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STABILITY THRESHOLD OF A BRITTLE SANDSTONE SUBJECT TO RECURRENT FREEZE-THAW CYCLES

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Abstract

The paper introduces a specific non-destructive technique in use to detect the occurrence of geomaterial instability subsequently causing damage, and a specific data reduction procedure to assess damage accumulation. The damaging process was detected by analysing the signal evolution of ultrasonic pulses propagating through some sandstone specimens subject to recurrent freeze-thaw cycles under constant static loading. It is assumed that the evolution of non-linear energy increase corresponds to the process of the damage extent. An input-output non-parametric technique and a non-linear analyser for data reduction procedure were chosen to portray the non-linear aspects of ultrasound propagation characteristics. Based on a multi-dimensional Fourier transform, the non-linear analyser permits to separate linear and non-linear parts. It can be used to monitor non-destructively and continuously the overall alteration or damage process of a brittle rock specimen at initial damage caused by freezing and thawing, so that damage mechanism could be quantitatively estimated by a dimensionless parameter, the so-called non-linearity ratio. The experimental results showed that rock frost damage in the form of crack sets is directly related to the increase of non-linearity ratio. It reveals early microstructural damages that appear before occurrence of macroscopic rupture.

Keywords: brittle rock, freeze-thaw, monitoring, non-destructive testing, stability threshold, ultrasound propagation

1. Introduction

Rockfall has been a serious threat to highways, railways and residential areas in many mountainous areas of the world. It was estimated that 10% of the world's population live in mountainous terrain and 36% of the land area of the world are mountainous [1]. Actually, rockfall has been occurring regularly all over the world. In some mountainous areas, rockfall can occur frequently; for example, Gardner [2] reported a total of 563 rockfalls during only 842 hours of field observation in Lake Louis area in Canada. Actually most of the rockfall events were unreported, and only those failing on highways, resident areas or causing damages or casualties will capture public attention. On February 10, 1996, the huge rockfall, that crushed a tunnel along highway 229 on Hokkaido Island of Japan and killed 20 people, captures much international attention to rockfall problem [3].

Rockfalls results from numerous, complex and interacting causes that may act over widely vary-

-ing time frames. Hence, it should never be considered that rockfall must have been generated by just the most recent and apparent changes. The most frequently documented rockfall causes and, more generally, slope movement causes can be classified on a mechanical basis into predisposition factors (background factors such as topography, vegetation, lithological parameters, and fracture geometry and density), preparatory factors (medium- to long-term changes in resistance or disturbing forces) and trigger factors (the short-to-medium term).

The role of rockfalls in the total denudation of mountain slopes is significant in cold mountains where vegetation is sparse and frost action is active [4]. Several weathering processes may contribute to rockfall generation in cold mountains. A number of studies suggest that many rockfalls originate from thawing of bedrock fractured during diurnal and/or seasonal freezing [5, 6, 7], while others highlight rockwall instability resulting from thermal stress in polar desert areas [8] or paraglacial unloading in recently deglaciated valleys [9]. The concurrent observation of rockfall activity and associated climate variables has demonstrated that the freeze-thaw penetration like the preparatory and trigger factor controls the maximum dimensions of detachable rock mass, while joint spacing on the rockface affects the size distribution of rockfall debris [7].

The limited magnitude of preparatory factors and their cumulative nature represent an intrinsic difficulty with respect to the characterisation of related phenomena. Moreover, in practice, defining the amount of a given preparatory factor necessary for inducing a rockfall is almost impossible. For this reason, the monitoring of preparatory factors in order to predict slope collapse is not sufficient. On the other hand, using trigger factors does not yield any better results since such factors sometimes arise suddenly and just prior to the rockfall, so that a very long period of monitoring is needed before their detection. One solution may stem from other techniques that do not directly concern rockfall causes, but rather associated phenomena, such as the initiation, propagation or reactivation of cracks within the rock.

