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1 **Energetically consistent Eddy-Diffusivity Mass-Flux**
2 **convective schemes. Part II: Implementation and**
3 **Evaluation in an oceanic context.**

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6 **Key Points:**

- 7 • The assumptions underlying the derivation of Mass Flux (MF) schemes are eval-
8 uated in the oceanic context using scaling analysis and LES simulations
9 • Based on LES results, a new closure for the turbulent transport of Turbulent Ki-
10 netic Energy (TKE) taking into account MF transport of TKE is proposed
11 • Guided by continuous energy budgets, an energy-conserving discretization is pro-
12 posed, and energy biases of inconsistent formulations are quantified

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Abstract

A convective vertical mixing scheme rooted in the Eddy-Diffusivity Mass-Flux (EDMF) approach is carefully derived from first principles in Part I. In addition, consistent energy budgets between resolved and subgrid scales when using an EDMF scheme are presented for seawater and dry atmosphere. In this second part, we focus on oceanic convection with the following objectives in mind: *(i)* justify in the oceanic context the assumptions made in Part I for the derivation of an MF scheme and a new TKE turbulent transport term *(ii)* show how continuous energy budgets can guide an energetically-consistent discretization *(iii)* quantify energy biases of inconsistent formulations, including double-counting errors due to inconsistent boundary conditions. The performance of the proposed energetically consistent EDMF scheme is evaluated against Large Eddy Simulations (LES) and observational data of oceanic convection. We systematically evaluate the sensitivity of numerical solutions to different aspects of the new formulation we propose. Notably, when compared to LES data, energetic consistency is key to obtaining accurate TKE and turbulent transport of TKE profiles. To further illustrate that the MF concept is a credible alternative to the traditional approaches used in the oceanic context (using an enhanced vertical diffusion or a counter gradient term) the proposed scheme is validated in a single-column configuration against observational data of oceanic convection from the LION buoy.

Plain Language Summary

In Earth system models, various important processes occur on scales that are too fine to be resolved with usual grid resolutions. Parameterizations have to be used to approximate the average effect of such processes on the scales resolved by a numerical model. The general objective of the proposed work is to approach the parameterization problem for boundary-layer turbulence and convective plumes in a “consistent” manner. Here the notion of consistency integrates various aspects: global energetic consistency, consistency with a particular averaging technique for the scale-separation, and the rigorous reduction of a physical system to a scale-aware parametric representation based on well-identified and justifiable approximations and hypotheses. An originality is to jointly consider energy budgets including a subgrid energy reservoir on top of the resolved energies allowing the proper coupling between the parameterization and the resolved fluid dynamics. In the first part of this work, we focused on theoretical aspects at the continuous level. In this second part, we look at the practical aspects of implementing the proposed concepts in an oceanic numerical model. This research paves the way toward an alternative methodology to parameterize oceanic convection across scales. Numerical simulations demonstrate the adequacy of the proposed parameterization.

1 Introduction

Oceanic vertical mixing parameterizations based on the Eddy-Diffusivity Mass-Flux (EDMF) concept have seen a growing interest in the past years (Giordani et al., 2020; Garanaik et al., 2024; Ramadhan et al., 2020). In the companion paper Perrot et al. (2024) (hereafter Part I), we provide a self-contained derivation from first principles of a convective mixing EDMF scheme. This type of closure involves separating vertical turbulent fluxes into two components: an eddy-diffusivity (ED) term that addresses local small-scale mixing in a near isotropic environment, and a mass-flux (MF) transport term that accounts for the non-local transport performed by vertically coherent plumes within the environment. Using the multi-fluid averaging underlying the MF concept, we review consistent energy budgets between resolved and subgrid scales for seawater and dry atmosphere, in anelastic and Boussinesq settings. We show that when using an EDMF scheme, closed energy budgets can be recovered if: *(i)* bulk production terms of turbulent kinetic energy (TKE) by shear, buoyancy and transport include MF contributions; *(ii)* bound-

63 ary conditions are consistent with EDMF, to avoid spurious energy fluxes at the bound-
 64 ary. Moreover we show that lateral mixing between plumes and environment (referred
 65 as entrainment or detrainment) induces a net production of TKE via the shear term, and
 66 such production is enhanced when horizontal drag increases. Throughout the theoret-
 67 ical development of the scheme, we maintain transparency regarding underlying assump-
 68 tions.

69 In this second part, we use Large Eddy Simulations of oceanic convection to: (i)
 70 evaluate the assumptions used in the derivation of the scheme using a conditional sam-
 71 pling of convective plumes, (ii) propose a new formulation for TKE transport and (iii)
 72 assess the sensitivity of the EDMF scheme to energetic consistency, transport of TKE,
 73 horizontal momentum transport and small plume area assumption. Additionally, we com-
 74 pare the scheme to the classical TKE+EVD mixing scheme used for oceanic deep-convection
 75 (Turbulent Kinetic Energy and Enhanced Vertical Diffusivity, Madec et al., 2019). We
 76 show how energy budgets derived in Part I can guide a consistent discretization of the
 77 TKE equation; and we quantify energy biases of inconsistent formulations, including double-
 78 counting errors due to inconsistent boundary conditions.

79 We consider a Single Column Model (SCM) of the ocean in Boussinesq approxi-
 80 mation (e.g. Tailleux & Dubos, 2024),

$$\begin{cases} \partial_t \bar{\mathbf{u}}_h = -\partial_z \overline{w' \mathbf{u}'_h} \\ \partial_t \bar{\theta} = -\partial_z \overline{w' \theta'} + \frac{\bar{\epsilon}}{c_p - \alpha g z} \\ \partial_t \bar{S} = -\partial_z \overline{w' S'} \end{cases}$$

81 where $\bar{\mathbf{u}}_h = (\bar{u}, \bar{v})$ is the horizontal velocity vector, $\bar{\theta}$ is conservative temperature, \bar{S} is
 82 salinity, and $\overline{w' \mathbf{u}'_h}$, $\overline{w' \theta'}$, $\overline{w' S'}$ are the corresponding vertical turbulent fluxes. Moreover
 83 $\bar{\epsilon}$ is the viscous dissipation of turbulence, c_p is the seawater specific heat capacity and
 84 α is the thermal expansion coefficient. The notation $\overline{(\cdot)}$ indicates that the model's vari-
 85 ables and fluxes are interpreted as *horizontal averages* over the numerical grid cell of their
 86 continuous counterpart. Thus we have a simple and exact correspondence to compare
 87 SCM and LES data. Although viscous heating is usually neglected in the ocean (McDougall,
 88 2003), we kept this term to obtain a closed energy budget.

89 The EDMF parameterization relies on a decomposition of the horizontal grid area
 90 into an isotropic environment where turbulence is assumed diffusive, and an averaged
 91 plume. Then the flux of any variable $X = u, v, \theta, S$ is closed according to the decom-
 92 position

$$\overline{w' X'} = \underbrace{-K_X \partial_z \bar{X}}_{\text{ED}} + \underbrace{a_p w_p (X_p - \bar{X})}_{\text{MF}} \quad (1)$$

93 where K_X is an eddy-diffusivity coefficient, a_p is the fractional area of the plume, w_p is
 94 the vertical velocity of the plume and X_p is the plume-related X quantity. In the present
 95 study, the eddy viscosity K_u and diffusivities $K_\phi = K_\theta = K_S$ in turbulent vertical
 96 fluxes are computed from a turbulence closure model based on a prognostic equation for
 97 the turbulent kinetic energy (TKE) $k = \overline{\mathbf{u}' \cdot \mathbf{u}'}/2$ and a diagnostic computation of ap-
 98 propriate length scales (a.k.a. 1.5-order turbulence closure, see Appendix A).

99 Total energy of the fluid in a Boussinesq approximation can be split into kinetic
 100 energy of the horizontal resolved flow $E_k = (\bar{\mathbf{u}}_h \cdot \bar{\mathbf{u}}_h)/2$ (usually referred as *mean* or
 101 *resolved kinetic energy*), residual turbulent (or subgrid) kinetic energy k , and $E_{i+p} =$
 102 $c_p \bar{\theta} - z \bar{b} + g z$, the sum of averaged internal energy and potential energy, where b is the
 103 buoyancy acceleration (see Tailleux and Dubos (2024) and Part I). We were able to de-

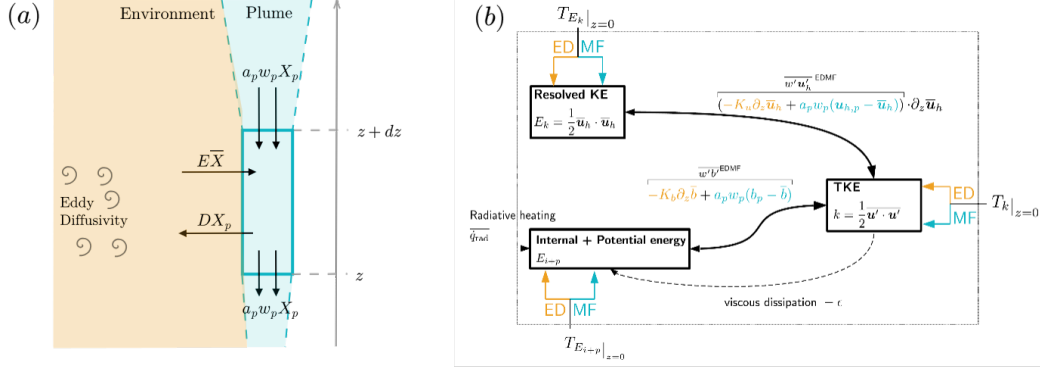


Figure 1: Schematic representation of (a) plume tracer budget, aside with eddy-diffusivity in the environment (b) energy budgets in EDMF.

104 rive closed energy budgets, including the TKE equation in Part I (see fig. 1(b))

$$\begin{cases}
 \partial_t E_k + \partial_z T_{E_k} & = -K_u (\partial_z \bar{\mathbf{u}}_h)^2 + a_p w_p (\mathbf{u}_{h,p} - \bar{\mathbf{u}}_h) \cdot \partial_z \bar{\mathbf{u}}_h \\
 \partial_t k + \partial_z T_k & = -K_\phi \partial_z \bar{b} + a_p w_p (b_p - \bar{b}) + K_u (\partial_z \bar{\mathbf{u}}_h)^2 - a_p w_p (\mathbf{u}_{h,p} - \bar{\mathbf{u}}_h) \cdot \partial_z \bar{\mathbf{u}}_h - \bar{\epsilon} \\
 \partial_t E_{i+p} + \partial_z T_{E_{i+p}} & = -(-K_\phi \partial_z \bar{b} + a_p w_p (b_p - \bar{b})) + \bar{\epsilon}
 \end{cases} \quad (2)$$

105 where

$$T_{E_k} = (-K_u \partial_z \bar{\mathbf{u}}_h + a_p w_p (\mathbf{u}_{h,p} - \bar{\mathbf{u}}_h)) \cdot \bar{\mathbf{u}}_h \quad (3)$$

$$T_{E_{i+p}} = -\partial_z (c_p (-K_\phi \partial_z \bar{\theta} + a_p w_p (\theta_p - \bar{\theta})) - z (-K_\phi \partial_z \bar{b} + a_p w_p (b_p - \bar{b}))) \quad (4)$$

106 Based on conditional sampling of convective plumes, we propose a new formulation for
 107 the transport of TKE, T_k , in section 2.

108 Finally plume-related equations can be derived using a two-fluid averaging proce-
 109 dure of the original unaveraged equations. Standard EDMF formulations – followed in
 110 this study – rely on two main assumptions (see e.g. Part I and Yano (2014)): (i) sta-
 111 tionarity of the plume, i.e. $\partial_t(a_p X_p) \ll \partial_z(a_p w_p X_p)$; (ii) small-area occupied by the
 112 plume, i.e. considering $a_p \ll 1$ while keeping order one contribution of mass-flux $a_p w_p$
 113 and source terms. Thus the generic plume equation assumes a balance between verti-
 114 cal plume advection, horizontal entrainment (E) of environment fluid into the plume or
 115 detrainment (D) of plume fluid into the environment, and potential additional sources
 116 (see fig. 1(a)),

$$\partial_z(a_p w_p X_p) = E\bar{X} - DX_p + S_{X,p}$$

117 The comprehensive plume model equations are given in Table 1. A description of the clo-
 118 sures are exposed in Part I.

119 The paper is organized as follows. In section 2 we derive a new closure for the tur-
 120 bulent transport of TKE. In section 3 we describe the two idealized LES test cases and
 121 assess the validity of hypotheses used in the derivation of the MF scheme. In section 4
 122 we derive a discretization that preserves the energetically consistent nature of EDMF
 123 equations. In section 5 we evaluate the SCM against LES and realistic data, assess the
 124 impact of different parameterization aspects, and quantify energy biases of inconsistent
 125 formulations, including double-counting errors due to inconsistent boundary conditions.
 126

$\partial_z(a_p w_p) = E - D$	Plume area conservation equation
$a_p w_p \partial_z \theta_p = E(\bar{\theta} - \theta_p)$	Plume temperature equation
$a_p w_p \partial_z S_p = E(\bar{S} - S_p)$	Plume salinity equation
$a_p w_p \partial_z \mathbf{u}_{h,p} = E(\bar{\mathbf{u}}_h - \mathbf{u}_{h,p}) + a_p w_p C_u \partial_z \bar{\mathbf{u}}_h$	Plume horizontal momentum equation
$a_p w_p \partial_z w_p = -bEw_p + a_p \left[aB_p + \frac{b'}{h}(w_p)^2 \right]$	Plume vertical velocity equation
$B_p = b_{\text{eos}}(\theta_p, S_p) - b_{\text{eos}}(\bar{\theta}, \bar{S})$	Buoyancy forcing term
$E = a_p C_\epsilon \max(0, \partial_z w_p)$	Lateral entrainment closure
$D = -a_p C_\delta \min(0, \partial_z w_p) - a_p w_p \delta_0 \frac{1}{h}$	Lateral detrainment closure

Table 1: Summary of the plume equations in the small area limit under the steady plume hypothesis. $C_u, a, b, b', \beta_1, \beta_2, \delta_0$ are dimensionless parameters. h is an estimate of the mixed layer depth, computed as the depth at which $w_p(z = -h) = 0$. "eos" stands for "equation of state".

2 A new closure for the transport of TKE

The redistribution terms of TKE are often little discussed in turbulence parameterization since they do not contribute directly to the vertically integrated energy budgets. However, they are of great importance in convective conditions where non-local transport dominates (Witek et al., 2011). For instance, TKE produced close to the surface due to destabilizing buoyancy fluxes is then transported by coherent plumes into the mixed layer. Taking into account MF transport of TKE is thus essential to achieve local energetic consistency, and model accurately TKE at any level z .

