersed in dense fluid confined in an annular space. The fluid force consists of inertia, friction and viscosity effects. The first two effects contribute to equation of motion of drill-string by: an added mass matrix, a stiffness and damping matrix proportional to the rotation speed of rotor. A modal damping matrix is derived from the third effect.

The aforementioned researchs considered the fluid at rest, but in drilling context, there are fluid flows inside and outside of the drill-string. Therefore these models are not appropriate to describe the influences of fluid on drill-string vibration.

Recently, Ritto [2] proposed a fluid - structure interaction model that takes into account the fluid flows in drilling based on Païdoussis model [1]. But this one is dedicated to a vertical well. The model presented hereafter in this paper focuses on the dynamic behavior of the drill-string taking into account the mud in a 3D-well. The Finite Element model uses Timoshenko beam elements. The fluid descending within the hollow drill-strings is assumed non-viscous while the fluid rising in the annular space is viscous. The effect of the trajectory of the well is also studied in order to develop a model as general as possible. The results of dynamic responses of the system show the important influences of the fluid on the drill-string vibration.

## 2 Beam Theory

The drill-string is modeled by beam elements with two nodes and six degrees of freedom (dofs) per node (Fig. 1). The material properties are constant for an element. The section and inertia characteristics of each element are averaged by those of two nodes at the extremities. The inertial effect due to rotation of beam section (Rayleigh) and the shear effect (Timoshenko) are taken in consideration. The equation of motion of drill-string is obtained by means of two steps. Firstly, the kinetic energy  $T$ , the strain energy  $U$  and the virtual work  $\delta W$  of external forces are calculated. Then, applying Lagrange’s equations leads to the equation

of motion. In order to clarify the calculation process of energy balance, the calculations for bending, longitudinal and torsional movements are presented separately.

## 2.1 Kinetic Energy of Beam Element

The elementary kinetic energy due to the longitudinal motion is:

$$T_1 = \frac{1}{2} \rho S \int_0^{l_e} \dot{w}^2 dz \quad (1)$$

The elementary kinetic energy due to the torsional motion is:

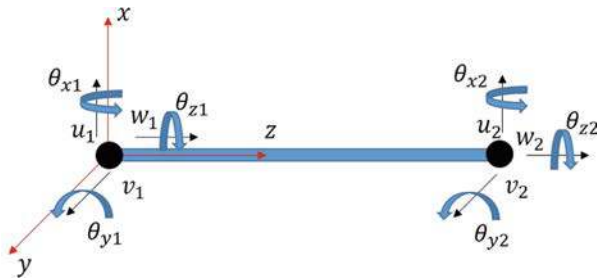
$$T_2 = \frac{1}{2} \rho I_p \int_0^{l_e} \dot{\theta}_z^2 dz \quad (2)$$

The elementary kinetic energy due to bending motion [3] is:

$$T_3 = \frac{1}{2} \rho S \int_0^{l_e} (\dot{u}^2 + \dot{v}^2) dz + \frac{1}{2} \rho I \int_0^{l_e} (\dot{\theta}_x^2 + \dot{\theta}_y^2) dz + \rho I_p \Omega \int_0^{l_e} \dot{\theta}_x \dot{\theta}_y dz \quad (3)$$

Where:

- $\rho$ : density (constant)
- $S$ : section (averaged)
- $I$ : second moment of beam section
- $I_p$ : polar moment of beam section
- $\Omega$ : rotation speed of drill-string
- $\theta_x, \theta_y$ : bending rotation angles about x-axis, y-axis.
- $\theta_z$ : torsional rotation angle
- $l_e$ : beam element length



**Fig. 1.** The 6 dofs per node of the beam element

The first integral of Eq. (3) is the classical expression of kinetic energy of a bending beam. The second integral is the secondary effect of rotary inertia (Timoshenko beam). The last integral represents the gyroscopic effect. When applying

the Lagrange equations, the first two integrals will provide the mass matrix, the third will provide the Coriolis matrix. The elementary vector of displacement  $\delta_e = [u, v, w, \theta_x, \theta_y, \theta_z]^T$  is expressed into nodal displacements, see Fig. 1 by using the classical shape functions matrix  $\mathbf{N}$  such as:

$$\delta_e = \mathbf{N}\delta_n \quad (4)$$

Where:  $\delta_n = [u_1, v_1, w_1, \theta_{x1}, \theta_{y1}, \theta_{z1}, u_2, v_2, w_2, \theta_{x2}, \theta_{y2}, \theta_{z2}]^T$ .