2. Existing techniques

The most common method of assessing rock deterioration caused by freezing and thawing is to measure the change in the flexural dynamic modulus of elasticity [10]. However, this technique is usually used in laboratory specimens. Acoustic emission, ultrasonic wave velocity can be used to detect freezing and thawing deterioration in rock [11, 12]. Acoustic emission is applied in a broad sense to the sounds which are internally generated in a material under stress. There is a wide variety of mechanisms known to be responsible for AE generation, ranging from dislocation motion to crack propagation. Therefore, the basic AE monitoring techniques are applicable to a wide class of materials and structures [13]. Unfortunately the interpretation of results still remains a delicate affair. This is due to the fact that the majority of AE events occur just prior to failure. The lack of significant AE activity in the initial stages of loading makes it more difficult to distinguish background noise from fracture related acoustic events. In this manner, the results show that ultrasonic wave velocity is not sensitive to early damage in rock caused by freezing and thawing; however, it is sensitive to significant damage caused at a later stage [14]. Modification of waveform during propagation through the material is a measure of the non-elastic components of deformation [15]. Attenuation may therefore be more sensitive than velocity to defects, but it is more complex to analyse [16]. In this paper, an input-output non-parametric procedure, based on ultrasonic pulse propagation and a non-linear analyser for data reduction, has been chosen to portray the stable behaviour of rock material subject to recurrent freeze-thaw cycles under constant static uniaxial loading. The experimental results evidence a threshold of change of its stable behaviour. This work describes a non-destructive technique based on ultrasound propagation characteristics to detect material instability threshold occurred in sandstone and a specific data reduction procedure to assess damage accumulation. It is assumed that the evolution of non-linear energy increase corresponds to the evolution of rock damage extent. The damaging process of the rock specimen has been detected by analysing the signal evolution of ultrasonic pulses propagating through the specimen which is subjected to freeze-thaw cycles under constant uniaxial load. A non-linear analyser based on a multidimensional Fourier transform permits to separate the linear and non-linear part of the mechanical response. The results, derived from non-linear characteristics of ultrasound propagation in some sandstone specimens in our laboratory, enable to evidence this threshold. On the other hand, signal evolution detected a significant difference at initial damage caused by freezing and thawing. It can be used to monitor non-destructively and continuously the overall alteration or damage process of sandstone so that damage mechanisms could be quantitatively estimated by a dimensionless parameter the so-called non-linearity ratio. This parameter may be used for an early detection of freezing and thawing deterioration in rock slopes.

3. Theoretical background

Frost damage results from complex coupled phenomena such as water flow behind the freezing front [17] and capillary pressures at the interconnections between nodal pores and capillaries [18, 19]. Damages occur primarily in nodal pores where the greatest increases of pressure are observed [18]. These damages generally take the form of microcrack networks. Thus, we can expect that frost durability of rock materials depends upon the resistance to tensional stresses induced by freezing of water in the porous media [20]. The final failure surfaces seem to result of a concentration of micro damages inside a non-zero thickness zone, which evolves to a macroscopic crack. Ultrasound propagation in a damaged rock material, unlike in an ideally perfect solid, is generally accompanied with attenuation and dispersion. Attenuation refers to the diminishing of wave intensity or wave amplitude as a wave propagates through a damaged medium, while dispersion refers to the shape distortion of a wave due to the frequency dependence of the effective wave (phase) velocity. Both attenuation and dispersion (shape velocity) are measurable quantities and the amount of change in attenuation and phase velocity can be correlated to the level of damage states. This feature is advantageously exploited in this ultrasonic non-destructive evaluation, relying on the mechanism of ultrasound propagation through rock materials. This provides a ready means for detecting and characterizing micro-cracking in damaged rock material in infrastructure. Techniques using ultrasonic waves are especially appealing because of the direct connection between the characteristics of the wave propagation and the damage states of a solid [21]. The present analysis only considers, in the framework of small perturbations, the propagation mechanism of a small wave through an elastic plastic solid presenting locally an unstable behaviour.

