



**HAL**  
open science

# **Solid and 3D beam finite element models for the nonlinear elastic analysis of helical strands within a computational homogenization framework**

Fabien Ménéard, Patrice Cartraud

## ► To cite this version:

Fabien Ménéard, Patrice Cartraud. Solid and 3D beam finite element models for the nonlinear elastic analysis of helical strands within a computational homogenization framework. *Computers & Structures*, 2021, 257, pp.106675. <10.1016/j.compstruc.2021.106675>. <hal-04667460>

**HAL Id: hal-04667460**

**<https://hal.science/hal-04667460v1>**

Submitted on 13 Nov 2024

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



Distributed under a Creative Commons CC BY-NC 4.0 - Attribution - Non-commercial use - International License

# Solid and 3D beam finite element models for the nonlinear elastic analysis of helical strands within a computational homogenization framework

Fabien Ménéard, Patrice Cartraud

*Institut de Recherche en Génie civil et Mécanique (GeM), UMR CNRS 6183 Ecole Centrale de Nantes, BP 92101, 44321 Nantes cedex 3, France*

---

## Abstract

This paper proposes a computational approach for studying the overall behaviour and local stress state of strand-type structures. This method is based on the homogenization theory of periodic beamlike structures, with the local problem posed on the strand axial period being solved using the finite element method. This approach fully utilises the strand's helical symmetry, thus minimising the size of the computational domain. Consequently, accounting for geometric complexity and contact interactions, which are of paramount importance for bending loads, is more straightforward. The numerical model mesh size can also be reduced thanks to the use of beam elements, and one objective of this paper is to assess the accuracy of such a model in comparison with solid element models and analytical results. These comparisons are performed on both single-layer and multi-layer strands. Results demonstrate the capability of the proposed computational approach to accurately capture the nonlinear bending behaviour stemming from the stick-slip transition as well as local stress distributions. As for the beam model, it apparently offers a very good compromise between accuracy and numerical efficiency.

*Keywords:* Homogenization, Helical symmetry, Strand, Cable, Bending, Contact, Finite element method

---

## 1. Introduction

1 Metal cables are widely used in various industrial fields due to their high tensile strength  
2 relative to bending stiffness. Applications extend from electrical power given their ability to  
3 transmit electrical current to civil engineering structures like bridges or anchored retaining  
4 walls. The components of metal cables commonly consist of an arrangement of helically-  
5 wound wires, in creating strands from an assembly of several layers of wires. The cable  
6 cross-section can be quite complex, with strand arrangements possibly involving simple and  
7 double helical paths, or simpler featuring a single straight strand, as is the case for the cables  
8 considered in this work.

9 Strand structural analysis raises modelling challenges due to the geometric complexity  
10 and nonlinearities resulting from friction between components, especially for a bending load.  
11

*Preprint submitted to Computers & Structures*

*July 6, 2021*

12 Given the objective of a service life design of these structures, it is therefore necessary to be  
13 able to accurately predict the overall behaviour and local stress state within the strand.

14 Strand behaviour can be studied by means of analytical approaches, in modelling indi-  
15 vidual components as curved beams and introducing some approximations and assumptions.  
16 In [1], a nonlinear formulation was established to take into account the radial contraction  
17 of the various components due to the Poisson effect. This formulation has been linearised  
18 in the single-layer strand case (see [2]). A concise formulation has been proposed in [3]  
19 that takes into account the stiffness matrix symmetry. A more recent model incorporating  
20 both radial contraction of the wires due to the Poisson effect and wire flattening effects was  
21 proposed in [4]. Most of the works dedicated to analytical models have been focused on the  
22 axial cable behaviour, while an interesting loading case is obtained when combining a tensile  
23 force with bending. Depending on the stress state within the cable, the contact between its  
24 components can be in either a stick or slip state, which leads to nonlinear bending stiffness.  
25 Such a consideration has been factored into the models proposed by Papailiou [5] and Foti  
26 [6], in which a nonlinear transition state is defined between the stick and slip states. Some  
27 analytical formulations provide stress estimations within wires for axial-torsional loads [1] as  
28 well as for bending with an initial tensile load [5]. However, addressing all mechanical and  
29 contact issues in an analytical formulation can prove to be cumbersome, especially when the  
30 strand cross-section is composed of many layers.

31 Over the last twenty years, several authors have developed numerical models that con-  
32 sider geometric complexity and contact interactions. Single-layer strands have been studied  
33 with solid finite element models in the cases of axial loading [7], pure bending [8] and tension-  
34 bending loading [9] and [10]. Multi-layer strands have been considered in [11], [12] and [13].  
35 The inclusion of contact interactions significantly increases computation time, especially  
36 when solid elements are being used. One way to reduce the computational cost is to use  
37 beam models. In [14], [15], [16] and [17], a small sliding assumption between strand com-  
38 ponents has been adopted, in turn leading to a low computation time. Contact resolution  
39 is done by a node to node approach using contact elements between strand layers. A finite  
40 sliding has been assumed in [18], [19] and [20] where contacts are solved by a beam-to-beam  
41 algorithm firstly developed in [21]. Another approach to decreasing the computational cost  
42 calls for taking advantage of helical symmetry in order to reduce the structural analysis to  
43 just a small portion of the cable. If the loading also fulfils helical symmetry, then a 2D  
44 model is an eligible option, as performed in [22] and [23] to study the axial behaviour of  
45 single strands. However, once bending is considered, a 3D model is required, e.g. [8].

46 The main objective of this work is to propose a computational approach based on the  
47 periodic homogenization method developed in [24] involving this time contact nonlinearities.  
48 The size of the computational domain is thus reduced to an axial strand period using specific  
49 periodic boundary conditions. In addition, two types of finite element models featuring  
50 solid and beam elements will be studied in order to compare the numerical results for axial,  
51 torsional and bending loadings. These comparisons will be drawn on the overall strand  
52 behaviour and local stress distributions, in focusing on bending with an initial tensile load.  
53 To validate the method, both single-and multi-layer strands will be examined and the results  
54 compared to the literature.

55 **2. Description of the mechanical problem**

56 *2.1. Multi-layer strand geometry*

57 The strand geometry considered herein consists of  $m$  concentric wire layers and a central  
 58 core, comprising respectively a helical and rectilinear mean line, see Figure 1a, with the  
 59 layers being wound around each other. In most strands, the winding angle on two successive  
 60 layers has opposite signs. Each layer  $j$  is defined by: its number of wires  $n_j$ , the wire radius  
 61  $R_{sj}$ , the mean layer radius  $R_{hj}$ , and its lay angle  $\alpha_j$ . The core of the strand is defined by  
 62 its radius, denoted  $R_c$ , see Figure 1b.

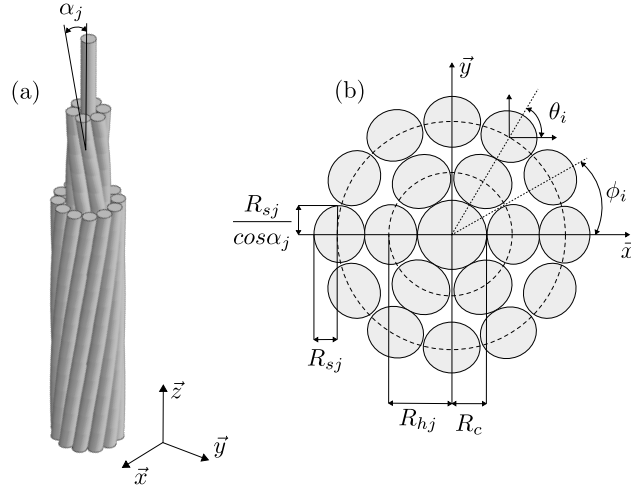


Figure 1: Representation over the length (a) and cross-section (b) of a strand composed of two layers and a core

63 The helix pitch length of a layer  $j$ , denoted  $p_j$ , is determined according to both the lay  
 64 angle and mean layer radius, such that:

$$p_j = \frac{2\pi R_{sj}}{\tan(\alpha_j)}. \quad (1)$$

65 As detailed in [18] and [25] for a  $z$ -axis strand, the mean line of a wire  $i$  belonging to a  
 66 layer  $j$  can be defined by the following parametric equations:

$$x_{ij} = R_{hj} \cos \left( \phi_i + \frac{2\pi(i-1)}{n_j} + \frac{\tan(\alpha_j)p_j t}{R_{hj}} \right), \quad (2)$$

$$y_{ij} = R_{hj} \sin \left( \phi_i + \frac{2\pi(i-1)}{n_j} + \frac{\tan(\alpha_j)p_j t}{R_{hj}} \right), \quad (3)$$

$$z_j = p_j t, \quad (4)$$

67 with  $t \in [0, 1]$  and  $\phi_i$  being the initial angular position of the wire in the strand section, see  
 68 Figure 1b.

69 *2.2. Contact interactions within a multi-layer strand*

70 During operations, various contact interactions appear between the strand components,  
 71 see Figure 2a, namely [7]:

- 72 • Intra-layer contacts involving a circumferential normal: contacts between wires of the  
 73 same layer established along contact lines, see Figure 2b.
- 74 • Inter-layer contacts involving a radial normal: contact between wires of two successive  
 75 layers. In the non-deformed state, these contacts can be considered as contact points,  
 76 see Figure 2c, i.e. in the case of overlapping layers. They are established along contact  
 77 lines in the case of contact between layer and core.

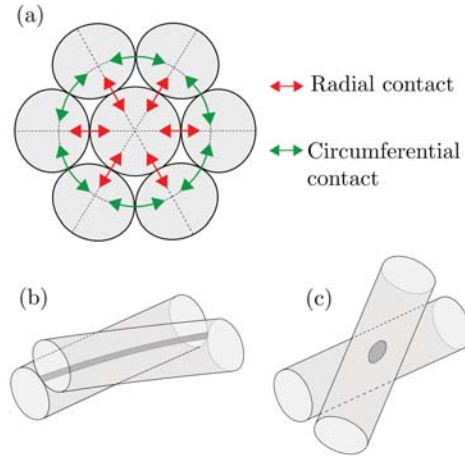


Figure 2: Representation of possible contact interactions within a strand section: radial and circumferential contact (a); contact line (b); and a contact point (c)

78 Depending on the pressure present between layers, the contact plays an important role,  
 79 in particular in the case of bending. Significant contact forces will in fact tend to stiffen  
 80 the structure. Conversely, for low intensities, the contact forces are negligible and allow  
 81 for relative displacement between components. Contact interactions therefore have a major  
 82 influence in the study of strand behaviour.

83 Contact problems are solved within the framework of general contact theory, according  
 84 to which normal contact is translated under the conditions of Hertz-Signorini and tangential  
 85 contact is approximated by Coulomb's Law with friction coefficient  $\mu$ , [26].

86 The relative sliding of strand wires varies depending on their angular position in the  
 87 section [27]. For bending loading, the first wire in a slip state occurs for the closest wire to  
 88 the bending axis on the outer layer. Subsequently, adjacent wires of the same layer also slip.  
 89 This phenomenon evolves from the outer layer to the inner layer until reaching a full slip.

90 The longitudinal slip  $u$  of a strand wire  $i$  in layer  $j$  in pure bending can be approximated  
 91 analytically by means of a loxodromic curve, such that:

$$u = R_{hj}^2 \frac{\cos^2 \alpha_j}{\sin \alpha_j} \kappa \cos \phi_i, \quad (5)$$

92 where  $\kappa$  is the strand curvature.

93 It is important to note that in certain cases, the manufacturing process or loading intensity  
 94 generates spaces between wires of the same layer or between wires of two adjacent  
 95 layers, see [4] and [28]. Under these circumstances, the contact surfaces can be heterogeneous  
 96 between layers or within a layer, thus modifying the final behaviour and sliding state of the  
 97 strand. In this work however, the initial deformations tied to the manufacturing process are  
 98 not taken into account, and a homogeneous contact is assumed.

### 99 2.3. Homogenization method for periodic beams

100 The homogenization method has already been employed in the past for strand or periodic  
 101 beam like structures ([24], [29], [30], [31], [32] and [33]). This method offers an efficient  
 102 and rigorous means for reducing the size of the Boundary Value Problem (BVP) domain,  
 103 thanks to the structural axial periodicity, which stems from the helical geometry of cable  
 104 components.

105 This method is suitable for the case of helical wire assembly, whereby the helix axis  
 106 is parallel to the slenderness direction or is itself a helix. The helical structure is then  
 107 considered as a slender 3D structure, with geometric heterogeneities repeating periodically  
 108 along its main axis, see Figure 3.

