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FLOW CHART FOR NUMERICAL COUPLING OF DYNAMIC PROBLEMS IN ELASTIC MEDIA

VINCENT MARTIN*

Laboratoire de Mécanique et d'Acoustique, CNRS, 13402 Marseille Cedex 20, France,

BERNARD PESEUX

Ecole Centrale de Nantes, 44072 Nantes Cedex 03, France

From the starting point of dynamic coupling problems in the harmonic field, such as fluid/fluid, fluid/structure, fluid/structure/fluid problems, a single procedure capable of resolving the coupling is shown when only one domain is under study. The notions of impedance transfer familiar in analytical methods in the dynamic harmonic field are introduced into numerical methods especially when transferring the structure to the fluid. The proposed work goes beyond the problems themselves and the methods currently applied to solve them (modal series, integral method, finite element method) and suggests how the ideas developed here could simplify greatly the way more complex problems might be tackled.

1. INTRODUCTION

The general purpose of the paper and a succinct description of the state of the art in the field of media coupling are first given. After defining a particular approach in this field and after emphasizing some of the ideas at the origin of the present work, one of which is of particular interest here, the stress is laid on the objective and the way it can be reached. Finally, the originality of the work is defined.

Over the past few years, several dynamic problems concerning fluid/structure coupling have been solved, by the authors among others, in the harmonic field. The physics of these problems and the numerical results can be found in the publications cited and shall not be reiterated here. Instead, a deeper insight into the way the calculations were carried out constitutes the purpose of this paper, aiming at defining a single flow chart valid for all the problems previously solved, each having had in the past its own specific technical approach. It appears that no work has yet been published in the area of the systematic organization of such calculations—a *fortiori* under the form of flow charts such as suggested here—whereas the coupling problems have received a lot of attention both for themselves and for the methods used to solve them. The work presented in this paper was entirely supported by the “Calcul des Structures” Research Group (GRECO).

Presented below are some works in the field of coupling of various media, especially fluid/structure, using numerical methods. It is possible to put light or heavy fluids into separate categories. Alternatively, the criterion of confined or infinite fluids could be adopted. Finally, the classification could depend on the methods used for analysis and resolution. With a numerical

* Current Address: IMT 13451 Marseille Cedex 20, France.

approach only the two latter classifications are appropriate but with a physical approach the first is probably the most reasonable. Nevertheless all these classifications are taken into account here to help situate some of the numerous authors. For coupling with heavy fluids, the structure is often the medium under study. R. Ohayon and R. Valid dealt with filled tanks by the finite element method (FEM) and showed how to formulate the problem in order to obtain symmetrical forms.¹ In the case of fluids of infinite extent—encountered essentially in the sea medium for ships—the medium near the structure may be described by the FEM and far from it by infinite elements, as developed by Bettess and Zienkiewicz.² Other work has been done in the field of hydrodynamics by Orsero and Armand.³ The method of singularities is also appropriate for this type of problem and may be combined with the FEM.⁴⁻⁶

For confined and external light fluids separated by a structure, Astley and Cummings used the modal series and the FEM both for the structure and for the external fluid near the structure.⁷ M. A. Hamdi used the boundary integral method (BIM) and the FEM to describe the acoustical field inside and outside an open-ended duct with elastic walls.⁸ For confined fluids the FEM with the modal series, and later the FEM for the fluid when associated with the FEM for the structure, were used by one of the authors.^{9, 10}

The problems dealt with in this paper have one point in common: the fields (and not the eigenvectors and the eigenvalues) are sought only in one specific domain, which is called the primary domain, and the variables attached to it are the primary variables. The secondary variables which describe the other domains are eliminated so that only the primary ones remain. At this stage the number of domains determines the number of steps necessary in the calculations. The next point to be emphasized regarding the organization of the calculations is the way in which the domains are connected as this property determines the procedure at certain stages of the chart. This fundamental aspect of the work described here requires some explanation. The way calculations are carried out varies enormously depending on whether all or only a part of a medium loads another. The first case where the entire medium bears on another will be called a direct transfer and the second situation (i.e. partial load) will usually necessitate a substructuring process in the matrices, called basic matrices, which describe the secondary medium. For example, when the flexible contour of a cavity containing a fluid belongs entirely to the interior contour of the cavity the transfer is direct (cf. Figure 1). On the contrary, the unbounded domain that loads the external contour of the cavity loads not only the flexible part of the contour but also its rigid part and a substructuring process is necessary. This notion of the transfer of one medium to another will now be seen to be closely linked to the methods used to describe the loading medium.

At this level, the physical nature of the media is given only to define the variables under study. From a matricial point of view the mathematical representation (variational, integral, differential) of the media with their geometry defines the type of processing (e.g. substructuring) appropriate

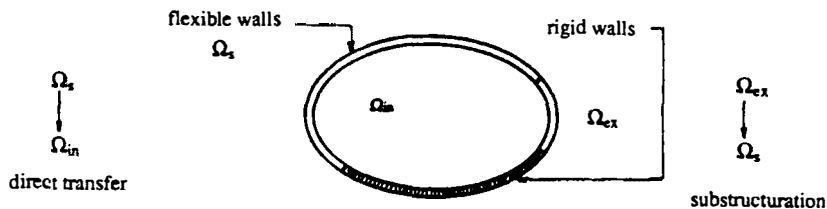


Figure 1. Geometrical configuration of a fluid-filled cavity with flexible boundary and external fluid load

for the transfer of impedances (boundary conditions of Dirichlet, Neumann, of impedance, of Sommerfeld) to the boundary of the primary domain. Finally, the third aspect of the chart: the basic matrices are built with the help of physical descriptions of the media and will be modified by the above processing. Thus our priorities are as follows: the geometrical configuration, i.e. the number of media and the number of interfaces in the problem (number of steps in the flow chart); the nature of the interfaces and the chosen representation in each domain (type of processing needed to transfer the impedances to the boundary of the primary domain); the physical behaviour inside the domains (building of the basic matrices). Conversely, the problems from which the study derives are described in the usual manner: differential equations for the mechanical behaviour of the media; choice of the modelling and matricial form; numerical organization of the calculations.

Following these ideas, the goal of the present paper is to emphasize the elements necessary to devise almost systematically the calculation procedure for complex coupling problems. Inside this organization the most important notion concerns the partial or total transfer of one medium to another. Each problem chosen is given here to throw light on each of the elements that are to contribute to the flow chart for calculations. First the quite classical case of transferring the fluid to the structure is analysed. Then Section 3 describes the problems where the fluid is the primary domain, and constitutes the major part of the paper as this approach is more original than the previous one. The single flow chart, the goal of this work, is illustrated with an imaginary complex problem that shows the usefulness and the efficiency of the flow charts obtained.

The originality of the work lies in the attempt to find a systematic procedure applicable to the class of problems under study, in clarification of the notion of direct or partial transfer of one medium to another and its connection with the methods of calculation chosen, and finally in implementation of cases where the fluid is the primary domain. In addition the basic matrices will be defined when the method chosen is the modal series representation as they are not known in this case.

2. COUPLING WITH THE STRUCTURE AS PRIMARY DOMAIN

Two coupling problems where the fluid(s) load(s) the structure are presented. Such an approach is not new and allows in the present context to emphasize some of the notions discussed in this paper. (The application of the notions to the case where the fluid is the primary domain will constitute Section 3). In the first case with two media, fluid and structure, the interest lies in the partial transfer of the fluid to the structure with the subsequent substructuring in the basic fluid matrices. The second case shows two fluids loading a structure on each side. Here the aim is in the parallel procedure with partial transfers. The reasons why the problems have been solved are briefly given but not the numerical results—which are published in the publications cited—as only the calculation procedure constitutes the present goal and has not yet been published.

Although the notations for each specific problem are explicated, we have attempted to present all the different problems in a similar manner as far as possible.

2.1. Problem of sand loaded by incompressible fluid

This problem arose from the study of the influence of sediment on the behaviour of ships at berth. Such a question was first seen from an academic point of view with an elementary experiment that was to validate the type of calculation to be used to modelize the real situation. At that time attention was focused on the sand of the seabed described as an elastic solid constituting the primary domain.¹¹

The geometrical configuration (Figure 2) has two domains: Ω_1 (solid), Ω_2 (fluid) deriving from a rigid open tank with an elastic solid at the bottom that supports a column of water. The boundary of Ω_2 is composed partly of the rigid walls of the tank, partly of the free surface (zero normal constraint) and partly of a prescribed velocity (excitation at the surface of the fluid). Finally, the interface Γ is only a portion of $\partial\Omega_2$, and the method used to describe the fluid Ω_2 , the BIM, results in a substructuring to obtain the impedance relation to be applied to Γ , allowing the problem in Ω_1 to be entirely defined.

The basic matrices and the interfacing matrices are obvious. The processing by substructuring necessary to bring Ω_2 to the interface Γ is now developed. For the sake of generality a source is imposed in Ω_1 and Ω_2 , although the original problem only dealt with the excitation at the surface of the fluid.

2.1.1. Equations of motion. The present problem has no loss of energy and the total operator is real.

(a) For the elastic solid the field of displacements \mathbf{u} is the unknown. It satisfies:

$$(\lambda + \mu)\nabla(\text{div } \mathbf{u}) + \mu \Delta \mathbf{u} + \rho_s \omega^2 \mathbf{u} + \mathbf{f}_s = \mathbf{0} \quad \text{in } \Omega_1$$

$$\mathbf{u} = \mathbf{0} \quad \text{on the rigid part of } \partial\Omega_1$$

λ , μ are the Lamé coefficients, ρ_s is the density of the structure, ω is the circular frequency.