Within the theory of plasticity, several postulates that guarantee the mechanical stability of frictional materials have been suggested based on either energy or wave propagation considerations. Mechanical stability is taken as the capability of geomaterials to sustain a given stress state. For a non-associated plastic flow rule, localisation can also appear during hardening. This type of instability, referred to as pre-failure flow instability. The phenomenon is treated as a material instability in the framework of bifurcation into localised modes theory due to Hill [22], Mandel [23], and Rudnicki and Rice [24].

Note the instability of a material can occur when the propagation of a small perturbation in the form of stress wave in a certain direction is impossible [25]. Based on the assumption that a stable material is able to propagate a small perturbation in the form of wave, Mandel proposed a necessary and sufficient condition for stability. He showed [23] that a wave propagates in a material with an elastic plastic matrix A , along the direction α , if and only if all the eigenvalues λ of the matrix B are real and positive. This phenomenon is used as a highly sensitive manifestation of geomaterial instability [26], and the possible appearance of strain localisation, frictional sliding or microcracking along a certain direction.

3.1. Proposed method

Volterra analysis is used here to deal with unknown non-linear systems. Although attempts have been made to identify the governing equation of SDOF systems [27], it is more usual to identify high order transfer functions characterizing the non-linear behaviour of the mechanical system. Volterra has described a series decomposition to express the system dynamic response $y(t)$ as a function of excitation $x(t)$. The response $y(t)$ can be expanded in Volterra series in the following way:

$$y(t) = \sum_{i=1}^{\infty} y_i(t) = \sum_{i=1}^{\infty} \int_{-\infty}^{+\infty} \dots \int_{-\infty}^{+\infty} h_i(\tau_1, \dots, \tau_i) \prod_{k=1}^i x(t - \tau_k) d\tau_k \quad (1)$$

These series result from the summation of multidimensional convolution products between excitation and Volterra kernels $h_i(\tau_1, \dots, \tau_i)$. The term y_1 of the series corresponds to the linear component of the response, whereas the other terms correspond to the non-linear components [28, 29]. Working out the Volterra kernels allows the characterisation of the system by separating linear and non-linear parts of the dynamic response.

It should be noted that the kernels are purely dependent on the governing differential equation, i.e. the system itself, and thence independent on excitation. If the system is linear, high order kernels are constant and equal to zero. Activation of non-linear phenomena gives rise to non-zero values in these high order kernels. For a non-linear system, appearance of supplementary non-linearities (such as instability, micro-cracking, damage, etc.) increases the amplitude of original high order kernels.

The proposed method aims to evaluate the significance of non-linear terms in the global response. If these non-linear aspects exhibit a sudden increase, it will be associated to the

occurrence of material instability causing additional non-linear behaviour.

3.2. Non-linearity ratio

A simple quantitative parameter on which a criterion could be applied is necessary to characterise the system response. The determination of N kernels for each measurement point by numerical calculations gives access to the different terms of the series, and to their energies. In the following developments, the pre-subscript l will refer to the measurement point l . The energy ${}_l E_m^i$ of the i^{th} component of $y_i(t)$ for the m^{th} excitation level assumes the form:

$${}_l E_m^i = \int_0^\infty {}_l y_i^{(m)}(t)^2 dt \quad (2)$$

By comparing these energies, the non-linear phenomenon significance may be evaluated. To quantify the non-linear contributions in the global response, ${}_l r_m$ the non-linearity ratio at point l for the m^{th} excitation is defined as follows:

$${}_l r_m = \frac{\sum_{i=2}^N {}_l E_m^i}{\sum_{i=1}^N {}_l E_m^i} = \frac{\text{non-linear energy}}{\text{total energy}} \quad (3)$$

According to Eq. (2), Volterra kernels can be introduced in the former equation. When the excitation is an impulse with magnitude a_m accruing at instant T , the non-linearity ratio is then written as:

$${}_l r_m = \frac{a_m^4 {}_l \eta_2 + \dots + a_m^{2N} {}_l \eta_N}{a_m^2 {}_l \eta_1 + a_m^4 {}_l \eta_2 + \dots + a_m^{2N} {}_l \eta_N} \quad {}_l \eta_i = \int_0^\infty {}_l h_i(t)^2 dt \quad (4)$$