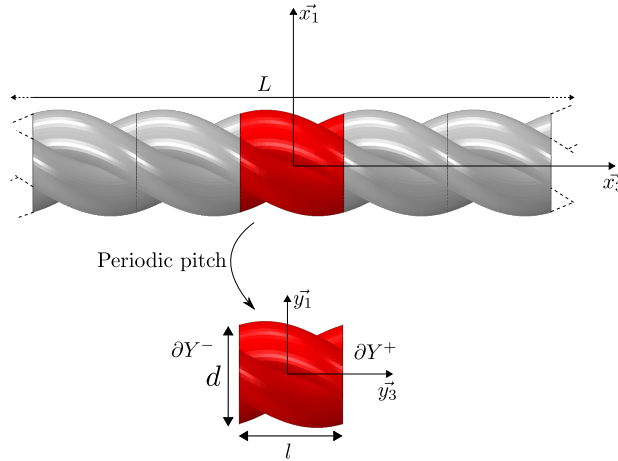


Figure 3: Strand representation with a periodic structure

109 In the case of a helical strand, the period is defined by the helical symmetry of the layer.  
 110 The axial period  $l_j$  of layer  $j$  is obtained by dividing the pitch, (1), by the number of wires  
 111  $n_j$  in the layer, such that:

$$l_j = \frac{p_j}{n_j}. \quad (6)$$

112 An example of a period is presented in Figure 3 for a 3-wire strand:  $n_j = 3$ . In order  
 113 to determine the length  $l$  of the axial strand period with several layers, a common period  
 114 between layers must first be defined, i.e.:

$$l = k_j \frac{p_j}{n_j} = k_{j+1} \frac{p_{j+1}}{n_{j+1}} = \dots = k_m \frac{p_m}{n_m}, \quad (7)$$

115 with  $k_j \in \mathbb{N}$ .

116 This homogenization approach is based on the asymptotic expansion method, in taking  
 117 into account that the initial 3D problem, posed on the cable structure, involves two small  
 118 parameters, defined as: 1) the ratio of the size of the axial period to the cable length, and 2)  
 119 the inverse of the cable slenderness. The latter parameter is also the ratio of cable diameter  
 120 to cable length. Without any restrictions imposed, these two parameters can be considered  
 121 equal and are denoted  $\epsilon$ :

$$\epsilon \simeq \frac{l}{L} \simeq \frac{d}{L}. \quad (8)$$

122 A microscopic scale  $y$  can now be introduced such that  $y = x/\epsilon$ .

123 The asymptotic expansion method consists of searching for the displacement solution  
 124 to the initial 3D problem in the form of an expansion in increasing powers of  $\epsilon$ . The  
 125 initial 3D problem is then decomposed into a series of microscopic 3D problems, posed on  
 126 the cable axial period, as well as a series of macroscopic 1D problems, see [31], [32] and  
 127 [33]. The lower-order macroscopic 1D problem is a Navier-Euler-Bernoulli-Saint-Venant  
 128 beam problem, with a homogenised behaviour obtained from the solution to the microscopic  
 129 problem. This problem can be expressed as follows, with more details available in [24]: Find  
 130 the displacement field  $y_3$  periodic  $u^{per}$ , strain  $\varepsilon$  and stress  $\sigma$  from the macroscopic strain  
 131 state corresponding to extension  $E^E$ , curvatures  $E^{F\alpha}$ , with  $\alpha = [1, 2]$ , and torsion rate  $E^T$ ,  
 132 such that:

$$\left\{ \begin{array}{l} div_y \sigma = 0, \\ \sigma = a(y) : e, \\ e_{\alpha\beta} = e_{y_{\alpha\beta}}(u^{per}), [\alpha, \beta] = [1, 2], \\ e_{13} = e_{y_{13}}(u^{per}) - y_2 E^T / 2, \\ e_{23} = e_{y_{23}}(u^{per}) + y_1 E^T / 2, \\ e_{33} = e_{y_{33}}(u^{per}) + E^E - y_\alpha E^{F\alpha}, \\ \sigma \cdot n = 0 \text{ on } \partial Y, \\ u^{per} \text{ periodic and } \sigma \cdot n \text{ anti-periodic,} \end{array} \right. \quad (9)$$

133 where  $a$  is the elastic modulus, and  $div_y$  and  $e_y$  the equilibrium and strain operators at the  
 134 microscopic scale. Periodic signifies  $l$ -periodic in variable  $y_3$ , while anti-periodic means that  
 135  $\sigma \cdot n$  are opposite on opposite sides  $\partial Y^+$  and  $\partial Y^-$  in the strand axis direction. Microscopic  
 136 problems with imposed macro-deformations (9) are solved using the finite element method  
 137 with specific periodic boundary conditions.

138 As explained previously, the lower-order macroscopic problem is a 1D beam problem.  
 139 This problem is derived from compatibility conditions which express that microscopic prob-

140 lems admit a solution. These conditions lead to a lower-order macroscopic problem which  
 141 turns out to be a beam problem involving internal forces which are the macroscopic axial  
 142 force  $N$ , the macroscopic bending moments  $M_\alpha$ , as well as the macroscopic torsional mo-  
 143 ment  $M_3$ . They are defined by both integrating microscopic stresses over the strand section  
 144 and averaging on the period length, i.e. [24]:

$$\begin{cases} N(x_3) = \langle \sigma_{33} \rangle, \\ M_\alpha(x_3) = \langle -y_\alpha \sigma_{33} \rangle, \\ M_3(x_3) = \langle -y_2 \sigma_{13} + y_1 \sigma_{23} \rangle, \\ \langle \cdot \rangle = \frac{1}{l} \int_Y \cdot dy_1 dy_2 dy_3. \end{cases} \quad (10)$$

145 The strain variables of this 1D macroscopic problem are the macroscopic strains  $E^E$ ,  
 146  $E^{F_1}$ ,  $E^{F_2}$  and  $E^T$  introduced previously. Thus, from the solution of the microscopic problem  
 147 (9), the homogenised behaviour can thus be written in the following form:

$$\begin{Bmatrix} N \\ M_1 \\ M_2 \\ M_3 \end{Bmatrix} = [a^{hom}] \begin{Bmatrix} E^E \\ E^{F_1} \\ E^{F_2} \\ E^T \end{Bmatrix}, \quad (11)$$

148 where  $[a^{hom}]$  defines the homogenised stiffness matrix. Each column of the homogenised  
 149 stiffness is obtained from the solution of microscopic problem (9), considering four elemen-  
 150 tary macroscopic strain states with only one non zero component of the macroscopic strain  
 151  $\{E^E, E^{F_1}, E^{F_2}, E^T\}$ .

152 The mathematical framework of the homogenization method is well established for per-  
 153 fectly bonded components, for which convergence results are available, see e.g. [32]. Its  
 154 extension to a structure comprising several components with contact interactions is ques-  
 155 tionable. In order to investigate this question, the microscopic problem (9) with periodic  
 156 boundary conditions have been solved for the axial period, but also for larger domains built  
 157 with three or five axial periods. For the most critical bending loading considered in the study  
 158 regarding the amplitude of the longitudinal slip, it has been found that the homogenised  
 159 behaviour was not sensitive to the size of the domain, since the deviation between results of  
 160 one and five axial periods is about 0.1 %. Moreover the local stress state obtained from the  
 161 solution of the microscopic problem on one axial period exhibits only negligible departure  
 162 from that obtained on the central axial period of larger domains. These results therefore  
 163 justify the extension of the classical framework of homogenisation theory to the case of  
 164 periodic structure with non-linear contact.

### 165 3. Finite element models

166 The helical strand study is presented here through three finite element analyses based  
 167 on models with solid or beam elements. The ABAQUS software has been used as an FEA  
 168 solver within the framework of linear elasticity and in considering small displacements and  
 169 small deformations. Moreover, the finite element solution takes contact nonlinearities into

170 account. The assumptions and construction of the relevant finite element models indicate  
 171 how to solve problems resulting from the homogenization method on a strand period.

172 *3.1. Geometry and meshing*

173 The geometry and mesh for these finite element models are generated using in-house  
 174 software developed in MATLAB. The strand period length used for modelling is defined  
 175 by equation (7). The coordinate system, correlated with the period, is such that the main  
 176 strand axis coincides with  $\vec{y}_3$ -axis, whereby the origin lies at the strand centre, see Figure 4.

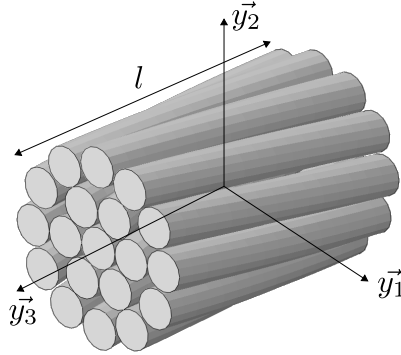


Figure 4: Depiction of the strand period, composed of two layers with:  $l = \frac{p_1}{n_1} = \frac{p_2}{n_2}$

177 *Solid element model*

178 In the case of a finite element model using solid elements, the first step consists of  
 179 building a wire section mesh with quadrangle elements in the plane  $(\vec{y}_1, \vec{y}_2)$ . By neglecting  
 180 the initial deformations, which may be due to the manufacturing process, the layer wire  
 181 section can be approximated, in  $(\vec{y}_1, \vec{y}_2)$ , by an elliptical section while the core section is  
 182 circular, see Figure 5a. Subsequently, the mean wire line is defined using equations (2),  
 183 (3) and (4). The wire section mesh is then translated along the mean component line  
 184 by applying a translation defined by means of the axial discretisation and a rotation  $\theta$  in  
 185 order to orient the wire section radius according to the strand section radius, see Figure  
 186 5b. Treatment of the contact by the finite element method is highly dependent on both the  
 187 axial and circumferential discretisation of each wire. The number of finite elements should  
 188 thus be sufficiently large to obtain a good representation of the contact surface and reduce  
 189 the geometric discontinuities. This last step entails building a connectivity table that allows  
 190 creating a 3D mesh of hexahedral elements, C3D8 in ABAQUS, see Figure 5c. The material  
 191 properties are defined in the elastic domain, as characterised by Young's modulus  $E$  and  
 192 Poisson's ratio  $\nu$ .

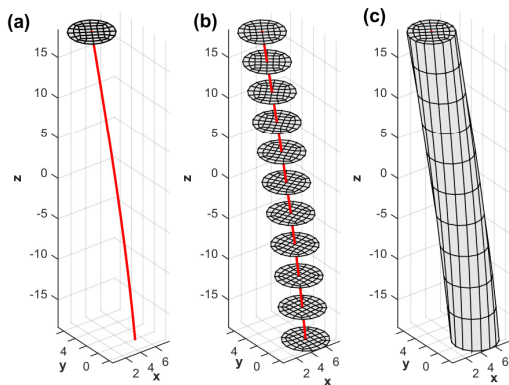


Figure 5: Generation of the mesh of a helical wire composed of solid elements: (a) generation of the mean line and surface mesh, (b) translation and rotation of the surface mesh along the mean line, and (c) building of the 3D mesh

193 *Beam element model*

194 For the finite element model composed of beam elements, the mesh is directly generated  
 195 from equations (2), (3) and (4). The cross-section perpendicular to the mean wire line  
 196 is defined in ABAQUS as being circular with radius  $R_{sj}$  for a layer wire and  $R_c$  for the central  
 197 core. Timoshenko beam elements are used to account for shear forces resulting from contact  
 198 interaction. The material properties are defined in the elastic domain, as characterised by  
 199 Young's modulus  $E$  and Poisson's ratio  $\nu$ .

200 *3.2. Contact modelling*

201 For the normal contact solution of the two models presented herein with either solid or  
 202 beam elements, linear penalisation method is used because of the large number of contact  
 203 involved in order to reduce computational time. Two methods were preliminary studied in  
 204 order to determine the penalty stiffness in the normal direction. One can use Hertz theory  
 205 [34] as mentioned in [14]. The second method consists to scale the representative underlying  
 206 element stiffness in contact as used in Abaqus, see [35]. The penalty stiffness increases  
 207 linearly from the initial penalty stiffness, equal to the representative underlying element  
 208 stiffness, to the final penalty stiffness evaluated at 10 times the initial penalty stiffness.  
 209 A comparison between those two methods in terms of contacts penetration, homogenised  
 210 stiffness and numerical convergence showed no significant deviation. Thus, the software  
 211 method is retained thereafter.

212 Tangential contact resolution is also performed based on the penalty method in regu-  
 213 larising Coulomb's Law, see Figure 6. The contact tangential stress  $\sigma_T$ , proportional to  
 214 displacements, can be written at increment  $i$  as follows:

$$\sigma_{T_i} = \sigma_{T_{i-1}} + \frac{\mu\sigma_N}{\gamma_{crit}} \gamma_i, \quad (12)$$

215 where  $\gamma_{crit}$  is the critical relative displacement between the components in contact, in de-  
 216 liningating the stick behaviour and slip behaviour. This expression is defined as a fraction of

217 the characteristic length  $l_e$  of the element surface in contact:

$$\gamma^{crit} = \beta l_e. \quad (13)$$

218 The value of  $\beta$  must be defined by the user. The choice of a low parameter  $\beta$  will tend  
 219 to generate more flexible behaviour, by decreasing the transition threshold from the slip-to-  
 220 stick state. In the case of a large parameter, Coulomb's Law will be better approximated  
 221 but may cause convergence problems.

