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A Directed Search Many Objective Optimization Algorithm Embodied with Kernel Clustering strategy

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Abstract. With the vast existence of multi-objective optimization problems to the scientific research and engineering applications, Many-objective Evolutionary Algorithms (MaOEAs) demand to systematically perpetuate population diversity and convergence distributions in the objective space with high dimensionality. To fulfill the balance in the relationship between convergence, distributions, and diversity, this paper proposes a directed search many-objective optimization algorithm embodied with kernel clustering strategy (DSMOA-KCS) in decision space where some mechanisms such as adaptive environmental selection which efficiently assimilates design for control of diversity and convergence in the distribution of the solutions in the decision scopes. DSMOA-KCS is a stochastic, multi-start algorithm using clustering to increase efficiency. DSMOA-KCS finds the starting point in the regions of interest. Then, it improves them by the directed search method. DSMOA-KCS is compared with several existing state-of-the-art algorithms (NSGA-III, RSEA, and MOEADPas) on many-objective problems with 5 to 30 objective functions using the Inverted Generational Distance (IGD) performance metric. DSMOA-KCS evaluation results illustrate that it is competitive and promising, performing better with some problems. Then, even distribution, convergence, and diversity are maintained.

Keywords: Many-objective Optimization · Kernel Cluster · Diversity and convergence · Evolutionary algorithm.

1 Introduction

Evolutionary Many-objective optimization has become a popular and influential research field in recent years. Its application to real-world problems has significantly been shown. That is to say, many practical problems can be defined as Many objective problems (MaOPs), such as time series learning [4] and engineering design [3]. Moreover, many optimization problems refer to multi-objective setbacks with more than three objectives.

$$\begin{array}{ll} Minimize & f(x) = (f_1(x), f_2(x), \cdots, f_M(x))^T\\ Subject & to & x \in \Omega \end{array}$$
(1)

where $x = (x_1, x_2, \ldots, x_n)^T$ is the n-dimensional decision vector in the decision space $\Omega, M > 3$ the number of objectives, and f(x) is the M-dimensional objective vector. While solving MaOPs with many Objective Evolutionary Algorithms, there is an incomparability of solutions that is caused due to the proportion of non-dominated solutions increasing significantly [7], [2]. This makes optimization using only the dominance relationship infeasible and challenging to maintain population diversity in a high-dimensional objective space.

Researchers presented many solutions, most of which fall into three categories. First, Pareto dominance-based MaOEAs use modified Pareto mechanisms to identify non-dominated solutions. 2-dominance and fuzzy dominance modify dominance definitions to sustain selection pressure. Divergent distance computations improve Pareto-based MaOEAs [9]. Zhang et al. used a knee-point-based selection approach [11]. Li et al. suggested a shift-based density estimation technique for many-objective optimization [8]. Indicator-based MaOEAs evaluate solutions and steer search processes. IGD is a popular indicator. Hyper Volume (HV) and R2 are also noteworthy. Decomposition-based MaOEAs decompose a MOP into SOPs or simple MOPs to be solved collectively. Certain MOEAs, such as RVEA [1] and MOEA/D [10], decompose a MOP into SOPs. NSGA-III [5], and SPEA, based on reference direction [6], decompose a MOP into multiple simpler MOPs by subdividing the objective space.

2 The Proposed Method

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The DSMOA-KCS evolutionary process is restarted when the best solution is reached; if not after a specific number of generations, it is denoted by parameters. Each evolution of DSMOA-KCS starts from a different initial population. Fig.1 shows how the number of leading chromosomes r changes over time.



Fig. 1: Flowchart of the proposed DSMOA-KCS

Fig.1's I parameter indicates the number of generations in which the best solutions are found and improved. Current and provided computing times are t and T. Every evolution involves chromosomal transmission. g indicates the number of chromosomes. Population size (p), crossover rate (c), mutation (m), elite count 1 (g), elite count 2 (r), and adaptive restarting condition (s) are methodically tweaked to improve directed sampling search outcomes. The crossover rate (c) is improved using local directed search, mutation, and environmental selection. A mating pool P' is formed by selecting the union P (Population size) and CSA (Corner Solution Archive) in algorithm 2. It specifies the convergence of solutions by randomly selecting two solutions from P' each time results are needed.