For $N=3$, the non-linearity ratio can be expressed as a function of a , so-called ${}_l \rho(a)$, assuming the form:

$${}_l \rho(a) = \frac{a^2 {}_l \eta_2 + a^4 {}_l \eta_3}{{}_l \eta_1 + a^2 {}_l \eta_2 + a^4 {}_l \eta_3} \quad (5)$$

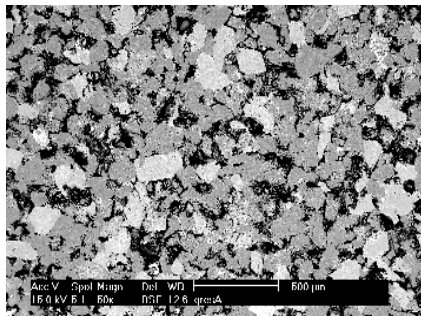
This function expresses the energetic part of non-linear components in the system response to an impulse excitation of magnitude a .

4. Experimental validation

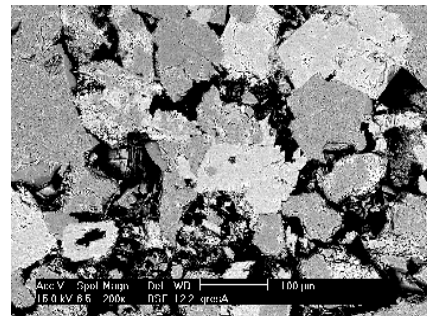
The proposed technique has been applied in laboratory on a brittle porous material such as Vosges sandstone subjected to recurrent freeze-thaw cycles under constant static uniaxial loading.

4.1. Sample description

The sandstone specimens investigated in this study are from the N-E of France. Figure 1 shows the appearance of the texture and porosity of the rock under SEM. The quartz grains content is more than 90 percent, and the feldspar content obtained by scanning electron microscopy is close to 5-10 percent with white mica. The observed grain size is between 100 and 300 μm (Fig.1a). Some physical and mechanical properties measured following ISRM suggested methods such as unit weight, porosity and unconfined compressive strength are 26.16 kN/m^3 , 17.5% (measured by mercury porosimetry), and 38 MPa respectively. The results of the mercury injection porosimetry and SEM image (Fig.1b) show that the pores frequently connect with each other.



a) texture: pores: black color; quartz grains: grey color; feldspar grains: bright color



b) porosity structure: black colors

Figure 1. SEM images of specimens of Vosges sandstone

4.2. Experimental equipment and freezing-thawing F-T cycles

With the experimental equipment, shown in Figure 2, ultrasound propagation measurements can be used to follow the effects of freezing upon a sandstone rock. Fully water-saturated specimens under vacuum conditions and jacketed with a thin rubber sleeve were placed under constant load. The freezing-thawing cycles have been carried out at the middle section of specimen by using a thermoelectric module (Peltier effect cell, heat pump). A Peltier effect cell is an assembly of semiconductor elements between two heat conducting pates. When an electric current is passed through such a device, is appeared a cold junction which absorbs heat, and a hot junction which evolves heat. It is sufficient to reverse the direction of the power supply current to change from cooling to heating and vice versa. The temperature range was -15°C to 20°C , and the temperature fall rate was 2°C per minute. These freezing conditions were chosen because they correspond to a hard frost, which leads to great rock damage. Since Newton's law of cooling applies to these F-T cycles as well, the temperature change will follow almost the same route at each cycle (Fig. 3).