(b) For the incompressible fluid, the unknown function is the velocity potential ϕ such that the velocity \mathbf{V} of the fluid satisfies:

$$\mathbf{V} = \nabla \phi$$

From Bernoulli's law, the pressure p is such that $\dot{p} + \rho g \dot{z} = -\rho \partial \dot{\phi} / \partial t$, g is the gravity, ϕ (or $\bar{\phi}$ where $\phi = \bar{\phi} e^{i\omega t}$) is the solution of the following equations:

$$\begin{aligned} \Delta \phi &= f_2 && \text{in } \Omega_2 \\ \partial_n \phi &= 0 && \text{on the rigid part of } \partial\Omega_2 \\ \partial_n \phi &= \mathbf{v} \cdot \mathbf{n} && \text{where the velocity is imposed, } n \text{ is directed outward } \Omega_2 \\ g \partial_n \phi - \omega^2 \phi &= 0 && \text{on the free surface} \end{aligned}$$

(c) The coupling at the interface Γ is written as

$$\begin{aligned} \boldsymbol{\sigma} \cdot \mathbf{n} &= p \mathbf{n} \\ \mathbf{u} \cdot \mathbf{n} &= U_n \end{aligned} \quad \text{that is} \quad \partial_n \phi - \frac{\omega^2}{g} \phi = -\frac{i\omega}{\rho g} \sigma_n$$

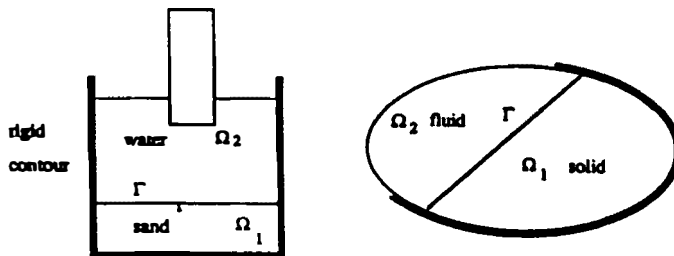


Figure 2. Original problem and equivalent geometrical configuration

where \bar{U}_n is the normal displacement of the fluid and σ the stress tensor. Indeed the continuity of displacements and stresses must be satisfied. σ_n is the normal component of $\sigma \cdot \mathbf{n}$ on Γ and ρ is the fluid density.

2.1.2. Methods. The elastic solid is described by the FEM and expressed in a variational form with the principle of virtual displacements. The fluid is described by the boundary integral method and needs to be written with an integral formulation.

(a) For the solid we have

$$\int_{\Omega_1} \delta\{\varepsilon\}^T \{\sigma\} \, d\Omega_1 - \rho_s \omega^2 \int_{\Omega_1} \delta\{u\}^T \{u\} \, d\Omega_1 - \int_{\partial\Omega_1} \delta\{u\}^T \{\sigma \cdot \mathbf{n}\} \, d\Gamma - \int_{\Omega_1} \delta\{u\}^T \{f_s\} \, d\Omega_1 = 0$$

Whatever the admissible $\delta \mathbf{u}$, $\{\varepsilon\}$ is the strain tensor written in the form of a vector with six components. Due to the boundary conditions, the integral on $\partial\Omega_1$ is calculated only on Γ with the help of an operator written $\{\sigma_n\} = \mathcal{L}(\partial_n \phi, \phi)$ which will be explained later.

The nodal approximation occurs in Ω_1 and the collocation method on Γ . With the subscript i indicating the nodal values of the fields and with the subscript c indicating the values of the fields at the points of collocation, we obtain the matricial form

$$\delta\{a_i\}^T \cdot ([K_s] - \rho_s \omega^2 [M_s]) \cdot \{a_i\} = \delta\{w_c\}^T \cdot [Q] \cdot \{\sigma_{nc}\} + \delta\{a_i\}^T \cdot \{f_{si}\}$$

$\{a_i\}$ is the vector of the nodal degrees of freedom, $[K_s]$ and $[M_s]$ are the stiffness and mass matrices for the solid respectively, w_c is the normal displacement at the centre of the boundary elements on Γ .

(b) For the fluid, the third Green formula with the outward normal leads to

$$-\frac{\phi}{2} + \int_{\partial\Omega_2} \phi \cdot \partial_n G \, d\Gamma = \int_{\partial\Omega_2} G \cdot \partial_n \phi \, d\Gamma - \int_{\Omega_2} G \cdot f_2 \, d\Omega_2$$

where the Green function G satisfies the operator Δ with radiation condition at infinity. The collocation method leads to the following matricial form for the fluid:

$$[D] \cdot \{\phi_c\} = [S] \cdot \{\partial_n \phi_c\} + \{f_{2c}\}$$

$[D]$ and $[S]$ are the double and simple layer matrix for the fluid, respectively.

2.1.3. Transfer of the fluid to the structure and interfacing. The goal is to obtain the form $\phi(\partial_n \phi)$ only on Γ . Indeed the operator $\mathcal{L}(\phi, \partial_n \phi)$ would give rise to another operator $\mathcal{L}'(\partial_n \phi)$, hence $\mathcal{L}'(u_n)$, and the problem in the solid would be entirely written in terms of displacements in Ω_1 . The impedance relation on the interface Γ is reached with the help of substructuring processing. On the whole of $\partial\Omega_2$ we have

$$[D] \begin{Bmatrix} \{\phi_B\} \\ \{\phi_\Gamma\} \end{Bmatrix} = [S] \begin{Bmatrix} \{v\} \\ k_0 \{\phi_{SL}\} \\ \{\partial_n \phi_\Gamma\} \end{Bmatrix} + \begin{Bmatrix} \{f_B\} \\ \{f_\Gamma\} \end{Bmatrix}$$

where $k_0 = \omega^2/g$ and $B = \partial\Omega_2 \setminus \Gamma$. (On the rigid boundary $\partial_n \phi = v_n = 0$)

Knowing that $SL \subset B$, $\{\phi_{SL}\}$ is written with $\{\phi_B\}$. Substructuring the resulting matrices leads to

$$[BB]\{\phi_B\} + [B\Gamma]\{\phi_\Gamma\} + [B\Gamma']\{\partial_n \phi_\Gamma\} = [BC]\{v\} + \{f_B\}$$

$$[\Gamma B]\{\phi_B\} + [\Gamma\Gamma]\{\phi_\Gamma\} + [\Gamma\Gamma']\{\partial_n \phi_\Gamma\} = [\Gamma C]\{v\} + \{f_\Gamma\}$$

and $\{\phi_B\}$ coming from the second equation, inserted in the first, results in the impedance relation sought:

$$\{\phi_\Gamma\} + [U]\{\partial_n\phi_\Gamma\} = [V]\{V\} + [T]\{f_B\} + [R]\{f_\Gamma\}$$

where

$$\begin{aligned} [R] &= (-[\Gamma B] \cdot [BB]^{-1} \cdot [B\Gamma'] + [\Gamma\Gamma])^{-1} \\ [U] &= [R](-[\Gamma B] \cdot [BB]^{-1} \cdot [B\Gamma'] + [\Gamma\Gamma]) \\ [V] &= [R](-[\Gamma B] \cdot [BB]^{-1} \cdot [BC] + [\Gamma C]) \\ [T] &= [R](-[\Gamma B] \cdot [BB]^{-1}) \end{aligned}$$

Inserting the impedance relation in the formulation of the primary domain results in

$$\delta\{a_i\}^T \cdot ([K_s] - \rho_s \omega^2 [M_s]) \cdot \{a_i\} = \delta\{w_c\}^T \cdot [Q] \cdot \left\{ -\frac{\rho g}{i\omega} \{\partial_n\phi_\Gamma\} - i\omega\rho \{\phi_\Gamma\} \right\} + \delta\{a_i\}^T \cdot \{f_{si}\}$$

or

$$\begin{aligned} \delta\{a_i\}^T ([K_s] - \rho_s \omega^2 [M_s]) \{a_i\} + \delta\{w_c\}^T [Q] (\rho g [I] + \omega^2 \rho [U]) \{w_c\} \\ = -i\omega\rho \delta\{w_c\}^T \{f'_{2c}\} + \delta\{a_i\}^T \{f_{si}\} \end{aligned}$$

Working in Ω_1 with the FEM, i.e. with a nodal approximation, the mesh on Γ seen from Ω_2 has to be harmonized with the mesh of Γ seen from Ω_1 , and the column vector of the unknowns at the points of collocation $\{w_c\}$ has to be expressed in terms of the nodal unknowns $\{a_i\}$.

Let $[VN]$ be the rectangular matrix of transformation such that $\{w_c\} = [VN] \{a_i\}$, this matrix is built from the co-ordinates of the collocation points (which are the centre of the facets that discretize the integral equation) and from the calculation of the values of the nodal interpolation functions at those points.

Let $[W]$ be the square matrix that establish a correspondence between each element on Ω_1 with a side on Γ and each facet on Γ . Then

$$\{a_c\} = [VN] \{a_i\}_\Gamma = [VN][W] \{a_i\}_{\Omega_1}$$

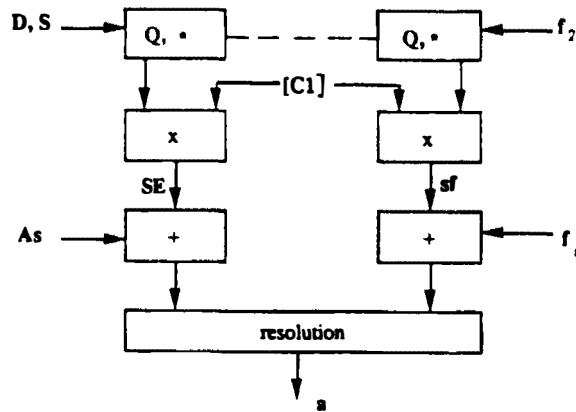


Figure 3. Flow chart corresponding to the equivalent geometrical configuration of Figure 2

Finally, the formulation in Ω_1 is

$$([A_s] + [SE])\{a_i\} = (\{f_{si}\} + \{sf\})$$

where

$$\begin{aligned} [A_s] &= ([K_s] - \rho_s \omega^2 [M_s]) \\ [SE] &= [C_1]^T [Q] (\rho g [I] + \omega^2 \rho [U]) [C_1] \quad \text{with} \quad [C_1] = [VN][W] \\ \{sf\} &= -i\omega \rho [C_1]^T [Q] ([V]\{v\} + [T]\{f_B\} + [R]\{f_\Gamma\}) \end{aligned}$$

2.1.4. Calculation procedures. This flow chart includes two steps as the problems involves two media but has a particular type of processing—shown with an asterix—due to the substructuring necessary because Γ is only a part of $\partial\Omega_2$ and Ω_2 is dealt with using the boundary integral method (Figure 3).