## 2.2 Strain Energy

The elementary strain energy may be decomposed by the energy associated to the longitudinal  $U_1$ , torsional  $U_2$  and bending  $U_3$  motions:

$$\begin{cases} U_1 = \frac{1}{2}ES \int_0^{l_e} \left(\frac{\partial w}{\partial z}\right)^2 dz, \\ U_2 = \frac{1}{2}GI_p \int_0^{l_e} \left(\frac{\partial \theta_z}{\partial z}\right)^2 dz, \\ U_3 = \frac{1}{2}EI \int_0^{l_e} \left[\left(\frac{\partial^2 u}{\partial z^2} + \frac{\partial^2 v}{\partial z^2}\right)^2\right] dz \end{cases} \quad (5)$$

Where  $G$  is the shear modulus. The stiffness matrix is built by using Eq. (5). The shear effect is added into the bending stiffness matrix by the quantity [3]:

$$a = \frac{12EI}{GS_r l_e^2} \quad (6)$$

where  $S_r$  is the reduced area of the cross-section.

**Longitudinal - Bending Coupling:** An axial force  $F_0$  modifies the lateral stiffness of the beam. A tension stiffens the beam, a compression weakens it and destroys it for the values of buckling critical loads. This effect is obtained by taking into account the nonlinear terms of the deformation field which is written by neglecting the shear effect and estimating that the axial deformations are small:

$$\epsilon_{zz} = \frac{\partial w}{\partial z} - x \frac{\partial^2 u}{\partial z^2} - y \frac{\partial^2 v}{\partial z^2} + \frac{1}{2} \left(\frac{\partial u}{\partial z}\right)^2 + \frac{1}{2} \left(\frac{\partial v}{\partial z}\right)^2 \quad (7)$$

The first term in (7) is the longitudinal deformation and the remains is the bending deformation. This one consists of linear (second and third terms) and non-linear (two lasts terms) deformation. The contribution of the axial force  $F_0$  is translated by strain energy:

$$U_{F_0} = \frac{F_0}{2} \int_0^{l_e} \left[ \left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2 \right] dz \quad (8)$$

**Torsional - Bending Coupling:** An axial torque  $C$  reduces the lateral stiffness of the beam whatever the direction of application [19,20]. This type of effort is circulatory and therefore non-conservative. Since a strain energy can not be established, it is necessary to calculate the virtual work of the two projections of the torque in the two axes related to the beam cross-section:

$$\delta W = C \int_0^{l_e} \left[ \frac{\partial v}{\partial z} \delta \left( \frac{\partial^2 u}{\partial z^2} \right) - \frac{\partial u}{\partial z} \delta \left( \frac{\partial^2 v}{\partial z^2} \right) \right] dz \quad (9)$$

### 3 Fluid - Structure Interactions

The fluid flows inside the drill-string and in the annular space induces the fluid forces acting on the structure. The characteristics of fluid flow, the properties of fluid influence the dynamic of drill-string within the well. The fluid - structure interactions model developed in [2] dedicated to vertical drilling has the following assumptions: incompressible fluid, non-viscous internal fluid, viscous external fluid; drill-string rotation not considered; drill-string - well concentric; linear variation of the hydrostatic pressure along the drill-string; constant velocity of the fluid. In reality the wellbore trajectory is rarely vertical. In order to reach the oil reservoir, it is sometimes necessary to drill a well of complex trajectory. In this work, a model based on [1,2] is developed taking into account the curvature of wellbore and the rotation speed of drill-string. Figure 2 shows the transformation from the global coordinate XYZ to the local coordinate xyz of point A on the center line of wellbore. It is achieved by respectively three rotations  $\psi$  around  $X$ ,  $\theta$  around  $Y_1$  and  $\phi$  around  $Z_2$ . Finally, the rotation matrix from global coordinate to local coordinate is:

$$\begin{pmatrix} \vec{I} \\ \vec{J} \\ \vec{K} \end{pmatrix} = \begin{bmatrix} \cos \theta \cos \phi & -\cos \theta \sin \phi & \sin \theta \\ \sin \psi \sin \theta \cos \phi + \cos \psi \sin \phi & -\sin \psi \sin \theta \sin \phi + \cos \psi \cos \phi & -\sin \psi \cos \phi \\ -\cos \psi \sin \theta \cos \phi + \sin \psi \sin \phi & \cos \psi \sin \theta \sin \phi + \sin \psi \cos \phi & \cos \psi \cos \theta \end{bmatrix} \begin{pmatrix} \vec{i} \\ \vec{j} \\ \vec{k} \end{pmatrix} \quad (10)$$

where  $(\vec{I}, \vec{J}, \vec{K})$  and  $(\vec{i}, \vec{j}, \vec{k})$  are the unit vectors of the global and local coordinates, respectively. In the undeformed state, the drill-strings are located in the center line of the well. The rotation matrix from undeformed configuration of drill-strings to its deformed state (Fig. 3) is given by:

$$\begin{pmatrix} \vec{i} \\ \vec{j} \\ \vec{k} \end{pmatrix} = \begin{bmatrix} 1 & -\theta_z & \theta_y \\ \theta_z & 1 & -\theta_x \\ -\theta_y & \theta_x & 1 \end{bmatrix} \begin{pmatrix} \vec{i}_3 \\ \vec{j}_3 \\ \vec{k}_3 \end{pmatrix} \quad (11)$$