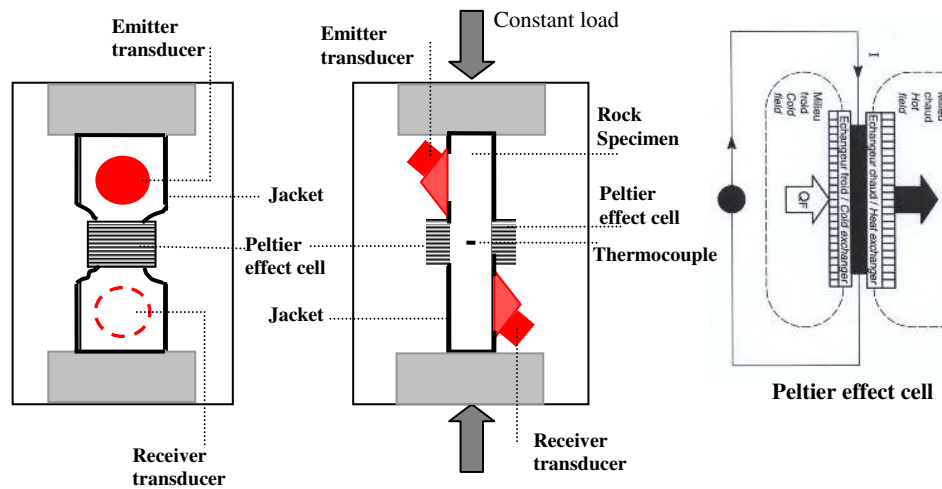


Figure 2. Experimental device of the ultrasound propagation and the F-T cycles.

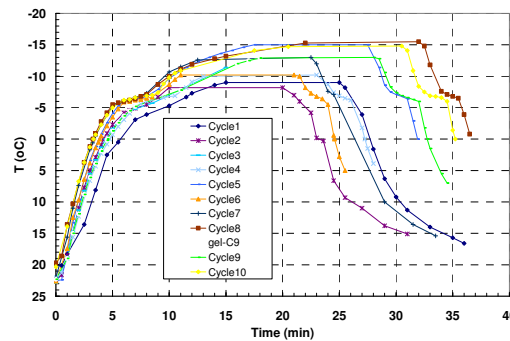


Figure 3. The freezing and thawing cycles and evolution of the temperature in a sandstone specimen during the cycles. Cycle description: 0-20min (20min): freezing; 20min-30min (10min): frozen; 0-20min (20min): thawed.

4.3. Ultrasound measurements during freezing-thawing F-T cycles

The pulse transmission technique, using scan piezoelectric transducer V150(0.10 MHz) has been applied as a non-destructive testing. A coupling agent was used to improve the wave transmissibility through the interface between the transducers and the rock specimen. The pulser section produces an electrical pulse to excite a piezoelectric transducer, which emits an ultrasonic pulse. The pulse travels through the specimen that is subjected to recurrent freeze-thaw cycles under constant static loading, to a second transducer acting as a receiver (Fig. 2). The transducer converts the pulse into an electric signal that makes available for non-linear analysis. Three input pulses were used at three different amplitude levels (Fig. 4). Response signal records at different F-T cycles have indicated the evolution of the ultrasonic pulse travelling through the specimen that is loaded by a constant static compressive load (Fig. 5). The experimental set-up continuously monitored the applied load, the temperature and the signals of ultrasonic waves.