Turbulent fluxes of TKE arise from the contribution of a TKE transport term, a pressure redistribution term and a viscous flux,

$$T_k = \frac{1}{2} \overline{w' \mathbf{u}' \cdot \mathbf{u}'} + \frac{1}{\rho_0} \overline{w' p'} - \nu \partial_z k \quad (5)$$

For oceanic flows, the viscous flux is negligibly small and will be omitted. We will assume that the pressure redistribution term can be incorporated into the transport term of TKE assuming proportionality, as it is usually done in CBL schemes (e.g. Mellor, 1973). In numerical models, TKE transport is usually parameterized via K-diffusion, namely

$$\partial_z \left(\overline{w' \frac{\mathbf{u}' \cdot \mathbf{u}'}{2}} \right) \simeq -\partial_z (K_k \partial_z k) \quad (6)$$

Alternatively, to derive a mass-flux based closure we can apply the plume/environment decomposition of the horizontal average to get the exact relation

$$\begin{aligned} \overline{w' \frac{\mathbf{u}' \cdot \mathbf{u}'}{2}} &= \sum_{i=e,p} \underbrace{a_i \frac{1}{2} \overline{\mathbf{u}'_i \cdot \mathbf{u}'_i w'_i}}_{\text{I}_i} + \underbrace{a_i (\mathbf{u}_i - \bar{\mathbf{u}}) \cdot \overline{\mathbf{u}'_i w'_i}}_{\text{II}_i} \\ &\quad + \underbrace{a_i (w_i - \bar{w}) \frac{1}{2} \overline{\mathbf{u}'_i \cdot \mathbf{u}'_i}}_{\text{III}_i} + \underbrace{a_i \frac{1}{2} \|\mathbf{u}_i - \bar{\mathbf{u}}\|^2 (w_i - \bar{w})}_{\text{IV}_i} \end{aligned} \quad (7)$$

143 where the subscript e denotes variables of the environment, and the subdomain moments
144 are defined as

$$\overline{w'_i u'_i} = \int_{A_i} (w - w_i)(u - u_j) dx dy \quad (8)$$

145 where A_i ($i = e, p$) is the area occupied by the environment or the plume. Terms of (7)
146 are interpreted as follow: I_i is an intra-subdomain turbulent TKE transport; II_i is a trans-
147 port of Reynolds stress by the coherent velocities; III_i is a transport of subdomain TKE
148 by the coherent velocities (*i.e.* transport of TKE by mass-flux); IV_i is a transport of con-
149 vective kinetic energy by coherent velocities. Based on LES simulations (see Sec. 3.3),
150 we found that: (i) I_p can be neglected, consistently with the small area limit; (ii) II_e and
151 II_p are almost compensating, thus the sum $II_e + II_p$ can be neglected. We can conveni-
152 ently reformulate the remaining terms as (see Appendix B for details):

$$III_e + III_p + IV_e + IV_p = a_p w_p \frac{1}{1 - a_p} \left(k_p + \frac{1}{2} \|\mathbf{u}_p - \bar{\mathbf{u}}\|^2 - k \right) \quad (9)$$

153 where sub-domain TKE are $k_i := 1/2 \overline{\mathbf{u}'_i \cdot \mathbf{u}'_i}$ ($i = e, p$) and TKE can be decomposed
154 as the sum of domain-averaged TKEs and sub-domain TKEs:

$$k = \frac{1}{2} a_e \|\mathbf{u}_e - \bar{\mathbf{u}}\|^2 + a_e k_e + \frac{1}{2} a_p \|\mathbf{u}_p - \bar{\mathbf{u}}\|^2 + a_p k_p \quad (10)$$

155 In EDMF closures, turbulence is assumed isotropic in the environment, thus we close $\frac{1}{2} \overline{\mathbf{u}'_e \cdot \mathbf{u}'_e w'_e}$
156 with K-diffusion, similar to the standard practice for TKE-only schemes. Then assum-
157 ing $\frac{1}{1 - a_p} \simeq 1$ (*i.e.* the small area limit) we have

$$\overline{w' \frac{\mathbf{u}' \cdot \mathbf{u}'}{2}} = \underbrace{-K_k \partial_z k}_{\text{ED}} + \underbrace{\overbrace{a_p w_p (k_p - k)}^{\text{Han \& Bretherton 2019}} + \overbrace{a_p w_p^3}^{\text{Witek et al. 2011}} + \frac{a_p w_p}{2} \|\mathbf{u}_{h,p} - \bar{\mathbf{u}}_h\|^2}_{\text{new EDMF}} \quad (11)$$

158 It is interesting to note that we can recover existing formulations from the proposed clo-
159 sure (11): if $a_p w_p = 0$ it boils down to the classical eddy-diffusivity closure; if $k_p = k$
160 and $\mathbf{u}_{h,p} = \bar{\mathbf{u}}_h$ the term $1/2 w_p^3$ proposed by Witek et al. (2011) is recovered; if TKE
161 is treated as a tracer transported by the plume then the formulation proposed by Han
162 and Bretherton (2019) is recovered. However, we should mention that these latter treat-
163 ment seems incorrect because $\overline{w' \mathbf{u}' \cdot \mathbf{u}' / 2}$ is not a second-order moment, but a third-order
164 moment which requires a proper treatment as seen in (7).

165 Finally, one still needs to provide a value for k_p . Without any assumption, its prog-
166 nostic equation reads (Tan et al., 2018, eq. (11))

$$\begin{aligned} \partial_t (a_p k_p) + \partial_z (a_p w_p k_p) &= -a_p \overline{w'_p \mathbf{u}'_{h,p}} \cdot \partial_z \mathbf{u}_{h,p} + a_p \overline{w'_p b'_p} \\ &+ E \left(k_e + \frac{1}{2} \|\mathbf{u}_e - \mathbf{u}_p\|^2 \right) - D k_p \\ &- \partial_z \left(a_p w'_p \frac{\mathbf{u}'_p \cdot \mathbf{u}'_p}{2} + a_p \mathbf{u}'_p \cdot \frac{1}{\rho_0} (\nabla p^\dagger)'_p \right) \\ &- a_p \epsilon_p \end{aligned}$$

167 Consistent with the neglect of subplume fluxes and temporal tendency in EDMF (see
168 Part I or Tan et al. (2018)), we propose as a first attempt to retain advection, entrain-
169 ment, detrainment and dissipation terms, which lead to the simplified form of the pre-

170 vious equation:

$$\begin{aligned}
 \partial_z(a_p w_p k_p) &= E \left(k_e + \frac{1}{2} \|\mathbf{u}_e - \mathbf{u}_p\|^2 \right) - Dk_p - a_p \epsilon_p \\
 &= E \left(\overbrace{\frac{1}{1-a_p} k - \frac{a_p}{1-a_p} \left(k_p + \frac{1}{2} \|\mathbf{u}_p - \bar{\mathbf{u}}\|^2 \right)}^{k_e} + \overbrace{\frac{1}{(1-a_p)^2} \frac{1}{2} \|\mathbf{u}_p - \bar{\mathbf{u}}\|^2}^{\frac{1}{2} \|\mathbf{u}_e - \mathbf{u}_p\|^2} \right) \\
 &\quad - Dk_p - a_p \epsilon_p \\
 &= E \frac{1}{1-a_p} \left(k - a_p k_p + \left[1 + \frac{a_p^2}{1-a_p} \right] \frac{1}{2} \|\mathbf{u}_p - \bar{\mathbf{u}}\|^2 \right) - Dk_p - a_p \epsilon_p
 \end{aligned}$$

171 where we have used the identity $\|\mathbf{u}_e - \mathbf{u}_p\|^2 = \frac{1}{(1-a_p)^2} \|\mathbf{u}_p - \bar{\mathbf{u}}\|^2$ and substituted k_e
 172 using (10). Using area conservation, we get the advective form

$$a_p w_p \partial_z k_p = E \frac{1}{1-a_p} \left(k - k_p + \left[1 + \frac{a_p^2}{1-a_p} \right] \frac{1}{2} \|\mathbf{u}_p - \bar{\mathbf{u}}\|^2 \right) - a_p \epsilon_p \quad (12)$$

173 Finally assuming $\frac{1}{1-a_p} \simeq 1$ and $a_p^2 \ll a_p$ (i.e. the small area limit) we have

$$a_p w_p \partial_z k_p = E \left(k - k_p + \frac{1}{2} \|\mathbf{u}_p - \bar{\mathbf{u}}\|^2 \right) - a_p \epsilon_p \quad (13)$$

174 where the closure for dissipation is taken as $\epsilon_p = c_\epsilon / l_\epsilon (k_p)^{2/3}$ (c_ϵ is a constant and l_ϵ
 175 is a length scale, see Appendix A). As a summary, the proposed closure of TKE trans-
 176 port is given by

$$\begin{cases} \overline{w' \mathbf{u}' \cdot \mathbf{u}'} = & -K_k \partial_z k + a_p w_p \left(k_p - k + \frac{1}{2} \|\mathbf{u}_p - \bar{\mathbf{u}}\|^2 \right) \\ a_p w_p \partial_z k_p = & E \left(k - k_p + \frac{1}{2} \|\mathbf{u}_p - \bar{\mathbf{u}}\|^2 \right) - a_p \epsilon_p \end{cases} \quad (14)$$

177 We will now use LES to evaluate EDMF assumptions and the new closure of TKE trans-
 178 port.

179 3 Test cases description and validation of formulation

180 In this section we describe the reference idealized cases that will be further used
 181 in the study; then we describe the LES model and the conditional sampling technique
 182 used to identify convective plumes; finally we evaluate EDMF assumptions and the new
 183 closure of TKE transport.

184 3.1 Description of idealized cases

185 The two idealized cases considered are reminiscent of typical convective conditions
 186 in the ocean (e.g. Marshall & Schott, 1999), where convection into a initially resting ocean
 187 of constant stratification $\Delta\theta = 1 \text{ K}/1000 \text{ m}$ (corresponding $N_0^2 = 1.962 \times 10^{-6} \text{ s}^{-2}$) is
 188 triggered by a surface cooling of $Q_0 = -500 \text{ W m}^{-2}$ (corresponding to a surface buoy-
 189 ancy loss of $B_0 = -2.456 \times 10^{-7} \text{ m}^2 \text{ s}^{-3}$). In both cases, salinity is kept uniform at $S =$
 190 32.6 psu . The first case (FC500) consists of free convection, where no wind stress is ap-
 191 plied. In the second idealized case (W005.C500) a uniform wind stress along the merid-
 192 ional direction, of magnitude $(u_*^a)^2 = 0.05 \text{ m}^2 \text{ s}^{-2}$, is applied. A summary of the pa-
 193 rameters for each case can be found in table 2. To characterize wind-shear effects, we
 194 introduce the Froude number (Haghsheenas & Mellado, 2019a)

$$Fr_* = \frac{u_*^a}{N_0 L_0} \quad (15)$$

Table 2: Idealized cases parameters

Case	Q_0 (W m ⁻²)	$(u_*^a)^2$ (m ² s ⁻²)	N_0^2 (s ⁻²)	t_f (h)	Fr_*
FC500	-500	0	1.962×10^{-6}	72	0
W005_C500	-500	0.05	1.962×10^{-6}	72	0.56

195 where the length scale $L_0 = (B_0/N_0^3)^{1/2}$ can be interpreted as an Ozmidov scale $(\epsilon/N^3)^{1/2}$
 196 (Garcia & Mellado, 2014) which is a measure of the smallest eddy size affected by a back-
 197 ground stratification N^2 in a turbulent field characterized by a viscous dissipation rate
 198 ϵ . After $t_f = 72$ h of simulation leading to a mixed layer depth h (defined as the depth
 199 at which the buoyancy flux is minimum) of several hundred meters, various non-dimensional
 200 numbers can be used to characterize the flow. Their values can be found in Tab. 3. The
 201 ratio of the mixed layer depth to the Obukhov length (Obukhov (1971) and Zheng et
 202 al. (2021) in the oceanic context) h/L_{Ob} , where

$$L_{Ob} = \frac{(u_*^o)^3}{-B_0}$$

203 is an estimate of the depth at which the production of TKE by turbulent shear is of the
 204 same order of magnitude as the production of TKE by buoyancy fluxes. Noting $w_* =$
 205 $(-B_0h)^{1/3}$ the convective velocity scale (Deardorff, 1970), we get

$$\frac{h}{L_{Ob}} = \left(\frac{w_*}{u_*}\right)^3 \quad (16)$$

206 We also recall that the oceanic friction velocity u_*^o satisfies $\rho_o(u_*^o)^2 = \rho_a(u_*^a)^2$. The Richard-
 207 son number at the mixed layer base,

$$Ri_h = \frac{N_0^2}{\left(\frac{u_*^o}{h}\right)^2}$$

208 measures the destabilization by surface shear stresses of a stably stratified water column.
 209 At $t_f = 72$ h, the case W005_C500 can be described by $h/L_{Ob} \simeq 5.7$ and $Ri_h \simeq 310$,
 210 which corresponds to a regime of strong deepening of the MLD according to Legay et
 211 al. (2023). Finally, for free convection cases (no wind) a convective Richardson number
 212 can be built as

$$Ri_* = \frac{N_0^2}{(w_*/h)^2} = \frac{N_0^2 h^{4/3}}{(-B_0)^{2/3}} = Ri_h \left(\frac{L_{Ob}}{h}\right)^{2/3}$$

213 It can be interpreted as follows. The time evolution of the mixed layer depth can be ac-
 214 curately described by the scaling (Turner, 1979; Van Roekel et al., 2018)

$$h \propto h_{\text{enc}} \quad (17)$$

215 where the *encroachment* depth is $h_{\text{enc}}(t) := \sqrt{2\frac{(-B_0)}{N_0^2}t}$. Then the ratio of the entrain-
 216 ment velocity $w_e = \frac{d}{dt}h$ to the convective velocity $w_* = (-B_0h)^{1/3}$ reads

$$\frac{w_e}{w_*} \propto Ri_*^{-1} \quad (18)$$

Table 3: Idealized cases non-dimensional parameters after 72 h of simulation

Case	h/L_{Ob}	Ri_h	Ri_*
FC500	∞	∞	97
W05_C500	5.7	310	97

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3.2 LES model description and conditional sampling

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The LES data have been generated by the non-hydrostatic model Méso-NH (Lac et al., 2018), using the Ocean-LES version developed by Jean-Luc Redelsperger. It is solving an anelastic Lipps-Hemler system adapted to the ocean, along with a linearized equation of state. The model uses a second-order Runge-Kutta time stepping and spatial discretization of advection operators is performed with a fourth-order centered scheme. Explicit subgrid scale closures are computed via a 3-D turbulence scheme based on a prognostic equation of the subgrid turbulent kinetic energy using a mixing-length scale, computed from the volume of a grid cell (Cuxart et al., 2000). The domain size is 1000 m on the vertical and 7.5 km \times 7.5 km on the horizontal, where doubly periodic conditions are applied. A resolution of 10 m on the vertical and 15 m on the horizontal is used. Each configuration is run for 72 h with a time-step of 10 s. To assess the quality of the simulations, we checked that the subgrid TKE was never exceeding 20% of the TKE explicitly resolved by the LES (Pope, 2004). Via analysis of the total TKE budget, we checked that a quasi-steady regime is reached after a few hours of simulation (e.g. Garcia & Melado, 2014). Moreover, at the end of the simulations, the typical size of coherent structures, which can be quantified by the horizontal integral length scale in the bulk of the mixed layer, is of the order $O(500 \text{ m}) \ll 7.5 \text{ km}$. This suggests that the horizontal domain is large enough to provide a satisfactory statistical sampling of turbulent structures.