2.2. Problem of cavity walls loaded by compressible fluids on each side

Having had to deal with the problem of guided acoustic waves in a duct with yielding walls and outer radiation¹²—which will be described in Section 3—we were also interested in the behaviour of the structure which closes the cavity.

The structure Ω_1 is modeled by the FEM. Both fluids (inside in Ω_2 , outside in Ω_3) load the structure and are described by the BIM (Figure 4). From an algorithmic point of view the calculations are carried out simultaneously as they need exactly the same operations for both fluids and the information subsequently brought to each side of the structure. The flow chart will show clearly the loads in parallel (Figure 5).

2.2.1. Equations of motion. Since the outer compressible fluid has a radiation at infinity with loss of energy, the problem is not conservative and the total operator is complex.

(a) For the thin elastic two-dimensional (2D) structure (plates), the normal displacement w is the unknown variable. The excitation f_1 is applied at the outer surface of Ω_1 and the plates are clamped at their edges:

$$\begin{aligned} D\Delta\Delta w - \rho_1 \omega^2 w &= (p_2 - p_3) - f_1 \quad \text{in } \Omega_1 \text{ (or on } \Gamma, \text{ the middle-plane of the plates)} \\ w = \partial_n w &= 0 \quad \text{on } \partial\Gamma \end{aligned}$$

p_2 and p_3 are the acoustic pressure inside and outside the cavity; n' is the normal to Γ directed outwards.

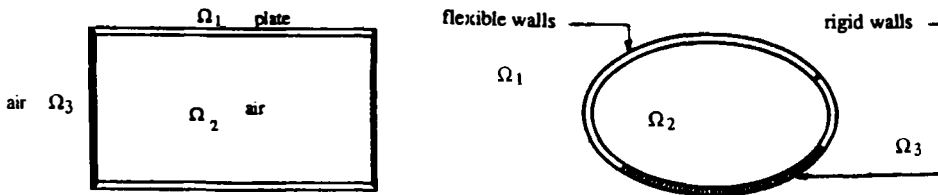


Figure 4. Original problem and equivalent geometrical configuration

(b) For the compressible fluid (gas) contained in the cavity the unknown variable is the acoustic pressure p_2 such that

$$(\Delta + k_2^2)p_2 = f_2 \quad \text{in } \Omega_2, \quad \partial_n p_2 = 0 \quad \text{on } \partial\Omega_2 \setminus \Gamma_2, \quad \partial_n p_2 = \rho^2 \omega^2 u_2$$

with Γ_2 the inner face of Ω_1 , k_2 the acoustic wave number, ρ_2 the density of the internal fluid, u_2 the normal displacement of the fluid particles on Γ_2 .

(c) For the compressible fluid (gas) outside the cavity the unknown variable is the acoustic pressure p_3 such that

$$\begin{aligned} (\Delta + k_3^2)p_3 &= f_3 \quad \text{in } \Omega_3, \quad \partial_n p_3 = 0 \quad \text{on } \partial\Omega_3 \setminus \Gamma_3 \\ \partial_n p_3 &= \rho_3 \omega^2 u_3 \quad \text{on } \Gamma_3, \quad \lim_{r \rightarrow \infty} r(\partial_r p_3 + ik_3 p_3) = 0 \quad \text{at infinity} \end{aligned}$$

with Γ_3 the external face of Ω_1 , k_3 the acoustic wave number, ρ_3 the density of the external fluid, u_3 the normal displacement of the fluid particles on Γ_3 .

(d) The coupling of the media requires continuity of normal displacements and normal stresses at the interfaces. The latter continuity is partially written in the equation of the plate.

$$\begin{aligned} u_2 &= w \quad \text{on } \Gamma_2, \quad u_3 = w \quad \text{on } \Gamma_3 \\ \boldsymbol{\sigma} \cdot \mathbf{n} &= p_2 \mathbf{n} \quad \text{on } \Gamma_2, \quad \boldsymbol{\sigma} \cdot \mathbf{n} = -p_3 \mathbf{n} \quad \text{on } \Gamma_3 \end{aligned}$$

2.2.2. *Methods.* The structure is analysed by the principle of virtual work giving¹³

$$\int_{\Gamma} \delta\{\varepsilon\}^T [D_p] \{\varepsilon\} d\Gamma - \rho_1 \omega^2 \int_{\Gamma} \delta w \cdot w d\Gamma = \int_{\Gamma} \delta w (p_2 - p_3) d\Gamma + \int_{\Gamma} \delta w \cdot f_1 d\Gamma$$

and the nodal approximation allows the matricial form

$$\delta\{a_i\}^T [A_1] \{a_i\} = \int_{\Gamma} \delta w (p_2 - p_3) d\Gamma + \delta\{a_i\}^T \{f_{1i}\}$$

Both internal and external fluids call for an integral representation. Let E be the point of Ω_2 and C of Ω_3 where acoustic sources $q_2 e^{i\omega t}$ and $q_3 e^{i\omega t}$ exist, and let G be the Green function satisfying the Helmholtz equation in the infinite space. For each fluid, we have in terms of acoustic pressure:

$$\begin{aligned} \int_{\partial\Omega_2} p_2(P) \frac{\partial G}{\partial n_P} dS(P) - \int_{\Gamma_2} G(M, P) \frac{\partial p_2}{\partial n_P} dS(P) + q_2 G(M, E) &= \begin{cases} 0 & \text{si } M \notin \Omega_2 \cup \partial\Omega_2 \\ \frac{1}{2} p_2(M) & \text{si } M \in \partial\Omega_2 \\ p_2(M) & \text{si } M \in \Omega_2 \end{cases} \\ - \int_{\partial\Omega_3} p_3(P) \frac{\partial G}{\partial n_P} dS(P) + \int_{\Gamma_3} G(M, P) \frac{\partial p_3}{\partial n_P} dS(P) + q_3 G(M, C) &= \begin{cases} 0 & \text{si } M \notin \Omega_3 \cup \partial\Omega_3 \\ \frac{1}{2} p_3(M) & \text{si } M \in \partial\Omega_3 \\ p_3(M) & \text{si } M \in \Omega_3 \end{cases} \end{aligned}$$

The internal fluid possesses an outward normal at its boundaries, the external fluid an inward normal

Each contour $\partial\Omega$ is discretized with plane facets, whose surfaces are $\partial\Omega_j$, supporting constant pressure and normal derivative of the pressure (this means that these two items are independent).

It results in the matricial form:

$$\begin{aligned} [D_2]\{p_2\} &= [S]\{\partial_n p_2\} - q_2\{G(M_i, E)\} \\ [D_3]\{p_3\} &= [S]\{\partial_n p_3\} + q_3\{G(M_i, C)\} \end{aligned}$$

The terms of the matrices are explicit only to emphasize the difference between internal and external basic fluid matrices:

$$S_{ij} = \int_{\partial\Omega_j} G(M_i, P) dS(P)$$

regardless of which side of the structure we consider, contrary to

$$D_{ij} = \begin{cases} -\frac{1}{2}\delta_{ij} + \int_{\partial\Omega_j} \frac{\partial G(M_i, P)}{\partial n_P} dS(P) & \text{in } \Omega_2 \\ +\frac{1}{2}\delta_{ij} + \int_{\partial\Omega_j} \frac{\partial G(M_i, P)}{\partial n_P} dS(P) & \text{in } \Omega_3 \end{cases}$$

which clearly depends on the side.

2.2.3. Transfer of the fluids to the structure and interfacing. Γ_2 is a part of $\partial\Omega_2$ and Γ_3 is a part of $\partial\Omega_3$. Thus substructurations are expected. A common and practical method in transferring the fluid to the structure, whose less prosaic form has been given in Section 2.1.3, is proposed here. It is in the more original case where the structure is transferred to the fluid that the development will be given again.

Due to the continuity of the normal velocities at the level of the plates we have

$$\{\partial_n p_2\}_{\Gamma_2} = \rho_2 \omega^2 [\text{VN}]\{a_i\}, \quad \{\partial_n p_3\}_{\Gamma_3} = \rho_3 \omega^2 [\text{VN}]\{a_i\}$$

bearing in mind that $\{a_i\}$ is the vector of nodal displacement of the structure using the FEM, and $[\text{VN}]$ the coupling matrix already described.