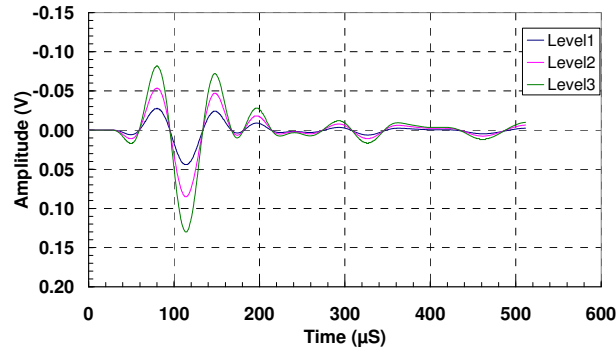
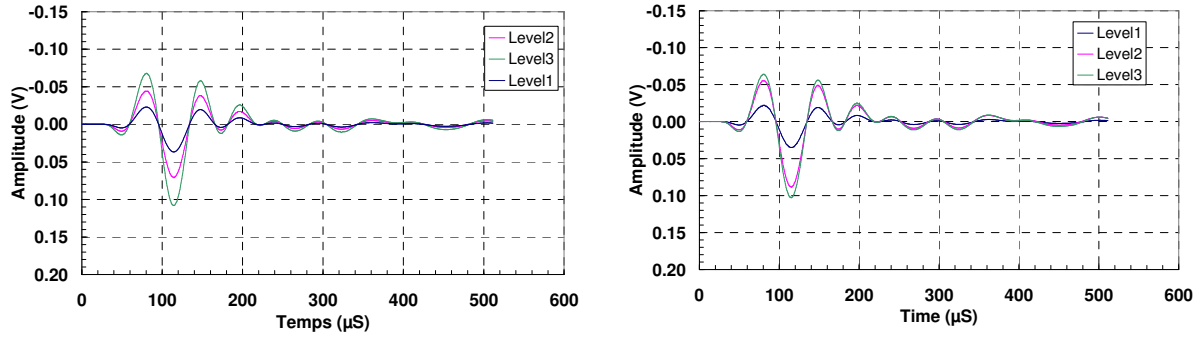


Figure 4. Wave amplitude (volts) versus time (sec) of the three proportional input ultrasonic pulses.



a) b)
Figure 5. Wave amplitude (volts) versus time (sec) of the three proportional: a) output ultrasonic pulses before the stability threshold ST, b) output ultrasonic pulses after the stability threshold ST.

4.4. Experimental results

The data processing technique provides the variation of non-linear energy normalised by total energy and non-linearity contrast as a function of F-T cycles and stress levels (Fig. 6 and 7). They have evidenced a material stability threshold ST that precedes significantly the occurrence of crack initiation. During F-T cycles, non-linearity ratio was measured at the beginning of the temperature stabilisation at the thawing stage. Non-linearity ratio measured at the normalised force amplitude of 38% (Fig.6) is relatively constant in the first two cycles. After two cycles, cracks apparently appear in

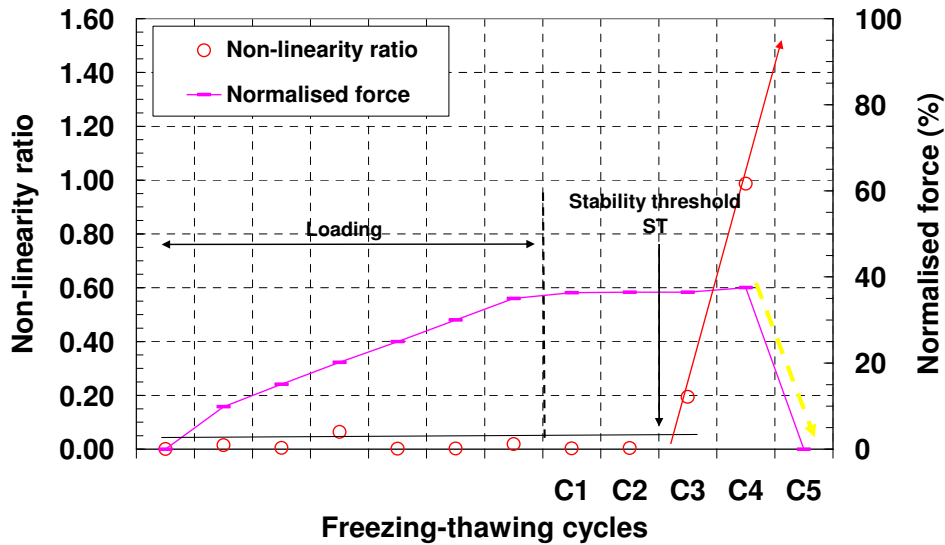


Figure 6. Graphical definition of the stability threshold ST for sandstone subject to recurrent freeze-thaw cycles C_n under constant static uniaxial loading: Normalised force= 38%, Temperature= 20°C.