Where  $\theta_x, \theta_y, \theta_z$  are respectively the rotations around  $x, y_1$  and  $z_2$  axis with the assumption that these angles are small.  $(\vec{i}_3, \vec{j}_3, \vec{k}_3)$  are the unit vectors of the deformed state coordinates. During the drilling, the filtered fluid is injected into the hollow drill-string with the constant axial speed  $U_i$  and then goes back into the annular space drill-string - bore-hole with the speed  $U_o$  loaded with rock

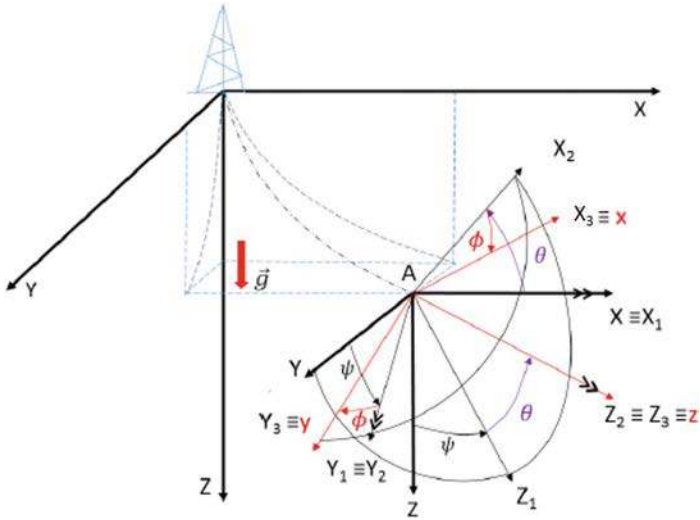


Fig. 2. Transformation from global coordinate to local coordinate

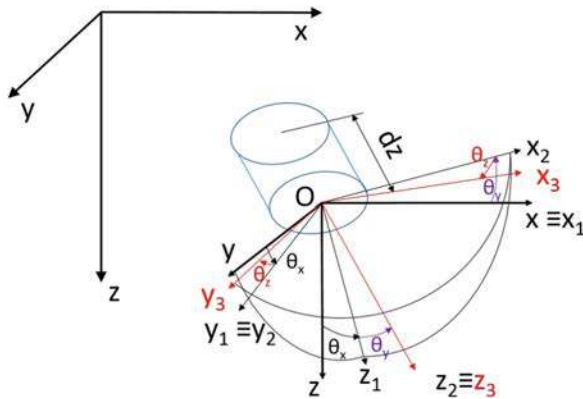


Fig. 3. Transformation from undeformed state to deformed state

debris, calories. The pressures inside and outside of the drill-string are respectively  $p_i, p_o$  and varied linearly along the drill-string. In the undeformed state, the axis of beam element coincides with the center line of well-bore. The Eq. (12) shows the forces associated with the drilling fluid acting on the structure. This equation is obtained by considering the forces balance of a drill-string element immersed in drilling-fluid. The detail of formulation is not shown in this paper. The calculation procedure is similar to that presented in [1,10,18].

$$\left\{ \begin{array}{l} F_{fx} = (M_f + \chi\rho_f A_o) \frac{\partial^2 u}{\partial t^2} + \left( \frac{1}{2} C_f \rho_f D_o U_o + k \right) \frac{\partial u}{\partial t} + (2M_f U_i - 2\chi\rho_f A_o U_o) \frac{\partial^2 u}{\partial t \partial z} \\ \quad + (M_f U_i^2 + \chi\rho_f A_o U_o^2 + A_i p_i - A_o p_o) \frac{\partial^2 u}{\partial z^2} + \left( A_i \frac{\partial p_i}{\partial z} - A_o \frac{\partial p_o}{\partial z} \right) \frac{\partial u}{\partial z} \\ \quad - M_f g (-\cos \psi \sin \theta \cos \phi + \sin \psi \sin \phi) + A_o \frac{\partial p_o}{\partial z} (-\cos \psi \sin \theta \cos \phi + \sin \psi \sin \phi) \\ F_{fy} = (M_f + \chi\rho_f A_o) \frac{\partial^2 v}{\partial t^2} + \left( \frac{1}{2} C_f \rho_f D_o U_o + k \right) \frac{\partial v}{\partial t} + (2M_f U_i - 2\chi\rho_f A_o U_o) \frac{\partial^2 v}{\partial t \partial z} \\ \quad + (M_f U_i^2 + \chi\rho_f A_o U_o^2 + A_i p_i - A_o p_o) \frac{\partial^2 v}{\partial z^2} + \left( A_i \frac{\partial p_i}{\partial z} - A_o \frac{\partial p_o}{\partial z} \right) \frac{\partial v}{\partial z} \\ \quad - M_f g (\cos \psi \sin \theta \sin \phi + \sin \psi \cos \phi) + A_o \frac{\partial p_o}{\partial z} (\cos \psi \sin \theta \sin \phi + \sin \psi \cos \phi) \\ F_{fz} = -A_i \frac{\partial p_i}{\partial z} + M_f g \cos \psi \cos \theta - \frac{1}{2} C_f \rho_f D_o U_o^2 - A_o \frac{\partial p_o}{\partial z} (1 - \cos \psi \cos \theta) \end{array} \right. \quad (12)$$