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To identify plumes, we use a velocity-based conditional sampling adapted from Pergaud et al. (2009), where the plume area is defined as

$$A_p(z, t) = \left\{ (x, y, z, t) \text{ such that } \bar{w}(z, t) - w(x, y, z, t) > m \times \max(\sqrt{w'^2}(z, t), \sigma_{\min}(z, t)) \right\} \quad (19)$$

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where the minimum standard deviation is chosen as $\sigma_{\min}(z, t) = 0.05/(-z) \int_z^0 \sqrt{w'^2}(z', t) dz'$. We checked that the further conclusions were not sensitive to m , and fixed $m = 1$. We do not use the tracer-based sampling of Couvreux et al. (2010) since it is valid only for small variations of the mixed layer depth. We neither utilize the "strong updraft" sampling of (Siebesma et al., 2007) since it assumes that a_p is a given constant. However, we checked that similar conclusions could be drawn from such samplings (not shown).

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3.3 Evaluation of the steady plume and small area hypotheses

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In this section, we directly evaluate the validity of the main assumptions of EDMF scheme, namely that plumes are stationary and that their area is negligible compared to the total grid area. Figs. 2 and 3 show that the plume temporal tendency terms are $O(10^{-1} - 10^{-2})$ smaller than plume advective terms which is consistent with the scaling in $1/(N_0 t)$ derived in Part I. This justifies the use of the steady plume hypothesis.

Figs. 4 and 5 show vertical profiles of temperature, vertical velocity, plume fractional area, and temperature flux for the idealized cases FC500 and W005_C500. Values of $a_p(z)$ range between 10% and 20% of the total area, as exposed in previous studies (e.g. Couvreux et al., 2010). Thus the assumption of modeling a small plume area is not really accurate. This justifies questioning the relevance of this assumption and considering the system described in 5.3, in which a_p is not considered small anymore. The convective

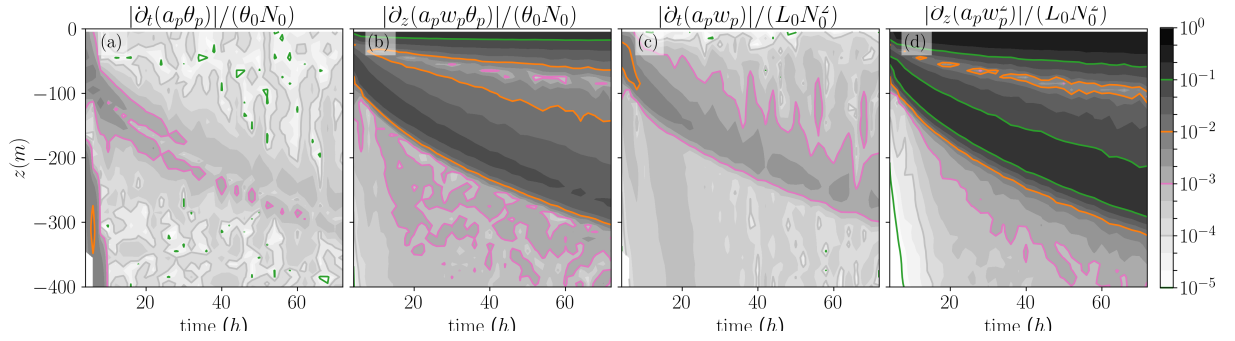


Figure 2: Temporal evolution of the normalized plume tendency $\partial_t(a_p X_p)$ and plume advection $\partial_z(a_p w_p X_p)$ terms ($X = \theta, w$), for the case FC500. Colored lines indicate thresholds to each power of ten.

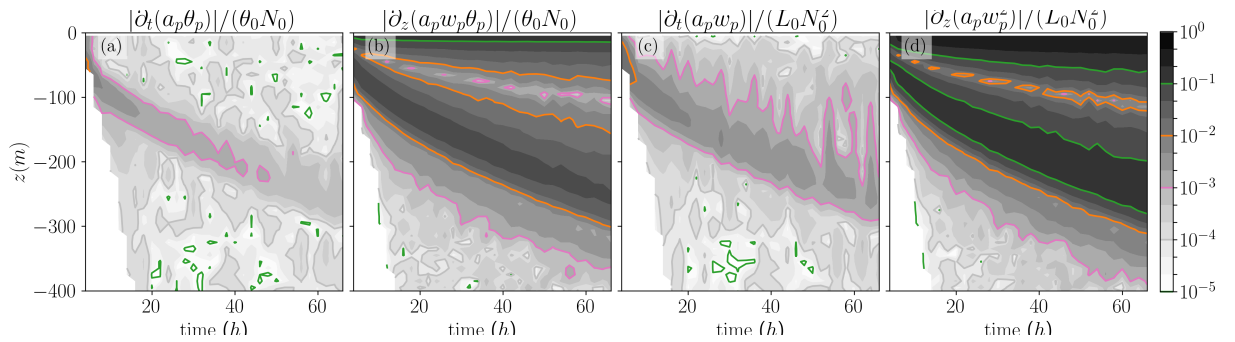


Figure 3: Same as figure 2 for the case W005_C500.

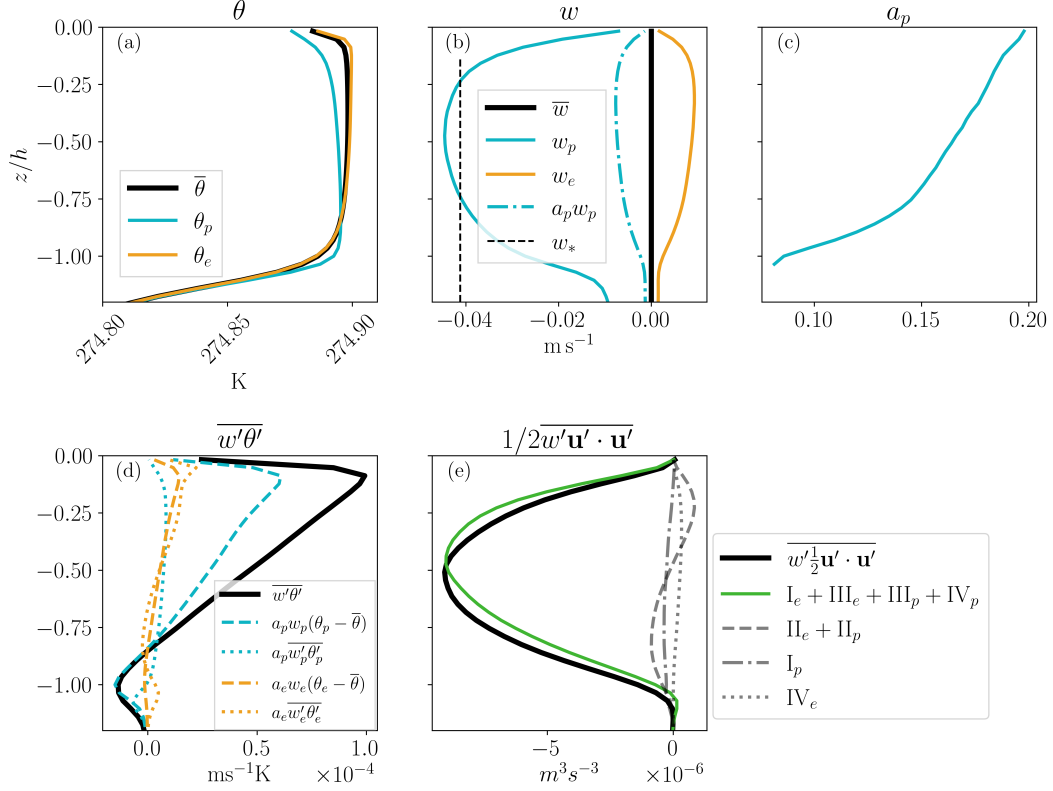


Figure 4: LES vertical profiles of (a) temperature, (b) vertical velocities, (c) plume fractional area, (d) temperature flux and (e) TKE flux for the FC500 case after 72 h of simulation. For each field, the black line represents a horizontal average over the whole grid cell, the blue line represents an average over the plume area and the orange line represents an average over the environment area. In panel (b) the blue dotted line represents $a_p w_p$, and the gray dashed line represents the value of the free convective velocity scale w_* . In panel (d), total flux is in black, plume fluxes in blue (MF is dashed and subplume is dotted), and environment fluxes in orange (same linestyles). In panel (e) are represented the total flux (black) and the contributions from the combined terms $I_e + III_e + III_p + IV_p$ (blue), $II_e + II_p$ (dashed gray), I_p (dash-dotted gray) and III_p (dotted gray) (see 2 for details).

256 velocity w_* is found to be a good estimate of the plume vertical velocity w_p (panel (b)).
 257 The contribution of the mass-flux term $a_p w_p (\theta_p - \bar{\theta})$ (panel (d)) to the total tempera-
 258 ture flux is increasing with depth, until reaching a quasi-perfect match in the entrain-
 259 ment layer. The rough validity of the assumption $a_p w_p (\theta_p - \bar{\theta}) \gg a_p \overline{w_p' \theta_p'}$, $a_e w_e (\theta_e - \bar{\theta})$
 260 is consistent with the rough validity of $a_p \ll 1$. The plume/environment decompo-
 261 sition of the vertical transport of TKE $1/2 \overline{w' \mathbf{u}' \cdot \mathbf{u}'}$ is presented in Figs. 4(e) and 5(e). The
 262 dominant terms exposed in eq. (9) explain well the total flux.

263 Thanks to LES, we have assessed the validity of hypotheses used to derive the contin-
 264 uous formulation of our energetically consistent EDMF scheme. We now turn to the
 265 discretization of such scheme.

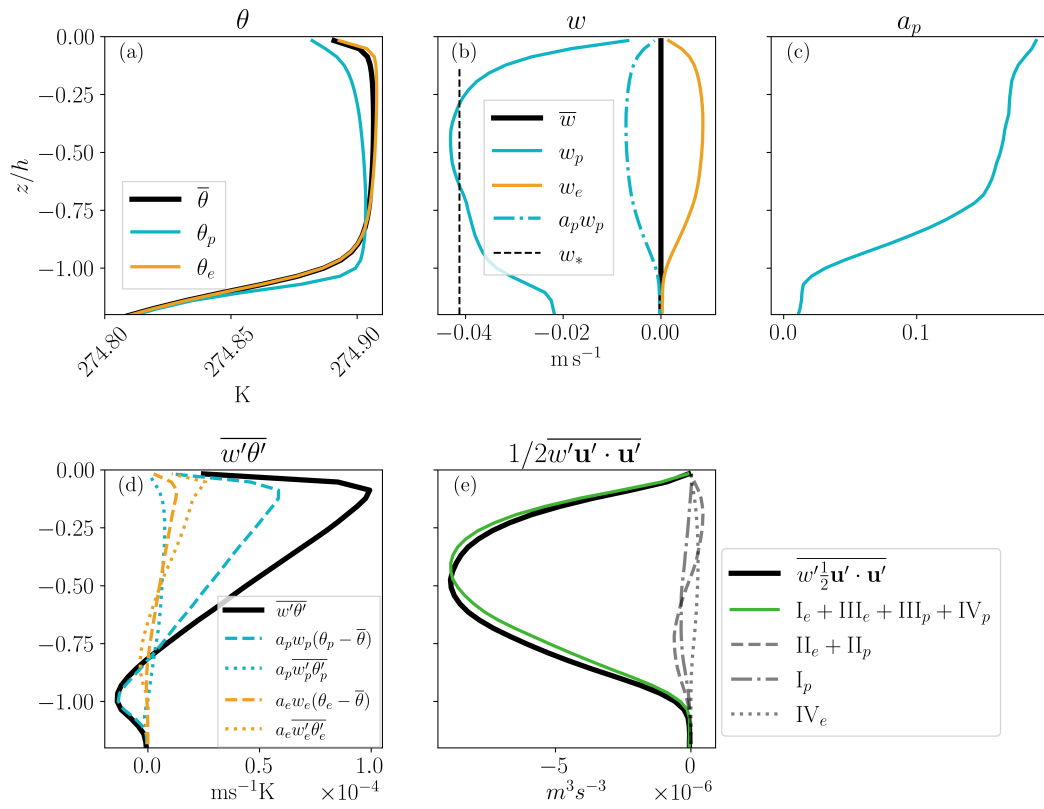


Figure 5: Same as figure 4 for the case W005_C500.

4 Discretization of energetically consistent EDMF equations

In this section, we derive a discretization that satisfies the conservative energy exchange described in Part I (see also the introduction in section 1) at a discrete level when using EDMF. This is achieved through discrete mean kinetic, turbulent kinetic, and potential energy budgets. This type of approach has already been proposed by Burchard (2002) in the ED case to derive an energy-conserving discretization method for the shear and buoyancy production terms in the TKE. In what follows, we generalize this approach to the EDMF case. This section is divided into 2 subsections, the first about the coupling between ED and MF schemes within the time-stepping algorithm and the second about an energy-conserving discretization of TKE production and destruction terms. For readers interested in a practical implementation of the proposed EDMF scheme in a numerical model, full details of a simple discretization of the MF equations are given in Appendix C including the way the mixed layer depth h is computed. The proposed discretization guarantees that w_p is strictly negative, that a_p is bounded between 0 and 1, and that the continuity and tracer equations are compatible, without the need for an iterative solution procedure.

