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Boundary Integral Representations in the Theory of Elastoplasticity for Domains with Edge and Corner Singularities

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Introduction

The present work is devoted to the boundary formulation for an isotropic body with edge and corner singularities subjected to a distribution of initial strains (or initial stresses). The elastic potentials and their derivatives are smooth almost everywhere except at the points on the surface where the densities are defined [6]. They can be discontinuous through the surface and a closer study shows that their behaviour depends on the property of the point on the surface (smooth or non-smooth) and on the regularity of the surface densities (displacements and tractions in the direct boundary element method). The behaviour of the potentials at smooth points is well known [12, 1, 3, 4, 7, 9, 15]. The purpose of this paper is to extend these results to non-smooth points. Additionally, a new representation of the boundary displacement gradient is given, which gives rise to a new Traction Boundary Integral Equation (TBIE).

Main results

First let us recall some basic results for smooth points. Let Ω be an open bounded domain with boundary S , where S is a Liapunov manifold ($S \in C^{1,\beta}$ with $\beta \in]0, 1]$). If $\tilde{u} \in C^{1,\alpha}(S)$ and $\tilde{t} \in C^{0,\alpha}(S)$ with $\alpha \in]0, \beta]$, the first and double layer potential and their derivatives can be uniformly extended to S [4, 8, 12]. The Hölder continuity of the manifold or of the surface densities is sufficient for numerical purposes. Actually a Dini's condition is enough if one looks for a necessary and sufficient condition [8, 13].

Here we consider the case of a piecewise Liapunov's manifold S . One assumes that Ω possesses the inner and outer cone property [8, 10]. Both elastic potentials can then be uniformly extended to S provided that $\tilde{u} \in C^{0,\alpha}(S)$ and $\tilde{t} \in L^\infty(S)$ with $\alpha \in]0, 1]$ [10, 11]. Some integral representations of the gradient of the displacement field are given in [3, 2, 9]. However, there exists no results concerning the displacement gradient in non-smooth domains. Furthermore, no regularity conditions on the surface densities are provided.

The main difficulty comes from the fact that the class $C^{1,\alpha}(S)$ cannot be defined over a Lipschitz region. the same problem arises when the trace on the boundary has to be defined in Sobolev spaces [5]. One can also note that from a physical point of view, the traction \tilde{t} is not continuous on S .

Let v be a vector field defined in Ω , $v \in C^{1,\alpha}(\Omega)$. The restriction of ∇v on the boundary S is then well defined and belongs to $C^{0,\alpha}(S)$ whereas neither the surface derivatives Dv nor $\partial_n v$ belongs to $C^{0,\alpha}(S)$. Accordingly, it will be assumed that:

$$\tilde{u}|_{S_i} \in C^{1,\alpha}(S_i) \text{ and } \tilde{t}|_{S_i} \in C^{0,\alpha}(S_i) \text{ where } S = \bigcup_{i=1}^{i=N} \tilde{S}_i \quad (1)$$

By introducing the notations $\tilde{u} = u|_S$ and $\tilde{t} = t_S$ and by the Hooke law, one can express the normal component of $\widetilde{\nabla}u$ which gives [3, 1]:

$$\widetilde{\partial_n u} = \frac{1}{\mu} \left(I - \frac{n \otimes n}{2(1-\nu)} \right) \cdot \tilde{t} - \frac{\nu}{1-2\nu} (\text{div}_S \tilde{u})n - n \cdot D\tilde{u} \quad (2)$$

One can then define $\widetilde{\nabla}u|_{S_i}$ on each S_i by $\widetilde{\nabla}u = D\tilde{u} + \widetilde{\partial_n u} \otimes n$ and introduce the "reconstituted boundary stress field" $\tilde{\sigma}$ given by $\tilde{\sigma} = C : \widetilde{\nabla}u$. Then the gradient of a displacement field satisfying the Navier-Cauchy equations in $R^3 \setminus S$ admits the following boundary integral representation when $\widetilde{\nabla}u \in C^{0,\alpha}(S)$:

$$u_{k,l}(x) = \int_S D_{ij} U_i^k(x,y) \tilde{\sigma}_{ij}(y) - \int_S T_i^k(x,y) \widetilde{\nabla}u_{il}(y) dS(y) + (\delta_{ki} - C_{ki}) \widetilde{\nabla}u_{il}(z) \quad (3)$$

where $z \in S$ and the integrals have to be understood in the principal value sense (for the definition of the free-term C , see [10]). One can also obtain the regularized form:

$$u_{k,l}(z) = \int_S D_{ij} U_i^k(z,y) [\tilde{\sigma}_{ij}(y) - \tilde{\sigma}_{ij}(z)] dS(y) - \int_S T_i^k(z,y) [\widetilde{\nabla}u_{il}(y) - \widetilde{\nabla}u_{il}(z)] dS(y) + \widetilde{\nabla}u_{kl}(z) \quad (4)$$