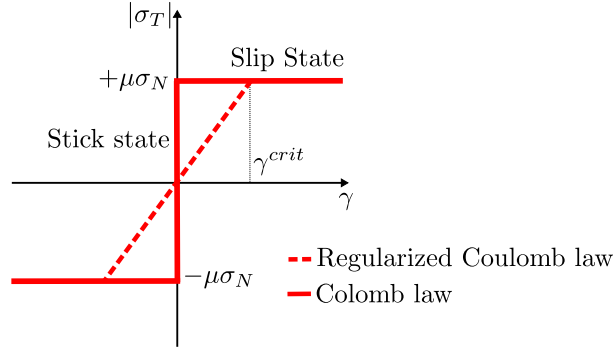


Figure 6: Tangential contact resolution with the penalty method in regularising Coulomb's Law from critical relative displacement  $\gamma^{crit}$

222 *Contact algorithm for solid elements*

223 The type of contact interactions differs depending on which model is being considered.  
 224 When taking strand geometry into account, a node-surface algorithm is chosen to solve  
 225 contact problems for solid elements. In addition, this approach enables reducing model size  
 226 and considerably simplifying the mesh, which avoids mesh matching at the contact interface.

227 *Contact algorithms for beam elements*

228 The beam-to-beam contact modelling is widely studied in the literature, see for example  
 229 [19], [21] [36] and [37] which provide developpments that are contained in the proposed  
 230 commercial code. Beam-to-beam contact can be used in the case of a helical strand, e.g. [18]  
 231 and [16]. Beam element nodes are located on the mean line, while the contact is established  
 232 on the section boundary. An initial formulation of the beam-to-beam contact, called cross  
 233 formulation, serves to solve the contact point problem by projecting the contact point onto  
 234 the beam elements, see Figure 7a. The radial formulation of the beam-to-beam contact  
 235 is used in the case of a contact line where contact conditions are evaluated in each node,  
 236 see Figure 7b. For this latter formulation, it is necessary to generate one additional node  
 237 at each section in contact. Combining these two formulations resolves all types of contact  
 238 present in a multi-layer strand under the small displacements assumption and with a regular  
 239 mesh. However, it has been shown in [38] that beam-to-beam contact is time-consuming for  
 240 a multi-layer strand, and moreover convergence is elusive especially for a bending load.

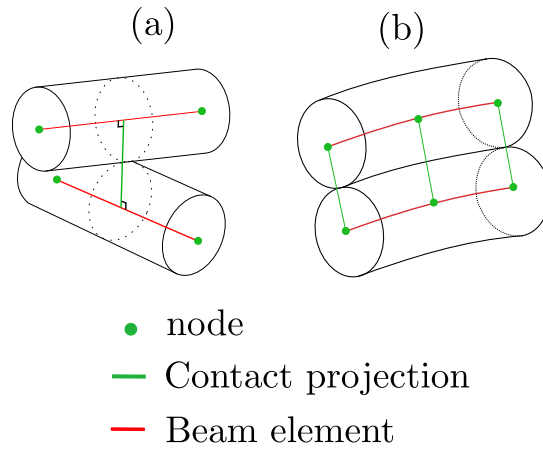


Figure 7: Contact formulation for beam-to-beam contact: (a) cross formulation, and (b) radial formulation

241 A second approach, inspired by the work reported in [38], is developed here to solve  
 242 contact problems with beam elements. The cross-section boundary of the beam elements  
 243 has been discretised using 4-node surface elements called SFM3D4 in ABAQUS. This surface  
 244 is solely dedicated to the contact and has neither stiffness nor thickness. In order to link  
 245 the contact surface with the beam, each node of the surface is rigidly constrained to the  
 246 displacement (by translation and rotation) of its projection node on the beam element.  
 247 The connection is carried out using a rigid beam ("Multipoint constraints"), see Figure 8.  
 248 The contact area discretisation is performed with a node-surface algorithm. Since contact  
 249 surfaces have no stiffness, it is necessary to combine the penalty method with the Lagrangian  
 250 method in order to limit contact surface penetration and define the contact stiffness. A fine  
 251 discretisation of the contact zone can be conducted using this approach without increasing  
 252 computation time.

253 It is important to note that the contact between beam elements assumes a constant  
 254 radius section, making it impossible to account for section contraction due to the Poisson's  
 255 effect and section deformation due to contact forces, see [18].

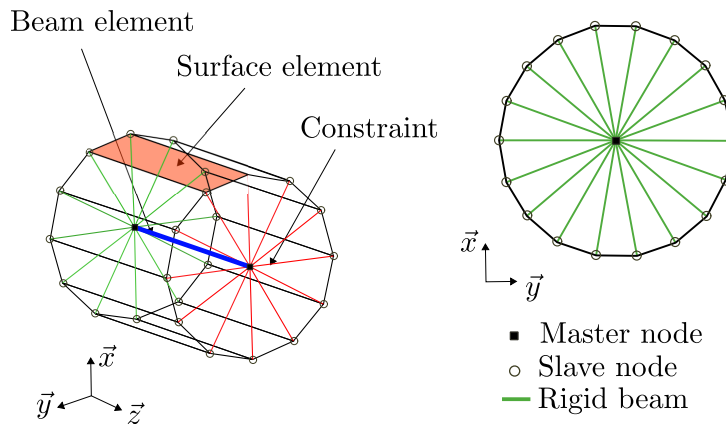


Figure 8: Connection between surface elements and beam elements using rigid beam elements

256 *Contact tracking approaches*

257 Regarding the slip between the various strand component, one can adopt small sliding  
 258 assumption as in [14], [15] and [16]. Contrary to finite sliding approach, with small sliding  
 259 method the contact relationships are established just once, at the beginning of the analysis,  
 260 thus leading to computational savings. However, small sliding tracking approach is not  
 261 available for beam-to-beam formulation and one objective of this paper is to compare the  
 262 numerical results obtained from different finite element models. Therefore, although finite  
 263 sliding tracking approach increases the calculation times, this method will be used in the  
 264 following.

### 265 3.3. Boundary conditions

266 Microscopic problems (9) are solved using the finite element method, with implementa-  
 267 tion being based on introducing additional nodes whose degrees of freedom are the macro-  
 268 scopic strains, see [39] and [40]. Other methods can also be employed, see e.g. [41]. In  
 269 our case the finite element solution corresponds to the total displacement field  $\vec{u}$ , i.e. the  
 270 sum of a periodic displacement field denoted  $\vec{u}^{per}$  and a displacement field related to the  
 271 macroscopic strains. It can be defined as:

$$\begin{cases} u_1 = u_1^{per} + \frac{1}{2}y_3^2 E^{F_1} - y_2 y_3 E^T, \\ u_2 = u_2^{per} + \frac{1}{2}y_3^2 E^{F_2} + y_1 y_3 E^T, \\ u_3 = u_3^{per} + y_3 E^E - y_\alpha y_3 E^{F_\alpha}, \quad \alpha = [1, 2]. \end{cases} \quad (14)$$

272 The periodic boundary conditions on  $u^{per}$  are taken into account through linear relations,  
 273 in connecting each degree of freedom of two opposite nodes belonging to the boundary  $\partial Y^+$   
 274 and  $\partial Y^-$  in the axial strand direction, see Figure 3.

275 In the case of solid elements, the translational degrees of freedom of the boundaries  $\partial Y^+$   
 276 and  $\partial Y^-$ , denoted  $U_i^+$  and  $U_i^-$ , are linked by the following equations [24]:

$$U_1^+ - U_1^- = l(\bar{y}_3 E^{F_1} - y_2 E^T), \quad (15)$$

$$U_2^+ - U_2^- = l(\bar{y}_3 E^{F_2} + y_1 E^T), \quad (16)$$

$$U_3^+ - U_3^- = l(E^E - y_\alpha E^{F_\alpha}), \quad \alpha = [1, 2], \quad (17)$$

277 with  $\bar{y}_3 = \frac{1}{2}(y_3^+ + y_3^-)$  and  $y_\alpha = y_\alpha^+ = y_\alpha^-$ .

278 In the case of beam elements, three other equations apply for the rotational degrees of  
 279 freedom of the boundaries  $\partial Y^+$  and  $\partial Y^-$ , denoted  $\theta_i^+$  and  $\theta_i^-$  [42]:

$$\theta_1^+ - \theta_1^- = lE^{F_1}, \quad (18)$$

$$\theta_2^+ - \theta_2^- = lE^{F_2}, \quad (19)$$

$$\theta_3^+ - \theta_3^- = lE^T. \quad (20)$$

280 In order to calculate the solution to problem (9) for an arbitrary macroscopic strain, two  
 281 reference nodes A and B are created with respectively one and three degrees of freedom:  
 282  $A = \{E^E\}$  and  $B = \{E^{F_1}; E^{F_2}; E^T\}$ .

283 It can be shown, see [39] and [40] in the case of homogenizing 3D periodic material, that  
 284 the problem to be solved can be written as follows:

$$[K] \left\{ \left\{ \begin{array}{c} \{U\} \\ E^E \\ E^{F_1} \\ E^{F_2} \\ E^T \end{array} \right\} \right\} = \left\{ l \left\{ \begin{array}{c} \{0\} \\ N \\ M_1 \\ M_2 \\ M_3 \end{array} \right\} \right\}, \quad (21)$$

285 where  $[K]$  is the stiffness matrix. The loading is therefore imposed in the form of a given  
 286 macroscopic strain, through the corresponding degrees of freedom. Computation of the  
 287 right-hand side then provides the macroscopic axial force and moments. From (11), one can  
 288 thus calculate the homogenised stiffness matrix  $[a^{hom}]$ .

289 As mentioned at the end of section 2, even if we have contact interaction, it is assumed  
 290 that periodic boundary conditions can be applied. They are two set of boundary conditions,  
 291 one for each components of the wire strand.

292 The solution to the problem (9) is defined up to a rigid body displacement, namely  
 293 through three translations along the three main axes and a rotation around the main axis  $\vec{y}_3$ .  
 294 It is necessary therefore to properly constrain this displacement to each strand component so  
 295 as to obtain a unique solution. For this type of problem, the two possibilities consist of either  
 296 using kinematic conditions [8] or solving the problem using an explicit integration scheme  
 297 [9]. To simplify the generation of finite element models, an alternative method has been  
 298 adopted here. Adding a viscous damping coefficient, as presented in [43], integrated into  
 299 the classical Newton-Raphson scheme enables dissipating rigid body displacements. This  
 300 viscous damping coefficient is evaluated throughout the simulation in order to guarantee a  
 301 ratio of damping energy to total energy of less than 5 %, thus ensuring a negligible influence  
 302 on the final solution.

#### 303 4. Single-layer strand validation

304 A single-layer strand analysis, focusing on 6 helical wires and a central core, will first  
 305 be presented to validate the models with solid and beam elements in a simple case. This  
 306 configuration has been the subject of numerous studies based on analytical models [1], [4], [5]  
 307 and numerical results [8], [9]. The various analyses pertain to the strand in tension, bending  
 308 and torsion, as well as in a combined tension-bending state to highlight the nonlinear nature  
 309 of the contact. The validation is based on the property that the solution of the microscopic  
 310 problems provides, with the exception of edge effects, the stress state that would be obtained  
 311 on a global structure subjected to a uniform strain state. Therefore, thanks to the periodic  
 312 boundary conditions, it is not necessary to perform this global analysis on a large length  
 313 of cable, but only to consider an axial period. One can then compare the solution of the

314 microscopic problems with analytical solutions that are valid for a uniform strain state. This  
 315 comparison will be made on the contact forces and stress fields.

#### 316 4.1. Case study

317 The strand considered here is single-layer and composed of 7 steel wires. The geometric  
 318 and material properties considered are listed in Table 1. To apply the periodic boundary  
 319 conditions, model length is  $l = 38.35$  mm, which corresponds to 1/6 of the layer pitch.

Table 1: Geometric and material properties of the single-layer strand

$R_s$ [mm]	$R_c$ [mm]	$\alpha$ [deg]	$E$ [GPa]	$\nu$
2.590	2.675	8.18	210	0.3

320 According to [4], lateral contact is neglected within the outer wire layer when two geo-  
 321 metric conditions are met, namely:

$$\left\{ \begin{array}{l} \xi_0 = \frac{R_s}{R_c} < 1 \\ \alpha < \alpha_{max} = \arccos \left( \sqrt{\frac{\tan^2(\pi/2 - \pi/n)}{(1 + \xi_0^{-1})^2 - 1}} \right) \end{array} \right. , \quad (22)$$

322 where  $\xi_0$  is the ratio between the wire radius of the outer layer and the core radius, and  
 323  $\alpha_{max}$  is the maximum angle defining the beginning of lateral contact within the outer layer.  
 324 In the present case, the conditions (22) are verified ( $\xi_0 = 0.97$  and  $\alpha_{max} = 11.86^\circ$ ), hence  
 325 a purely radial contact within the strand can be assumed. The contact surface between the  
 326 outer layer and the core reduces to a contact line. A friction coefficient  $\mu = 0.3$  is set for  
 327 tangential contact interactions.