4.1 Coupling ED and MF schemes

In the EDMF approach, the usual vertical diffusion/viscous subgrid terms are completed by an advective term so that the following equation must be advanced in time (with $X = u, v, \theta, S$):

$$\partial_t \bar{X} = \partial_z (K_X \partial_z \bar{X}) - \partial_z (a_p w_p (X_p - \bar{X})) \quad (20)$$

This amounts to couple a boundary layer scheme which provides K_X and a convection scheme which provides $a_p w_p$ and X_p . The numerical treatment of such coupling can be approached in 2 ways: either by integrating the 2 schemes sequentially or in parallel. For the numerical experiments discussed in Sec. 5 we chose a *boundary layer-then-convection* strategy corresponding to the following temporal integration for the single-column model (leaving aside the Coriolis and solar penetration terms, and for $\phi = \theta, S$)

ED step

$$\begin{aligned} \phi^{n+1,*} &= \phi^n + \Delta t \partial_z (K_\phi(k^n, b^n) \partial_z \phi^{n+1,*}) \\ \mathbf{u}_h^{n+1,*} &= \mathbf{u}_h^n + \Delta t \partial_z (K_u(k^n, b^n) \partial_z \mathbf{u}_h^{n+1,*}) \\ b^{n+1,*} &= b_{\text{eos}}(\phi^{n+1,*}) \end{aligned}$$

MF step

$$\begin{aligned} [a_p, w_p, \phi_p, \mathbf{u}_{h,p}, k_p, B_p] &= \text{MF}(\phi^{n+1,*}, b^{n+1,*}, \mathbf{u}_h^{n+1,*}) \\ \phi^{n+1} &= \phi^{n+1,*} - \Delta t \partial_z (a_p w_p (\phi_p - \phi^{n+1,*})) \\ \mathbf{u}_h^{n+1} &= \mathbf{u}_h^{n+1,*} - \Delta t \partial_z (a_p w_p (\mathbf{u}_{h,p} - \mathbf{u}_h^{n+1,*})) \end{aligned}$$

TKE update

$$k^{n+1} = k^n + \Delta t \partial_z (K_k(k^n, b^n) \partial_z k^{n+1}) + \mathcal{F}_k(b^{n+1}, \mathbf{u}_h^{n+1}, \mathbf{u}_h^n, a_p, w_p, \mathbf{u}_{h,p}, k_p, B_p)$$

where the MF(.) function represents the computation of mass-flux quantities as described in Appendix C and \mathcal{F}_k contains the TKE transport and forcing terms whose discrete expression will be given in next subsection. In oceanic models, the "ED step" is classically computed using an Euler backward scheme. With the proposed approach, the convection scheme takes as input a state already updated by the boundary layer scheme (and by the solar penetration and non-solar surface heat flux which are applied during the "ED step") The convection scheme therefore uses a state whose static stability is representative of the current time-step and external forcing. Ultimately, with the proposed ap-

300 proach, the various stages can be expressed directly as follows

$$\begin{aligned}\phi^{n+1} &= \phi^n + \Delta t \partial_z (K_\phi \partial_z \phi^{n+1,*} - a_p w_p (\phi_p - \phi^{n+1,*})) \\ [a_p, w_p, \phi_p] &= \text{MF}(\phi^{n+1,*})\end{aligned}$$

301 which reflects the fact that the ED part and the MF part are properly synchronized in
302 time, as they consider the same mean fields. In the case where stratification is stable through-
303 out the column, the mass flux scheme returns a zero fraction a_p and we simply obtain
304 $\phi^{n+1} = \phi^{n+1,*}$.

305 On the other hand, the approach of simultaneously considering the ED and MF
306 parts in a single tridiagonal problem (a.k.a. the *boundary layer-and-convection* strategy,
307 e.g. Giordani et al. (2020)) would lead to

$$\begin{aligned}\phi^{n+1} &= \phi^n + \Delta t (K_\phi \partial_z \phi^{n+1} - a_p w_p (\phi_p - \phi^{n+1})) \\ [a_p, w_p, \phi_p] &= \text{MF}(\phi^n)\end{aligned}$$

308 In this case, the mass flux variables are computed using mean fields at time n . Indeed
309 ϕ_p has been computed using ϕ^n while it is applied at time $n+1$. However, this strat-
310 egy has the advantage of being unconditionally stable while the *boundary layer-then-convection*
311 strategy is conditionally stable¹. Based on the sequential integration of ED and MF com-
312 ponents, we now turn to the spatial discretization of an energetically coherent EDMF
313 scheme.

314 4.2 Energy consistent discretization of TKE dissipation and production 315 terms

316 We consider the standard grid arrangement used in oceanic models which are usu-
317 ally discretized on a Lorenz grid in the vertical (density is located in the center of the
318 cells on the vertical). We consider N grid cells in the vertical with thickness $\Delta z_j = z_{j+1/2} -$
319 $z_{j-1/2}$ ($z_{1/2} = -H$ and $z_{N+1/2} = 0$ the surface) such that $\sum_{j=1}^N \Delta z_j = -H$. Tradi-
320 tionally, turbulent quantities like turbulent kinetic energy k and eddy diffusivities K_X
321 are naturally located on the interfaces at $z_{j+1/2}$ to avoid interpolations when comput-
322 ing the vertical gradients of the turbulent fluxes. For the discrete values, not to inter-
323 fere with the grid indices, the subscript p for the plume quantities is now a superscript
324 such that plume quantities are now noted $X_{j+1/2}^p = X_p(z = z_{j+1/2})$. In the follow-
325 ing, we consider that the plume quantities and k are discretized at cell interfaces and the
326 mean quantities are discretized at cell centers.

327 The turbulent flux $\overline{w'X'}$ (for $X = u, b, \theta$) are discretized at cell interfaces as

$$(\overline{w'X'})_{j+1/2} = \mathcal{J}_{j+1/2}^X = -K_{j+1/2}^X \frac{(\delta_z X)_{j+1/2}^{n+1,*}}{\Delta z_{j+1/2}} + (a^p w^p)_{j+1/2} (X_{j+1/2}^p - X_j^{n+1,*})$$

328 with $(\delta_z X)_{j+1/2}^{n+1,*} = X_{j+1}^{n+1,*} - X_j^{n+1,*}$ and $[a^p, w^p, X^p] = \text{MF}(X^{n+1,*})$.

329 4.2.1 Discrete mean kinetic energy equation

330 Multiplying the discrete equation for u_j by $u_j^{n+1/2} = (u_j^{n+1} + u_j^n)/2$ we obtain

$$\frac{(u_j^{n+1})^2 - (u_j^n)^2}{2\Delta t} = -\frac{u_j^{n+1/2}}{\Delta z_j} \left[\mathcal{J}_{j+1/2}^u - \mathcal{J}_{j-1/2}^u \right]$$

¹ The stability constraint is given by $-a_p w_p \leq \frac{\Delta z}{2\Delta t} \left(1 + \sqrt{1 + 8 \frac{K\Delta t}{\Delta z^2}} \right)$ which is obtained through standard Von Neumann stability analysis.

331 After some simple algebra² the mean kinetic energy equation can be expressed as

$$\frac{(u_j^{n+1})^2 - (u_j^n)^2}{2\Delta t} + \frac{1}{\Delta z_j} \left(u_{j+1/2}^{n+1/2} \mathcal{J}_{j+1/2}^u - u_{j-1/2}^{n+1/2} \mathcal{J}_{j-1/2}^u \right) = \frac{1}{2\Delta z_j} \left\{ (\delta_z u)_{j+1/2}^{n+1/2} \mathcal{J}_{j+1/2}^u + (\delta_z u)_{j-1/2}^{n+1/2} \mathcal{J}_{j-1/2}^u \right\} \quad (21)$$

332 where the terms in curly brackets on the right-hand side contribute to the discrete ex-
 333 pressions of the ED and MF TKE production terms by shear while the flux divergence
 334 term corresponds to the $\partial_z T_{E_k}$ term in (2). The vertically integrated discrete budget reads

$$\begin{aligned} \sum_{j=1}^N \Delta z_j \left(\frac{(u_j^{n+1})^2 - (u_j^n)^2}{2\Delta t} \right) + u_{N+1/2}^{n+1/2} \mathcal{J}_{N+1/2}^u - u_{1/2}^{n+1/2} \mathcal{J}_{1/2}^u \\ = \frac{1}{2} \left((\delta_z u)_{N+1/2}^{n+1/2} \mathcal{J}_{N+1/2}^u + (\delta_z u)_{1/2}^{n+1/2} \mathcal{J}_{1/2}^u \right) + \sum_{j=1}^{N-1} (\delta_z u)_{j+1/2}^{n+1/2} \mathcal{J}_{j+1/2}^u \end{aligned}$$

335 Boundary condition for the momentum flux are such that the *numerical flux* equals the
 336 *physical flux*, i.e. $\mathcal{J}_{N+1/2}^u = -\tau_x/\rho_0$ and $\mathcal{J}_{1/2}^u = 0$, where τ_x is the surface momentum
 337 stress. For conciseness we assumed a no stress condition at the bottom, and vanishing
 338 gradients of momentum at the surface and the boundary as in (Burchard, 2002) (but they
 339 could have been kept without additional complications). The vertically integrated dis-
 340 crete budget is thus simplified into

$$\sum_{j=1}^N \Delta z_j \left(\frac{(u_j^{n+1})^2 - (u_j^n)^2}{2\Delta t} \right) - u_{N+1/2}^{n+1/2} \frac{\tau_x}{\rho_0} = \sum_{j=1}^{N-1} (\delta_z u)_{j+1/2}^{n+1/2} \mathcal{J}_{j+1/2}^u \quad (22)$$

341 The term on the right-hand side corresponds to a transfer between the mean and tur-
 342 bulent kinetic energies.

343 4.2.2 Discrete mean internal and potential energy equation

344 We consider the energy $c_p \bar{\theta} - z \bar{b}$ which satisfies the discrete evolution equation

$$\frac{(c_p \theta - z b)_j^{n+1} - (c_p \theta - z b)_j^n}{\Delta t} = -\frac{c_p}{\Delta z_j} \left[\mathcal{J}_{j+1/2}^\theta - \mathcal{J}_{j-1/2}^\theta \right] + \frac{z_j}{\Delta z_j} \left[\mathcal{J}_{j+1/2}^b - \mathcal{J}_{j-1/2}^b \right] + \epsilon_j$$

345 After some simple algebra³, the discrete energy equation can be expressed as

$$\begin{aligned} \frac{(c_p \theta - z b)_j^{n+1} - (c_p \theta - z b)_j^n}{\Delta t} + \frac{1}{\Delta z_j} \left[\left(c_p \mathcal{J}_{j+1/2}^\theta - z_{j+1/2} \mathcal{J}_{j+1/2}^b \right) - \left(c_p \mathcal{J}_{j-1/2}^\theta - z_{j-1/2} \mathcal{J}_{j-1/2}^b \right) \right] \\ = -\frac{1}{2\Delta z_j} \left\{ \Delta z_{j+1/2} \mathcal{J}_{j+1/2}^b + \Delta z_{j-1/2} \mathcal{J}_{j-1/2}^b \right\} + \epsilon_j \quad (23) \end{aligned}$$

346 where the terms in curly brackets on the right-hand side contribute to the discrete ex-
 347 pressions of the ED and MF TKE buoyancy production/destruction terms while the flux
 348 divergence term corresponds to the $\partial_z T_{E_{i+p}}$ term in (2).

349 Assuming boundary conditions such that no buoyancy/heat flux through the bot-
 350 tom we get

² We multiply $\mathcal{J}_{j+1/2}^u$ by $u_j^{n+1/2} = -\frac{1}{2}(\delta_z u)_{j+1/2}^{n+1/2} + u_{j+1/2}^{n+1/2}$ and $\mathcal{J}_{j-1/2}^u$ by $u_j^{n+1/2} = \frac{1}{2}(\delta_z u)_{j-1/2}^{n+1/2} + u_{j-1/2}^{n+1/2}$

³ We multiply $\mathcal{H}_{j+1/2}^b$ by $z_j = -\frac{1}{2}\Delta z_{j+1/2} + z_{j+1/2}$ and $\mathcal{H}_{j-1/2}^b$ by $z_j = \frac{1}{2}\Delta z_{j-1/2} + z_{j-1/2}$

$$\begin{aligned}
 \sum_{j=1}^N \Delta z_j \left(\frac{(c_p \theta - zb)_j^{n+1} - (c_p \theta - zb)_j^n}{\Delta t} \right) + c_p \mathcal{J}_{N+1/2}^\theta + \frac{1}{2} \Delta z_{N+1/2} \mathcal{J}_{N+1/2}^b \\
 = - \sum_{j=1}^{N-1} \Delta z_{j+1/2} \mathcal{J}_{j+1/2}^b + \sum_{j=1}^N \Delta z_j \epsilon_j \quad (24)
 \end{aligned}$$

351 where we prescribe that the numerical flux equals the "physical" surface flux, i.e. $c_p \mathcal{J}_{N+1/2}^\theta +$
 352 $\frac{1}{2} \Delta z_{N+1/2} \mathcal{J}_{N+1/2}^b = c_p \overline{w' \theta'}_0 + g \frac{\Delta z_{N+1/2}}{2} (\alpha \overline{w' \theta'}_0 - \beta \overline{w' S'}_0)$. Note however that in-
 353 consistent boundary condition can lead to additional unphysical energy fluxes (see 5.1.2).

354 4.2.3 Constraints for discrete energetic consistency

355 In the same spirit as Burchard (2002), the discrete energy budgets (21) and (23)
 356 can be used to derive an energy-conserving discretization of the shear and buoyancy pro-
 357 duction terms for turbulent kinetic energy. As stated above the TKE variable is natu-
 358 rally located at cell interfaces. A discrete expression for its evolution equation is given
 359 by :

$$\frac{k_{j+1/2}^{n+1} - k_{j+1/2}^n}{\Delta t} = - \frac{1}{\Delta z_{j+1/2}} [\mathcal{G}_{j+1} - \mathcal{G}_j] + \text{Sh}_{j+1/2} + \mathcal{B}_{j+1/2} - \epsilon_{j+1/2}$$

360 The turbulent transport of TKE consistent with (14) is

$$\mathcal{G}_j = -K_j^k \frac{k_{j+1/2}^{n+1} - k_{j-1/2}^{n+1}}{\Delta z_j} + (a^p w^p)_{j+1/2} (k_{j+1/2}^p - k_{j-1/2}^p) + \frac{(a^p w^p)_{j+1/2}}{2} \left((w_{j+1/2}^p)^2 + (u_{j+1/2}^p - u_j^{n+1,*})^2 \right)$$

361 Using (22) and (24), the constraints for energetic consistency between the mean and tur-
 362 bulent energies at the discrete level lead to

$$\begin{aligned}
 \sum_{j=1}^{N-1} \Delta z_{j+1/2} \text{Sh}_{j+1/2} &= - \sum_{j=1}^{N-1} (\delta_z u)_{j+1/2}^{n+1/2} \mathcal{J}_{j+1/2}^u \\
 \sum_{j=1}^{N-1} \Delta z_{j+1/2} \mathcal{B}_{j+1/2} &= \sum_{j=1}^{N-1} \Delta z_{j+1/2} \mathcal{J}_{j+1/2}^b \\
 \sum_{j=1}^{N-1} \Delta z_{j+1/2} \epsilon_{j+1/2} &= \sum_{j=1}^N \Delta z_j \epsilon_j
 \end{aligned}$$

363 Such constraints are satisfied for the following choice of discretizations

$$\begin{cases}
 \text{Sh}_{j+1/2} &= \frac{(\delta_z u)_{j+1/2}^{n+1/2}}{\Delta z_{j+1/2}} \left(K_{j+1/2} \frac{(\delta_z u)_{j+1/2}^{n+1,*}}{\Delta z_{j+1/2}} - (a_p w_p)_{j+1/2} (u_{j+1/2}^p - u_j^{n+1,*}) \right) \\
 \mathcal{B}_{j+1/2} &= -K_{j+1/2}^b (N^2)_{j+1/2}^{n+1,*} + (a_p w_p)_{j+1/2} (b_{\text{eos}}(\phi_{j+1/2}^p) - b_{\text{eos}}(\phi_j^{n+1,*})) \\
 \epsilon_j &= \frac{\Delta z_{j+1/2} \epsilon_{j+1/2} + \Delta z_{j-1/2} \epsilon_{j-1/2}}{\Delta z_j}
 \end{cases} \quad (25)$$

364 further assuming that $\epsilon_{1/2} = \epsilon_{N+1/2} = 0$. Considering the discretizations (25), the
 365 total energy budget of the water column reads

$$\begin{aligned}
 \sum_{j=1}^N \Delta z_j \frac{(c_p \theta - zb + u^2/2)_j^{n+1} - (c_p \theta - zb + u^2/2)_j^n}{\Delta t} + \sum_{j=1}^{N-1} \Delta z_{j+1/2} \frac{k_{j+1/2}^{n+1} - k_{j+1/2}^n}{\Delta t} \\
 = u_{N+1/2}^{n+1/2} \frac{\tau_x}{\rho_0} + \left(\frac{\alpha g \Delta z_{N+1/2}}{2} + c_p \right) \overline{w' \theta'}_0 - \frac{\beta g \Delta z_{N+1/2}}{2} \overline{w' S'}_0 - \mathcal{G}_N + \mathcal{G}_1 \quad (26)
 \end{aligned}$$

366 We check in the following section that this budget is indeed recovered in our SCM code.

5 SCM evaluation and parameterization impacts

In this section, we test on the two idealized cases the sensitivity of the scheme to the formulation of TKE transport, energetic consistency, boundary conditions, mass-flux transport of horizontal momentum, and small plume area assumption. We also quantify energy biases of inconsistent formulations, including double-counting errors due to inconsistent boundary conditions. The constants c_m, c_e, c_k used in the ED terms are the same as the constants used in the TKE equation of the LES model. The parameters used for the plume equations closures have been chosen as $\beta_1 = 0.99, \beta_2 = 1.99, a = 1., b = 1., b' = 0.75, C_u = 0.5, a_p^0 = 0.2, \delta_0 = 1.125$. A careful tuning and uncertainty quantification of the parameters, using for instance statistical method (e.g. Souza et al., 2020; Couvreux et al., 2021), is left for future studies. Sensitivity to time step and number of levels is presented in Appendix D.