where  $A_o$  is the beam cross-section,  $D_o$  is the external diameter of the beam,  $\rho_f$  is the fluid density.  $C_f, k$  are viscous damping coefficients of the fluid,  $\chi$  is the added mass coefficient of fluid in the annular space.

$$\chi = \frac{D_{ch}^2 + D_o^2}{D_{ch}^2 - D_o^2} \quad (13)$$

$D_{ch}$  is the internal diameter of well-bore. The viscous damping coefficient  $k$  is a function of drill-string/well-bore geometry, fluid properties and rotation speed of drill-string [1]:

$$k = \frac{2\sqrt{2}}{\sqrt{S}} \left[ \frac{1 + \left( \frac{D_o}{D_{ch}} \right)^3}{\left( 1 - \left( \frac{D_o}{D_{ch}} \right)^2 \right)^2} \right] \rho_f A_o \Omega \quad (14)$$

$S = \Omega D_o^2 / 4\nu$  is the Stokes number,  $\Omega$  is the rotation speed of drill-string.

The discretization of Eq. (12) by using finite elements method and shape functions in Eq. (4) gives the added mass, damping, stiffness elementary matrix and vectors of fluid forces associated to drilling fluid:

$$[\mathbf{M}_e]^f = \int_0^1 (M_f + \chi\rho_f A_o U_o) (N_u^T(\xi) N_u(\xi) + N_v^T(\xi) N_v(\xi)) l_e d\xi \quad (15)$$

$$\begin{aligned}
[\mathbf{C}_e]^f &= \int_0^1 \left( \left( \frac{1}{2} C_f \rho_f D_o U_o + k \right) (N_u^T(\xi) N_u(\xi) + N_v^T(\xi) N_v(\xi)) l_e \right) d\xi \\
&+ \int_0^1 (2M_f U_i - 2\chi \rho_f A_o U_o) \left( \left[ \frac{\partial N_u(\xi)}{\partial \xi} \right]^T N_u(\xi) + \left[ \frac{\partial N_v(\xi)}{\partial \xi} \right]^T N_v(\xi) \right) d\xi
\end{aligned} \tag{16}$$

$$\begin{aligned}
[\mathbf{K}_e]^f &= \int_0^1 \left[ \begin{array}{c} (M_f U_i^2 + \chi \rho_f A_o U_o^2 + A_i p_i - A_o p_o) \\ \left( \left[ \frac{\partial^2 N_u(\xi)}{\partial \xi^2} \right]^T N_u(\xi) + \left[ \frac{\partial^2 N_v(\xi)}{\partial \xi^2} \right]^T N_v(\xi) \right) \frac{1}{l_e} \\ + \left( A_i \frac{\partial p_i}{\partial z} - A_o \frac{\partial p_o}{\partial z} \right) \left( \left[ \frac{\partial N_u(\xi)}{\partial \xi} \right]^T N_u(\xi) + \left[ \frac{\partial N_v(\xi)}{\partial \xi} \right]^T N_v(\xi) \right) \end{array} \right] d\xi
\end{aligned} \tag{17}$$

$$\mathbf{f}_f^x = \int_0^1 \left[ \begin{array}{c} -M_f g (-\cos \psi \sin \theta \cos \phi + \sin \psi \sin \phi) \\ + A_o \frac{\partial p_o}{\partial z} (-\cos \psi \sin \theta \cos \phi + \sin \psi \sin \phi) \end{array} \right] N_u^T(\xi) l_e d\xi \tag{18}$$

$$\mathbf{f}_f^y = \int_0^1 \left[ \begin{array}{c} -M_f g (\cos \psi \sin \theta \sin \phi + \sin \psi \cos \phi) \\ + A_o \frac{\partial p_o}{\partial z} (\cos \psi \sin \theta \sin \phi + \sin \psi \cos \phi) \end{array} \right] N_v^T(\xi) l_e d\xi \tag{19}$$

$$\mathbf{f}_f^z = \int_0^1 \left[ \begin{array}{c} M_f g \cos \psi \cos \theta - A_i \frac{\partial p_i}{\partial z} - \frac{1}{2} C_f \rho_f D_o U_o^2 \\ + A_o \frac{\partial p_o}{\partial z} (1 - \cos \psi \cos \theta) \end{array} \right] N_w^T(\xi) l_e d\xi \tag{20}$$

where  $l_e$  is the beam element length,  $\xi = z/l_e$ .