2.1 Directed search sampling and guiding solutions

Firstly this paper takes into consideration unconstrained Many objective problems.

$$\begin{array}{ll}
Min & F(x) \\
x \in \mathbb{R}^n
\end{array} \tag{2}$$

where F is defined as the vector of objective functions $F : \mathbb{R}^n \to \mathbb{R}^k$, $F(x) = (f_1(x), \ldots, f_k(x))^T$, and each objective $f_i : \mathbb{R}^n \to \mathbb{R}$ is smooth. The optimality of MaOEAs is defined by the dominance of a vector $y \in \mathbb{R}^n$ dominated by vector $x \in \mathbb{R}^n (x \prec y)$ with respect to MaOP if $f_i(x) < f_i(y)$ or else y is dominated by x. Point $x \in \mathbb{R}^k$ is a Pareto point if there exists no $y \in \mathbb{R}^n$ that dominates x. The set of all Pareto optimal solutions is symbolized by P. The F(p) is the Pareto front. Using the Jacobian of F at point x is illustrated by

$$J(x) = \begin{pmatrix} \nabla f_i(x)^T \\ \vdots \\ \nabla f_k(x)^T \end{pmatrix} \in \mathbb{R}^{k \times n}$$
(3)

where $\nabla f_i(x)$ symbolizes the gradient of the objective f_i . If the MaOEAs are differentiable then this necessitates condition for Pareto optimality of unconstrained MaOPs. Assumption point $x_0 \in \mathbb{R}^n$ with rank $(j(x_0)) = k$ is given and vector $d \in \mathbb{R}^k$ showing the desired search direction in objective space. Then the search direction $v \in \mathbb{R}^n$ in the decision space is sought such that for $y_0 := x_0 + hv$, where $h \in \mathbb{R}_+$ is the step size that represents the movement from x_0 in the direction v and as shown in equation 4 below with y_0 representing the movement from x_0 in direction of V:

$$\lim_{h \searrow 0} \frac{f_i(y_0) - f_i(x_0)}{h} = \{ \nabla f_i(x_0), v \} = d_i, i = 1, \cdots, k, if ||v|| = 1$$
(4)

In this research paper |.| represents the 2-norm. F in equation 4 is stated as matrix vector notations as

$$J(x_0)v = d \tag{5}$$

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Solving linear equations yields the search direction v. Considering the number of decision variables is substantially higher than the number of objectives for a particular MaOP, n >> k is uncertain, implying the solution is not unique; hence, the algorithm chooses

$$v_{+} := J(x_{0})^{+}d \tag{6}$$

where $J(x_0)^+ \in \mathbb{R}^{k \times n}$ symbolizes the pseudo inverse of $J(x_0)$ with candidate solution of x_0 obtaining a new solution by $x_1 = x_0 + hv$ where $v \in \mathbb{R}^n$ is the vector satisfying equation 5. v_{+} is the solution of equation 5 with the nominal Euclidean norm. With h, progress in direction d is expected to be significant in the objective space. Guiding solutions are aimed to help MaOEAs accelerate convergence. DSMOA-KCS divides search space using w'. Identify the solutions closest to the ideal point of the objective space, then draw a line from the lower upper bound points to the solution in the choice space. DSMOA-KCS ends with a calculation. Encoding, crossover, mutation, and evaluation are not included because they are problem-dependent. As directed search sampling implies, this study selects elite individuals, elite count 1 (q) and elite count 2 (r), to integrate and strengthen the exploration and exploitation capabilities of the suggested algorithm to assess individual quality. DSMOA-KCS generates possible solutions by sampling a problem space. Local searches from good sample points can provide local optima. Points lead to unknown, potentially better local optima. Algorithm 1 shows the algorithm's highest level of abstraction after we have examined its inspiration and ideas.

Algorithm 1: Framework of proposed DSMOA-KCS

	Input: $F : \mathbb{R}^n \to \mathcal{R}$.					
	$a, \hat{b} \in \mathbb{R}^n$: bottom, and upper bounds					
1	return Value $opt \in \mathbb{R}^n$: global minimum candidate					
2	$i \leftarrow 1, N \leftarrow 126, \lambda \leftarrow 0.5, opt \leftarrow \infty$					
3	new, unclustered, reduced, clustered $\leftarrow \{\}$					
4	while stopping criteria is false do					
5	new \leftarrow new \cup generate N sample from $[a, b]$ distributed uniformly					
6	merged \leftarrow sort clustered \cup new by ascending order regarding F					
7	$\text{last} \leftarrow i \cdot N \cdot \lambda$					
8	reduced \leftarrow select $[0, \ldots, last]$ element from merged					
9	$x^* \leftarrow \text{select } [0] \text{ element from reduced}$					
10	$opt \leftarrow minimum \text{ of } \{opt, x^*\}$					
11	clustered, unclustered \leftarrow cluster reduced					
12	$new \leftarrow \{\}$					
13	while size of unclustered > 0 do					
14	$x \leftarrow \text{pop from unclustered}$					
15	$x^* \leftarrow \text{local search over } F \text{ from } x \text{ within } [a, b]$					
16	$opt \leftarrow minimum \text{ of } \{opt, x^*\}$					
17	cluster x^*					
18	if x^* is not clustered then					
19	create cluster from $\{x^*, x\}$					
20	end					
21	end					
22	$i \leftarrow i + 1$					
23	23 end					
24	24 return opt.					
	-					