5.1 Impact of the energetic consistency and comparison to EVD

5.1.1 Bulk production

In this section, we evaluate three different configurations of the SCM against LES data. First, an EDMF scheme in which an ED parameterization of the TKE equation is used (referred to as "EDMF-inconsistent"), namely

$$\partial_t k + \partial_z (-K_k \partial_z k) = -K_\phi \partial_z \bar{b} + K_u (\partial_z \bar{u}_h)^2 - \bar{\epsilon} \quad (27)$$

This configuration is not energetically consistent since the contributions of MF to the buoyancy and shear production of TKE are missing (Part I). It would be the result of an independent coupling of TKE and MF schemes. The second configuration consists of the previously detailed EDMF scheme in which the TKE equation consistently includes the contribution of MF terms to energy transfers (referred to as "EDMF-Energy") as detailed in (2). Finally, an ED closure based on a TKE-scheme, along with enhanced vertical diffusivity to $10 \text{ m}^2 \text{ s}^{-1}$ at depths where $N^2 < 0$ is tested and implemented for reference. This is the standard option in NEMO (Madec et al., 2019), and we will refer to it as "ED+EVD". The comparison of the three aforementioned configurations are plotted for the two LES cases on figures 6 and 7. ED+EVD necessarily produce unstable (near-neutral) temperature profiles (fig. 6(a) and 7(a)) when the associated temperature flux is positive, since by design ED+EVD can only allow down-gradient fluxes. Moreover ED+EVD fails to reproduce the correct magnitude of the so-called vertical entrainment zone (e.g. Garcia & Mellado, 2014), in which penetrative convection generates negative temperature flux and sharpens the temperature gradients at the base of the mixed layer. The lack of penetrative convection is known to reduce the deepening rate (e.g. chap. 6, Garratt, 1994), thus producing an important bias of a hundred meters regarding the mixed layer depth compared to LES. However in this region the negative flux of temperature could have been represented by an ED-only closure, since positive gradient of temperature are expected (e.g. (Bretherton & Park, 2009)) ; the observed bias of the entrainment flux is not due to the usage of an ED closure, but to the detailed formulation of such a downgradient scheme. On the other hand, EDMF-inconsistent and EDMF-energy equally perform in representing these profiles. The absence of a noticeable effect of the energetic consistency on the temperature mean and flux profiles is a consequence of the small value of the eddy-diffusivity fluxes (dashed lines) in the mixed layer. When considering the TKE profile (fig. 6(c) and 7(c)), ED+EVD can model the correct order of magnitude, however, the TKE does not penetrate enough. EDMF-inconsistent fails to reproduce TKE due to energetic inconsistency. Indeed, looking at temperature and velocity fluxes allows us to infer that the losses of resolved energy due to buoyancy and shear are dominated by the MF contributions. However, such contributions are not included as sources of TKE for the EDMF-inconsistent scheme, leading to the very low levels of TKE observed in the simulation. EDMF-energy can reproduce accurate profiles of TKE. The main discrepancies arise close to the surface and at the base of the mixed layer. How-

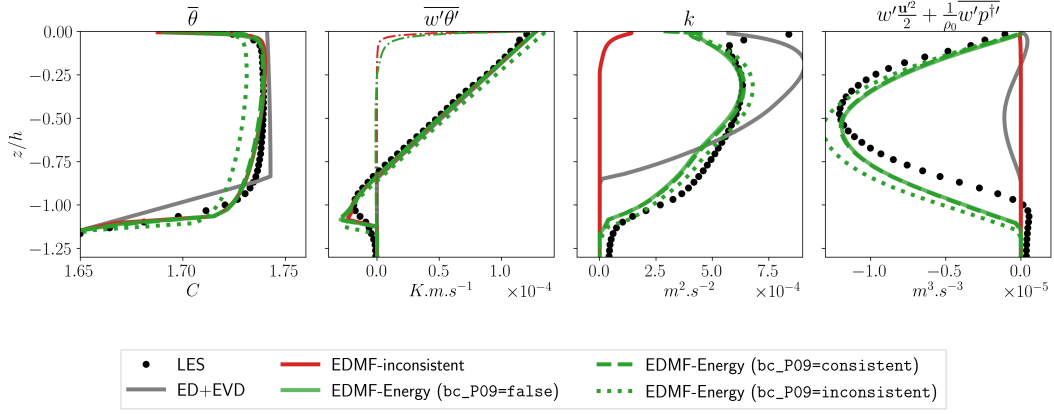


Figure 6: Mean and turbulent profiles for the case FC500 case after 72h of simulation. LES data (black dots), ED+EVD scheme (gray line), EDMF-inconsistent (red line), EDMF-energy (green line), EDMF-energy-bc_P09=consistent (green dashed) and EDMF-energy-bc_P09=inconsistent (green dotted) are represented, along with the ED contribution to the temperature fluxes for EDMF-inconsistent (dashed-dotted red) and EDMF-energy (dash-dotted green).

417 ever energetic consistency ensures that these discrepancies are only due to modelling choices
 418 (such as TKE boundary conditions) and cannot be the result of inconsistent energetics.
 419 Regarding the vertical transport of TKE (fig. 6(d) and 7(f)), EDMF-inconsistent is not
 420 able to reproduce the LES profile. ED+EVD displays the correct shape, however, the
 421 magnitude is approximately ten times less than the reference profile. EDMF-energy ac-
 422 curately reproduces the profile, with a shift downwards of a few percent of the mixed layer
 423 depth h .

424 In Fig. 8, we represent the vertically integrated energy budget of the SCM for the
 425 case W005_C500 (FC500 is similar), namely the quantity

$$\int_{-H}^0 \partial_t (E_k + k + E_{i+p}) dz + [T_{E_k} + T_k + T_{E_{i+p}}]_{z=-H}^{z=0} \quad (28)$$

426 where the different energy reservoirs and fluxes are presented on equations (2)-(4). As
 427 expected, EDMF-energy conserves energy up to round-off errors ($\pm 10^{-12} \text{m}^3 \text{s}^{-3}$) whereas
 428 EDMF-inconsistent exhibit an energy loss of about $10^{-5} \text{m}^3 \text{s}^{-3}$ corresponding to

$$\int_{-H}^0 (-a_p w_p (b_p - \bar{b}) + a_p w_p (\mathbf{u}_{h,p} - \bar{\mathbf{u}}_h) \cdot \partial_z \bar{\mathbf{u}}_h) dz \quad (29)$$

429 which scales with $B_0 h$.

430 5.1.2 Boundary conditions

431 In Part I we showed that inconsistent choices of boundary conditions for plume and
 432 mean temperature can lead to energy biases. Here we numerically evaluate the impact
 433 of three different plume/mean choices. The default choice for our scheme is

$$(\text{bc_P09=false}) \begin{cases} \theta_p|_0 & = \bar{\theta}|_0 \\ -K_\phi \partial_z \bar{\theta}|_0 & = \overline{w'\theta'}|_0 \end{cases} \quad (30)$$

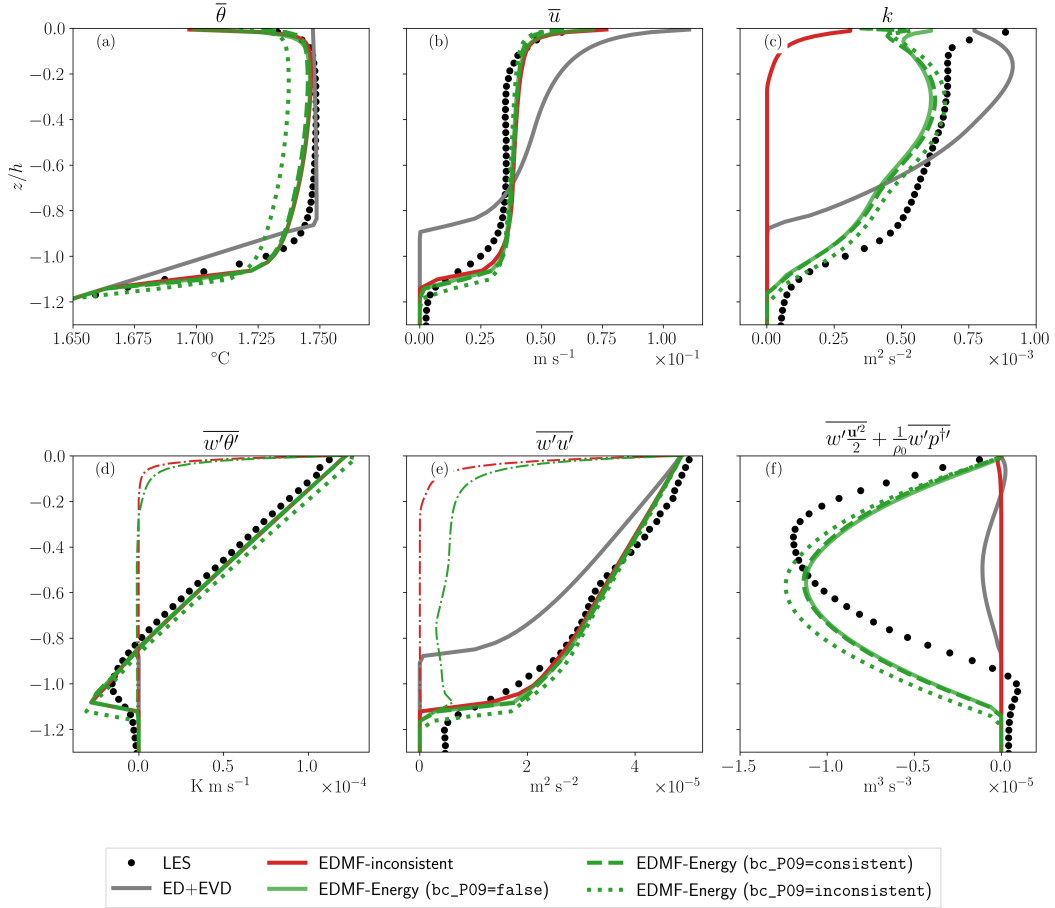


Figure 7: Same as figure 6 for the case W005_C500.

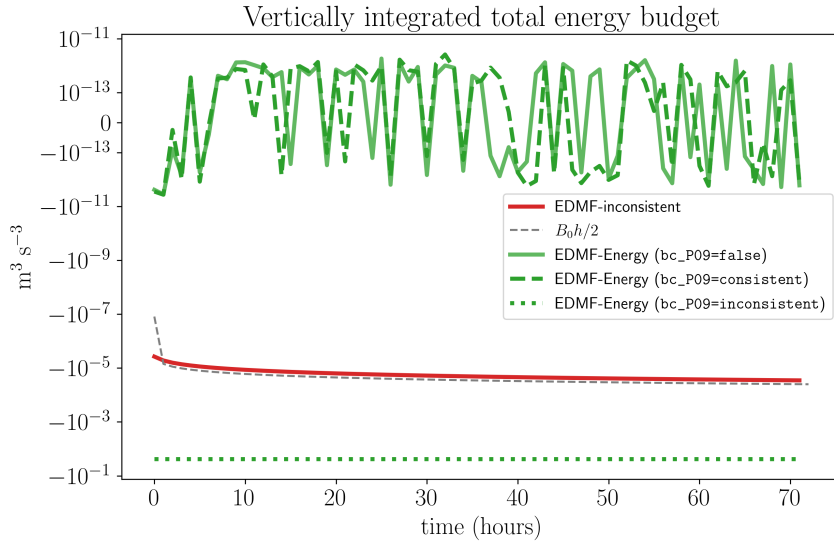


Figure 8: Time series of the vertically integrated energy budget (28) for the case W005_C500 (see text for details).

434 which result in the physical energy flux $T_{E_{i+p}}|_0 = c_p \overline{w'\theta'}|_0$. The boundary condition
 435 proposed in (Pergaud et al., 2009), adapted from (Soares et al., 2004),

$$(\text{bc_P09=inconsistent}) \begin{cases} \theta_p|_0 = \bar{\theta}|_0 + \beta_{\text{P09}} \frac{\overline{w'\theta'}|_0}{\sqrt{k}|_0} \\ -K_\phi \partial_z \bar{\theta}|_0 = \overline{w'\theta'}|_0 \end{cases} \quad (31)$$

436 where $\beta = 0.3$ is a numerical constant. These boundary conditions is equivalent to pre-
 437 scribe the unphysical energy flux $T_{E_{i+p}}|_0 = c_p \overline{w'\theta'}|_0 + c_p \overline{w'\theta'}|_0 \frac{a_p|_0 w_p|_0 \beta_{\text{P09}}}{\sqrt{k}|_0}$ where the
 438 second term of the r.h.s. is a spurious source of energy. This bias is due to an inconsis-
 439 tent partitioning of the physical boundary flux $c_p \overline{w'\theta'}|_0$ into ED and MF fluxes. Such
 440 bias can be corrected by modifying the ED flux,

$$(\text{bc_P09=consistent}) \begin{cases} \theta_p|_0 = \bar{\theta}|_0 + \beta_{\text{P09}} \frac{\overline{w'\theta'}|_0}{\sqrt{k}|_0} \\ -K_b|_0 \partial_z \bar{\theta}|_0 = \overline{w'\theta'}|_0 - a_p|_0 w_p|_0 (\theta_p|_0 - \bar{\theta}|_0) = \left(1 - \frac{a_p|_0 w_p|_0 \beta_{\text{P09}}}{\sqrt{k}|_0}\right) \overline{w'\theta'}|_0 \end{cases} \quad (32)$$

441 leading to the physical energy flux $T_{E_{i+p}}|_0 = c_p \overline{w'\theta'}|_0$.