As for $\{p_2\}$, $\{p_3\}$, all values on $\partial\Omega_2$ and $\partial\Omega_3$ are calculated although, finally, only the values applied to Γ_2 and Γ_3 will be extracted. This procedure constitutes the prosaic form of the substructuration.

$$\begin{aligned} \{p_2\}_{\partial\Omega_2} &= [D_2]^{-1}([S]\{\partial_n p_2\}_{\Gamma_2} - q_2\{G(M_i, E)\}) \\ \{p_3\}_{\partial\Omega_3} &= [D_3]^{-1}([S]\{\partial_n p_3\}_{\Gamma_3} + q_3\{G(M_i, C)\}) \end{aligned}$$

On an element with n , outward normal, the pressure applied is $\{p\} = \{p_2 - p_3\}$. Let us write

$$\{p_2\}_{\Gamma_2} = [P_2]\{p_2\}_{\partial\Omega_2}, \quad \{p_3\}_{\Gamma_3} = [P_3]\{p_3\}_{\partial\Omega_3}$$

where $[P_2]$ and $[P_3]$ make it possible to extract the pressure at the centre of the facets identical to the plate elements. Writing $[Q]$ the diagonal matrix of the facet' surfaces, the generalized force applied to the structure is

$$\begin{aligned} \{\varphi_h\} &= [\text{VN}]^T [Q] ([P_2]\{p_2\}_{\partial\Omega_2} - [P_3]\{p_3\}_{\partial\Omega_3}) \\ \{\varphi_h\} &= [\text{VN}]^T [Q] ([P_2]([D_2]^{-1}[S]\{\partial_n p_2\}_{\Gamma_2} - q_2[D_2]^{-1}\{G(M_i, E)\}) \\ &\quad - [P_3]([D_3]^{-1}[S]\{\partial_n p_3\}_{\Gamma_3} - q_3[D_3]^{-1}\{G(M_i, C)\})) \end{aligned}$$

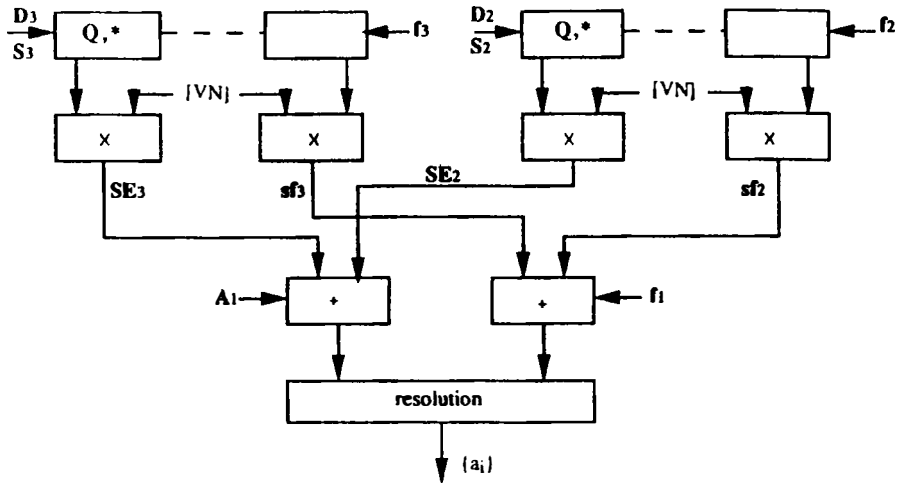


Figure 5. Flow chart corresponding to the equivalent geometrical configuration of Figure 4

and the matricial equation for the loaded structure processed by FEM is

$$\begin{aligned}
 & ([A_1] - \rho_2 \omega^2 [VN]^T [Q] [P_2] [D_2]^{-1} [S] [VN] \\
 & \quad + \rho_3 \omega^2 [VN]^T [Q] [P_3] [D_3]^{-1} [S] [VN]) \{a_i\} \\
 & = q_2 ([VN]^T [Q] [P_2] [D_2]^{-1}) \{G(M_i, E)\} \\
 & \quad - q_3 ([VN]^T [Q] [P_3] [D_3]^{-1}) \{G(M_i, C)\}
 \end{aligned}$$

or

$$\begin{aligned}
 & (-\omega^2 (\rho_1 [M_1] + \rho_2 [M_2] - \rho_3 [M_3]) + [K_1]) \{a_i\} = \{sf_2\} + \{sf_3\} \\
 & = ([A_1] + [SE_2] + [SE_3]) \{a_i\}
 \end{aligned}$$

$[M_1]$ and $[K_1]$ are the structural mass and stiffness matrices, respectively.

The added mass matrices are complex and dependent on the pulsation ω . Having solved the loaded structure, the available $\{a_i\}$ allows us to find, for example, the internal pressure by

$$p_2(P) = \langle D_2(P) \rangle \{p_2\} - \langle S(P) \rangle \{\partial_n p_2\} + q_2 G(P, E)$$

2.2.4. Calculation procedures. The basic matrices and the development of the substructuring are described in Section 2.1. As only a part of the boundaries of each fluid loads the structure, here again substructurations are needed and their prosaic form is shown above. The parallel procedure implemented to solve the problem actually calls the subroutines for both fluids at the same point in the calculation program. Indeed both fluids are taken into consideration in the same way (Figure 5).

3. COUPLING WITH THE FLUID AS PRIMARY DOMAIN

Three coupling problems where the fluid is the domain of interest are presented. The first is elementary since the basic matrices deriving from the FEM are obvious and the transfer of the

structure to the fluid is direct. In fact, this first case constitutes an introduction to the more original situation where the structure loads the fluid. The second problem is a coupling between two fluids, one of them being the primary domain. In addition to the definition of the basic matrices when using the modal method, the example shows clearly the close relation between geometrical and numerical transfers as the latter is total while the former is partial. Finally a fluid/structure/fluid coupling gives the series procedure in presence of the FEM and the BIM and of one direct and one partial transfer. As previously, the reasons why each of the problems proposed has been tackled will be briefly indicated, without developing the numerical results which have already been published. However, some of the numerical results will be mentioned here as illustrations. Indeed only the numerical processing itself constitutes the purpose of this article.

3.1. Problem of guided acoustic waves in a duct with yielding walls

In order to discover whether the interaction between flexural and dilatational waves in a fluid-filled rectangular duct might be the reason why a dispersion in the speed of the plane wave inside the duct was observed experimentally, a calculation was carried out by the FEM both in the structure and in the fluid. The numerical results of the coupling problem agreed qualitatively with the phenomenon observed, and it appeared that the usual hypothesis of local reaction of the walls did not apply in all frequency ranges (Reference 10 and illustration at the end of this paragraph in Figure 8).

The geometrical configuration (Figure 6) has two domains: Ω_1 (fluid) and Ω_2 (structure). The boundary of Ω_1 is formed partly by rigid walls and partly by flexible walls. In the latter part the inside of Ω_2 is the boundary. The whole system has no outside load and the outside of Ω_2 is entirely free (free surface condition, i.e. $\sigma \cdot n = 0$). Ω_1 constitutes the primary domain and the boundary condition (BC) will be described numerically on the inside of Ω_2 , interface between the two media.

Due to the method used (FEM for both domains), the basic matrices are straightforward and we focus here on the definition and building of interfacing matrices.

Originally, the excitation took place inside the cavity but here an excitation on the outer surface of the plate is also considered.

3.1.1. Equations of motion. (a) For the compressible gas inside the cavity, the unknown function is the acoustic pressure p_1 :

$$\begin{aligned} (\Delta + k_1^2)p_1 &= f_1 && \text{in } \Omega_1 \\ \partial_n p_1 &= 0 && \text{on } \partial\Omega_1 \setminus \Gamma \\ \partial_n p_1 &= \rho_1 \omega^2 u_1 && \text{on } \Gamma \end{aligned}$$

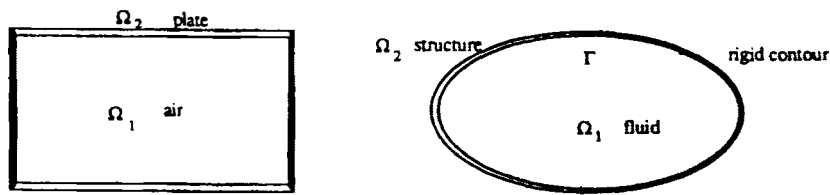


Figure 6. Original problem and equivalent geometrical configuration

where u_1 is the normal displacement of the fluid particles at the level of interface Γ with the structure. The normal is directed outward Ω_1 .

(b) The structure is a thin plate whose unknown is the normal displacement w . The plate is clamped on its boundary $\partial\Gamma$, the equation is written on Γ ($\equiv \Omega_2$) and we envisage a mechanical source f_2 applied to the external face of Γ .

$$(D\Delta\Delta - \omega^2\rho_2)w = p_1 - f_2 \quad \text{in } \Gamma, \quad w = \partial'_n w = 0 \quad \text{on } \partial\Gamma$$

\mathbf{n}' is the normal to $\partial\Gamma$, not to be confused with \mathbf{n} .

(c) The coupling at the interface is written with

$$u_1 = w, \quad \boldsymbol{\sigma} \cdot \mathbf{n} = p_1 \mathbf{n} \quad \text{on } \Gamma$$

3.1.2. Methods The fluid domain Ω_1 and the structure Γ ($\equiv \Omega_2$) are described by the FEM and thus are written in the variational form with the help of the principle of virtual forces for the fluid and of the principle of virtual displacements for the structure, leading in the fluid to

$$\int_{\Omega_1} (-\mathbf{grad} \delta p_1 \cdot \mathbf{grad} p_1 + k_1^2 \delta p_1 \cdot p_1) d\Omega_1 + \int_{\Gamma} \delta p_1 \partial_n p_1 d\Gamma = \int_{\Omega_1} \delta p_1 \cdot f_1 d\Omega_1$$

where the nodal approximation in Ω_1 results in the matricial form

$$\delta \{p_{1i}\}^T [A_1] \{p_{1i}\} + \rho_1 \omega^2 \int_{\Gamma} \delta p_1 w d\Gamma = \delta \{p_{1i}\}^T \{f_{1i}\}$$

and, in the structure to

$$\int_{\Gamma} \delta \{\varepsilon\}^T [D_f] \{\varepsilon\} d\Gamma - \rho_2 \omega^2 \int_{\Gamma} \delta w \cdot w d\Gamma = \int_{\Gamma} \delta w \cdot p_1 d\Gamma - \int_{\Gamma} \delta w \cdot f_2 d\Gamma$$

where the nodal approximation gives the matricial form

$$\delta \{a_i\}^T [A_2] \{a_i\} = \int_{\Gamma} \delta w \cdot p_1 d\Gamma - \delta \{a_i\}^T \{f_{2i}\}$$

3.1.3. Transfer of the structure to the fluid and interfacing. As the primary domain is Ω_1 , the structure is transferred to the boundary $\partial\Omega_1 \equiv \Gamma$. The structure is described as

$$\delta \{a_i\}^T [A_2] \{a_i\} = \delta \{a_i\}^T [C_1] \{p_{1i}\} - \delta \{a_i\}^T \{f_{2i}\}$$

i.e.

$$\{a_i\} = [A_2]^{-1} ([C_1] \{p_{1i}\} - \{f_{2i}\})$$

and the matricial equation of the internal fluid loaded by the structure is

$$[A_1] \{p_{1i}\} + \rho_1 \omega^2 [C_1]^T [A_2]^{-1} ([C_1] \{p_{1i}\} - \{f_{2i}\}) = \{f_{1i}\}$$

or

$$([A_1] + [SE_1]) \{p_{1i}\} = \{f_{1i}\} + \{sf_1\}$$

While the matrices $[A_1]$ and $[A_2]$ are well known, we need to emphasize the building of $[C_1]$. In the variational form of the fluid, the nodal approximation of the term $\int_{\Gamma} \delta p_1 \cdot w d\Gamma$ gives

$$\langle \delta p_{1i} \rangle \int_{\Gamma} \{N_{if}\} \langle N_{js} \rangle d\Gamma \{a_j\}$$

and the integral provides the term c_{1ij} of the matrix $[C_1]$. In the FEM, where the total integral is the sum of elementary integrals, the form is

$$\sum_e \langle \delta p_{1i} \rangle_e \int_{\Gamma_e} \{N_{if}\}_e \langle N_{js} \rangle_e d\Gamma \{a_j\}_e$$

3.1.4. *Calculation procedures.* The flow chart consists of the steps shown in Figure 7.