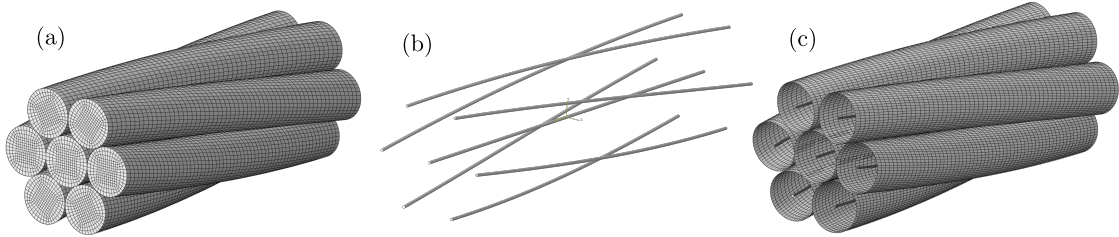


Figure 9: Display of the various meshes of the strand: (a) solid element model composed of 102,907 nodes and 92,400 C3D8 elements, (b) beam element model with 427 nodes and 420 B31 elements, and (c) beams with a surface element model containing 26,047 nodes, 420 B31 and 25,200 SFM3D4 elements

328 The various meshes are displayed in Figure 9. According to [9], mesh size may influence  
 329 both global and local quantities such as strand stiffness, contact distribution and contact  
 330 pressure between layers. Here, the element size has been chosen thanks a convergence  
 331 study of two quantities of interest: the global homogenised stiffness and the contact normal  
 332 transmission through its line load distribution over the core layer interface. The latter

333 is computed for a tensile test. Three mesh densities: coarse, medium and fine have been  
 334 considered and the medium one was found to provide the best compromise between accuracy  
 335 and computation time.

336 By taking into account the rigid body constraints connecting the beam elements to the  
 337 surface elements displayed in Figure 9.c, the number of independent degrees of freedom on  
 338 the surface beam model is equal to that of the beam model with beam-to-beam contact.

#### 339 4.2. Single-layer strand under a tensile load

340 An initial validation proposed consists of studying the strand under a tensile strain. For  
 341 this purpose, an elongation of 0.1 % is applied to the strand. Figure 10a shows the strand's  
 342 linear behaviour in tension. The stiffness results of the proposed models are compared with  
 343 those of the theory developed by [1] and [4] with a relative difference of less than 2 %. The  
 344 helical geometry of the outer layer implies a tension-torsion coupling, see Figure 10b. A  
 345 comparison with results from the analytical models detailed in [3] and [4] shows a small  
 346 relative difference of less than 2 %. Numerical models composed of beam elements seem to  
 347 be slightly stiffer than the numerical model composed of solid elements.

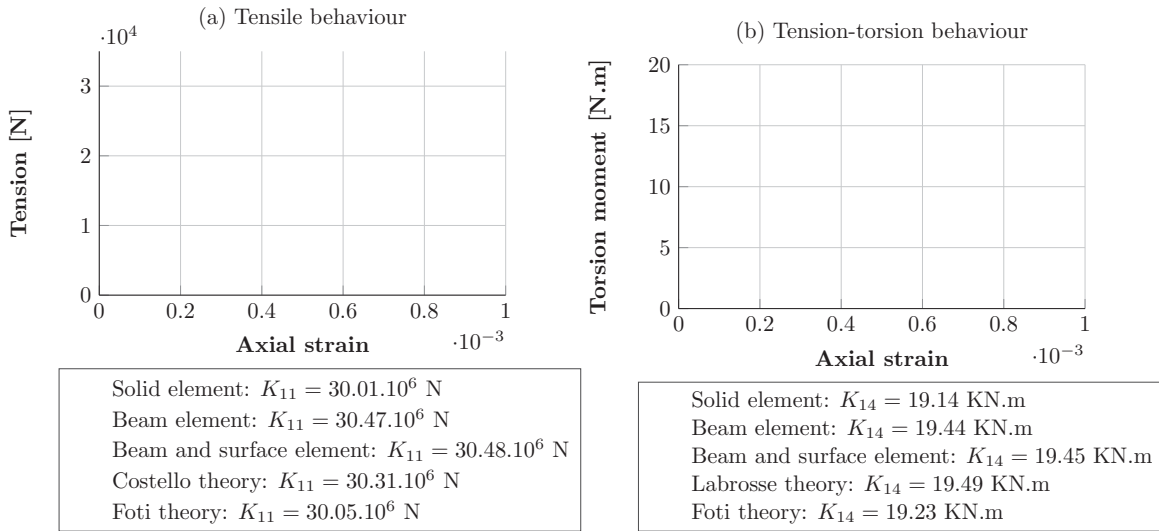


Figure 10: Single-layer strand behaviour under a tensile strain

348 The small difference in stiffness observed between numerical and analytical results can  
 349 be explained by the contact problem solution. The influence of the friction coefficient during  
 350 a tensile loading is negligible because small or no slip is present between strand components  
 351 [44]. Only normal contact plays an important role in the tensile case. Contact forces  
 352 summed over the contact line between the core and a layer wire are in good agreement with  
 353 the theory developed in [5] and [14], with a relative difference of less than 2 %, see Figure  
 354 11a. This comparison validates application of the penalty method to solve normal contact  
 355 problems for models with solid and beam elements. Use of the augmented Lagrangian

356 method, coupled with the penalty method, also yields good results for the model featuring a  
 357 beam with surface elements. The line load distribution in the normal direction is represented  
 358 in Figure 11b. The contact surface discretisation with solid elements involves a periodic  
 359 distribution of the contact forces. In the presence of a node-to-node contact, the contact  
 360 force is indeed maximised, whereas when a node comes into contact with a surface, the  
 361 contact force is minimised. Hence, the overall strand stiffness is reduced due to this variation  
 362 along the contact line. A finer discretisation is necessary to approximate a contact line,  
 363 therefore increasing the computation time [9]. The contact line load distribution is better  
 364 approximated with beam element models.

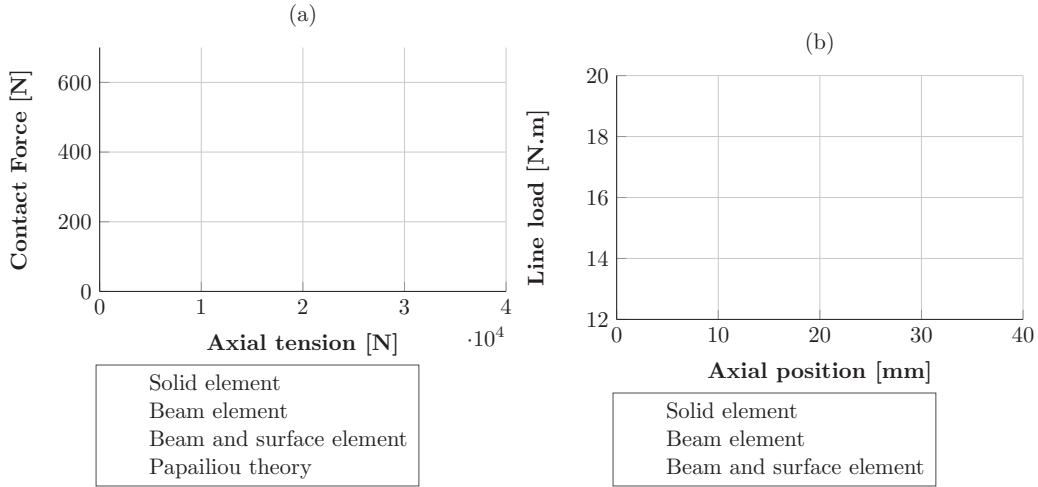


Figure 11: (a) Evolution in contact forces summed over the strand length between the core and a layer wire vs. axial tension, and (b) line load distribution between the core and a layer wire

365 The axial stress field, obtained in the central section of each numerical model, is displayed  
 366 in Figure 12. As observed for the model with solid elements, axial stress concentrations result  
 367 from contact. For models with beam elements, this phenomenon has not been captured since  
 368 the axial stress is determined at each integration node of the section from beam theory. The  
 369 maximum axial stress occurs at the central core, while the minimum stress is located on the  
 370 outer layer wires. These wires work in bending, which therefore reduces their contribution to  
 371 axial stiffness. Table 2 shows good agreement between the numerical models and Costello's  
 372 analytical model. Costello's model is based on curved beams and is thus closer to the beam  
 373 finite element model than to the solid element model.

Table 2: Layer wire axial stress of a single-layer strand subjected to a tensile strain:  $E^E = 0.001$ .

Model	$\sigma_{33max}$ [MPa]	$\sigma_{33min}$ [MPa]
Solid	204.9	185.0
Beam	207.8	203.7
Beam with surface	207.7	203.7
Costello	207.1	197.2

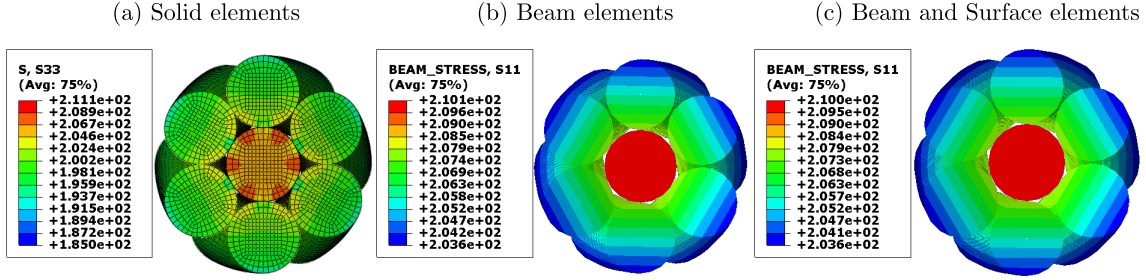


Figure 12: Axial stress field  $\sigma_{33}$  at  $y_3 = 0$  for the single-layer strand subjected to a tensile strain:  $E^E = 0.001$

### 374 4.3. Single-layer strand under a torsional load

375 A torsion angle of  $0.1 \text{ rad.m}^{-1}$  is now imposed through the periodic boundary conditions.  
 376 Figure 13 shows the linear behaviour in strand torsion. The numerical models are slightly  
 377 stiffer in torsion than the models proposed by [1] and [4], with a relative difference of less  
 378 than 4 %. A verification conducted on the torsion-tension coupling indicates that numerical  
 379 results restore the stiffness matrix symmetry, i.e.  $K_{14} = K_{41}$ .

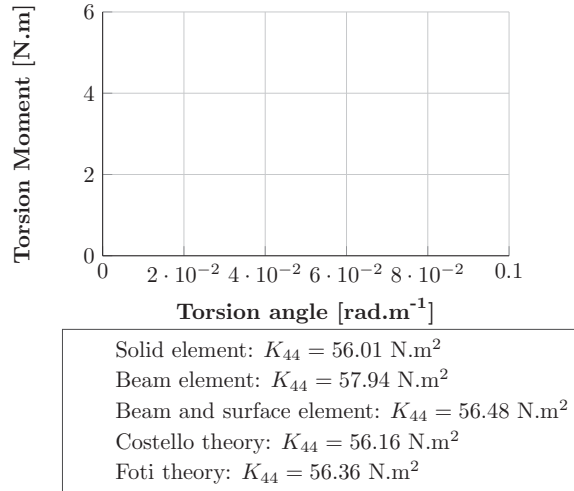


Figure 13: Single-layer strand behaviour under a torsional load

380 Figure 14 depicts the axial stress field obtained on the central section for a torsional  
 381 strain. Due to the helical geometry of the outer layer, the wires are subjected to tension,  
 382 bending and torsion. The maximum axial stress occurs in the outer fibre of each wire in  
 383 the radial direction. The axial stress in the core equals zero since the core is not subjected  
 384 to any elongation. In this configuration, the contact pressure exerted by the outer layer on  
 385 the core is negligible. The maximum and minimum stress values are listed in Table 3. The  
 386 lower stress values for the model with solid elements result from a more flexible torsion-  
 387 tension behaviour. As regards the models with beam elements, numerical models tend to  
 388 approximate the analytical model of [1] by relying on curved beams. The shear stresses for

389 the central section are shown in Figure 15. The maximum and minimum shear stress values  
 390 are located on the central core, which has the largest radius. A comparison of the maximum  
 391 and minimum values of the numerical models and Costello's theory is drawn in Table 4,  
 392 which reveals a good level of agreement.