442 On figures 6 and 7 we expose the result of these different boundary conditions on
 443 mean and turbulent quantities after 72h of simulation. For the EDMF-energy scheme,
 444 solutions computed using `bc_P09=false` and `bc_P09=consistent` coincide, whereas `bc_P09=inconsistent`
 445 exhibit a cool bias of $\simeq 0.01^\circ\text{C}$ on temperature profiles (fig. 6(a) and 7(a)) due to the
 446 spurious flux imposed at the surface. Since buoyancy in the mixed layer scales with $N^2 h$
 447 (Garratt, 1994) this bias is expected to increase with the mixed layer depth. Finally it
 448 induces bias of a few percents on the TKE and TKE flux profiles. On figure 8 vertically
 449 integrated total energy budgets are represented against time for these different bound-
 450 ary conditions for the case W005_C500. "`bc_P09=false`" and "`bc_P09=consistent`" equally
 451 performs at conserving energy up to round-off errors ($\pm 10^{-12} \text{m}^3 \text{s}^{-3}$), whereas "`bc_P09=inconsistent`"
 452 exhibits an important bias of $10^{-1} \text{m}^3 \text{s}^{-3}$.

5.2 Impact of TKE transport

454 In fig. 10 and 9 we compare the new TKE transport parameterization exposed in
 455 sec. 2 to a standard ED+EVD formulation, and to the MF closures proposed by (Witek
 456 et al., 2011) and by (Han & Bretherton, 2019).

$$\overline{w' \frac{\mathbf{u}' \cdot \mathbf{u}'}{2}} = \underbrace{-K_k \partial_z k}_{\text{ED}} + \underbrace{\overbrace{a_p w_p (k_p - k)}^{\text{Han \& Bretherton 2019}} + \overbrace{a_p w_p^3}^{\text{Witek et al. 2011}} + \frac{a_p w_p}{2} \|\mathbf{u}_{h,p} - \bar{\mathbf{u}}_h\|^2}_{\text{new EDMF}} \quad (33)$$

457 Moreover, note that we use a different plume TKE equation than the one used in Han
 458 and Bretherton (2019),

$$a_p w_p \partial_z k_p = \underbrace{\overbrace{E (k - k_p)}^{\text{Han \& Bretherton 2019}} + E \left(\frac{1}{2} \|\mathbf{u}_p - \bar{\mathbf{u}}\|^2 \right)}_{\text{new EDMF}} - a_p \epsilon_p \quad (34)$$

459 The choice of TKE transport seems to have a negligible impact on temperature, tem-
 460 perature fluxes, velocity and momentum fluxes. When looking at TKE fluxes (fig. 9(d)
 461 and 10(f)), all parameterizations reproduce the bell-shape but exhibit a consistent shift
 462 downwards of approximately $0.1h$. Haghshenas and Mellado (2019b) mention that the
 463 local maximum at the base of the mixed layer is caused by shear production of TKE and
 464 corresponds to a upwards transport from the mixed layer base into the mixed layer in-
 465 terior. None of the parameterizations are able to capture this effect. Regarding the min-
 466 imum of the flux, ED+EVD and Han & Bretherton parameterizations result in biases

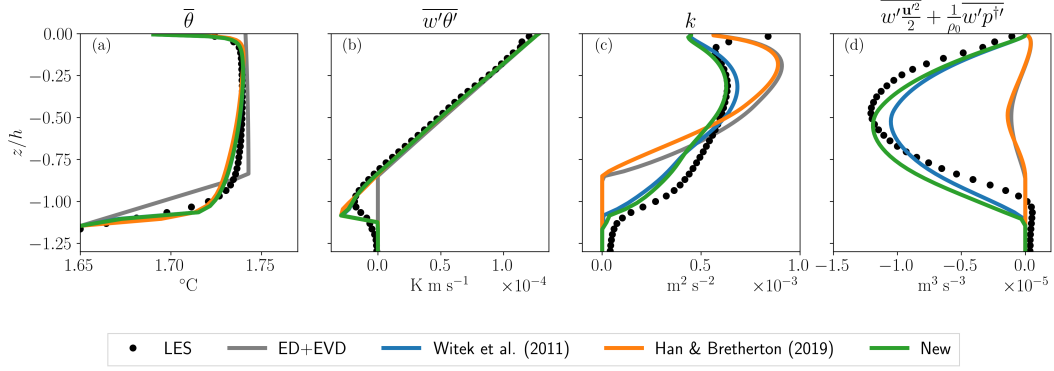


Figure 9: Vertical profiles of (a) temperature, (b) temperature flux, (c) turbulent kinetic energy and (d) turbulent transport of TKE for the FC500 case after 72h of simulation. LES data (black dots) are ED+EVD scheme (gray line) represented, as well as three energetically consistent EDMF scheme differing only by their parameterization of TKE transport: Witek et al. 2011 (blue line), Han & Bretherton 2019 (orange line) and the new formulation (green line).

467 of 90% and 80% biases compared to LES, whereas Witek et al. has the correct value and
 468 the new parameterization present a 10% overestimate of the maximum. We will see that
 469 this overestimation can be corrected if the assumption of small a_p is relaxed, see sec. 5.3.
 470 Transport of TKE has a direct effect on the profile of TKE (fig. 9(c) and 10(c)). The
 471 surface production of TKE seems not enough redistributed on the vertical for ED+EVD
 472 and Han & Bretherton schemes, leading to insufficient penetration of TKE inside the lower
 473 half of the mixed layer. The more realistic TKE transports of Witek et al. and of the
 474 new scheme lead to a better penetration of the TKE inside the mixed layer. The sur-
 475 face TKE bias, also present in the free convective case on fig. 6 may be related to bound-
 476 ary condition choices that should be further investigated. Our implementation of Han
 477 & Bretherton reproduces well the shape of TKE plotted on their figure 4; but their im-
 478 plementation exhibits a clear lack of production of TKE.

479 5.3 Impact of horizontal momentum transport and small plume area

480 In section 3.3, analysis of LES revealed that the assumption $a_p \ll 1$ was not fully
 481 satisfied. This small-area assumption can be relaxed with no additional complexity if the
 482 subplume fluxes $w'_p \phi'_p$ are still neglected. A summary of the EDMF-Energy parameter-
 483 ization in such a regime is presented in Tab. 4. Moreover we examine the impact of the
 484 mass-flux on horizontal momentum transport by comparing simulations in which $\overline{w' \mathbf{u}'_h}^{MF} =$
 485 0 (a choice made in several studies, e.g Tan et al. (2018); Cohen et al. (2020); Giordani
 486 et al. (2020)) to simulations where the horizontal velocity plume model exposed in tab.
 487 1 leads to $\overline{w' \mathbf{u}'_h}^{MF} \neq 0$. The cross-comparison of small-area impact and MF horizon-
 488 tal momentum impact leads to four EDMF configurations, presented on fig. 11 for the
 489 case W005_C500.

490 Relaxing the small area assumption mainly affects the minimum of the TKE trans-
 491 port, reducing by 10% which roughly corresponds to the rescaling coefficient $\sigma = 1/(1-$
 492 $a_p)$. As a consequence TKE is less redistributed on the vertical, leading to slightly higher
 493 TKE close to the surface and reduced TKE in the interior. Other mean fields and fluxes
 494 do not seem affected. However, the influence of the small plume area would be more clearly
 495 seen in the gray zone of convection, i.e. when the horizontal size of convective structures
 496 approach the horizontal size of the LES domain (Honnert, 2022). For 3D implementa-

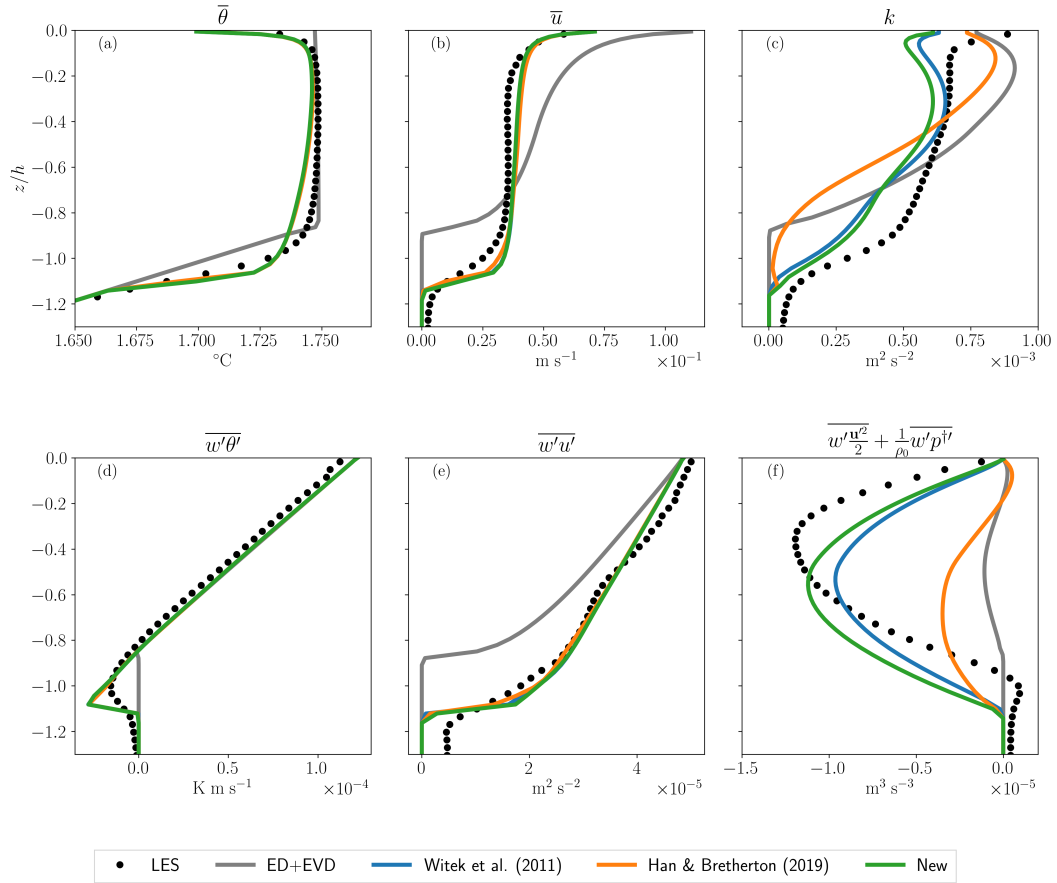


Figure 10: Same as figure 10 for the case W005_C500.

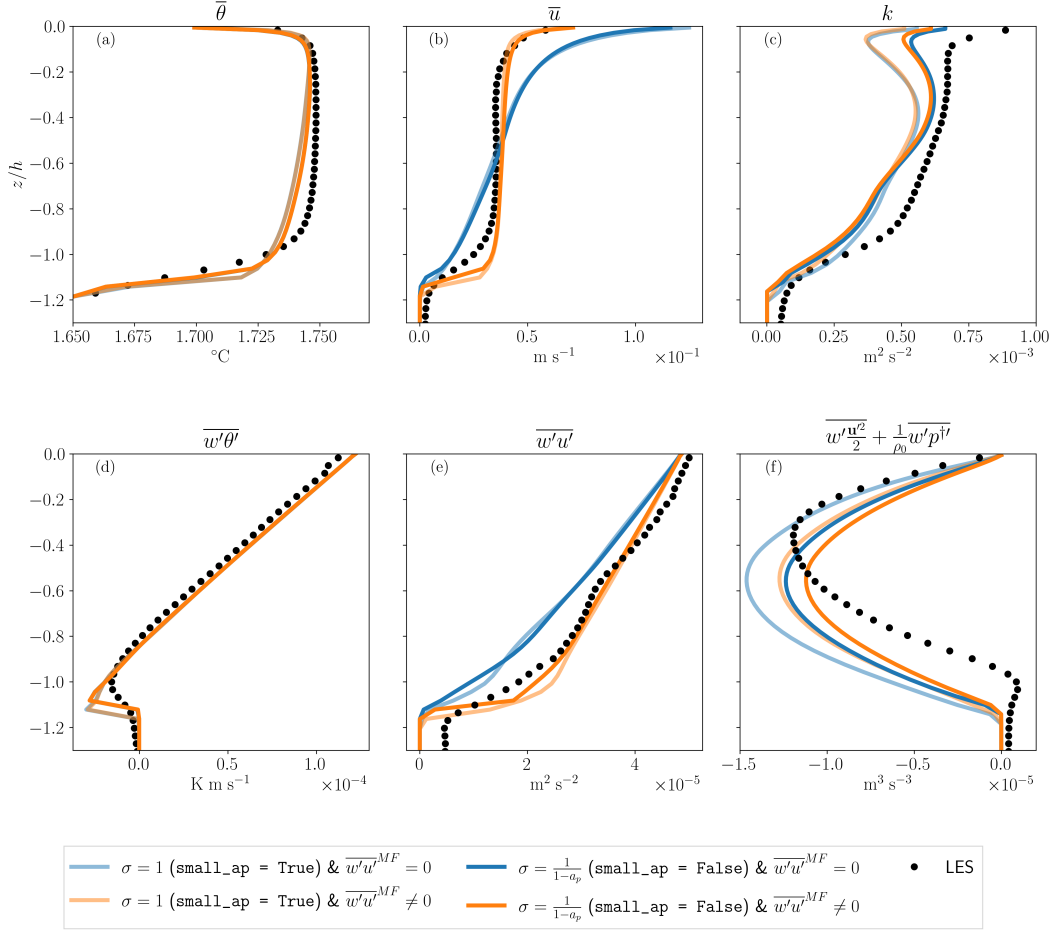


Figure 11: Mean and turbulent profiles for the W005_C500 case after 72h of simulation. Different configurations of EDMF-energy are used: small plume area assumption (transparent lines) or not (solid lines); MF on momentum flux (orange lines) or not (blue lines).

497 tions of the parameterization at resolution lying within the gray zone, the dependency
 498 of parameters to resolution should be assessed.

499 The main effect of MF on horizontal velocities is to increase the momentum flux,
 500 which in turn produces more mixing of the zonal velocity compared to the velocities com-
 501 puted with ED-only momentum flux. According to the reference LES the well-mixed pro-
 502 file is more realistic. Configurations with MF momentum flux exhibit more negative min-
 503 ima of TKE flux, which in turn produces higher levels of TKE. However this impact is
 504 less important than that of the small-area assumption. This can be explained by the fact
 505 that W005_C500 depicts a convection-dominated regime; thus TKE is mainly produced
 506 by buoyancy, and the flux of TKE is dominated by the w_p^3 term.

