

HOW TO MODIFY THE TREE OF SHAPES OF AN IMAGE: CONNECTED OPERATORS WITHOUT GRADIENT INVERSION

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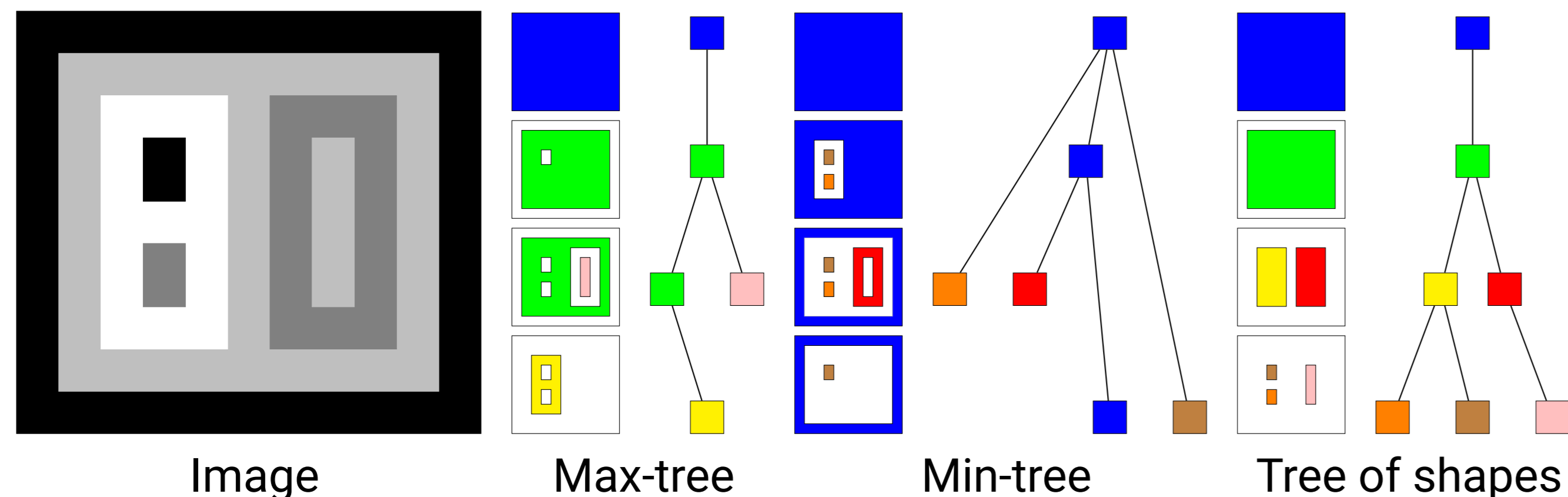
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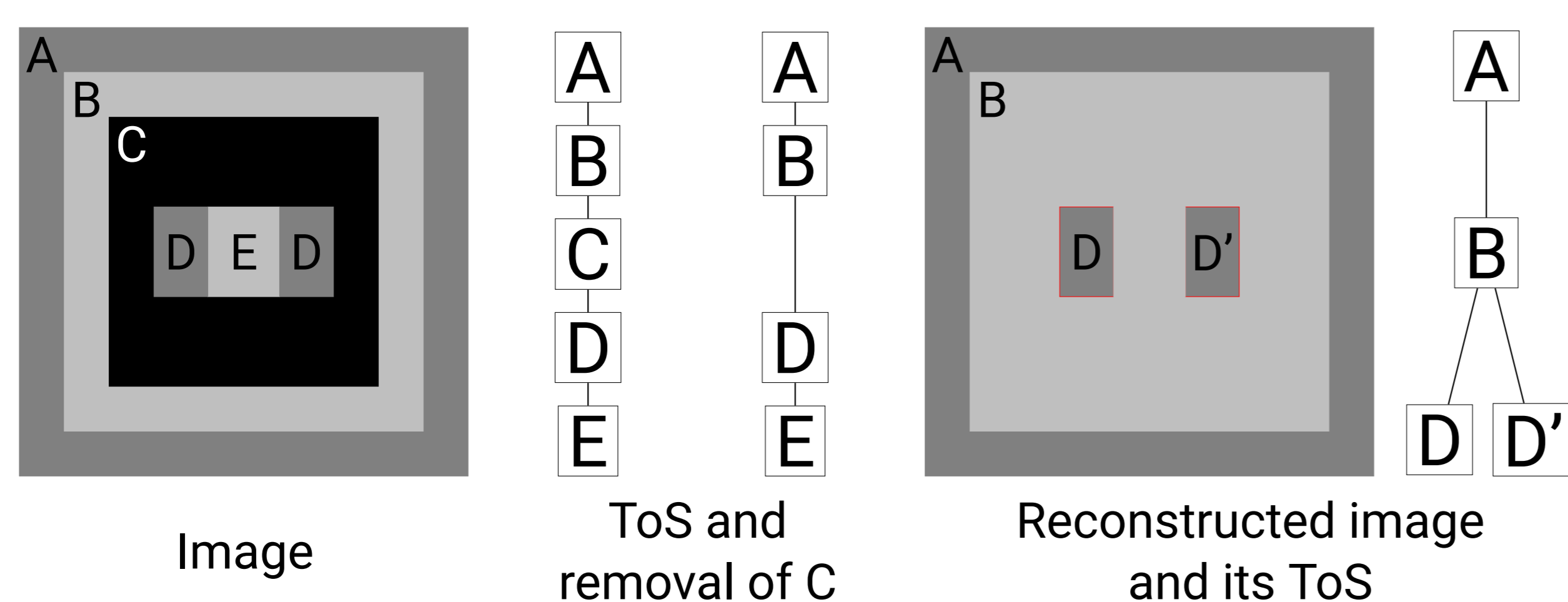