Table 3: Layer wire axial stress of a single-layer strand subjected to a torsional strain:  $E^T = 0.1 \text{ rad.m}^{-1}$

Model	$\sigma_{33max}$ [MPa]	$\sigma_{33min}$ [MPa]
Solid	23.47	5.40
Beam	30.48	0.60
Beam and surface	30.38	0.01
Costello	30.20	0.00

Table 4: Layer wire shear stress of a single-layer strand subjected to a torsional strain:  $E^T = 0.1 \text{ rad.m}^{-1}$

Model	$\sigma_{13max}$ [MPa]	$\sigma_{13min}$ [MPa]
Solid	23.58	-23.58
Beam	20.10	-20.10
Beam and surface	20.16	-20.16
Costello	20.28	-20.28

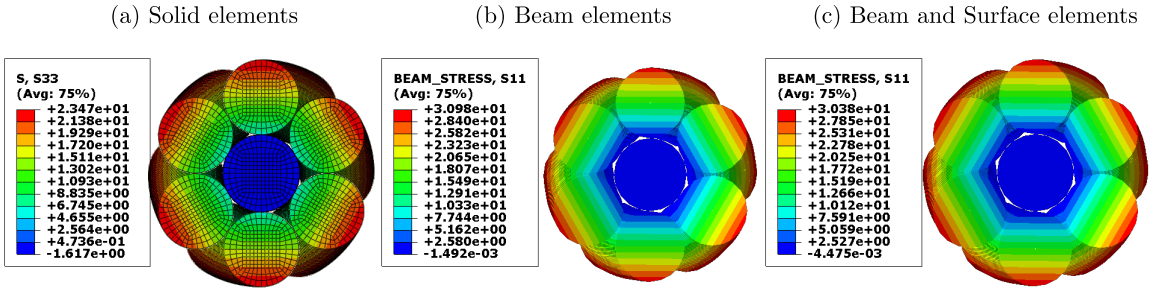


Figure 14: Axial stress field  $\sigma_{33}$  at  $y_3 = 0$  for the single-layer strand subjected to a torsional strain:  $E^T = 0.1 \text{ rad.m}^{-1}$ .

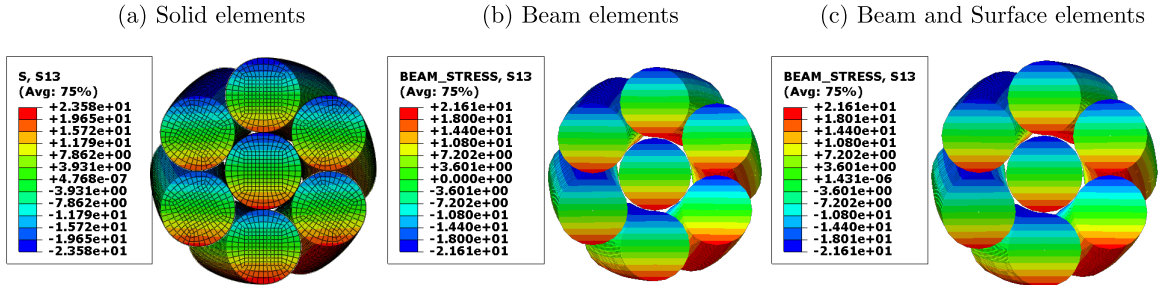


Figure 15: Shear stress field  $\sigma_{13}$  at  $y_3 = 0$  for the single-layer strand subjected to a torsional strain:  $E^T = 0.1 \text{ rad.m}^{-1}$ .

393 4.4. Single-layer strand under a bending load

394 Let's now consider a pure bending load. A curvature of  $0.1 \text{ m}^{-1}$  is applied to the strand  
 395 around the  $\vec{y}_1$ -axis. The behaviour obtained for the numerical models is compared with  
 396 Papailiou's theory, see Figure 16, with a relative difference of less than 0.5 %. In the case  
 397 of pure bending, normal contact stresses are negligible, and each layer wire is free to slip  
 398 [8]. Due to the very low frictional stresses, the influence of the friction coefficient is also  
 399 negligible. Strand behaviour in pure bending is therefore linear and bending stiffness is  
 400 minimal. Figure 17 shows that the longitudinal sliding of layer wires is in good agreement  
 401 with the equation of a loxodromic curve, see Eq.(5).

402 The axial stress field, see Figure 18, indicates that each wire is in pure bending. As  
 403 demonstrated in [8], when the cable is subjected to pure bending, all the wires are also  
 404 subjected to pure bending. Maximum stress occurs in the core by virtue of having the  
 405 largest radius. The maximum and minimum stresses in a layer wire are listed in Table 5.  
 406 Once again, the numerical values are very similar to those from the theory proposed by  
 407 Papailiou [5].

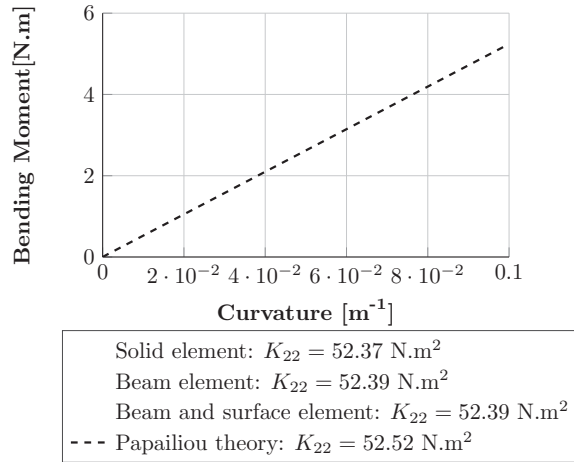


Figure 16: Single-layer strand behaviour in bending subjected to a curvature  $E^{F_1} = 0.1 \text{ m}^{-1}$

Table 5: Layer wire axial stress of a single-layer strand subjected to pure bending:  $E^{F_1} = 0.1 \text{ m}^{-1}$

Model	$\sigma_{33max}$ [MPa]	$\sigma_{33min}$ [MPa]
Solid	51.97	-51.97
Beam	53.71	-53.70
Beam with surface	53.69	-53.11
Papailiou	53.84	-53.84

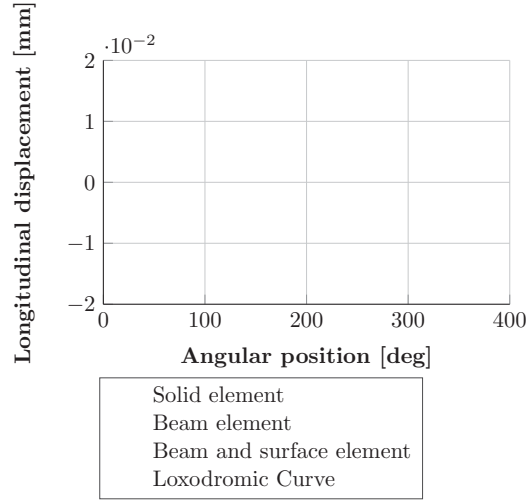


Figure 17: Longitudinal slip along  $\vec{y}_3$  of a layer wire for a curvature  $E^{F_1} = 0.1 \text{ m}^{-1}$ .

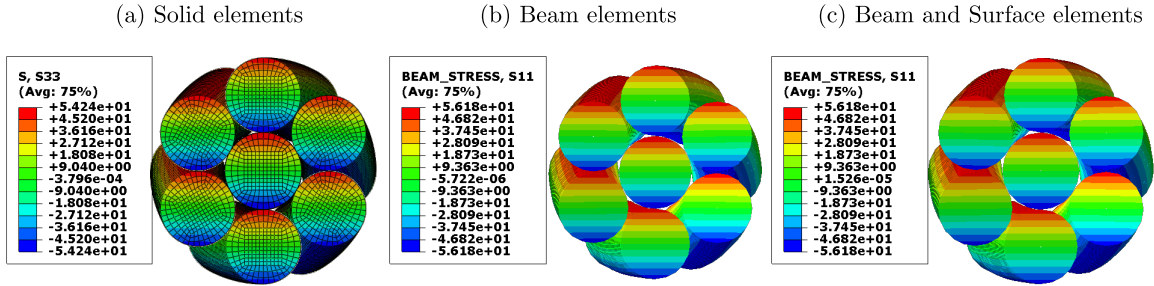


Figure 18: Axial stress field  $\sigma_{33}$  at  $y_3 = 0$  for the single-layer strand subjected to pure bending:  $E^{F_1} = 0.1 \text{ m}^{-1}$

408 4.5. Single-layer strand under a bending load with an initial tensile load

409 To highlight contact nonlinearity, a study of the strand subjected to a tension-bending  
 410 loading will now be presented. The strand is initially loaded in tension by an elongation of  
 411 0.1%, then a curvature of  $0.1 \text{ m}^{-1}$  is applied around the  $\vec{y}_1$ -axis.

412 The overall behaviour obtained for each numerical model is presented in Figure 19. This  
 413 behaviour can be separated into several regions, depending on curvature intensity. Initially,  
 414 the bending stiffness is maximised because all strand components are maintained by the  
 415 normal contact forces produced by tensile loading. In this case, the layer wires and core  
 416 behave as a single solid, and the cross-section undergoes overall bending, see Figure 20. The  
 417 bending stiffnesses obtained by numerical models in the stick state are lower than the bending  
 418 stiffness computed by analytical models, see Table 6. This finding is explained by the fact  
 419 that analytical models are based on a perfect geometry and assumed to contain perfectly  
 420 bonded components at the initial state, in excluding any sliding. On the other hand, small  
 421 slips are always present in numerical models that tend to decrease strand stiffness in the stick

422 state. To more closely approximate analytical models, it would be necessary to refine the  
 423 mesh of the contact areas, see [9], which would greatly increase computation time, especially  
 424 for solid elements. One can also check that the discrepancy between solid element model  
 425 and analytical solution reported in Table 6 is close to that obtained in [9]. From a critical  
 426 curvature, i.e. around  $0.015 \text{ m}^{-1}$ , layer wires close to the bending axis start to slip. Each  
 427 wire then undergoes pure bending, and bending stiffness decreases to its minimum value,  
 428 which corresponds to that obtained in the previous section, see Figure 21.

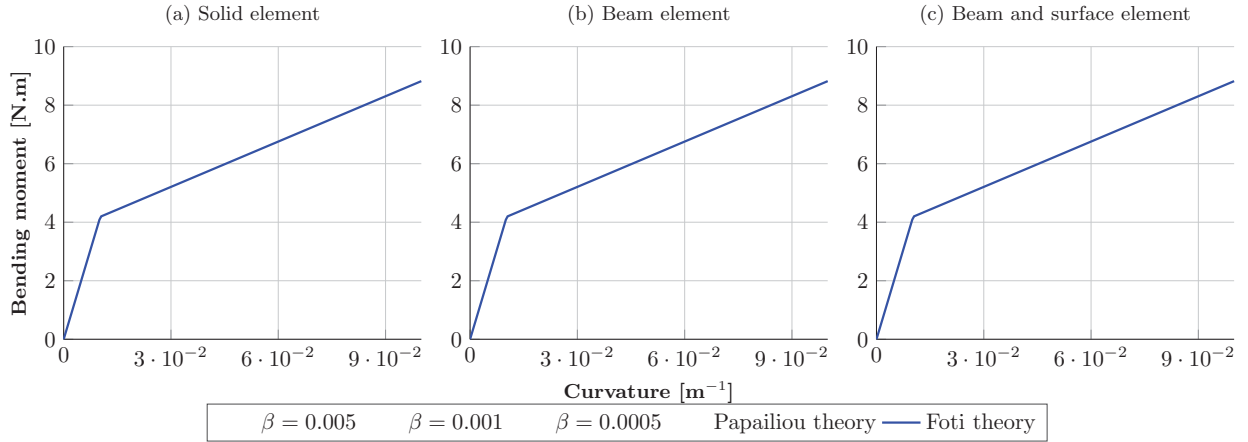


Figure 19: Single-layer strand behaviour in bending with an initial tensile load for several values of  $\beta$ .

Table 6: Single-layer strand bending stiffness under bending with an initial tensile load for both the stick and slip states, with  $\beta = 0.0005$ .

Model	$K_{22}$ stick state [ $\text{N.m}^2$ ]	$K_{22}$ slip state [ $\text{N.m}^2$ ]
Solid	320.55	52.30
Beam	411.00	51.83
Beam and surface	378.58	52.99
Papailiou	409.43	52.52
Foti	408.54	51.63

429 A strong influence from the numerical coefficient  $\beta$  on numerical behaviour is obtained.  
 430 With the default value of  $\beta$  given in ABAQUS, i.e.  $\beta = 0.005$ , stick state behaviour is poorly  
 431 reproduced. It is therefore necessary to define the lowest possible coefficient  $\beta$  to approximate  
 432 Coulomb's Law while ensuring solution convergence. In the present case, convergence of the  
 433 moment-curvature curve is obtained for  $\beta = 0.0005$ .

434 For the stick behaviour, the axial stress distribution is shown in Figure 20. As explained  
 435 previously for the beam model, axial stress varies linearly across the cross-section, while for  
 436 a model with solid elements, the effect of contact forces can be observed. Stress values in  
 437 the outer layer of the numerical models are listed in Table 7 and compared with Papailiou's

438 theoretical output in the stick state. Given that some slip occurs in the numerical models,  
 439 the maximum stress values are slightly lower than those of the analytical model.