We have here the simplest diagram in which the interfacing matrix has been obtained explicitly. However, the interface is not quite clear in this elementary example since Γ represents Ω_2 as well as the inner surface of $\partial\Omega_2$. The ambiguity here comes from the formulation of the thin structure in which the integration along the thickness is calculated explicitly, inserted in the bending coefficient, and thus disappears.

3.2. Problem of multimode acoustic guided wave

The propagation of a multimode wave was simulated in order to control sources which were to reproduce a given vibratory field in a guide. The Helmholtz equation described the harmonic field. Near the sources, the numerical solution was calculated by the FEM, as the expansion of the solution on the series of eigenfunctions of the duct was not adequate when the sources were of complicated geometry and vibratory shape. Beyond the vicinity of the sources, the wave was established on certain propagated modes. At the common boundary of the two domains, we connected the numerical solution and the truncated expansion of the series of propagated modes and, consequently, the conditions of reflection at the end of the duct were described in an integro-differential form on the fictitious boundary (Reference 9 and illustration at the end of this section in Figure 12).

The present configuration has two volumes in a cavity, (Figure 9) each filled with the same fluid. The cavity is open on part of its contour and limited by rigid walls elsewhere. The compressible fluid in Ω_1 is analysed by the FEM while the particular geometry of Ω_2 enables us to use a modal series satisfying the BC on the rigid part of $\partial\Omega_2$. On its open part an impedance form is given which reveals the presence of the fluid outside the cavity. Actually the interface between

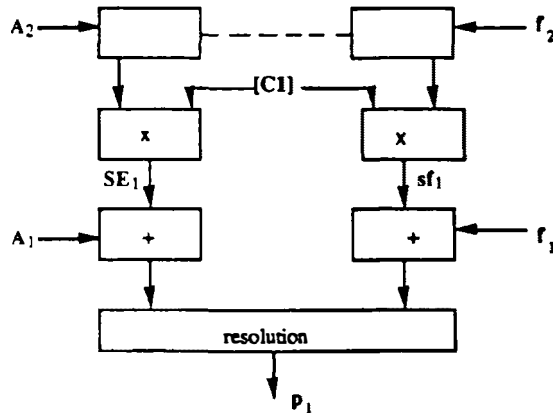


Figure 7. Flow chart corresponding to the equivalent geometrical configuration of Figure 6

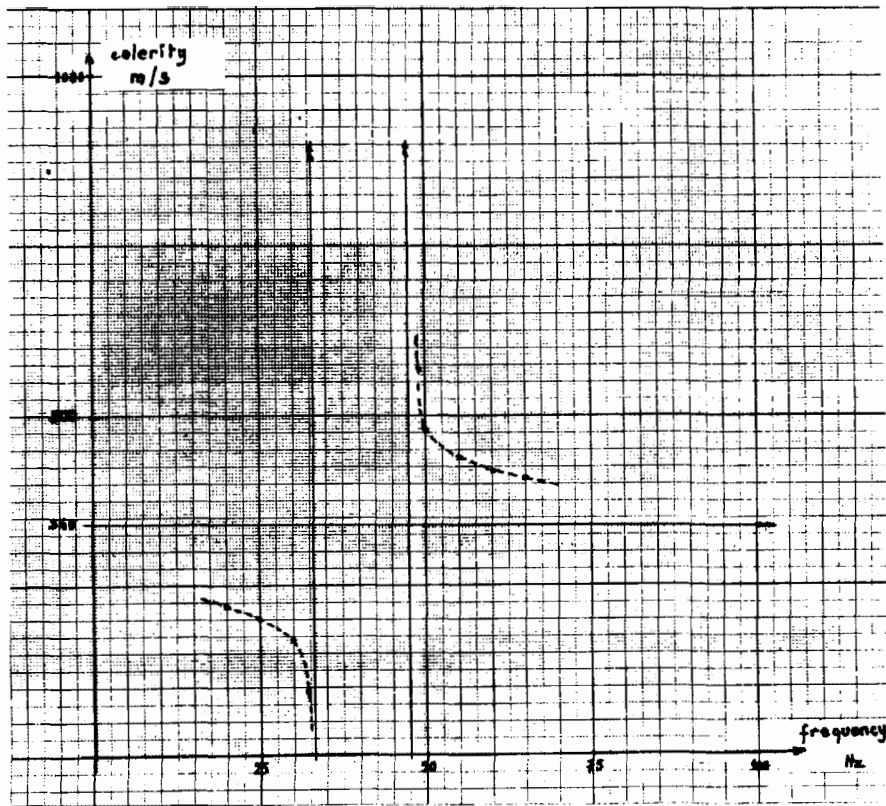


Figure 8. Numerical graph of celerity versus frequency [extracted from V. Martin, Comptes Rendus de l'Académie des Sciences, Paris, T 306, Serie II, 1988, p. 1-4]

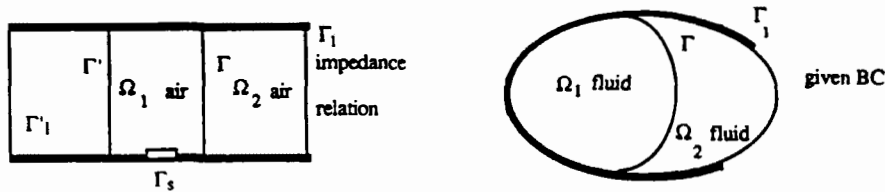


Figure 9. Original problem and equivalent geometrical configuration

Ω_1 and Ω_2 constitutes only a part of $\partial\Omega_2$ but, from a numerical point of view, we will show that it is possible to transfer Ω_2 to the interface Γ without resorting to substructuring processing, i.e. as if $\partial\Omega_2$ were identical to Γ .

The way by which Ω_2 is transferred to Γ is straightforward and the interfacing matrices are now familiar. Conversely, the basic matrices can only be obtained by analogy with Section 2.1 where Ω_2 was dealt with by the BIM. Once again for the sake of generality a source in Ω_2 is added to that of the original problem which was on the boundary of Ω_1 .

3.2.1. Equations of motion. Due to the radiation through Γ_1 , the problem is not conservative and the equations are as follows.

(a) For the compressible gas inside the domain Ω_1 , the unknown function is the acoustic pressure p_1 :

$$\begin{aligned} (\Delta + k^2)p_1 &= 0 & \text{in } \Omega_1 \\ \partial_n p_1 &= 0 & \text{on } \partial\Omega_1 \setminus (\Gamma \cup \Gamma' \cup \Gamma_s) \\ \partial_n p_1 &= f_1 & \text{on } \Gamma_s \end{aligned}$$

(b) For the compressible gas inside the domain Ω_2 , the unknown function is the acoustic pressure p_2 :

$$\begin{cases} (\Delta + k^2)p_2 = f_2 & \text{in } \Omega_2 \\ \partial_n p_2 = 0 & \text{on } \partial\Omega_2 \setminus (\Gamma \cup \Gamma' \cup \Gamma_l \cup \Gamma_i) \\ \partial_n p_2(x) + ik\beta(x)p_2(x) = 0 & \text{on } \Gamma_l \text{ and on } \Gamma_i \end{cases}$$

(c) The coupling is written on the interfaces Γ and Γ' :

continuity of the velocities, thus when the gas in Ω_1 and Ω_2 are the same: $\partial_n p_1 = \partial_n p_2$

continuity of the normal stresses: $p_1 = p_2$

3.2.2. *Methods.* The domain Ω_1 is processed using the FEM and thus the problem is written here in the variational form (weak formulation). Ω_2 is dealt with by the modal series. In Ω_1 we have

$$\int_{\Omega_1} (-\mathbf{grad} \delta p_1 \cdot \mathbf{grad} p_1 + k^2 \delta p_1 \cdot p_1) d\Omega + \int_{\Gamma} \delta p_1 \cdot \partial_n p_1 d\Gamma = - \int_{\Gamma_s} \delta p_1 f_1 d\Gamma$$

where Γ represents itself and Γ' . A nodal approximation leads to

$$\delta \{p_{1i}\}^T [A_1] \{p_{1i}\} + \int_{\Gamma} \delta p_1 \partial_n p_1 d\Gamma = \delta \{p_{1i}\}^T \{f_{1i}\}$$

In Ω_2 , we have (to the left of the abscissa of the source in Ω_2):

$$p_g(x, S) = \sum_{\sigma=1}^{\infty} (a_{\sigma} e^{-ik_{\sigma}x} \Phi_{\sigma}(S) + b_{\sigma} e^{+ik_{\sigma}x} \Phi_{\sigma}(S))$$

A modal approximation occurs with a truncation of the series of modes, leading to

$$p_g(x, S) = (\{a_{\sigma}\}^T [e^{-ik_{\sigma}x}] + \{b_{\sigma}\}^T [e^{+ik_{\sigma}x}]) \{\Phi_{\sigma}\}$$

where both matrices involved are diagonal and will be written $[e^{-x}]$ and $[e^{+x}]$, respectively.