## 4 Quasi Static and Dynamic Calculations

### 4.1 Quasi Static Calculation

Before calculating dynamic responses of the system, the quasi-static position of the drill-string confined in the well-bore should be calculated. The equilibrium position is described by Eq. (21):

$$\mathbf{K} \boldsymbol{\delta} = \mathbf{F}_s + \mathbf{F}_c(\boldsymbol{\delta}) \tag{21}$$

where  $\mathbf{F}_s$  is the static force vector including gravity force and fluid force,  $\mathbf{F}_c$  is the contact force vector. The *quasi-static* term is used to describe the equilibrium position in the case where the rotating drill-string is pushed into the well. This generates in  $\mathbf{F}_c$  the normal, tangential components and the friction torque. The expression of normal force is given by [15, 16]

$$\mathbf{F}_{cn} = \begin{cases} - \left( K_c(P) P + C_c(P) \dot{P} \right) [u/r, v/r, 0]^T & \text{if } P > 0 \\ \mathbf{0} & \text{if } P \leq 0 \end{cases} \tag{22}$$

where  $P = \sqrt{u^2 + v^2} - j_0$ ,  $r = \sqrt{u^2 + v^2}$ ,  $j_0$  is the clearance between drill-string and bore-hole. To avoid numerical instability, the contact stiffness and damping is regularized by the arctan function as in [16] (Fig. 4):

$$K_c(P) = \frac{k_c}{2} \left[ \frac{2}{\pi} \arctan(\pi\lambda P) + 1 \right]; C_c(P) = \frac{c_c}{2} \left[ \frac{2}{\pi} \arctan(\pi\lambda P) + 1 \right] \quad (23)$$

where  $k_c$  and  $c_c$  are maximum values of contact stiffness and damping,  $\lambda$  is the regularization parameter. The contact force in tangent direction related to the normal component of contact force by the classical Coulomb law:

$$\mathbf{F}_{ct} = -\mu(v_g) \left( K_c(P) P + C_c(P) \dot{P} \right) [-v/r, u/r, 0]^T \quad (24)$$

where  $\mu(v_g)$  is the friction coefficient which depends on the sliding speed  $v_g$  of the drill-string on the walls well-bore and static and dynamic friction coefficients [15,16]. The friction torque is written as:

$$\mathbf{T}_f = -\mu(v_g) \left( K_c(P) P + C_c(P) \dot{P} \right) R_e \vec{z} \quad (25)$$

where  $R_e$  is the outer radius of drill-string. The quasi-static position is obtained by solving Eq. (21) with Newton-Raphson scheme.

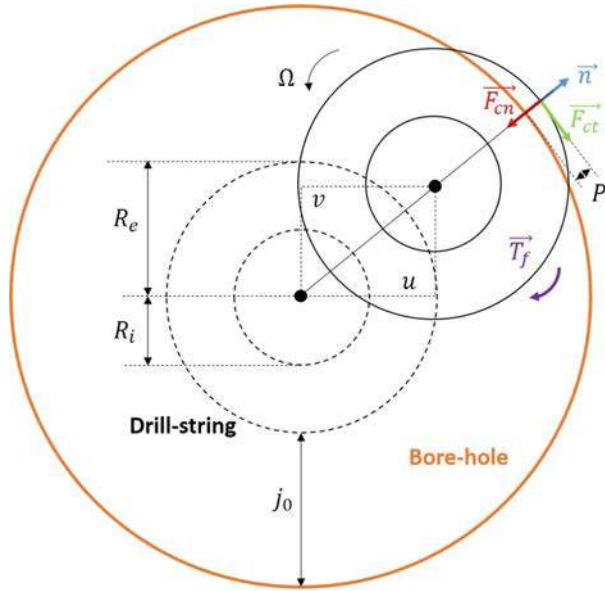


Fig. 4. Contact scheme between drill-string and bore-hole

## 4.2 Dynamic Response Calculation

During the drilling, the interaction between the drill-bit and the formation is taken into account by an axial force **WOB** and an axial torque **TOB** at the bit.