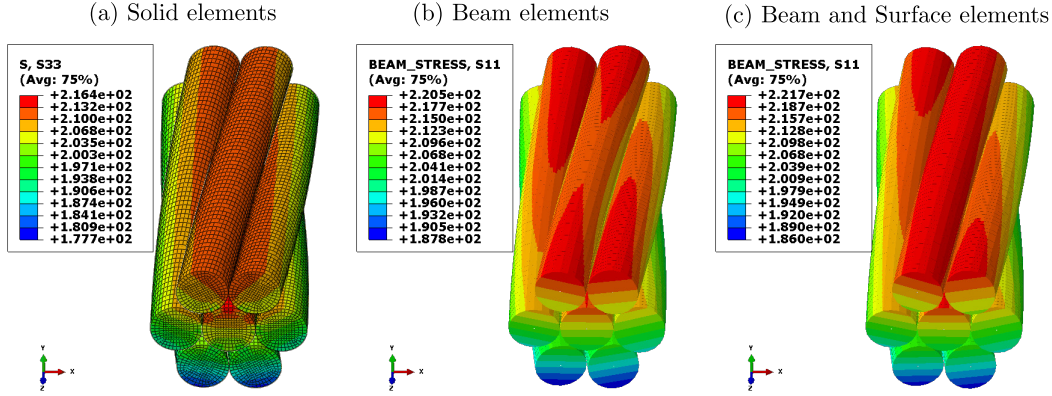


Figure 20: Axial stress field  $\sigma_{33}$  for the single-layer strand under bending with initial tensile load:  $E^E = 0.001$  and  $E^{F1} = 0.00875 \text{ m}^{-1}$ , in the stick state

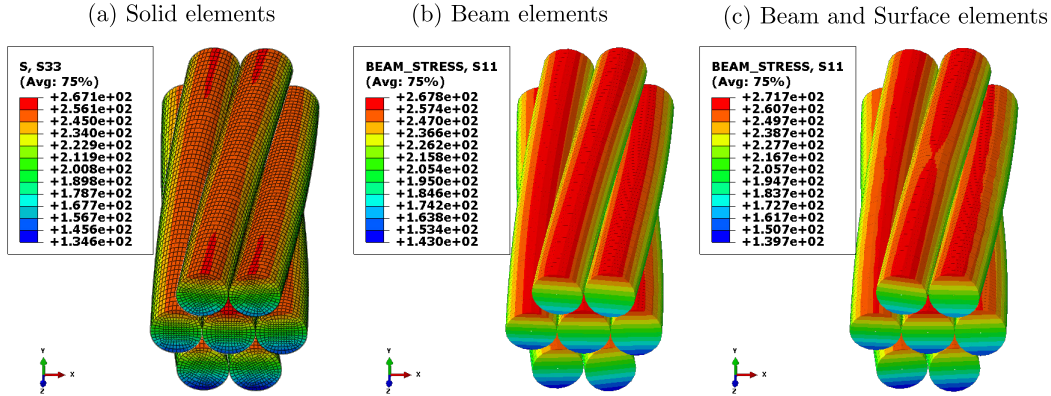


Figure 21: Axial stress field  $\sigma_{33}$  for the single-layer strand under bending with an initial tensile load:  $E^E = 0.001$  and  $E^{F1} = 0.1 \text{ m}^{-1}$ , in the slip state

440 For sliding behaviour, as observed in Figure 21, each wire undergoes pure bending. The  
 441 maximum stress is located on the strand core due to its larger radius. The numerical values  
 442 are listed in Table 8 and compared to the analytical results stemming from Papailiou's  
 443 theory, in showing a good level of agreement.

444 The computational efficiency of each numerical model is presented in Table 9. For the  
 445 single-layer strand, the beam model significantly reduces model size and computation time  
 446 by a factor of 30, in comparison with the solid element model. Beam elements associated  
 447 with surface elements slightly improve contact modelling and reduce computation time by  
 448 a factor of 16, in comparison with the solid element mesh. The beam-to-beam contact  
 449 solution seems to be more efficient than a beam with surface elements in solving the line  
 450 contact problem.

Table 7: Layer wire axial stress of a single-layer strand under bending with an initial tensile load:  $E^E = 0.001$  and  $E^{F1} = 0.00875 \text{ m}^{-1}$  in the stick state

Model	$\sigma_{33max}$ [MPa]	$\sigma_{33min}$ [MPa]
Solid	213.1	177.7
Beam	220.4	187.8
Beam and surface	221.7	186.0
Papailiou	221.4	192.7

Table 8: Layer wire axial stress of a single-layer strand under bending with an initial tensile load:  $E^E = 0.001$  and  $E^{F1} = 0.1 \text{ m}^{-1}$  in the slip state

Model	$\sigma_{33max}$ [MPa]	$\sigma_{33min}$ [MPa]
Solid	257.3	134.6
Beam	267.8	143.0
Beam and surface	268.2	139.7
Papailiou	276.1	139.6

Table 9: Computation time comparison between finite element models for a tensile and bending loading (i7-6700HQ CPU 2,60 GHz with 8 Go Ram)

Model	CPU time [s]
Solid	1395
Beam	46.4
Beam with surface	89

## 451 5. Multi-layer strand validation

452 Let's now consider a simple multi-layer strand, composed of 2 helical wire layers and  
 453 a cylindrical circular core. Two contact types are present, in the form of a contact line  
 454 between the core and the first layer, while contact points are also present between the first  
 455 and second layers. The axial strand behaviour is analysed through a tensile loading, and  
 456 contact nonlinearities will be highlighted through a loading in both tension and bending.

### 457 5.1. Case study

458 The strand currently studied is composed of 19 steel wires, including 12 on the outer  
 459 layer, 6 on the inner layer and one central wire for the core. The geometry used and material  
 460 properties are presented in Table 10.

Table 10: Geometric and material properties of the multi-layer strand

Layer	$R$ [mm]	$\alpha$ [deg]	$E$ [GPa]	$\nu$
Core	2.675	-	210	0.3
1	2.590	-8.24	210	0.3
2	2.590	8.18	210	0.3

461 The multi-layer strand chosen here satisfies the two geometric conditions, see Eq. (22),  
 462 making it possible to neglect the lateral contact between wires of the same layer, i.e.:

- 463 • For the inner layer:  $\xi_0 = 0.97 < 1$  and  $\alpha_{max} = 11.86^\circ$ .
- 464 • For the outer layer:  $\xi_0 = 0.33 < 1$  and  $\alpha < \alpha_{max} = 17.20^\circ$ .

465 Therefore, only the radial contact between the core and the inner layer and between the  
 466 inner layer and the outer layer is taken into account. The former is established along a  
 467 contact line while the latter is a series of contact points. A friction coefficient  $\mu = 0.3$  is  
 468 chosen for all contact interactions.

469 The various meshes are shown in Figure 22. Using the same wire geometric properties  
 470 than the single layer strand, 60-element axial discretisation is selected for each model pre-  
 471 sented here. By taking into account the rigid body equations connecting beam elements to  
 472 surface elements, the number of independent degrees of freedom for both beam models is  
 473 the same.

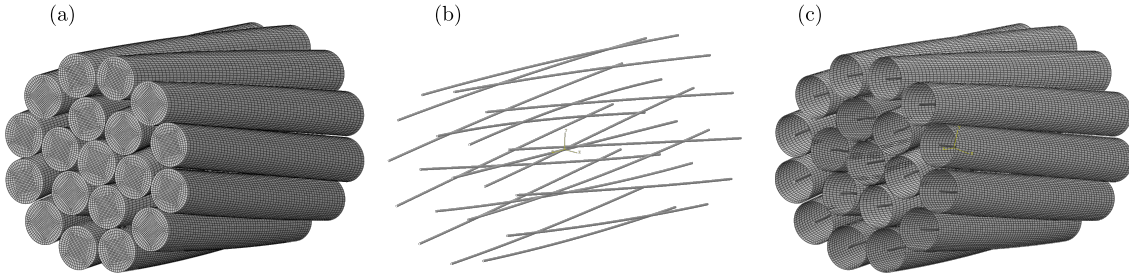


Figure 22: Display of the various meshes of the multi-layer strand: (a) solid element model composed of 520,391 nodes and 478,800 C3D8 elements, (b) beam element model with 1,159 nodes and 1,140 B31 elements, and (c) beams with surface elements model containing 70,699 nodes, and 1,140 B31 and 68,400 SFM3D4 elements

## 474 5.2. Multi-layer strand under a tensile load

475 For the first analysis, an axial deformation of 0.1% is applied to the strand. A comparison  
 476 of numerical simulations with analytical models from [1] and [4] reveals a very good level  
 477 of agreement, see Table 11. It can be noticed that despite opposite lay angles for the inner  
 478 and outer layers, the larger number of wires in the outer layer produces a tension-torsion  
 479 coupling.

Table 11: Tension and tension-torsion stiffness from the multi-layer strand at  $E^E = 0.001$

Model	$K_{11}$ [ $10^7$ N]	$K_{14}$ [KN.m]
Solid	7.97	56.25
Beam	8.11	56.97
Beam and surface	8.12	57.05
Costello	8.16	57.71

480 For a tensile strain, only normal contact influences strand behaviour. As for the single-  
 481 layer strand, see 11a, a very good level of agreement has been obtained with Papailiou's  
 482 model for contact forces. The contact pressure generated by the outer layer on the inner  
 483 layer greatly influences contact pressure between the latter and the core. However, the line  
 484 load distribution differs from one model to another, see Figures 23 and 24. The contact force  
 485 distribution between the core and the inner layer depends on the contact points between  
 486 the inner and outer layers. For models with beam elements, the maximum values of normal  
 487 contact forces between the core and inner layer are in good agreement with the axial con-  
 488 tact positions between the inner and outer layers. For the model with solid elements, this  
 489 observation is less obvious. As in the case of the single-layer strand, the contact resolution  
 490 for a model with solid elements will strongly depend on the mesh. Therefore, one of the  
 491 advantages of 3D models, i.e. the ability to take contact forces into account for stress com-  
 492 putations, is offset by the requirement of a very fine mesh for obtaining accurate stresses in  
 493 the vicinity of contact zones.

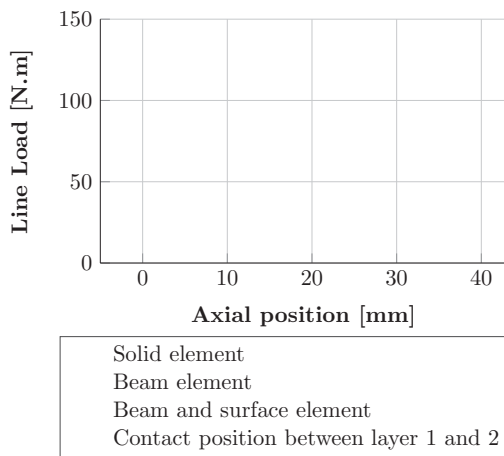


Figure 23: Line load distribution between the inner layer wire and the core

494 The axial stress field for a tensile loading is shown in Figure 24 for the inner layer and  
 495 in Figure 25 for the outer layer. Each model reveals the impact of a contact point on the  
 496 axial stress distribution within the outer layer. When the outer layer wire is not in contact  
 497 with an inner layer wire, the wire section works in bending, with a maximum stress at the  
 498 lower fibre of the wire in the radial direction. Conversely, when the outer layer wire is in  
 499 contact with an inner layer wire, the wire is still subjected to a bending moment, such that  
 500 the maximum stress lies on the upper fibre of the wire in the radial direction. Therefore,  
 501 the outer layer wire section undergoes local bending with a moment whose sign changes as  
 502 a function of the contact position between the two layers.

503 The minimum and maximum axial stress values are listed in Table 12. In the outer  
 504 layer, the maximum stress values obtained from numerical models are consistent with one  
 505 another. Significant differences appear between solid element and beam element results with  
 506 respect to minimum stress values. These differences can also be seen for the minimum and

507 maximum stresses in the inner layer. In this inner layer, the stress distribution is similar  
 508 to that observed for the single-layer strand, with higher stress variations in the contact  
 509 areas between the outer and inner layers. For the maximum stress value in the outer layer,  
 510 numerical results from the various models are again consistent. The minimum stress value  
 511 in the outer layer differs between the solid element models and beam element models. As  
 512 noted earlier, this difference stems mainly from the capability of the solid element model to  
 513 account for the local stress concentrations due to contact. Compressive stresses can also be  
 514 locally observed in elements making contact with the inner layer in the solid element model.