3.2.3. *Transfer of the 'modal' fluid to the 'numerical' fluid (Figure 10).* On the right of the source, the pressure is written by

$$p_d(x, S) = \{c_{\sigma}\}^T [e^{-ik_{\sigma}x}] \{\Phi_{\sigma}\} + \{d_{\sigma}\}^T [e^{+ik_{\sigma}x}] \{\Phi_{\sigma}\}$$

or

$$p_d(x, S) = \{c_{\sigma}\}^T [e^{-x}] \{\Phi_{\sigma}\} + \{d_{\sigma}\}^T [e^{+x}] \{\Phi_{\sigma}\}$$

leading to the longitudinal derivative (∂_x) of the pressure:

$$\partial_x p_d(x, S) = \{c_{\sigma}\}^T [-ik_{\sigma} e^{-x}] \{\Phi_{\sigma}\} + \{d_{\sigma}\}^T [ik_{\sigma} e^{+x}] \{\Phi_{\sigma}\}$$

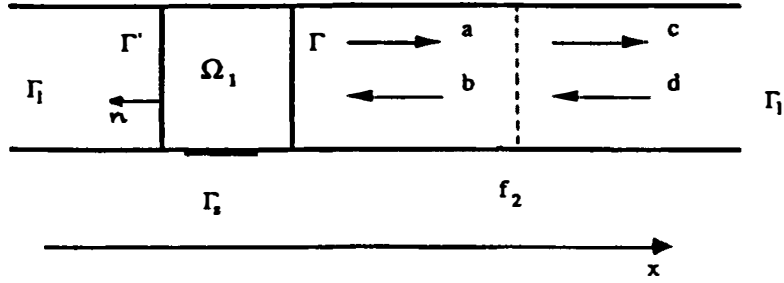


Figure 10. Propagation of models in the duct

Given the impedance relation on Γ_1 :

$$\partial_x p_d(x, S) + ik\beta(S)p_d(x, S) = 0, \quad x = l$$

results in

$$[-ik_\sigma \Lambda_\sigma \delta_{\mu\sigma} + ik\beta_{\mu\sigma}][e^{-l}]\{c\} = [-ik_\sigma \Lambda_\sigma \delta_{\mu\sigma} - ik\beta_{\mu\sigma}][e^{+l}]\{d\}$$

where

$$\beta_{\mu\sigma} = \beta_{\sigma\mu} = \int_S \beta(S) \Phi_\sigma \Phi_\mu dS \quad \text{and} \quad \Lambda_\sigma = \int_S \Phi_\sigma^2 dS$$

The matricial form $[\xi]\{c\} = [\delta]\{d\}$, with the symmetrical matrices $[\xi]$ and $[\delta]$ may be diagonal in some particular cases. Those cases which also lead to the information sought are now described.

At the abscissa $x = f$ of the source f_2 , the continuity of pressure p and jump of velocity $\partial_x p$ must be verified. Thus,

$$p_g(x, S) = \{a_\sigma\}^T [e^{-x}]\{\Phi_\sigma\} + \{b_\sigma\}^T [e^{+x}]\{\Phi_\sigma\}$$

leads to

$$\{b\} = [\delta]^{-1}[\xi] \left(\{a\} - [e^{+f}] \left\{ \frac{1}{2ik_\sigma} f_\sigma \right\} \right) - [e^{-f}] \left\{ \frac{1}{2ik_\sigma} f_\sigma \right\}$$

with

$$\{f_\sigma\} = \int \left\{ \frac{\Phi_\sigma}{\Lambda_\sigma} \right\} f dS \quad \text{and} \quad [e^{\pm f}] = [e^{\pm ik_\sigma x f}]$$

Now the pressure is expressed with the amplitudes of modes $\{a\}$ only:

$$p_g(x, s) = \{\Phi_\sigma\}^T ([e^{-x}] + [e^{+x}][\delta]^{-1}[\xi])\{a\} + \{\Phi_\sigma\}^T [e^{+x}][e^{-f}] \\ + [\delta]^{-1}[\xi][e^{+f}] \left\{ -\frac{1}{2ik_\sigma} f_\sigma \right\}$$

or

$$p_g(x, S) = \{\Phi_\sigma\}^T [a]\{a\} + \{\Phi_\sigma\}^T [e^{+x}]\{\mathcal{F}\}$$

and similarly,

$$\partial_x p_g(x, S) = \{\Phi_\sigma\}^T ([-ik_\sigma e^{-x}] + [ik_\sigma e^{+x}][\delta]^{-1}[\xi])\{a\} - \{\Phi_\sigma\}^T [ik_\sigma e^{+x}]\{\mathcal{F}\}$$

or

$$\partial_x p_g(x, S) = \{\Phi_\sigma\}^T [\mathcal{D}] \{a\} + \{\Phi_\sigma\}^T [\partial_x e^{+x}] \{\mathcal{F}\}$$

With the amplitudes of modes of p_g in $\{\mathcal{P}\}$ and those of $\partial_x p_g$ in $\{\partial\mathcal{P}\}$ we finally obtain the impedance relation sought. Indeed

$$\{\mathcal{P}\} = [\mathcal{J}] \{a\} + [e^{+x}] \{\mathcal{F}\}, \quad \{\partial\mathcal{P}\} = [\mathcal{D}] \{a\} + [\partial_x e^{+x}] \{\mathcal{F}\}$$

resulting in

$$\{\partial\mathcal{P}\} = [\mathcal{D}] [\mathcal{J}]^{-1} \{\mathcal{P}\} - ([\mathcal{D}] [\mathcal{J}]^{-1} [e^{+x}] - [\partial_x e^{+x}]) \{\mathcal{F}\}$$

The matrices $[\mathcal{D}]$ and $[\mathcal{J}]$ are diagonal and the matricial form obtained is that obtained by the integral representation. This allows us to define, by analogy, the basic matrices $[\mathcal{D}]$ and $[\mathcal{J}]$ satisfying

$$[\mathcal{D}] \{\mathcal{P}\} = [\mathcal{J}] \{\partial\mathcal{P}\} - \{\mathcal{F}\}$$

Now in the general case where $[\mathcal{D}]$ and $[\mathcal{J}]$ are nonsymmetrical, they produce the following steps. Let

$$\begin{aligned} [S_1] &= [e^{+f}] [\delta]^{-1} [\xi] + [e^{-f}], & [S_2] &= [e^{+f}] [\delta]^{-1} [\xi] - [e^{-f}] \\ [A] &= [e^{-f}] ([S_1] [S_2]^{-1} - [I])^{-1} ([S_1] [S_2]^{-1} + [I]) [e^{-f}] \\ [\mathcal{F}] &= [e^{-f}] ([S_1] [S_2]^{-1} - [I])^{-1} [S_1] [S_2]^{-1} \left\{ \frac{1}{ik_\sigma \Lambda_\sigma} \int_\Gamma \Phi_\sigma f_2 dS \right\} \end{aligned}$$

then

$$\{b\} = [\mathcal{J}] \{a\} + \{\mathcal{F}\}$$

and the basic matrices are

$$[\mathcal{D}] = [\partial_x e^{-x}] + [\partial_x e^{+x}] [A], \quad [\mathcal{J}] = [e^{-x}] + [e^{+x}] [A]$$

The coupling between Ω_1 and Ω_2 occurs by matching the pressure and its normal derivative on the fictitious boundary Γ . The same occurs on the left of Ω_1 . The variational form in Ω_1 is thus

$$\delta \{p_{1i}\}^T [A_1] \{p_{1i}\} + \delta \{p_{1i}\}^T \int_\Gamma \{N_i\} \langle \Phi_\sigma \rangle ds \{\partial\mathcal{P}\} = \delta \{p_{1i}\}^T \{f_{1i}\}$$

that is

$$\begin{aligned} [A_1] \{p_{1i}\} + [C_1]^T [\mathcal{D}] [\mathcal{J}]^{-1} [\Lambda]^{-1} [C_1] \{p_{1i}\} &= \{f_{1i}\} \\ + [C_1]^T ([\mathcal{D}] [\mathcal{J}]^{-1} [e^{+x}] - [\partial_x e^{+x}]) \{\mathcal{F}\} & \end{aligned}$$

or

$$([A_1] + [SE_1]) \{p_{1i}\} = \{\{f_{1i}\} + \{sf_{1i}\}\}$$

3.2.4. Calculation procedures. The main information added by this configuration is the definition of the basic matrices and, despite the fact that the interface is only a part of $\partial\Omega_2$, no substructuring processing is required here. Indeed the representation chosen in Ω_2 satisfies all boundary conditions except on Γ . This remark leads to the differentiation between the geometrical and the numerical partitions of $\partial\Omega_2$. It is the latter, closely linked to the numerical representation, which determines whether processing is necessary at the first stage. The illustration is given in Figure 12.