the rock matrix and modify the geometry of the porous media and non-linearity ratio increases. At the third thaw, porosity increases due to newly created microcracks. Since the water volume remains constant and the porosity increases, relative water saturation in the rock decreases and the water distribution throughout the rock becomes discontinuous. These phenomena are revealed by the sudden increase in non-linearity ratio during thawing. Moreover, an increase in the constant load (Fig.7) from 38% to 52.5% (normalised force), leads to a shift of the stability threshold. At the first thaw the non-linearity ratio increases because of the new cracks that have been generated during freezing stage or by the increase of the uniaxial stress level. It could be concluded that the changes of the non-linearity ratio during the F-T cycles correspond to the existence of the damage period revealed at the thawing stage. The damage phase is established from non-linearity of the wave propagation while it is not visible from the wave velocity in the first F-T cycles (See Fig.5, the time of arrival of the output ultrasonic waves before and after the ST is the same).

The F-T cycles simulated in these experiments represent conditions of natural continental climates. The early appearance of rock damage in the sample studied in the laboratory suggests that similar behaviour will be observed after a few F-T cycles in the natural environment, if the rocks are water saturated. This creation of cracks under frost action corresponds to the first appearance of a weathering zone in the natural environment.

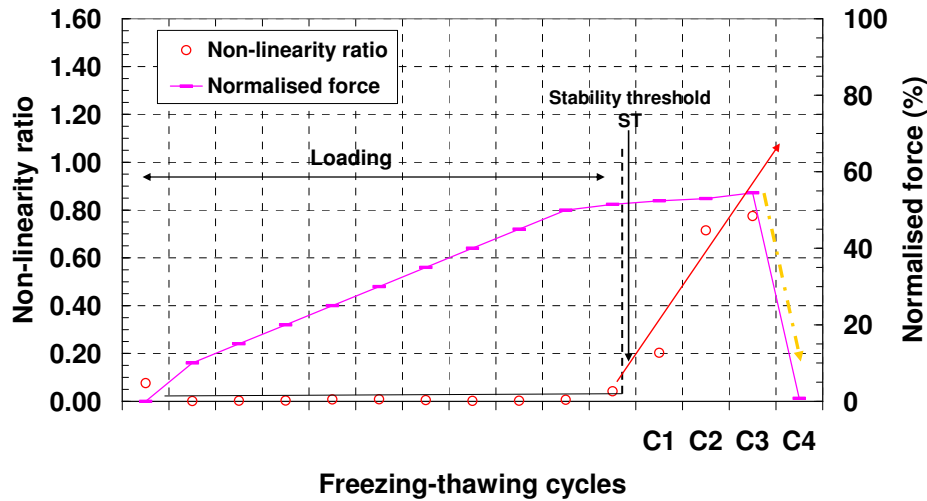


Figure 7. Graphical definition of the stability threshold ST for sandstone subject to recurrent freeze-thaw cycles C_n under constant static uniaxial loading: Normalised force= 52.5%, Temperature= 20°C.

5. Conclusion

This paper demonstrates that the highly sensitive detection of material instability occurred in brittle porous rock could provide a very useful early warning for the security of work at high risk for environment. As a rockfall cause, freezing and thawing of the water filling rock mass pores and discontinuities may provoke ruptures. A non-destructive evaluation of material instability could offer an efficient field monitoring before failure initiation in order to reduce risk of imminent failures by giving advanced and sufficient warning for remedial measures to be designed. The application of ultrasound scanning to inspection and monitoring of rock slopes relies on the fact that during the process of micro-cracking, the brittle or quasi brittle rock may be locally unstable and hence modifies and partially obstructs the propagation characteristics of ultrasound pulses. One of the advantages of using non-linearity monitoring technique is the possibility to observe damage processes during the entire load history without any disturbance to the structures. Measurement can be performed from remote locations, adding another distinct advantage. The authors hope that the presented method will be applied in the near future on the other rock materials and will eventually be used commonly to monitoring of rockfalls.

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