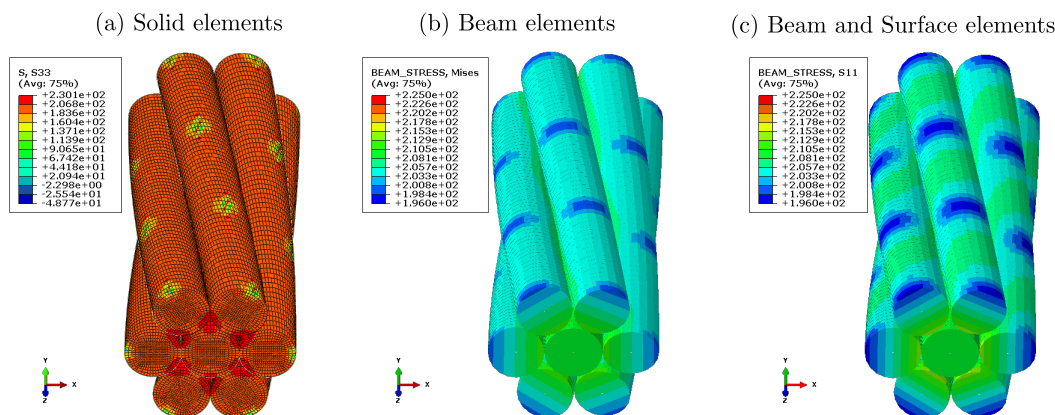


Figure 24: Axial stress field  $\sigma_{33}$  in the inner layer and the core for the multi-layer strand under a tensile load:  $E^E = 0.001$ , thus highlighting contact position between the inner and outer layers

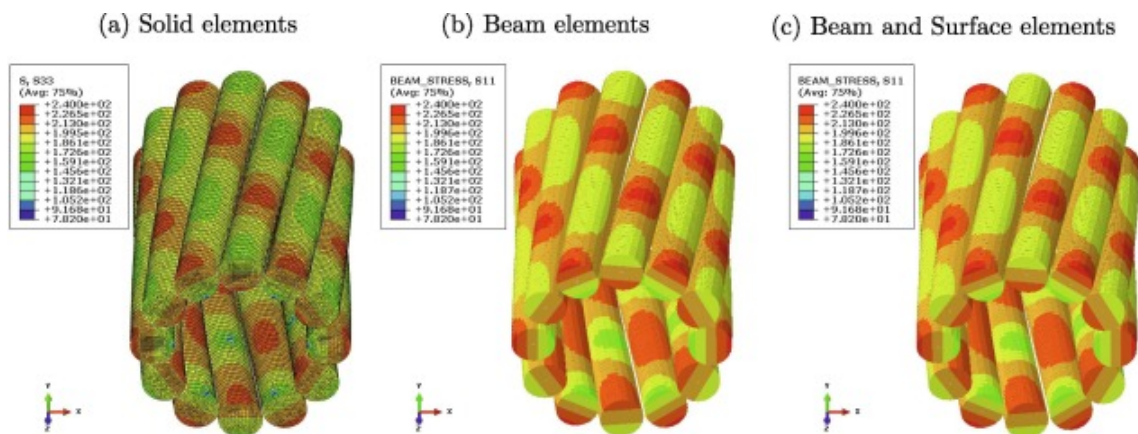


Figure 25: Axial stress field  $\sigma_{33}$  in the outer layer for the multi-layer strand under a tensile load:  $E^E = 0.001$

515 Axial stress values given by Costello's model are given in Table 12 for information pur-  
 516 poses. Although the beam model results lie close to Costello's theory for the inner layer, a  
 517 departure can be noticed in the outer layer; this discrepancy originates from contact points,  
 518 which are not taken into account in Costello's model.

Table 12: Layer wire axial stress of a multi-layer strand subjected to a tensile strain:  $E^E = 0.001$ . (\*) Contact line assumption between inner and outer layer.

Model	Outer layer		Inner layer	
	$\sigma_{33\max}$ [MPa]	$\sigma_{33\min}$ [MPa]	$\sigma_{33\max}$ [MPa]	$\sigma_{33\min}$ [MPa]
Solid	233.0	78.2	230.1	-48.8
Beam	234.5	175.3	211.5	199.8
Beam and surface	232.9	177.6	214.0	197.2
Costello*	205.1	201.0	207.1	197.6

519 *5.3. Multi-layer strand under a bending load with an initial tensile load*

520 The strand is now subjected to both a tensile and bending strain. The contacts are  
 521 initialised by an axial strain of 0.1%; next, a curvature of  $0.1 \text{ m}^{-1}$  is applied to the structure  
 522 around the  $\bar{y}_1$ -axis.

523 The bending behaviour is presented in Figure 26. Three phases can be identified: first  
 524 the stick state, then slipping of the outer layer, and lastly slipping of the inner layer. Ac-  
 525 cording to Papailiou’s theory [5], the critical curvature of sliding is defined by a mean critical  
 526 curvature for each wire layer. The analytical bending moment is therefore piecewise linear.  
 527 For numerical models, wire sliding is evaluated throughout the simulation, which leads to  
 528 smoothing the behaviour.

529 The influence of coefficient  $\beta$  can once again be noticed. For low values of  $\beta$ , numerical  
 530 behaviour tends to be stiffer in the stick state, approaching the maximum stiffness estimated  
 531 by Papailiou’s theory, see Table 13. These results converge when  $\beta$  reaches a value of 0.0005.  
 532 The various contact models also influence bending behaviour. The results obtained from  
 533 both the beam-to-beam contact model and the beam with surface elements model lie close  
 534 to the analytical values. As in the case of the single-layer strand, the discrepancy between  
 535 solid element model and analytical solution is large. However, the numerical models are in  
 536 good agreement with the bending stiffness in the total sliding state, see Table 13.

537 A cyclic loading in bending, with an amplitude of  $0.2 \text{ m}^{-1}$ , is proposed in Figure 27,  
 538 thereby highlighting the hysteresis cycle of the strand. The difference between analytical and  
 539 numerical results tends to increase during both the discharge and second charge phases. A  
 540 good level of agreement can be observed between the finite element results and the analytical  
 541 model.

Table 13: Multi-layer strand bending stiffness under bending with an initial tensile load for the stick and slip state with  $\beta = 0.0005$ .

Model	$K_{22}$ stick state [N.m <sup>2</sup> ]	$K_{22}$ slip state [N.m <sup>2</sup> ]
Solid	1738.4	139.5
Beam	3366.7	148.9
Beam and surface	2912.9	140.3
Papailiou	3306.8	140.8

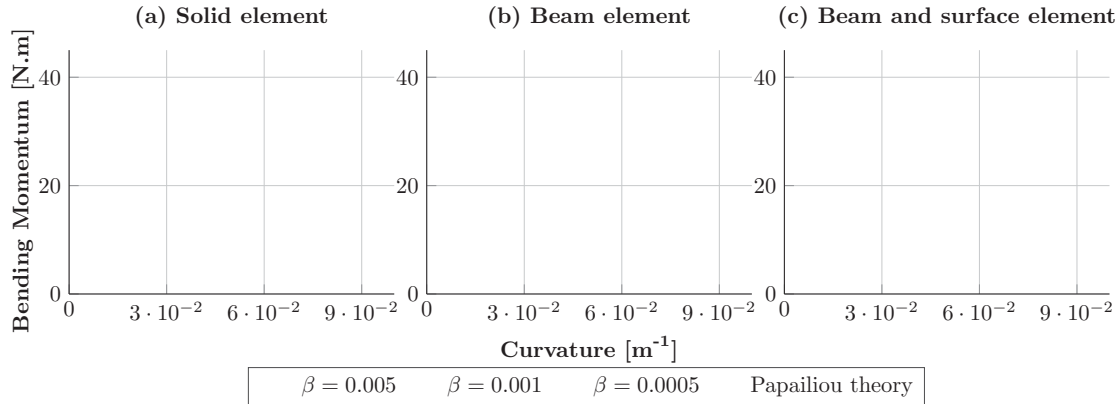


Figure 26: Multi-layer strand behaviour in bending with an initial tensile load for several values of  $\beta$ .

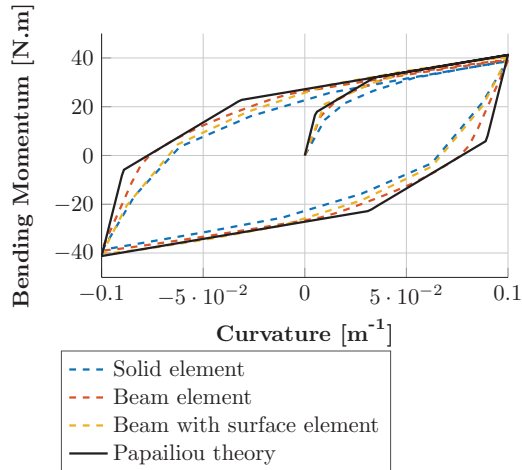


Figure 27: Hysteresis strand behaviour with a curvature amplitude of  $0.2 \text{ m}^{-1}$ : Comparison between the numerical model with  $\beta = 0.005$  and Papailiou's theory

542 Figure 29 shows the axial stress field in the various layers for a sliding behaviour. As in  
 543 the case of the single-layer strand, it can be remarked that all strand wires are subjected to  
 544 pure bending. The variation between maximum and minimum stresses therefore increases  
 545 with a sliding behaviour, see Table 15. The two beam element models output the same  
 546 results. As opposed to beam element models, the axial stresses with the solid element model  
 547 are locally greater for a contact position in the inner layer.

548 The axial stress values obtained by Papailiou's theory are given for information in Tables  
 549 14 and 15. The beam models agree with the theory for stresses within the inner layer.  
 550 However, Papailiou's theory, which does not consider contact points, does underestimate  
 551 the axial stresses in the outer layer.

552 As indicated in Table 16, the computation time associated with beam models is very low  
 553 compared to that of solid element models. In comparison with the beam-to-beam contact

554 model, use of the beam with surface elements model reduces computation time by a factor  
 555 of 5 and seems to be more efficient in solving the contact point problem.

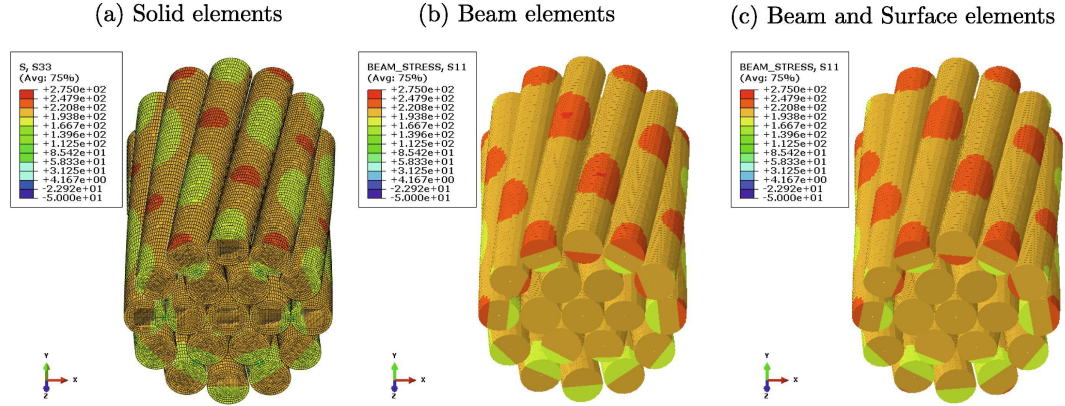


Figure 28: Axial stress field  $\sigma_{33}$  for the multi-layer strand under bending with an initial tensile load:  $E^E = 0.001$  and  $E^{F_1} = 0.005 \text{ m}^{-1}$ , in the stick state

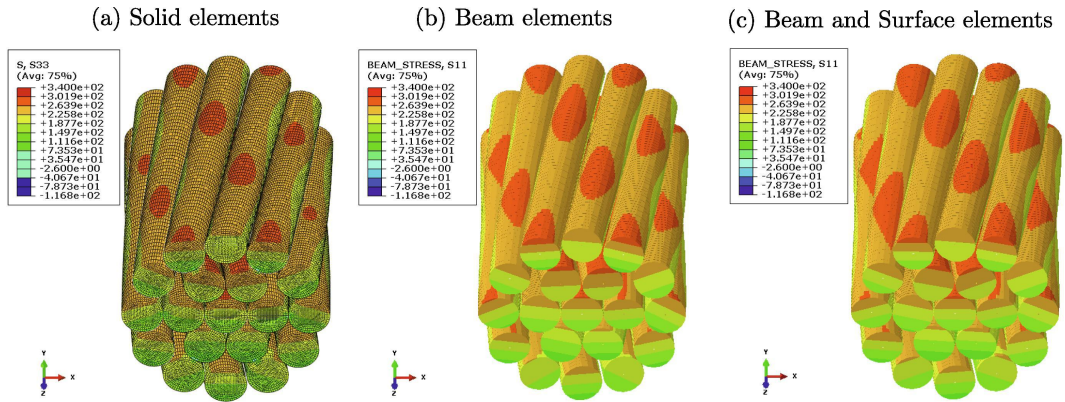


Figure 29: Axial stress field  $\sigma_{33}$  for the multi-layer strand under bending with an initial tensile load:  $E^E = 0.001$  and  $E^{F_1} = 0.1 \text{ m}^{-1}$ , in the stick state

Table 14: Layer wire axial stress of a multi-layer strand under bending with an initial tensile load:  $E^E = 0.001$  and  $E^{F_1} = 0.005 \text{ m}^{-1}$ , in the stick state (\*) Contact line assumption between inner and outer layers.