2.2 Environmental selection

Algorithm 2 outlines environmental selection. The reproduction by corner solution Archive uses guiding solutions to speed up and balance convergence and variety. P and E are unioned to find non-dominated solutions. N is greater than the stated number of non-dominated answers. Each individual of the parent population P will execute a crossover with randomly selected guiding solutions, generating |P| candidate solutions. A mutation is conducted on solutions to get an intermediate offspring population, joined with a guiding solution set to form a combined population. The environmental selection algorithm 2 uses kernel matrix calculation.

Algorithm 2: Environmental selection process

Input: P (population), E (Expectant Population), N (population size), CSA (Corner Solution Archive), $z^*(ideal point)$, z^{nad} (nadir point) $\mathbf{1} \ P \leftarrow \text{Non-dominated} \ P \stackrel{\cdot}{\cup} E$ 2 if |P| > N then Normalization (CSA, Z^*, Z^{nad}) з Normalization (P, Z^*, Z^{nad}) 4 $L \leftarrow \text{Kernel Matrix calculation } (P, CSA)$ 5 $A \leftarrow Mutation/crossover operation (L, N)$ 6 $P \leftarrow P(A)$ 7 8 \mathbf{end} Output: P.

Matrix L incorporates convergence and population diversity. CSA (Corner Solution Archive) differentiates inside and outside Pareto space, finds corner solutions of objective functions, and sorts solutions in levitating order of objective space. First, environmental selection produces offspring after union selection with the original population. Pareto dominance, crowding distance, and normalized solutions are combined. If (|P| > N), the non-dominated front is measured as a critical front, and solutions are picked from the Corner Solution Archive (CSA). The kernel matrix L illustrates population convergence and diversity. Equation 7 is used to calculate $L_x y$.

$$L_{xy} = q(x)s(x,y)q(y) \tag{7}$$

where $x, y \in P, q(x)$ shows the solution x quality and S(x, y) is the similarity between x and y defined by equation 8 below

$$S(x,y) = exp(-cos(x,y))$$
(8)

where cos(x, y) is cosine of angles between solutions x and y. The quality of q(x) of solution x is calculated basing on its convergence, as illustrated in equations 9 and 10.

$$q(x) = \begin{cases} con_1(x)x \in \text{outside space} \\ 2^* \max_{p \in P} (con_1(P))x \in \text{inside space} \end{cases}$$
(9)

$$on_1(x) = \frac{con(x)}{\max_{p \in P}(con(P))}$$
(10)

where con_1 is the normalized convergence. Outside space and inside space describe the different areas of the objective space. If $\sqrt{\sum_{i=1}^{M} f_i(x)^2} \le t$ the solution x belongs to the inside space or otherwise to the outside space. The threshold t is set to $t = \max\left\{\sqrt{\sum_{i=1}^{M} f_i(x)^2} | x \in CSA\right\}$. Corner Solution Archive (CSA)

c

differentiates the outside space and inside space. This paper uses the approximation method to initiate the CSA regarding the value of k, leading in two situations. k = 1: to find the corner solutions of objective $i = 1, 2, \ldots, M$ proposed algorithm classifies the solutions in ascending order of objective value f_i so we get sorted lists and add the first $\lceil \frac{N}{3M} \rceil$ solutions of each list into the CSA. 1 < K < M: With consideration of k = M - 1 an approximation method to attain CSA is used. with any objective $i = 1, 2, \ldots, M$, solutions are sorted in ascending criteria of $\sqrt{\sum_{j=1,k\neq i}^{M} (f_j(x))^2}$ and attain M sorted lists. The initial solutions of each list is selected in the CSA. With the above two situations, $|CSA| = \lceil \frac{N}{3M} \rceil \times M + \lceil \frac{2N}{3M} \rceil \times M \approx N$ is obtained. Calculating the Kernel matrix is done after calculating the cosine of the angle between every two solutions in the population and the quality q(x) of each solution x. Row vector q then accommodates the qualities of all solutions, after which a quality matrix Q is generated as the product of q^T and q. Decisively, Q is multiplied with L in an element-wise manner to revise and output L.