Motivations

The tree of shapes (ToS) [1] is a **hierarchical data structure** that models a grey-level image via the inclusion of its **level lines**. It is often described as the self-dual version of the min- and max-trees.



- The **modification** (shifting of the altitude of one or several nodes) of the tree of shapes allows to design **connected operators**.
- If carried out uncarefully, the resulting structure **may not be the ToS of the modified image** and **this image may present differential inconsistencies**.



The proposed modifications rules guarantee that:

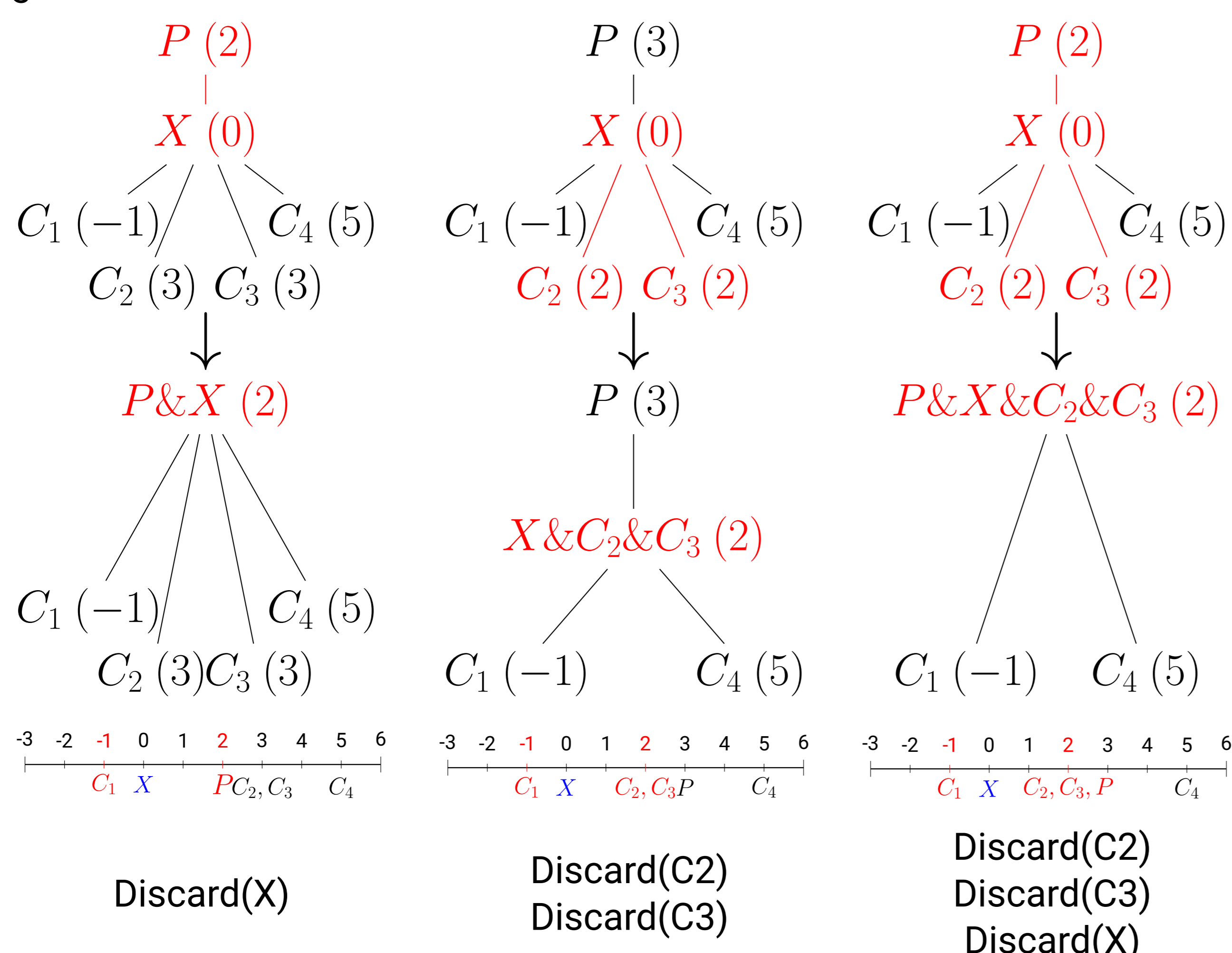
- the modified tree of shapes is **still the tree of shapes** of the modified image;
- the **signs of the gradients** of the resulting image are not inverted.

Short range altitude shifting

The *Discard* function is the **atomic operation** merging a node to its parent. The **pixels** of the discarded node are assigned to its parent and its **children** become children of the parent.

Short range shift: modify the altitude of a node to a target altitude within that of its **nearest neighbouring nodes** (parent and children).

The short range shifting of node X with a target altitude of 2 can lead to the following configurations:



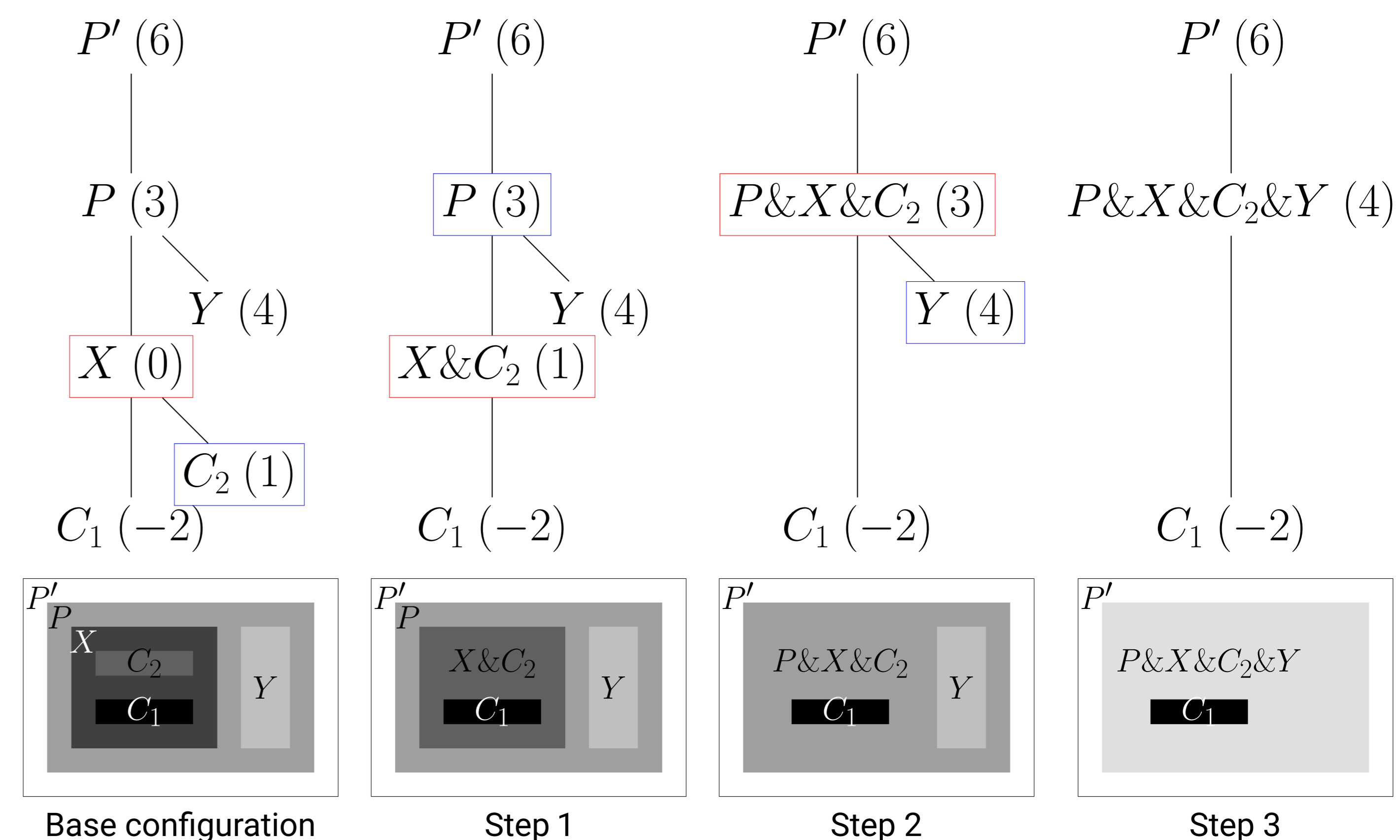
References

- [1] Caselles, V., Coll, B., Morel, J. : Topographic maps and local contrast changes in natural images. International Journal of Computer Vision, 33:5–27 (1999)
- [2] Synchrotron microtomography, SOLEIL ANATOMIX, project #20211303, courtesy S. Almagro Work funded by Région Normandie (thesis grant RIN), Partenariats Hubert Curien (Sakura, 49674RK) and Agence Nationale de la Recherche (ANR-20-CE45-0011, ANR-22-CE45-0034, ANR-23-CE45-0015). ANATOMIX is an EQUIPEX funded by the Investments for the Future program of the Agence Nationale de la Recherche (ANR-11-EQPX-0031).

Long range altitude shifting

Long range shift: modify the altitude of a node to **any target altitude** within the grey level range of the image.

- **Gradually** progress toward the target altitude by applying **successive short range shifts** until the target altitude is reached.
- Results in the **progressive flattening** of the surrounding regions.



Tree filtering

A tree of shapes can be **filtered** by selecting unwanted nodes with respect to a **criterion** and modifying them following a **modification policy**.