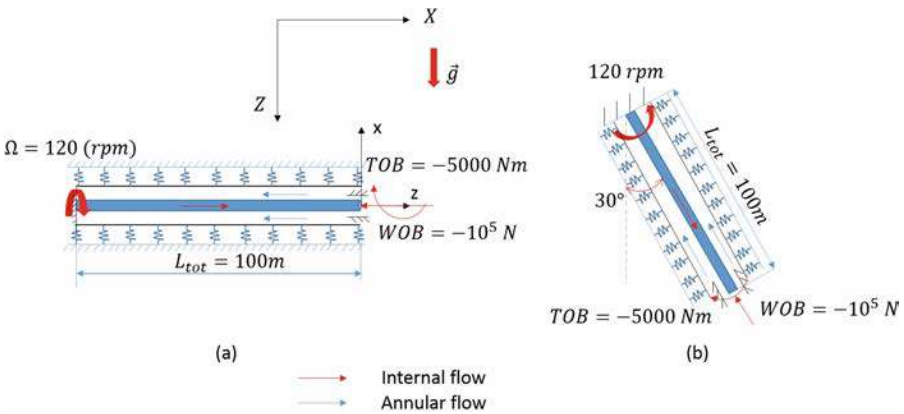
In this work, their values are considered constant. After assembly by considering the *quasi-static* computation, the final discretization equation is written as:

$$([M] + [M_f]) \ddot{\delta} + ([C] + [C_f]) \dot{\delta} + ([K] + [K_f]) \delta = F_s + F_c + WOB + TOB + F_{imp} \quad (26)$$

where  $[M]$ ,  $[C]$  and  $[K]$  are classical mass, damping and stiffness matrices.  $[M_f]$ ,  $[C_f]$  and  $[K_f]$  are added mass, damping, stiffness matrices related to fluid.  $F_{imp}$  is the force due to the taking into account the rotation speed of drill-string.  $F_{imp} = -[M] \ddot{\delta}_{imp} - [C] \dot{\delta}_{imp} - [K] \delta_{imp}$ , where:  $\ddot{\delta}_{imp}$ ,  $\dot{\delta}_{imp}$ ,  $\delta_{imp}$  are acceleration, velocity and displacement vectors which contain only imposed degree of freedom. The solving of Eq. (26) is carried out by using the fourth order Runge-Kutta scheme with adaptive time step [17].

## 5 Numerical Application in Time Domain

This section is dedicated to study influences of the fluid on the vibration of the drill-string for different trajectories of the well. Two test cases are considered: one for a horizontal well and the other for a well inclined by  $30^\circ$  with respect to the vertical axis. Consider a hollow beam in-plane of 100 m length, internal radius  $R_i$  and external radius  $R_e$ . The beam is confined inside a horizontal well-bore (Fig. 5(a)) and an inclined well-bore (Fig. 5(b)) which are modeled by the elastic stops. The clearance between drill-string and well-bore is  $j_0$ . The FE model consists of 101 nodes, so 606 degrees of freedom (dofs). The beam is subjected to gravity force and a speed of rotation of 120 rpm is imposed at the first (uppest) node. The fluid is conveyed inside the pipe-beam with velocity  $U_i$  and then after leaving the end of the beam it is pushed in the opposite direction with velocity  $U_o$ . During the rotation, a constant force of  $-10^5 N$  and a constant torque of  $-5000 Nm$  are applied to the lowest node (drill-bit). The



**Fig. 5.** Test cases for dynamic computation: horizontal (a) and inclined (b) wells

**Table 1.** Materials properties

$E(N/m^2)$	2.03E11
$\rho(kg/m^3)$	7900
$\nu$	0.3
$\rho_f(kg/m^3)$	1200
$\nu_f(m^2/s)$	1E-5
$C_f$	0.0125
$Q(m^3/s)$	1E-2

**Table 2.** Contact modeling parameters

$R_e(m)$	8.0E-2
$R_i(m)$	5.0E-2
$j_0(m)$	1.5E-2
$k_c(N/m)$	1E7
$c_c(Ns/m)$	1E4
$\lambda(m^{-1})$	7E7
$\mu_s$	0.3
$\mu_d$	0.2
$V_{ref}(m/s)$	0.033