507 5.4 Realistic case: Hymex/ASICS-MED campaign

508 We now move to more realistic situations corresponding to a sequence of strong con-
 509 vective events which were documented in the Northwestern Mediterranean during the
 510 winter 2013 of the HyMeX/ASICS-MED experiment at the LION buoy. These events
 511 corresponds to localized *oceanic deep convection* (Marshall & Schott, 1999) for which the
 512 water column is typically mixed over thousands meters, sometime down to the seafloor.

$\sigma = \frac{1}{1 - a_p}$ $\overline{w'\theta'} = \sigma a_p w_p (\theta_p - \bar{\theta}) - K_\phi \partial_z \bar{\theta}$ $\overline{w'S'} = \sigma a_p w_p (S_p - \bar{S}) - K_\phi \partial_z \bar{S}$ $\overline{w'\mathbf{u}'_h} = \sigma a_p w_p (\mathbf{u}_{h,p} - \bar{\mathbf{u}}_h) - K_m \partial_z \bar{\mathbf{u}}_h$	Rescaling coefficient Vertical turbulent flux of temperature Vertical turbulent flux of salt Vertical turbulent momentum flux
$\partial_z(a_p w_p) = E - D$ $a_p w_p \partial_z \theta_p = \sigma E (\bar{\theta} - \theta_p)$ $a_p w_p \partial_z S_p = \sigma E (\bar{S} - S_p)$ $a_p w_p \partial_z \mathbf{u}_{h,p} = \sigma E (\bar{\mathbf{u}}_h - \mathbf{u}_{h,p}) + a_p w_p C_u \partial_z \bar{\mathbf{u}}_h$ $a_p w_p \partial_z w_p = -\sigma b E w_p + a_p \left[a B_p + \sigma \frac{b'}{h} (w_p)^2 \right]$ $B_p = b_{\text{eos}}(\theta_p, S_p) - b_{\text{eos}}(\bar{\theta}, \bar{S})$ $a_p w_p \partial_z k_p = \sigma E \left((k - k_p) + (1 + a_p^2 \tilde{\sigma}) \frac{1}{2} \ \mathbf{u}_p - \mathbf{u}\ ^2 \right) - a_p \epsilon_p$ $E = a_p C_\epsilon \max(0, \partial_z w_p)$ $D = -a_p C_\delta \min(0, \partial_z w_p) - a_p w_p \delta_0 \frac{1}{h}$	Plume area conservation equation Plume temperature equation Plume salinity equation Plume horizontal momentum equation Plume vertical velocity equation Buoyancy forcing term Plume related TKE Lateral entrainment closure Lateral detrainment closure
$\partial_t k - \partial_z (K_k \partial_z k) = K_m (\partial_z \bar{\mathbf{u}}_h)^2 - K_\phi \partial_z \bar{b}$ $- \sigma a_p w_p ((\mathbf{u}_{h,p} - \bar{\mathbf{u}}_h) \cdot \partial_z \bar{\mathbf{u}}_h - (b_p - \bar{b}))$ $- \partial_z \left(\sigma a_p w_p \left[k_p - k + \frac{1}{2} \ \mathbf{u}_p - \mathbf{u}\ ^2 \right] \right)$ $- \bar{\epsilon}$	ED related TKE production terms MF related TKE production terms MF related TKE transport term TKE dissipation

Table 4: Summary of the plume equations without the small plume area assumption, i.e. without assuming $a_p \ll 1$. Small-area assumption is recovered by imposing $\sigma = 1$ and $\tilde{\sigma} = 0$.

513 These winter events are common in the Mediterranean sea as well as in the Labrador and
 514 Greenland seas, and are of great importance for deep water formation and their over-
 515 all influence on basin scale dynamics. The ASICS-MED experiment was also used in Giordani
 516 et al. (2020) and we use a similar setup here (similar vertical grid as well as similar ini-
 517 tial and surface boundary conditions). The experiments are performed with a SCM sim-
 518 ilar to (1) but including additional Coriolis and solar penetration (using a standard Jerlov
 519 law) terms. We use a nonlinear equation of state. We also include penalization terms
 520 in the SCM nudging w_p to zero at the ocean floor to account for the effect of the bot-
 521 tom (which is at a depth of 2400 m at the LION buoy). Thanks to the penalization term
 522 a no-slip boundary condition is imposed at the bottom and a no-gradient condition is
 523 imposed for tracers. The vertical grid resolution ranges from 1 m near the surface to
 524 150 m near the bottom located at $z = -2400$ m. Parameters of the TKE scheme are
 525 set to the standard NEMO values, $\mathbf{c} = (c_m, c_\epsilon, c_k) = (0.1, \frac{\sqrt{2}}{2}, 0.1)$.

526 A series of 30-days numerical simulations were carried out starting from January
 527 15, 2013. The surface boundary conditions are shown in Fig. 12. In particular, very strong
 528 cooling events occurred during the period of interest. Two simulations were made sys-
 529 tematically with an eddy-diffusivity term activated. A first simulation was done with En-
 530 hanced Vertical Diffusion (referred to as ED+EVD) which is the standard practice for
 531 climate simulations using NEMO, a second one using a mass flux scheme on tracers, dy-
 532 namics, and with the additional terms for energetic consistency in the TKE equation (re-
 533 ferred to as EDMF-energy). To get a more concrete idea of the improvements brought
 534 about by the mass flux scheme over the usual practice for NEMO applications (ED+EVD),
 535 we show in Fig. 12 (bottom panel) the temporal evolution of the mixed layer depth h_{mixl}
 536 computed from mooring data and single-column numerical simulations. h_{mixl} is defined
 537 as the depth where the following criterion is met

$$\int_{h_{\text{mixl}}}^{z_{\text{ref}}} \partial_z b_{\text{eos}}(\theta, S = 38.5 \text{ psu}) dz = \frac{g}{\rho_0} \rho_c \quad (35)$$

538 with $z_{\text{ref}} = 300$ m and $\rho_c = 0.01 \text{ kg m}^{-3}$. This choice of reference level is consistent
 539 with the one used in Houpert et al. (2016) also with LION buoy observational data. More-
 540 over, the mixed layer depths obtained by applying (35) on the data (see red curve in Fig.
 541 12) are directly comparable with the ones shown in Waldman et al. (2017) (see their Fig.
 542 4). We had to consider a constant salinity in the buoyancy calculation because the salin-
 543 ity data from the LION buoy are noisy in the vertical and did not allow for a robust di-
 544 agnostic. The bottom panel in Fig. 12 illustrates the fact that the mixed layer depth is
 545 significantly better represented by the EDMF-Energy scheme than by the ED+EVD ap-
 546 proach. Moreover, a direct comparison with temperature and salinity from mooring data
 547 is shown in Fig. 13 at different times. In particular several phases can be identified dur-
 548 ing the experiment (e.g. Coppola et al., 2017; Waldman et al., 2017): (i) in the period
 549 15-25 January 2013 winter convection starts to deepen the mixed layer down to around
 550 -800 m to the point of eroding the Levantine intermediate waters (Estournel et al., 2016)
 551 (ii) in the period 26-29 January 2013 the mixed layer keeps thickening to the depth of
 552 the western Mediterranean deep water (≈ -1250 m) (iii) in the period 4-9 February
 553 2013 a new intense convective event associated with a strong Mistral event contributes
 554 to deepen the mixed layer down to the bottom (reached in 9 February). This is followed
 555 by a restratification phase involving horizontal processes that cannot be represented in
 556 our SCM formalism which explains why we do not analyze solutions beyond February
 557 9.

558 6 Discussion and conclusion

559 In this work, we used idealized Large Eddy Simulations of oceanic convection to
 560 analyze hypotheses and outputs of a novel Eddy-Diffusivity Mass-Flux parameterization
 561 in a Single-Column Model configuration, derived in Part I (Perrot et al., 2024). This en-
 562 ergetically consistent parameterization is rather simple to implement, whether in a code

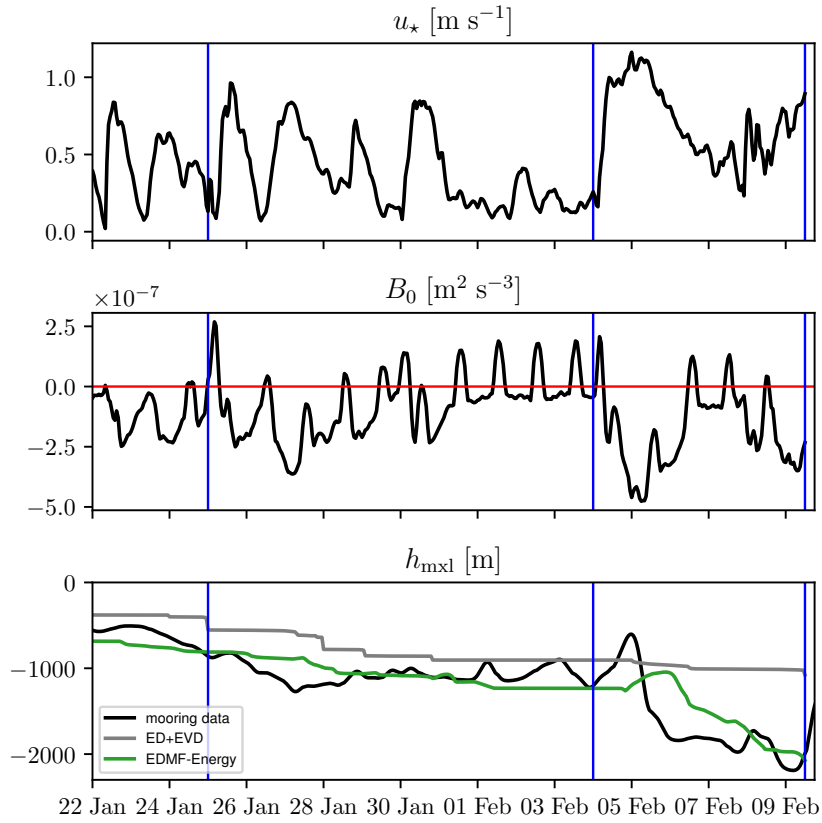


Figure 12: Time series of the friction velocity u_* (m s^{-1} , top panel) and surface buoyancy flux B_0 ($\text{m}^2 \text{s}^{-3}$, middle panel) computed from atmospheric forcings. Time series of mixed layer depth h_{mixl} (m, bottom panel) obtained from observations at the LION buoy (black line) and from single column numerical experiments using ED+EVD (solid gray line) and EDMF-Energy (solid green line). The vertical blue lines correspond to the dates at which the vertical temperature and salinity profiles derived from observations and numerical simulations are compared in Fig. 13.

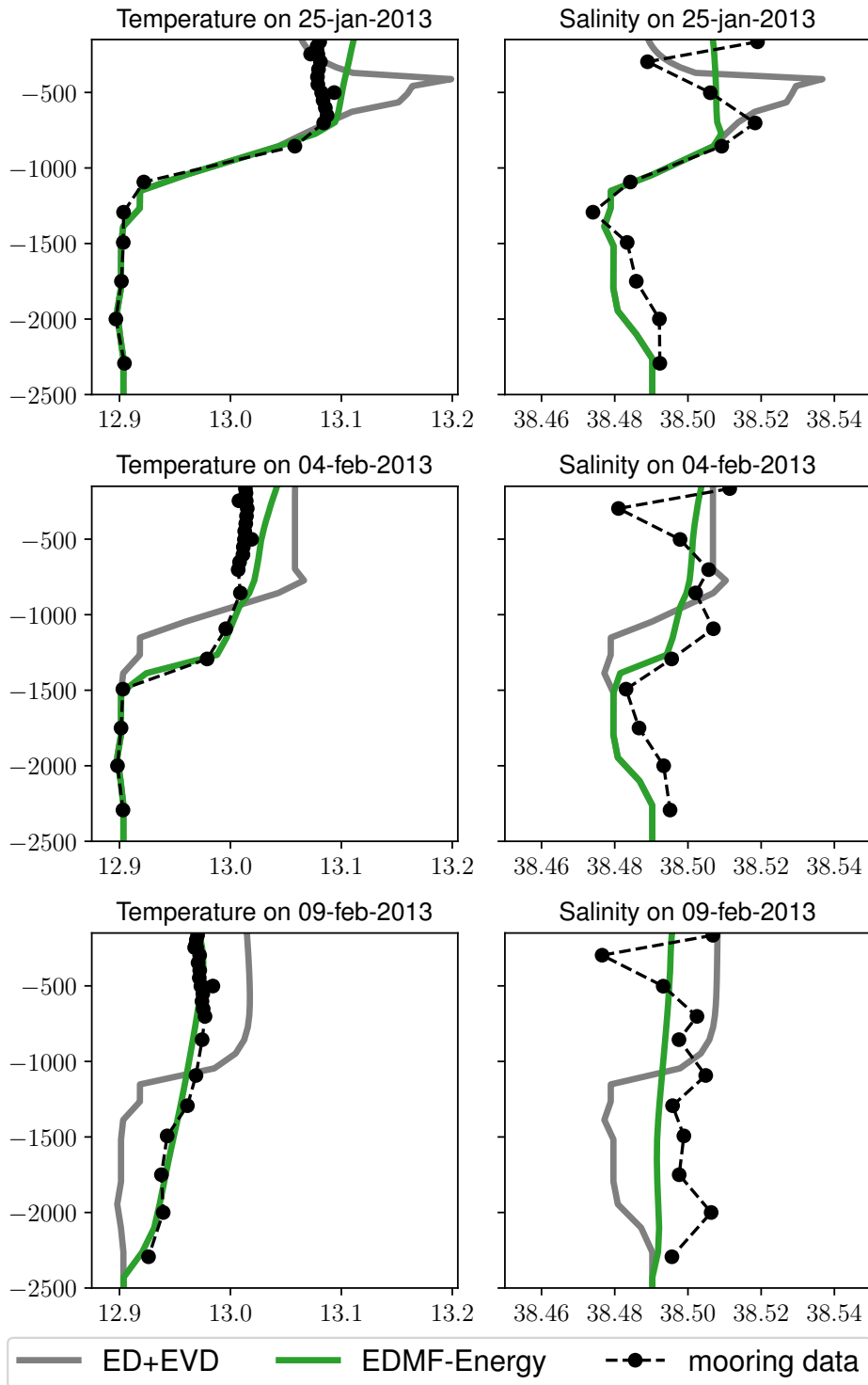


Figure 13: Temperature ($^{\circ}\text{C}$, left panels) and salinity (psu, right panels) profiles obtained from single column experiments at the location of the LION buoy using an eddy-diffusivity closure with enhanced vertical diffusion (ED+EVD, solid gray lines) and energetically-consistent EDMF (EDMF-Energy, solid green lines). Results from numerical experiments are compared to observations from the LION buoy (dashed black lines) for 4 dates represented on the Fig. 12 by vertical blue lines.