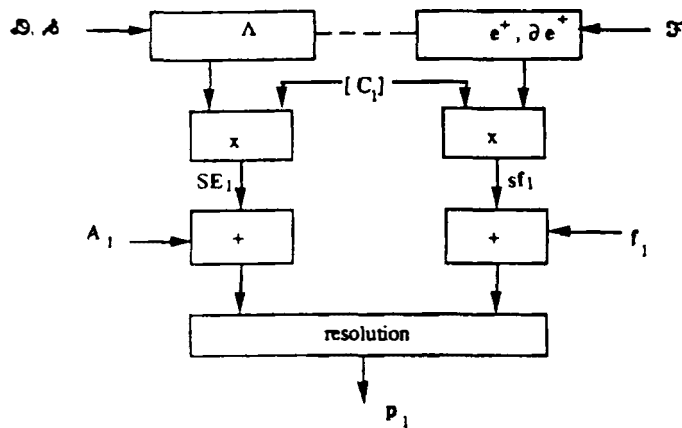


Figure 11. Flow chart corresponding to the equivalent geometrical configuration of Figure 9

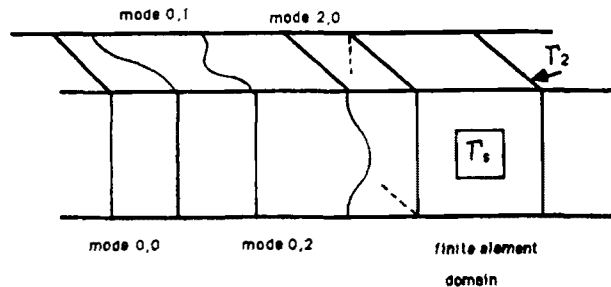


Figure 12. Combination of FEM and modal series in an acoustical guide (extracted from Reference 9). Infinite 3-D duct with a square source on the wall

Mode	Relative value (208 nodes)	Relative value (1,225 nodes)
0,0 ...	1.14 + i 0.90	1.06 + i 0.95
0,1 ...	0.89 + (i 1.)	0.95 + (i 1.)
0,2 ...	0.77 + (i 1.)	0.77 + (i 1.)
2,0 ...	0.56 + (i 1.)	1.12 + (i 1.)

The relative value is the real part (resp. the imaginary part) of the result obtained from the FEM with 208 and 1225 nodes, divided by the real (rep., the imaginary part) of the analytical solution. An exact numerical solution is then $1 + i$. The imaginary part of modes 1, 2 and 3 is necessarily 1, because there is no imaginary part either in the numerical solution nor in the analytical one.

3.3. Problem of guided acoustic wave in a duct with yielding walls and external radiation

Having obtained information about the speed against frequency for acoustic waves guided between flexural walls (cf. Section 3.1), it was of interest to observe the influence of the external radiation of the walls. It appears that the external load inevitably modifies the acoustic pressure level but does not bring significant changes to the dispersion of the wave, at least for light external fluids (Reference 12 and illustration at the end of this paragraph in Figure 15).

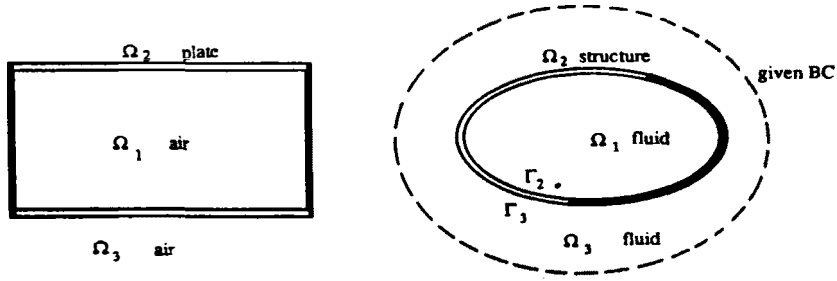


Figure 13. Original problem and equivalent geometrical configuration

The geometrical configuration (Figure 13) shows three domains: Ω_1 (inner fluid), Ω_2 (structure), Ω_3 (outer fluid load). The latter is fictitiously limited by a sphere of sufficiently large radius to write the local condition of outgoing waves along the normal to the boundary. The Green function satisfies the propagation in the outer fluid with this local condition. Now two interfaces exist: the interface between Ω_3 and Ω_2 is only a part of $\partial\Omega_3$ as Ω_3 loads the flexible structure as well as the rigid walls; the interface between Ω_2 and Ω_1 is identical to that in Section 3.1.

Both the compressible inner fluid Ω_1 and the structure Ω_2 are described by the FEM and Ω_3 is given an integral representation.

The interfacing matrices and the basic matrices are well known. The presence of a third medium adds steps to the flow chart obtained with the previous configurations. Each medium has its own source, for the sake of generality.

3.3.1. Equations of motion. Since the compressible fluid has, as a boundary condition, radiation at infinity, the global problem is not conservative and the total operator is complex. The equations of the problem are as follows:

(a) For the compressible fluid (gas) contained inside the cavity the unknown function is the acoustic pressure p_1 such that

$$\begin{aligned} (\Delta + k_1^2)p_1 &= f_1 && \text{in } \Omega_1 \\ \partial_n p_1 &= 0 && \text{on } \partial\Omega_1 \setminus \Gamma_1 \text{ (}\Gamma_1 \text{ inner surface of } \Omega_2\text{)} \\ \partial_n p_1 &= \rho_1 \omega^2 u_1 && \text{on } \Gamma_1 \end{aligned}$$

with u_1 normal displacement of the fluid particles on Γ_1

(b) The equation of the thin elastic structure is 2-D, the normal displacement w is the unknown variable. The excitation f_2 is applied on the outer surface of Ω_2 and the plates are clamped on their edges:

$$\begin{aligned} D\Delta\Delta w - \rho_2 \omega^2 w &= (p_1 - p_3) - f_2 && \text{in } \Omega_2 \text{ (or on } \Gamma, \text{ the middle-plane of the plates)} \\ w = \partial_n w &= 0 && \text{on } \partial\Gamma \end{aligned}$$

(c) For the compressible fluid (gas) outside the cavity, the unknown function is the acoustic pressure p_3 such that:

$$\begin{aligned} (\Delta + k_3^2)p_3 &= f_3 && \text{in } \Omega_3 \\ \partial_n p_3 &= 0 && \text{on } \partial\Omega_3 \setminus \Gamma_3 \text{ (}\Gamma_3 \text{ external face of } \Omega_2\text{)} \\ \partial_n p_3 &= \rho_3 \omega^2 u_3 && \text{on } \Gamma_3 \\ \lim_{r \rightarrow \infty} r (\partial_r p_3 + ik_3 p_3) &= 0 && \text{at infinity} \end{aligned}$$

with u_3 the normal displacement of the fluid particles on Γ_3

(d) The coupling of the media is written

$$\begin{aligned} u_1 = w \quad \text{on } \Gamma_1 \quad u_3 = w \quad \text{on } \Gamma_3 \\ \sigma \cdot n = p_1 n \quad \text{on } \Gamma_1, \quad \sigma \cdot n = -p_3 n \quad \text{on } \Gamma_3 \end{aligned}$$

3.3.2. *Methods.* Both the fluid domain Ω_1 and the structure Γ ($\equiv \Omega_2$) are described by the FEM and thus require a variational form. The unbounded fluid domain Ω_3 is processed by the BIM and needs an integral representation.

For the fluid inside, the complementary virtual work principle leads to

$$\int_{\Omega_1} (-\mathbf{grad} \delta p_1 \cdot \mathbf{grad} p_1 + k_1^2 \delta p_1 \cdot p_1) d\Omega + \int_{\Gamma} \delta p_1 \partial_n p_1 d\Gamma = \int_{\Omega_1} \delta p_1 \cdot f_1 d\Omega$$

and the nodal approximation results in

$$\delta \{p_{1i}\}^T [A_1] \{p_{1i}\} + \rho_1 \omega^2 \int_{\Gamma_1} \delta p_1 w d\Gamma = \delta \{p_{1i}\}^T \{f_{1i}\}$$

The structure is analysed by the principle of virtual work giving

$$\int_{\Gamma} \delta \{\varepsilon\}^T [D_p] \{\varepsilon\} d\Gamma - \rho_2 \omega^2 \int_{\Gamma} \delta w \cdot w d\Gamma = \int_{\Gamma} \delta w (p_1 - p_3) d\Gamma + \int_{\Gamma} \delta w \cdot f_2 d\Gamma$$

and the nodal approximation allows the matricial form:

$$\delta \{a_i\}^T [A_2] \{a_i\} = \int_{\Gamma} \delta w (p_1 - p_3) d\Gamma + \delta \{a_i\}^T \{f_{2i}\}$$

The outside unbounded fluid Ω_3 has the integral representation:

$$\frac{1}{2} p_3(x) + \int_{\partial\Omega_3} p_3 \partial_n G d\Gamma = \int_{\Gamma_3} G \partial_n p_3 d\Gamma + \int_{\Omega_3} G f_3 d\Omega$$

(n inwards directed normal) where $x \in \partial\Omega_3$ and G is the Green function for the Helmholtz equation with Sommerfeld condition at infinity. Processing by collocation on each facet leads to

$$[D_3] \{p_{3c}\} = \rho_3 \omega^2 [S_3] \{u_{3c}\} + \{f_{3c}\}$$

3.3.3. *Transfer of the external fluid and of the structure to the inner fluid.* Ω_3 loads Ω_2 , itself loading Ω_1 . The transfer of Ω_3 to Γ_3 takes into account the fact that Γ_3 is a part of $\partial\Omega_3$ and processing is needed. From

$$[D] \begin{Bmatrix} \{p_B\} \\ \{p_{\Gamma}\} \end{Bmatrix} = [S] \{\partial_n p_{\Gamma}\} + \begin{Bmatrix} \{f_B\} \\ \{f_{\Gamma}\} \end{Bmatrix}$$

where $B \equiv \partial\Omega_3 \setminus \Gamma$, it results in, exactly as in Section 2.1:

$$\{p_{\Gamma_3}\} + [U] \{\partial_n p_{\Gamma_3}\} = [T] \{f_B\} + [R] \{f_{\Gamma}\}$$