A similar reasoning allows us to obtain an integral representation on the boundary S of the gradient displacement field for a problem with initial strains ε^p when $\widetilde{\nabla}u_{ij}(y) \in C^{0,\alpha}(S)$ and $\varepsilon_{ij}^p(y) \in C^{0,\alpha}(\Omega)$. For the numerical

implementation, an integration by parts can be performed in the first integral. This boundary stress representations gives then rise to a TBIE in which the boundary integro-differential kernels are $U(z, y)$ and $T(z, y)$.

Conclusion

An original boundary integral representation for the displacement gradient has been established in the case of domains with edges and corners. These results have also been generalized for problems with initial strains or stresses and are helpful to determine the associated boundary integral equations of the problem.

At regular points, the simple and double layer potentials can be considered separately when studying the limit behaviour of the displacement gradient. On the contrary, the situation is more intricate at singular points. Since neither the surface derivative Du nor the stress vector t can be assumed to belong to $C^{0,\alpha}(S)$, it turns out that introducing the "reconstituted boundary displacement gradient" is an appropriate way to obtain a boundary integral representation and a new TBIE.

References

- [1] BALAS J., SLADEK J., SLADEK V., *Stress Analysis by Boundary Element Methods*, Studies in Applied Mechanics, Elsevier, 1989.
- [2] BONNET M., BUI H.D., Regularization of the displacement and traction for 3D elastodynamics using indirect methods, In *Avances in Boundary Element Techniques*, eds. J.H. Kane, G. Maier, N. Tosaka, S.N. Atluri, Sringer Verlag, 1993.
- [3] BONNET M., *Equations Intégrales et Eléments de Frontière*, Applications en mécanique des solides et des fluides, CNRS Editions, Eyrolles, 1995.
- [4] CHEN G., ZHOU J., *Boundary Element Methods*, Computational Mathematics and Applications, Academic Press, 1992.
- [5] COSTABEL M., Boundary integral operators on Lipschitz domains : elementary results, *SIAM J. Math. Anal.*, **19**, pp 613-626, 1988.
- [6] COSTABEL M., WENDLAND W., Strong ellipticity of boundary integral operators, *J. Reine Angew. Math.*, **372**, pp 34-63, 1986.
- [7] CRUSE T.A., SOWITO W., On the Somogiana stress identity in elasticity, *Comput. Mech.*, **11**, pp 1-10, 1993.
- [8] DAUTRAY R., LIONS J.L., *Analyse Mathématique et Calcul Numérique*, Vol. 2 : l'Opérateur de Laplace, Masson, 1989.
- [9] GUIGGIANI M., Hypersingular formulation for boundary stress evaluation, *Engng. Anal. with Bound. Elem.*, **13**, pp. 169-179, 1994.
- [10] HARTMANN F., The Somogiana identity on piecewise smooth surfaces, *J. of Elasticity.*, **11**, pp. 403-423, 1981.
- [11] HARTMANN F., Elastic potentials on piecewise smooth surfaces, *J. of Elasticity.*, **12**, pp. 31-50, 1982.
- [12] KUPRADZE V.D., GEGELIA T.G., BASHELEISHVILI M.O., BURCHULADZE T.V., *Three-dimensional problems of the mathematical theory of elasticity and thermoelasticity*, North-Holland, 1989.
- [13] MIKHLIN S.G., PRÖSSDORF S., *Singular Integral Operators*, Springer, 1986.
- [14] POLCH E.Z., CRUSE T.A., HUANG C.-J., Traction BIE solutions for flat cracks, *Comput. Mech.*, **2**, pp. 253-267, 1987.
- [15] SLADEK V., SLADEK J., LE VAN A., Completely regularized integral representations and integral equations for anisotropic bodies with initial strains, *Z.A.M.M.*, to appear, 1998.