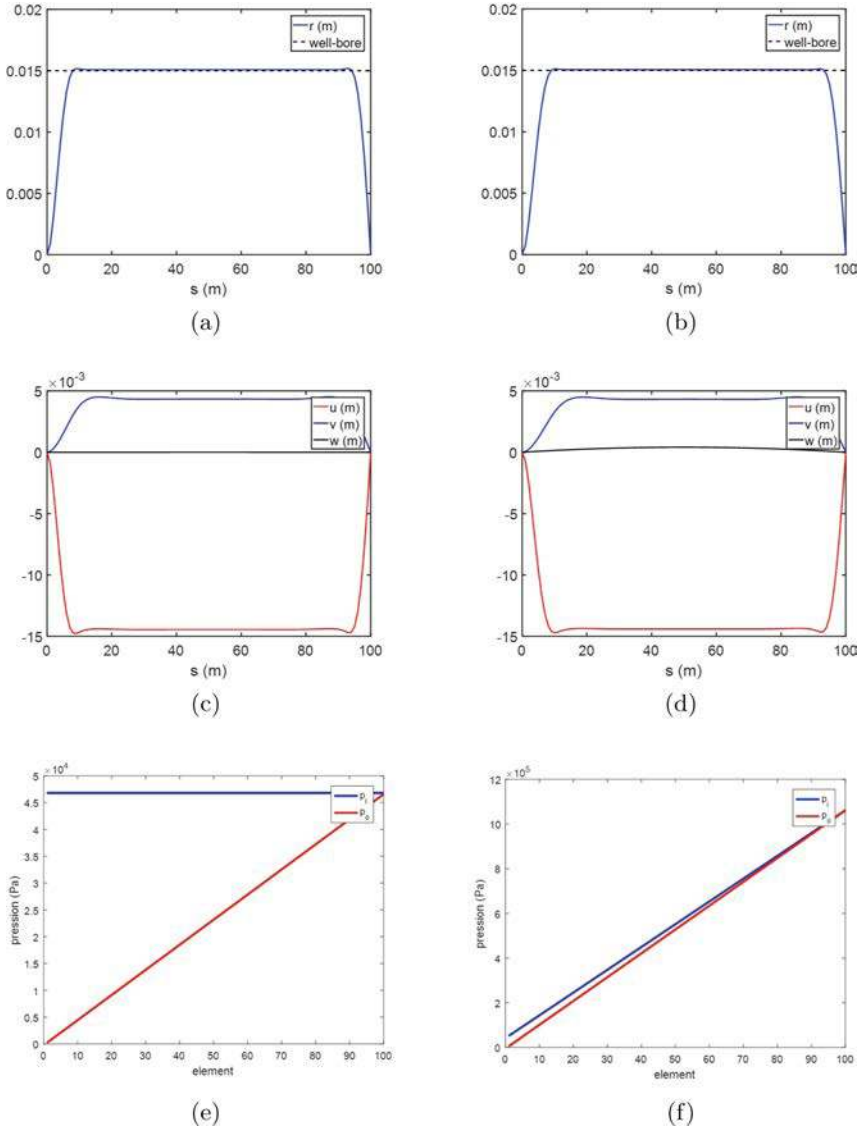
beam material and fluid properties are given in Table 1. The geometry of drill-string and parameters used for modeling of drill-string - well-bore contact are given in Table 2.

### 5.1 Quasi-Static Position

Concerning the boundary conditions for the computation of static equilibrium position, the first node is embedded while the last node is blocked three translational movements and the rotation around z-axis. Under gravity effect, the beam is bent. Figure 6 shows the deflection of the beam in the radial direction (a, b) and in three directions  $x, y, z$  (c, d). Since the beam is subjected to the friction efforts in tangent direction defined in local coordinate system, the two beams in two test cases are bent out of plane, the displacements along the  $y$  axis are non-zero. As showed in Fig. 6(e, f), the hydrostatic pressures of inclined beam are more important than those of horizontal beam. This means that the fluid forces acting on the inclined beam, deforms slightly the beam along its axial direction (Fig. 6(d)).

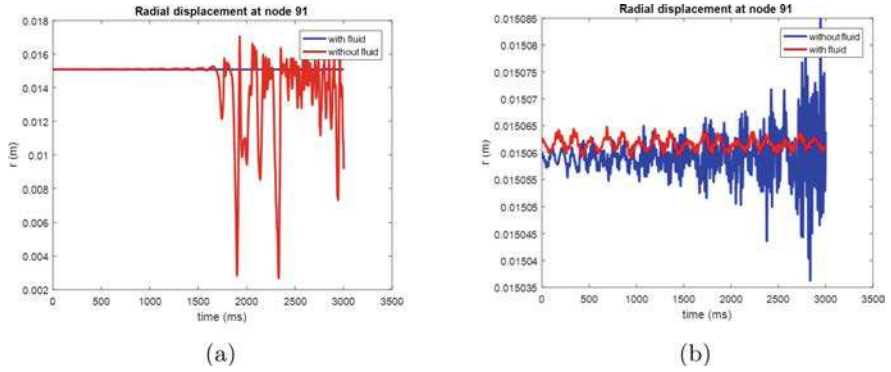
### 5.2 Nonlinear Dynamic Response in the Time Domain

The dynamic response of the structure is computed from the equilibrium position. Figure 7 shows the comparison of radial displacements at node 91 where there are some interesting phenomena, with and without fluid-structure interaction, for the horizontal and inclined drill-strings. After the quasi-static computation, most of the drill-string is in contact with well-bore in both two test cases. The presence of the fluid greatly reduces the radial displacement especially in the case of the horizontal drill-string. For the inclined beam, the lateral vibration is smaller than that of the horizontal beam. The latter can be explained by the fact that the effect of gravity is reduced when the drill-string is inclined. The friction between the beam and the well-bore is therefore less important. This is the reason why the effect of the fluid in the case of the inclined beam is