All matrices have the same form as in Section 2.1 except for the change of sign in $[U]$ due to the inward normal. Bearing in mind from Section 3.1 that

$$\int_{\Gamma} \delta w p_1 = \{\delta a_i\}^T [C_{i1}] \{p_{1i}\}$$

we obtain

$$([A_2] - \rho_3 \omega^2 [C_2]^T [Q] [U] [C_2]) \{a_i\} = [C_1] \{p_{1i}\} - [C_2]^T [Q] ([T] \{f_B\} + [R] \{f_r\}) + \{f_{2i}\}$$

or

$$([A_2] + [SE_2]) \{a_i\} = [C_1] \{p_{1i}\} + (\{sf_2\} + \{f_{2i}\})$$

The transfer of the loaded structure to the inner fluid gives:

$$[A_1] \{p_{1i}\} + \rho_1 \omega^2 [C_1]^T ([A_2] + [SE_2])^{-1} ([C_1] \{p_{1i}\} + \{f_{2i}\} + \{sf_2\}) = \{f_{1i}\}$$

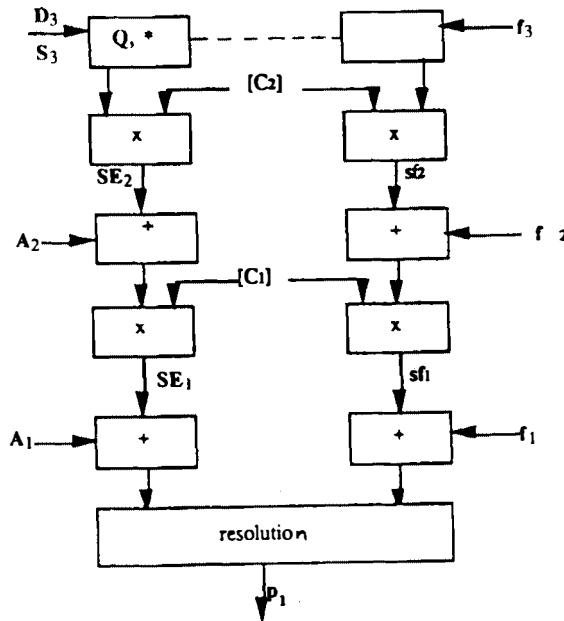


Figure 14. Flow chart corresponding to the equivalent geometrical configuration of Figure 13

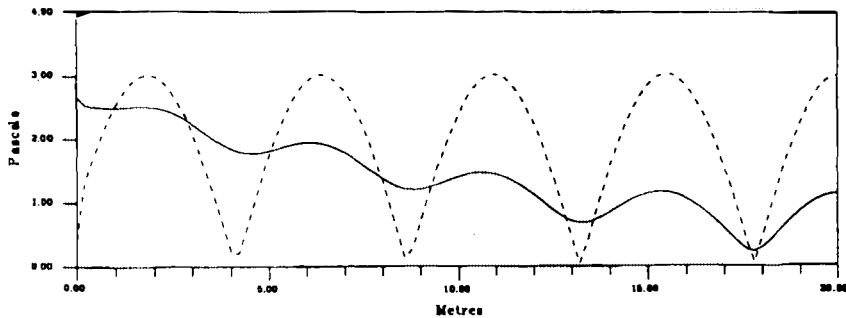


Figure 15. Numerical field of acoustic pressure inside a guide with and without external load (extracted from Reference 12). Absolute value of the inside pressure against the distance from the source without (a) and with (b) external load for a frequency in the supersonic zone

or

$$([A_1] + \rho_1 \omega^2 [C_1])^T ([A_2] + [SE_2])^{-1} [C_1] \{p_{1i}\} = -\rho_1 \omega^2 [C_1]^T ([A_2] + [SE_2])^{-1} (\{f_{2i}\} + \{sf_2\}) + \{f_{1i}\}$$

written

$$([A_1] + [SE_1]) \{p_{1i}\} = \{sf_1\} + \{f_{1i}\}$$

3.3.4. Calculation procedures. The number of steps reflects the three media and their loads in series. As Γ_3 is only a part of $\partial\Omega_3$, processing occurs at the first stage. Subsequently, this is no longer necessary due to the fact that all Γ_1 loads Ω_1 . The interfacing matrices and the basic matrices are well known (Figure 14). The illustration is given in Figure 15.

4. GENERALIZATION OF THE CALCULATION PROCESS

With the help of a fictitious industrial example, it is now shown how the elements extracted from the previous problems can be used to elaborate directly the calculation procedures in the case of an actual complex problem. The less common case of preponderant fluid is envisaged.

The context is the vibro-acoustic modelling of an aircraft in order to study the acoustic field in the passengers' cabin. It is assumed that a preliminary study has led to the conclusion that it is possible to ignore the vibration of the wings, the floor, the cockpit and the rear of the plane. On the contrary, the external load, the fuselage and the rear bulkhead all have an important influence on the internal acoustic field. The static pressure in the cabin is almost the same as the atmospheric pressure, while it is weak in the hold and neglected behind the rear bulkhead (the problem, almost totally invented only for the sake of flow chart use, must certainly not be taken to describe a real aircraft). Finally, the object under study is as follows (see Figure 16).

The only effort that has to be made is in the equivalent geometrical configuration to show how the various media are related. The external fluid loads both rigid and flexible structures. The rigid

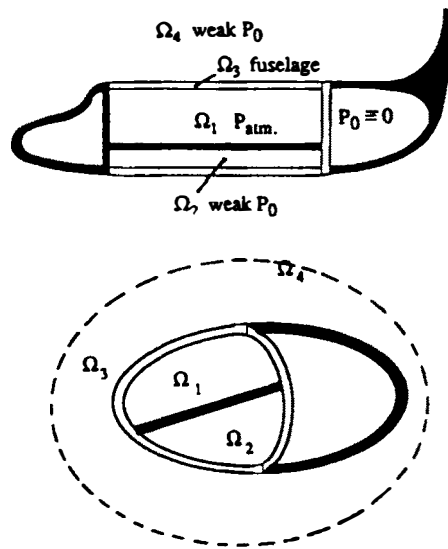


Figure 16. Original problem and equivalent geometrical configuration

floor, which separates the internal cavity of the passengers' cabin and that of the hold, is attached to the vibrating rear bulkhead and to the vibrating fuselage. The fuselage itself is linked to the rigid parts of the total structure. The equivalent geometrical configuration would thus be as in Figure 16.

At this stage, it is observed that the primary domain Ω_1 is loaded by the structure Ω_3 , whose vibratory behaviour is influenced by the hold and by the external fluid. It is clear that only a part of $\partial\Omega_4$ loads Ω_3 , that only a part of $\partial\Omega_2$ loads Ω_3 , that only a part of $\partial\Omega_3$ loads Ω_1 . The flow chart can already be drawn.

The mathematical representations of the media are as follows: FEM for the structure and for the passengers' cabin; BIM for the external fluid and the hold. The BIM and FEM will not modify the partial geometrical transfers and substructurations in the basic matrices are expected. The calculation procedure is as Figure 17.

The excitation may be due to sources in Ω_4 . It is also possible to insert a natural excitation in terms of forces in a structure described in terms of displacements, and a natural excitation in terms of displacements in the cabin described in terms of pressure (force), eventually to add excitation in Ω_2 .

Every component of the diagram has been explicated here and it seems that no other element need be mentioned to reach our objective, i.e. to proceed directly from a complex problem of coupling to the appropriate flow chart. The example given will hopefully convince the reader of the veracity of this assertion.

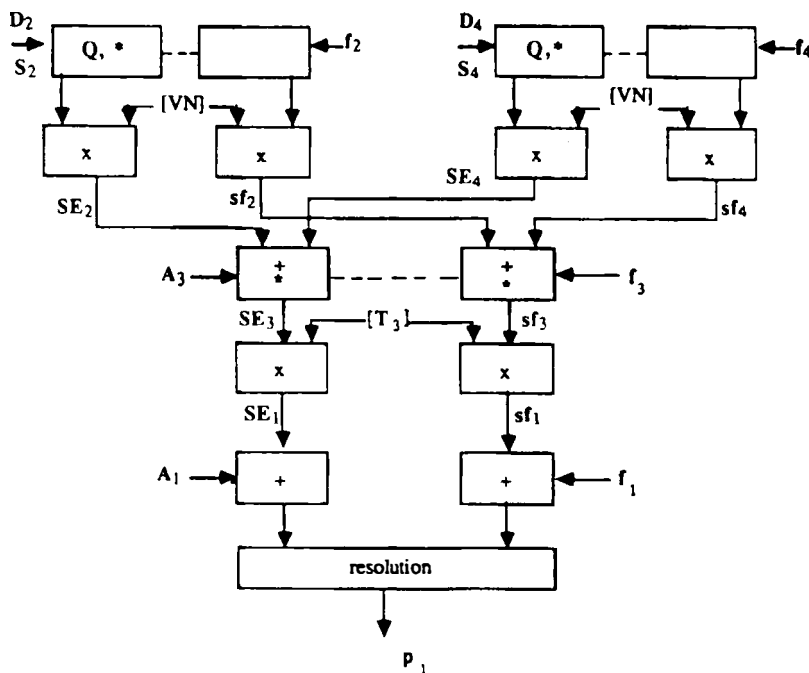


Figure 17. Flow chart corresponding to the equivalent geometrical configuration of Figure 16

5. CONCLUSION

The single procedure for calculations proposed here concerns linear coupling problems of media, fluid and structure in particular, with a privileged domain.

The numerical studies are intended to describe acoustic or vibratory fields in the dynamic harmonic range and do not deal with eigenvalues and eigenvectors. Within this framework, calculation procedures and interface characteristics have been brought into evidence. The usefulness of the work presented lies in the direct definition of the complex calculation procedure simply from the equivalent geometrical configuration of the problem. The only effort to be made now is in the drawing of the equivalent geometry when media are intricately mixed as shown in the fictitious industrial example. Moreover, the close relationship between the geometrical configuration and mathematical representation chosen, regarding the definition of total or partial transfer of one medium, has been emphasized.

Using the flow chart obtained and the current methods (FEM, BIM, Modal Series), we have worked on the case of essential excitations—here only the natural excitations were described—in a fluid/structure problem where the fluid is the medium of interest and where the structure is excited by a displacement.¹⁴ The fact that here also the charts require no modifications, adds to the general applicability of the procedure.

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