3 Experimental Results and Analysis

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The proposed algorithm is compared to state-of-the-art algorithms NSGA-III, RSEA, and MOEADPas on a set of benchmark problems with 5, 10, 13, and 15 objective functions.



Fig. 2: Pareto front distributions of optimal solutions comparisons of DTLZ1

Fig.2 shows the parallel coordinate plots and regular Pareto fronts of (a) DSMOA-KCS, (b) MOEADPaS, (c) NSGA-III, (d) RSEA, and (e) as DTLZ1's True Pareto Front. The x - axis symbolizes the objective numbers, and the y - axis represents the objective values. It can also be seen intuitively in Fig.2 that the convergence value ranges from 1-2 for the algorithms DSMOA-KCS, RSEA, and NSGA-III with corresponding consistent values of the True Pareto Front.

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Problem	Μ	MOEADPaS	NSGA-III	RSEA	DSMOA-KCS
DTLZ1	5	5.1984e+0 (7.83e+0) -	6.3430e-2 (1.63e-4) -	6.3430e-2 (1.63e-4) -	6.3395e-2 (1.28e-4)
DTLZ1	10	1.6379e+1 (1.10e+1) -	1.3664e-1 (1.28e-2) -	1.4493e-1 (2.49e-2) =	1.1553e-1 (1.62e-2)
DTLZ1	13	2.5126e+1 (6.74e+0) -	1.4782e-1 (3.77e-2) -	1.9635e-1 (4.02e-2) -	1.3470e-1 (7.78e-3)
DTLZ1	15	2.3236e+1 $(1.04e+1)$ -	1.5283e-1 (3.52e-2) =	1.9237e-1 (2.04e-2) -	1.4904e-1 (2.87e-2)
DTLZ2	5	3.0698e-1 (9.93e-2) -	1.9489e-1 (1.94e-5) =	2.4765e-1 (1.88e-2) -	1.9489e-1 (1.69e-5)
DTLZ2	10	1.1756e+0 (1.51e-1) -	4.6852e-1 (3.22e-2) -	5.1935e-1 (2.09e-2) -	4.6908e-1 (3.10e-2)
DTLZ2	13	1.2822e+0 (1.62e-7) -	5.4978e-1 (4.02e-2) -	6.1536e-1 (2.74e-2) -	5.4591e-1 (3.56e-2)
DTLZ2	15	1.2896e+0 (6.47e-8) -	5.7042e-1 (3.26e-2) -	6.6712e-1 (2.07e-2) -	5.6608e-1 ($3.43e-2$)
DTLZ4	5	3.3055e-1 (5.13e-2) -	2.6457e-1 (1.08e-1) =	2.6000e-1 (2.68e-2) -	2.3413e-1 (8.92e-2)
DTLZ4	10	5.3467e-1 (1.91e-2) -	4.7282e-1 (3.63e-2) -	5.3787e-1 (1.03e-2) -	4.7219e-1 (3.58e-2)
DTLZ4	13	6.1679e-1 (1.94e-2) -	5.5299e-1 (4.00e-2) -	6.0308e-1 (1.02e-2) -	5.4235e-1 (3.75e-2)
DTLZ4	15	6.3863e-1 (1.10e-2) -	5.5156e-1 (3.51e-2) -	6.5844e-1 (1.29e-2) -	5.4496e-1 (3.70e-2)
DTLZ6	5	1.4736e+0 (6.78e-16) -	9.9562e-1 (1.03e-1) +	1.1655e+0 (6.88e-2) -	1.0009e+0 (1.38e-1)
DTLZ6	10	6.5482e+0(1.38e+0) -	1.6124e+0 (1.93e-1) -	2.0422e+0 (1.43e-1) -	1.5907e + 0 (1.87e-1)
DTLZ6	13	1.0572e+1 (9.99e-3) -	1.9119e+0 (3.98e-1) -	2.2510e+0 (1.45e-1) -	$1.8563e{+}0$ (3.96e-1)
DTLZ6	15	1.0674e+1 (2.93e-3) -	1.7980e+0 (3.46e-1) =	2.3528e+0 (1.66e-1) -	$1.7410\mathrm{e}{+0} (2.30\mathrm{e}{-1})$
WFG1	5	9.3854e-1 (1.16e-1) -	4.3655e-1 (3.98e-3) -	4.6716e-1 (1.08e-2) -	4.3487e-1 (3.46e-3)
WFG1	10	2.4955e+0 (1.31e-1) -	1.1086e+0 (8.50e-2) -	1.0764e+0 (3.13e-2) -	1.066e + 0 (3.14e-2)
WFG1	13	2.9502e+0 (1.55e-1) -	1.6082e + 0 (6.93e-2) =	1.6072e+0 (3.02e-2) -	1.5841e + 0 (5.43e-2)
WFG1	15	2.7066e+0 (2.19e-1) -	1.6281e+0 (9.06e-2) -	1.6015e+0 (3.69e-2) -	$1.5543e{+0} (1.09e{-1})$
WFG2	5	1.0322e+0 (2.11e-1) -	4.7210e-1 (1.77e-3) -	4.9358e-1 (1.27e-2) -	4.7062e-1 (2.13e-3)
WFG2	10	1.4558e+1 $(4.97e+0)$ -	1.2569e+0(1.15e-1) -	1.1006e+0(3.73e-2) -	1.0876e + 0(1.48e - 1)
WFG2	13	2.3162e+1 (4.45e+0) -	1.7446e+0 (1.20e-1) -	1.6880e+0 (1.19e-1) -	$1.59795e{+}0$ (1.28e-2)
WFG2	15	2.4836e+1 $(5.73e+0)$ -	1.5650e + 0 (9.09e-2) =	2.0002e+0 (3.05e-1) -	$1.5585e{+0}$ (8.52e-2)
WFG6	5	1.6458e+0 (4.56e-1) -	1.1632e + 0 (1.95e-3) +	1.3021e+0 (3.55e-2) -	$1.1630\mathrm{e}{+0} (1.84\mathrm{e}{-3})$
WFG6	10	1.5090e+1 $(3.58e+0)$ -	4.7755e+0 (1.22e-2) -	4.9503e+0 (1.02e-1) -	4.7755e+0 (1.24e-2)
WFG6	13	2.1633e+1(3.87e+0) -	7.6367e+0 (3.49e-1) -	7.4523e + 0 (1.93e-1) +	7.7059e+0 (3.93e-1)
WFG6	15	2.6594e+1 $(4.04e+0)$ -	8.4332e+0 (6.92e-1) -	9.3531e+0 (3.07e-1) -	8.4029e + 0 (5.08e-1)
WFG7	5	1.7394e+0 (1.78e-1) -	1.1770e+0 (5.14e-4) -	1.3236e+0 (3.93e-2) -	$1.16822e{+}0$ (5.15e-4)
WFG7	10	1.8234e+1 $(1.81e+0)$ -	4.7855e+0 (1.72e-2) -	4.9491e+0 (1.05e-1) -	4.7788e + 0 (5.82e-2)
WFG7	13	2.5556e+1(1.68e+0) -	7.2118e+0 (1.72e-1) -	7.3819e+0 (1.97e-1) -	$7.2083e{+0}(1.63e{-1})$
WFG7	15	2.9696e+1(1.63e+0) -	8.1386e+0 (1.04e-1) +	9.4909e+0 (3.05e-1) -	8.2267e+0 (1.83e-1)
+/-/=		0/32/0	2/26/6	1/30/1	29/3/0