Model	Outer layer		Inner layer	
	$\sigma_{33\text{max}}$ [MPa]	$\sigma_{33\text{min}}$ [MPa]	$\sigma_{33\text{max}}$ [MPa]	$\sigma_{33\text{min}}$ [MPa]
Solid	258.8	67.0	247.5	-48.9
Beam	248.6	169.7	214.7	201.8
Beam and surface	243.7	172.7	213.8	201.2
Papailiou*	221.2	194.5	215.9	199.8

Table 15: Layer wire axial stress of a multi-layer strand under bending with an initial tensile load:  $E^E = 0.001$  and  $E^{F1} = 0.1 \text{ m}^{-1}$ , in the slip state (\*) Contact line assumption between inner and outer layers.

Model	Outer layer		Inner layer	
	$\sigma_{33\text{max}}$ [MPa]	$\sigma_{33\text{min}}$ [MPa]	$\sigma_{33\text{max}}$ [MPa]	$\sigma_{33\text{min}}$ [MPa]
Solid	318.9	-71.3	338.0	-116.8
Beam	299.4	166.2	283.6	122.3
Beam and surface	300.9	162.6	287.0	112.3
Papailiou*	276.1	139.6	290.9	125.2

Table 16: Computation time comparison between finite element models for a tensile and bending load (8-core Intel Xeon (Haswell) E5-2680v3 CPUs 2.50GHz with 128 Go Ram)

Model	CPU time [min]
Solid	367
Beam	38
Beam and surface	7

## 556 6. Conclusion

557 This paper has proposed a new computational approach for studying the overall be-  
558 haviour and local stress state of strand-type structures. This method is based on the ho-  
559 mogenization theory of periodic structures, with the local problem posed on the strand axial  
560 period being solved using the finite element method. This approach fully utilises the strand’s  
561 helical symmetry, thus minimising the size of the computational domain. Consequently,  
562 accounting for geometric complexity and contact interactions, which are of paramount im-  
563 portance for bending loads, is more straightforward. The size of the numerical model can  
564 also be reduced thanks to the use of beam elements, and one objective of this paper has  
565 been to assess the accuracy of such a model, in comparison with solid element models and  
566 analytical results.

567 An initial validation case study was performed on a single-layer strand; it was shown  
568 that for each loading case, the various numerical models, with both solid elements and beam  
569 elements, reproduced the single-layer strand behaviour in close agreement with several an-  
570 alytical models. The three numerical models all led to very similar results. The differences  
571 stem mainly from the contact model, which differs from one model to another. For a model  
572 with solid elements, a finer mesh becomes necessary in the vicinity of contact zones in order  
573 to obtain accurate stresses. This configuration increases computation time, especially with  
574 sections containing a large number of wires. Beam models provide a very good compromise  
575 between accuracy and numerical efficiency. In addition, the use of a contact surface associ-  
576 ated with the beam elements appears to be more efficient than beam-to-beam contact. This  
577 formulation facilitates contact detection and improves solution convergence even in the case  
578 of large sliding. Nevertheless, beam modelling does not allow taking local contact forces into  
579 account during stress computations.

580 For the second example, a multi-layer strand was studied to show the influence of a second  
581 layer of wires on strand behaviour. In addition to the contact line between the core and the  
582 inner layer, a contact point distribution exists between the inner and outer layers. It has  
583 also been demonstrated that the overall behaviour in tension and tension-bending obtained  
584 by the analytical model can be reproduced by each numerical model. In contrast, regarding  
585 local axial stresses, a discrepancy was found between numerical and analytical results; the  
586 numerical models indeed suggest a significant influence of contact points between the inner  
587 and outer layers on the axial stress distribution. Stress concentrations occur at each contact  
588 location, thereby locally increasing the maximum stress value and decreasing the minimum  
589 stress value. In this case, analytical models prove to be inadequate since they fail to take  
590 contact points into account.

## 591 7. Acknowledgments

592 This work has received support from France Energies Marines, along with State subsidi-  
593 dies managed by the National Research Agency, as part of the Investments for the Future  
594 program, under the reference ANR-10-IEED-0006-28.

## References

- [1] G. Costello, *Theory of wire rope*, Springer-Verlag, 2nd edition, 1997.
- [2] S. Ghoreishi, *Modélisation analytique et caractérisation expérimentale du comportement de câbles synthétiques*, Ph.D. thesis, École Centrale de Nantes (France), 2005.
- [3] M. Labrosse, *Contribution à l'étude du rôle du frottement sur le comportement et la durée de vie des câbles monocouches.*, Ph.D. thesis, École Centrale de Nantes (France), 1998.
- [4] F. Foti, L. Martinelli, Modeling the axial-torsional response of metallic strands accounting for the deformability of the internal contact surfaces: Derivation of the symmetric stiffness matrix, *International Journal of Solids and Structures* 171 (2019) 30–46.
- [5] K. Papailiou, On the bending stiffness of transmission line conductors, *IEEE Transactions on Power Delivery* (1997) 1576–1588.
- [6] F. Foti, L. Martinelli, An analytical approach to model the hysteretic bending behavior of spiral strands, *Applied Mathematical Modelling* 40 (2016) 6451–6467.
- [7] F. Foti, A. de Luca di Roseto, Analytical and finite element modelling of the elastic–plastic behaviour of metallic strands under axial–torsional loads, *International Journal of Mechanical Sciences* 115–116 (2016) 202–214.
- [8] W. Jiang, A concise finite element model for pure bending analysis of simple wire strand, *International Journal of Mechanical Sciences* 54 (2012) 69–73.
- [9] D. Zhang, M. Ostojca-Starzewski, Finite Element Solutions to the Bending Stiffness of a Single-Layered Helically Wound Cable With Internal Friction, *Journal of Applied Mechanics* 83 (2016) 031003.
- [10] Y. Yu, Z. Chen, H. Liu, X. Wang, Finite element study of behavior and interface force conditions of seven-wire strand under axial and lateral loading, *Construction and Building Materials* 66 (2014) 10–18.
- [11] S. Kmet, E. Stanova, G. Fedorko, M. Fabian, J. Brodniansky, Experimental investigation and finite element analysis of a four-layered spiral strand bent over a curved support, *Engineering Structures* 57 (2013) 475–483.
- [12] R. Judge, Z. Yang, S. Jones, G. Beattie, Full 3D finite element modelling of spiral strand cables, *Construction and Building Materials* 35 (2012) 452–459.

- [13] J. Wu, The finite element modeling of spiral ropes, *International Journal of Coal Science & Technology* 1 (2014) 346–355.
- [14] R. Baumann, P. Novak, Efficient computation and experimental validation of ACSR overhead line conductors under tension and bending, *Cigre Science Engineering* 9 (2017) 5–7.
- [15] C. Yu, S. Yin, B. Dong, Y. Bao, The Analysis of 91-Wire Strand Tensile Behavior Using Beam Finite Element Model, *IOP Conference Series: Materials Science and Engineering* 381 (2018) 012115.
- [16] F. Bussolati, M. Guiton, Y. Poirette, M. Martinez, P. Guidault, O. Allix, A new fully-detailed finite element model of spiral strand wire ropes for fatigue life estimate of a mooring line, *International Conference on Ocean, Offshore and Arctic Engineering* 38 (2019) 10.
- [17] I. Páczelt, R. Beleznai, Nonlinear contact-theory for analysis of wire rope strand using high-order approximation in the FEM, *Computers & Structures* 89 (2011) 1004–1025.
- [18] S. Lalonde, R. Guilbault, F. Légeron, Modeling multilayered wire strands, a strategy based on 3D finite element beam-to-beam contacts - Part I: Model formulation and validation, *International Journal of Mechanical Sciences* 126 (2017) 281–296.
- [19] D. Durville, Contact-friction modeling within elastic beam assemblies: an application to knot tightening, *Computational Mechanics* 49 (2012) 687–707.
- [20] P. D. T.D. Vu, D. Durville, Finite element simulation of the mechanical behavior of synthetic braided ropes and validation on a tensile test, *International Journal of Solids and Structures* 58 (2015) 106–116.
- [21] G. Z. P. Wriggers, On contact between three-dimensional beams undergoing large deflections, *Communications in Numerical Methods in Engineering* 13 (1997) 429–438.
- [22] A. Frikha, P. Cartraud, F. Treysède, Mechanical modeling of helical structures accounting for translational invariance. Part 1: Static behavior, *International Journal of Solids and Structures* 50 (2013) 1373–1382.
- [23] N. Karathanasopoulos, G. Kress, Two dimensional modeling of helical structures, an application to simple strands, *Computers & Structures* 174 (2016) 79–84.
- [24] P. Cartraud, T. Messager, Computational homogenization of periodic beam-like structures, *International Journal of Solids and Structures* 43 (2006) 686–696.
- [25] E. Stanova, G. Fedorko, M. Fabian, S. Kmet, Computer modelling of wire strands and ropes Part I: Theory and computer implementation, *Advances in Engineering Software* 42 (2011) 305–315.
- [26] P. Wriggers, *Computational Contact Mechanics*, Springer, Berlin, Heidelberg, 2006.
- [27] F. Foti, L. Martinelli, Mechanical modeling of metallic strands subjected to tension, torsion and bending, *International Journal of Solids and Structures* 91 (2016) 1–17.
- [28] K. Feyrer, *Wire Ropes: Tension, Endurance, Reliability.*, Springer, Berlin, Heidelberg, 2007.
- [29] D. Boso, M. Lefik, B. Schrefler, A multilevel homogenised model for superconducting strand thermo-mechanics, *Cryogenics* 45 (2005) 259–271.
- [30] B. S. M. Kamiński, Probabilistic effective characteristics of cables for superconducting coils, *Computer Methods in Applied Mechanics and Engineering* 188 (2000) 1–16.
- [31] N. Buannic, P. Cartraud, Higher-order effective modeling of periodic heterogeneous beams. I. Asymptotic expansion method, *International Journal of Solids and Structures* (2001) 38:7139–7161.
- [32] A. Kolpakov, Calculation of the characteristics of thin elastic rods with a periodic structure, *Journal of Applied Mathematics and Mechanics* 55 (1991) 358–365.
- [33] J. Kim, M. Cho, E. Smith, An asymptotic analysis of composite beams with kinematically corrected end effects, *International Journal of Solids and Structures* 45 (2008) 1954–1977.
- [34] R. B. W.C. Young, *Roark’s Formulas for Stress and Strain*, McGraw-Hill, 7 edition, 2002.
- [35] M. Smith, *ABAQUS/Standard User’s Manual*, Version 6.14, Simulia, 2009.
- [36] K. S. A. Konyukhov, Geometrical covariant approach for contact between curves representing beam and cable type structures, *PAMM* 8 (2008) 10299 – 10300.
- [37] P. W. A. Gay Neto, P.M. Pimenta, Self-contact modeling on beams experiencing loop formation, *Comput. Mech.* 55 (2015) 193–208.
- [38] F. Bussolati, *Modèle multi-échelle de la fatigue des lignes d’ancrage câblées pour l’éolien offshore flottant*, Ph.D. thesis, Université Paris-Saclay (France), 2019.

- [39] J. Michel, H. Moulinec, P. Suquet, Effective properties of composite materials with periodic microstructure: a computational approach, *Computer Methods in Applied Mechanics and Engineering* 172 (1999) 109–143.
- [40] H. Magoaric, S. Bourgeois, O. Débordes, Elastic plastic shakedown of 3D periodic heterogeneous media: a direct numerical approach, *International Journal of Plasticity* 20 (2004) 1655–1675.
- [41] S. Yi, L. Xu, G. Cheng, Y. Cai, FEM formulation of homogenization method for effective properties of periodic heterogeneous beam and size effect of basic cell in thickness direction, *Computers & Structures* 156 (2015) 1–11.
- [42] N. Buannic, P. Cartraud, G. L’Hostis, Homogénéisation de structures ou matériaux constitués de poutres, *Colloque national en calcul des structures*, Giens, France (1999).
- [43] D. D. Tjahjanto, A. Tyrberg, J. Mullins, Bending Mechanics of Cable Cores and Fillers in a Dynamic Submarine Cable, volume 38, Trondheim, Norway.
- [44] W. Jiang, M. Warby, J. Henshall, Statically indeterminate contacts in axially loaded wire strand, *European Journal of Mechanics - A/Solids* 27 (2008) 69–78.

- Helical strands analysis is addressed rigorously with homogenization theory
- Strand response is non-linear due to contact interactions between its components
- Different models with solid and beam finite elements are used
- Numerical results are compared to those coming from analytical models