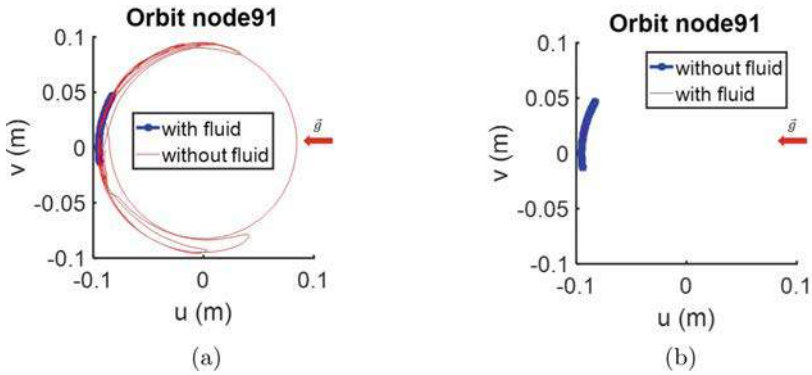


**Fig. 6.** Quasi-static position for the horizontal drill-string (a, c, e) and for the inclined drill-string (b, d, f): (a, b) radial displacement, (c, d) displacements in the  $x, y, z$  directions, (e, f) internal and annular hydrostatic pressures.

not very important like in the case of the horizontal beam. Figure 8 shows the orbits of contact points at node 91 in horizontal Fig. 8(a) and inclined Fig. 8(b) drill-strings. During the rotation, friction excites the vibration of the beam. It rolls on well-bore. If the beam rotates without fluid, amplitude of rolling angle

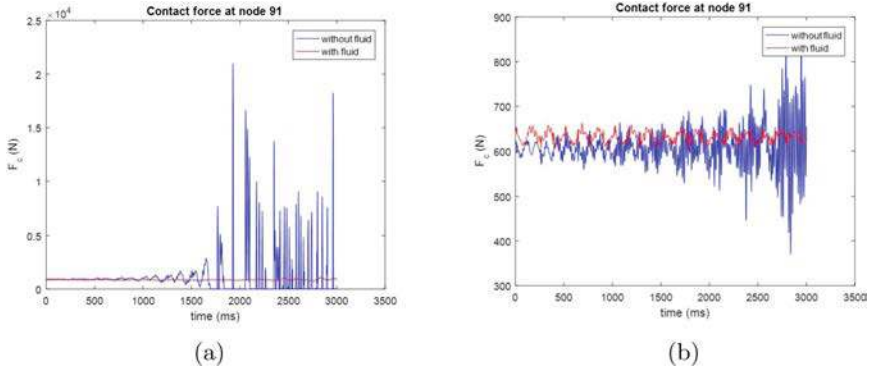


**Fig. 7.** Drill-string dynamic responses at node 91 with and without fluid: (a) horizontal drill-string, (b) inclined drill-string



**Fig. 8.** Orbit of contact point at node 91 with and without fluid: (a) horizontal drill-string, (b) inclined drill-string

is bigger than with fluid. In particular, in the case of horizontal drill-string, the contact point at node 91 can make a full rotation around the well-bore. When the drill-string rotates in the well with fluid, the rolling angle of the beam is decreased significantly. Figure 9 shows the contact forces at node 91 with and without fluid for horizontal Fig. 9(a) and inclined Fig. 9(b) drill-string. For the horizontal beam, we can see that if there is not fluid in the well, the node 91 sometimes loses the contact. It lifts off, comes back and hit the well-bore. So the contact force increases sharply. The presence of fluid provides the fluid forces that keep the drill-string always in contact with well-bore. The value of contact force is therefore decreased significantly. In the case of inclined drill-string, this one is always in contact with well-bore even if there is not fluid in the well. But the fluid reduces also the amplitude of contact force.



**Fig. 9.** Contact force at node 91 with and without fluid: (a) horizontal drill-string, (b) inclined drill-string

## 6 Conclusions

A fluid - structure interaction model taking into account the inner and annular fluid flow and the curvature of well-bore is developed in this work. A 3D beam with constant speed of rotation imposed at the top is considered. The non-linear dynamic responses due to the contact between drill-string and well-bore are obtained by using fourth order Runge-Kutta method with adaptive time step. Influences of fluid - structure interaction model on the vibration of drill-string are tested for two different well-bore trajectories. The results showed that the fluid has an important influence on the lateral vibrations of drill-string. It reduces the lateral displacement, the rolling angle of drill-string and therefore decreases the contact force. However, the actual fluid - structure interaction model takes into account only the concentric drill-string - well-bore. In reality major part of drill-string lays on the well-bore at equilibrium position. In the future work, the more general fluid model including the eccentric drill-string - well-bore should be implemented.

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