Table 1: IGD Values obtained by MOEADPaS, NSGA-III, RSEA, and DSMOA-KCS

As shown in Fig.2, DSMOA-KCS shows a uniform and better distribution, not forgetting diversity compared to the other algorithms regarding the DTLZ True Pareto Front. As can be seen in Table I, IGD results show that DSMOA-KCS attains the best IGD values of 29 out of 32 benchmark instances of DTLZ1, DTLZ2, DTLZ4, & DTLZ6 and WFG1, 2, 6, & 7 all with 5, 10, 13, and 15 objectives, respectively. MOEADPaS attains the least results, NSGA-III on 2 and RSEA on 1. DSMOA-KCS is suitable for resolving problems that are not time-critical but sufficiently tricky.

4 Conclusion

In order to solve the problem of equity between convergence and diversity, this paper proposes a directed search sampling method and Environmental selection procedures to act as guiding solutions for the execution of crossover together with parent solutions and seconded to reproduce guiding solutions for the execution of the offspring. While also taking into consideration the improvement of the offspring. A modified single cluster linkage is constructed to handle multiple MaOPs, and DSMOA-KCS is compared with recent algorithms on the WFG and DTLZ benchmarks, with the number of objectives ranging from 5 to 15. The 8 Michael Aggrey Okoth, Ronghua Shang, Weitong Zhang, and Licheng Jiao

results reveal that the proposed DSMOA-KCS significantly outperforms other algorithms in certain problem instances; however not better in all the problem instances than other algorithms.

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