Algorithm 1: Generic tree of shapes modification

Input: $\mathfrak{T}(Nodes, Relations), Alt$

Parameters: $Criterion : Node \rightarrow \{True, False\}, Policy : Node \rightarrow Value$

Output: $\mathfrak{T}(Nodes, Relations), Alt$

- 1 **foreach** $X \in Nodes$ **such that** $Criterion(X) = True$ **do**
- 2 $v := Policy(X)$
- 3 Apply **Long range shift** ($Nodes, Relations, Alt, X, v$)

Applications

This filtering strategy allows the definition of various **connected filters**, for example:

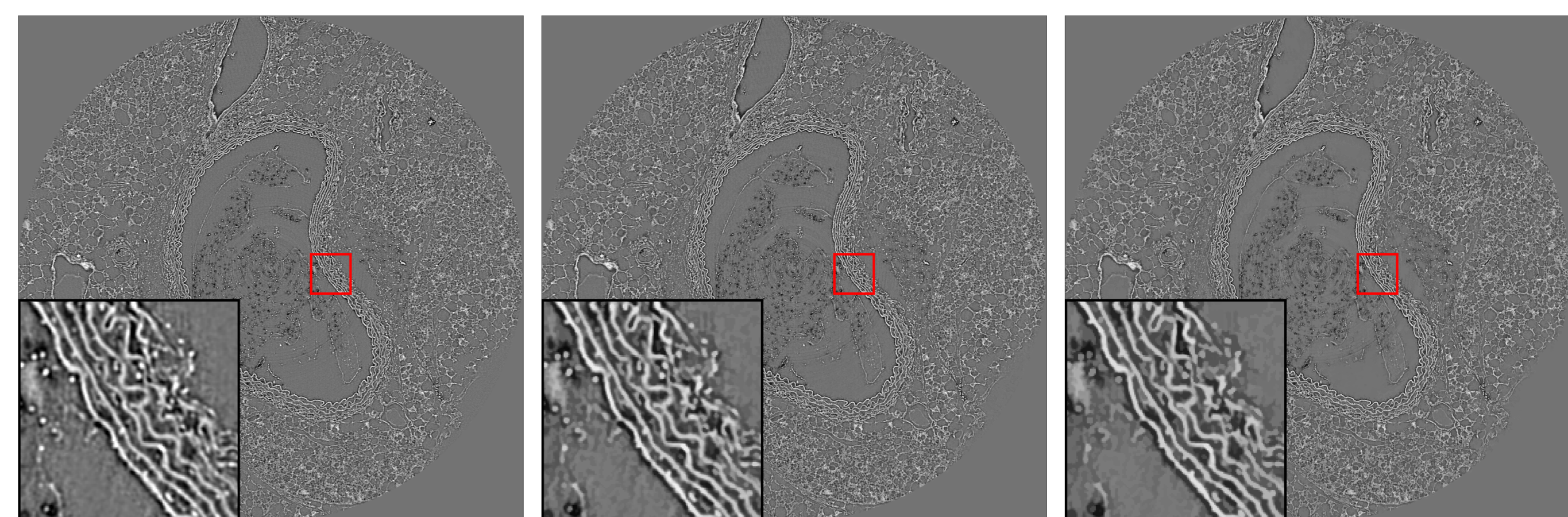
Area opening, removing nodes of the image with **small proper part**.

- **Criterion:** Proper part lower than n .
- **Policy:** take the altitude of the parent through a **long rang shift**.



Tree simplification, to alleviate the impact of the **noise** in the structure.

- **Criterion:** True for **any** node of the tree.
- **Policy:** take the **mean altitude** of the node's neighborhood (parent and children).



Biological image [2]
4.2 · 10⁶ pixels,
ToS: 1.2 · 10⁶ nodes

$n = 5$
SSIM = 0.981
ToS: 4.7 · 10⁵ nodes

$n = 10$
SSIM = 0.910
ToS: 2.3 · 10⁵ nodes