563 with an existing "non-energetically consistent" EDMF scheme or, more generally, in any
 564 code relying on a prognostic TKE equation. The MF terms are obtained by solving a
 565 straightforward system of ODEs and take the form of vertical advection terms in the mean
 566 equations (see Appendix C for practical details). The proposed approach can also be ap-
 567 plied in the case where the ED closure does not use TKE. In this case, it would require
 568 to add a prognostic or diagnostic TKE equation (even if it does not interact with the
 569 ED term) to enforce energetic consistency.

570 Using vertical-velocity based conditional sampling of LES, we found that station-
 571 arity of the mean convective plume is well satisfied. However even if plume typical ra-
 572 dius is small compared to the size of the horizontal domain, plumes typically cover 15
 573 % to 20% of the horizontal domain. Consequently, the assumption of small plume area
 574 can be criticized, and we provided and implemented a set of equations without such an
 575 hypothesis. Based on LES we also proposed a new EDMF closure for the turbulent trans-
 576 port of TKE which generalizes two existing approaches. We expose the subtleties of nu-
 577 merical implementation of the SCM, including fully consistent discrete energy budgets
 578 based on their continuous counterparts. This study highlights how energy considerations
 579 can unambiguously guide discretizations. Finally we performed a series of sensitivity ex-
 580 periment to different modelling aspects regarding energetic consistency, boundary con-
 581 ditions, transport of TKE, transport of horizontal momentum by MF and small plume
 582 area assumption. Temperature and flux of temperature profiles do not appear to signif-
 583 icantly be affected by such choices, and the SCM reproduces LES profiles compared to
 584 EVD. Modelling plume horizontal velocities distinct from the mean seems necessary to
 585 properly reproduce horizontal velocities and its fluxes. The usage of an energetically con-
 586 sistent TKE equation and along with the new formulation of TKE transport is key to
 587 obtaining realistic TKE and turbulent transport of TKE profiles. We also quantify the
 588 energy loss of inconsistent formulations, due to either inconsistent TKE equation or in-
 589 consistent boundary conditions. Surprisingly, removing the assumption of small plume
 590 area only affects the transport of TKE. All these findings are based on idealized cases,
 591 where in particular ED fluxes are small explaining the negligible retroaction of energetic
 592 consistency and TKE on the solutions. Thus a further assessment of global 3D simula-
 593 tions is a necessary next step to understand the implications of energetic consistency.
 594 To further illustrate that the MF concept is a credible alternative to the traditional ap-
 595 proaches used in the oceanic context (using e.g. an enhanced vertical diffusion) the pro-
 596 posed scheme is validated in a single-column configuration against observational data
 597 of oceanic convection from the LION buoy.

598 To implement and then assess the impact of this energetically consistent param-
 599 eterization on realistic 3D oceanic simulations a calibration of the remaining "free" pa-
 600 rameters must be achieved (Hourdin et al., 2017; Couvreur et al., 2021). It should be
 601 performed on parameters whose universality can sometimes be statistically assessed (Souza
 602 et al., 2020), and should be mathematically and physically constrained as much as pos-
 603 sible. We believe that designing energetically consistent parameterization is a way to achieve
 604 more realistic models before their tuning.

605 **Appendix A Mixing length and dissipation computations**

606 For the oceanic applications detailed in this article, we have chosen a formulation
 607 of eddy-diffusivity and viscosity close to that used in the NEMO ocean model (Madec
 608 et al., 2019). The eddy-viscosity and diffusivity are classically assumed to be related to
 609 TKE by

$$\begin{aligned} K_u &= c_m l_m \sqrt{k} \\ K_\phi &= K_u (\text{Pr}_t)^{-1} \end{aligned}$$

610 with l_m a mixing length scale, Pr_t the non-dimensional turbulent Prandtl number, and
 611 c_m is a constant ($c_m = 0.1$ in NEMO). The mixing length l_m is calculated in two steps

612 by considering separately the length scales l_{up} and l_{dwn} associated respectively to up-
 613 ward and downward movements : (1) l_{up} and l_{dwn} are initialized assuming $l_{\text{up}} = l_{\text{dwn}} =$
 614 $\sqrt{2k}\tau_{\text{ed}}$ with τ_{ed} a characteristic time equal to $1/N = (\partial_z \bar{b})^{-1}$ (2) a physical limitation
 615 is used to guarantee that l_{up} and l_{dwn} do not exceed the distance to the top and the bot-
 616 tom, this limitation amounts to controlling the vertical gradients of l_{up} and l_{dwn} such
 617 that they are not larger than the variations of depth (*e.g.* Madec et al., 2019). Once l_{up}
 618 and l_{dwn} are computed the mixing length is taken as $l_m = \min(l_{\text{up}}, l_{\text{dwn}})$. The turbu-
 619 lent Prandtl number is modelled by $\text{Pr}_t = \min(\text{Pr}_t^{\text{max}}, \max(\text{Ri}/\text{Ri}_c, 1))$ with $\text{Ri} = N^2/\|\partial_z \bar{\mathbf{u}}_h\|^2$,
 620 $\text{Pr}_t^{\text{max}} = 10$ and $\text{Ri}_c = 0.2$.
 621 The viscous dissipation is parameterized as $\bar{\epsilon} = \frac{c_\epsilon}{l_\epsilon} k^{3/2}$, where $c_\epsilon = \sqrt{2}/2$ is a numeri-
 622 cal constant and the dissipation length is $l_\epsilon = \sqrt{l_{\text{up}} l_{\text{dwn}}}$ (*e.g.* Gaspar et al., 1990).

623 Appendix B Third Order Moment computations

624 By definition we have

$$k = \overbrace{\frac{1}{2} a_e \|\mathbf{u}_e - \bar{\mathbf{u}}\|^2 + a_e k_e}^{a_e k_e^t} + \overbrace{\frac{1}{2} a_p \|\mathbf{u}_p - \bar{\mathbf{u}}\|^2 + a_p k_p}^{a_p k_p^t} \quad (\text{B1})$$

$$\implies k_p^t - k_e^t = \frac{1}{1 - a_p} (k_p^t - k) \quad (\text{B2})$$

625 thus

$$\text{III}_e + \text{III}_p + \text{IV}_e + \text{IV}_p = a_p w_p \left(k_p + \frac{1}{2} \|\mathbf{u}_p - \bar{\mathbf{u}}\|^2 \right) + a_e w_e \left(k_e + \frac{1}{2} \|\mathbf{u}_e - \bar{\mathbf{u}}\|^2 \right) \quad (\text{B3})$$

$$= a_p w_p \left(k_p + \frac{1}{2} \|\mathbf{u}_p - \bar{\mathbf{u}}\|^2 - k_e - \frac{1}{2} \|\mathbf{u}_e - \bar{\mathbf{u}}\|^2 \right) \quad (\text{B4})$$

$$= a_p w_p (k_p^t - k_e^t) \quad (\text{B5})$$

$$= \frac{a_p}{1 - a_p} w_p (k_p^t - k) \quad (\text{B6})$$

626 Appendix C Discretization of mass-flux equations

627 We start from the standard grid arrangement used in oceanic models which are usu-
 628 ally discretized on a Lorenz grid in the vertical (density is located in the center of the
 629 cells on the vertical). We consider N grid cells in the vertical with thickness $\Delta z_j = z_{j+1/2} -$
 630 $z_{j-1/2}$ ($z_{1/2} = -H$ and $z_{N+1/2} = 0$ the surface) such that $\sum_{j=1}^N \Delta z_j = -H$. For the
 631 discrete values, not to interfere with the grid indices, the subscript p for the plume quan-
 632 tities is now a superscript such that plume quantities are now noted $X_{j+1/2}^p = X_p(z =$
 633 $z_{j+1/2})$. In the following, we consider that the plume quantities and k are discretized at
 634 cell interfaces and the mean quantities \bar{X} are discretized at cell centers and are inter-
 635 preted in a finite-volume sense (*i.e.* $\bar{X}_j = \frac{1}{\Delta z_j} \int_{z_{j-1/2}}^{z_{j+1/2}} \bar{X}(z) dz$). We start from the
 636 mass-flux equations given in Tab. 1 with $\tilde{b}' = b'/h$ but in conservative form (except for
 637 the vertical velocity and TKE plume equations) :

$$\partial_z (a_p w_p) = E - D \quad (\text{C1})$$

$$\partial_z (a_p w_p \phi_p) = E \bar{\phi} - D \phi_p \quad (\text{C2})$$

$$\partial_z (a_p w_p \mathbf{U}_p) = E \bar{\mathbf{U}} - D \mathbf{U}_p \quad (\text{C3})$$

$$w_p \partial_z w_p = -(E/a_p)(b w_p) + a B_p + \tilde{b}' w_p^2 \quad (\text{C4})$$

$$a_p w_p \partial_z k_p = E \left(k - k_p + \frac{1}{2} (\bar{\mathbf{u}}_p - \bar{\mathbf{u}})^2 \right) - a_p (\epsilon_\nu)_p \quad (\text{C5})$$

638 where the equation for horizontal momentum has been manipulated to have the same
 639 form as the ϕ_p equation by taking $\mathbf{U}_p = \mathbf{u}_{h,p} - C_u \bar{\mathbf{u}}_h$ and $\bar{\mathbf{U}} = (1 - C_u) \bar{\mathbf{u}}_h$. The ad-
 640 vective form is used for the w_p equation to make the computation of w_p independent of
 641 a_p (with the closure hypothesis given in Tab. 4 for E , E/a_p is independent of a_p); the
 642 motivations for this will become clearer later. The mass-flux equations correspond to a
 643 first-order nonlinear set of ODEs. There are a whole lot of methods for solving such ini-
 644 tial value problems. We present here a simple method combining explicit (Euler) and
 645 semi-implicit (Crank-Nicolson) steps as the use of more advanced methods did not pro-
 646 duce significantly different results. In the following, we describe the different steps for
 647 the resolution starting from known initial values $X_{N+1/2}^p$ at the surface and advancing
 648 downward.

649 C1 Initial conditions

650 The discrete form of the boundary conditions given in eq. (30) are obtained by a
 651 linear extrapolation of $\bar{\phi}_N$ and $(\bar{\mathbf{u}}_h)_N$ toward the surface.

$$\begin{aligned}
 w_{N+1/2}^p &= -w_{\min}^p \\
 \phi_{N+1/2}^p &= \frac{(2\Delta z_N + \Delta z_{N-1})\bar{\phi}_N - \Delta z_N \bar{\phi}_{N-1}}{\Delta z_N + \Delta z_{N-1}} \\
 \mathbf{U}_{N+1/2}^p &= (1 - C_u) \frac{(2\Delta z_N + \Delta z_{N-1})(\bar{\mathbf{u}}_h)_N - \Delta z_N (\bar{\mathbf{u}}_h)_{N-1}}{\Delta z_N + \Delta z_{N-1}}
 \end{aligned} \tag{C6}$$

652 Since the TKE k is already discretized at cell interfaces the boundary condition for k_p
 653 does not require an extrapolation. In particular the condition on ϕ_p leads to the follow-
 654 ing value of the B_p term in the topmost grid cell :

$$B_N^p = \Delta z_N \left(\frac{\bar{b}_N - \bar{b}_{N-1}}{\Delta z_N + \Delta z_{N-1}} \right) = \frac{\Delta z_N}{2} (N^2)_{N-1/2}$$

655 meaning that using the condition (C6) allows to trigger convection as soon as the Brunt-
 656 Väisälä frequency is negative. Indeed a negative value of B_N^p in the RHS of the w_p -equation
 657 (C4) leads to a positive value of $(\partial_z w_p)_N$ and thus larger negative values of w_p when go-
 658 ing downward.

659 C2 w_p -equation and mixed layer depth

660 The w_p -equation (C4) using the entrainment E given in Tab. 4 can be formulated
 661 as

$$\partial_z w_p^2 + b\beta_1 \min(\partial_z w_p^2, 0) = 2aB_p + 2\tilde{b}' w_p^2$$

662 which can be discretized in a straightforward way as

$$\begin{aligned}
 \tilde{\beta} \left[(w^p)_{j+1/2}^2 - (w^p)_{j-1/2}^2 \right] &= 2a\Delta z_j B_j^p + (\tilde{b}' \Delta z_j) \left[(w^p)_{j+1/2}^2 + (w^p)_{j-1/2}^2 \right] \\
 B_j^p &= b_{\text{eos}}(\phi_{j+1/2}^p) - b_{\text{eos}}(\bar{\phi}_j)
 \end{aligned} \tag{C7}$$

663 where $\tilde{\beta} = 1 + b\beta_1$ if $aB_j^p + \tilde{b}'(w^p)_{j+1/2}^2$ is negative and $\tilde{\beta} = 1$ otherwise. Knowing
 664 $w_{j+1/2}^p$, it is easily found that

$$(w^p)_{j-1/2}^2 = \frac{(\tilde{\beta} - \tilde{b}' \Delta z_j)(w^p)_{j+1/2}^2 - 2a\Delta z_j B_j^p}{\tilde{\beta} + \tilde{b}' \Delta z_j} \tag{C8}$$

665 Once this quantity falls below a certain threshold $(w_{\min}^p)^2$, the plume is considered evanes-
 666 cent. In the oceanic context we consider $w_{j-1/2}^p = -\sqrt{(w^p)_{j-1/2}^2}$ for the rest of the cal-
 667 culations to guarantee that $w_{j-1/2}^p$ is strictly negative. The upwinding used to compute

668 B_p in (C7) in addition to the fact that the w_p -equation does not depend on a_p avoid the
 669 need for an iterative process to solve the mass-flux equations.

670 The mixed layer depth h we consider to scale the parameter b' is the depth at which
 671 the discrete value of w^p equals w_{\min}^p . In practice, as soon as $(w^p)_{j-1/2}^2$ given by (C8) is
 672 smaller than $(w_{\min}^p)^2$ we use (C8) to compute the distance Δz_j^* below $z_{j+1/2}$ where the
 673 plume vertical velocity exactly equals w_{\min}^p :

$$\Delta z_j^* = \frac{(w^p)_{j+1/2}^2 - (w_{\min}^p)^2}{2aB_j^p + \tilde{b}' \left((w^p)_{j+1/2}^2 + (w_{\min}^p)^2 \right)}$$

674 leading to $h = z_{j+1/2} - \Delta z_j^*$. To compute Δz_j^* we assume that $\tilde{\beta} = 1$ because the bot-
 675 tom of the plume is reached only if we are in the detrainment zone.

676 C3 Continuity and tracer equations

677 The entrainment E_j and detrainment D_j rates given in Tab. 4 (taken from Rio et
 678 al. (2010)) discretized on a grid cell j correspond to

$$\begin{aligned} \Delta z_j E_j &= \frac{1}{2} \left(a_{j+1/2}^p + a_{j-1/2}^p \right) \beta_1 (\delta_z w^p)_j^+ \\ \Delta z_j D_j &= \frac{1}{2} \left(a_{j+1/2}^p + a_{j-1/2}^p \right) \left[-\beta_2 (\delta_z w^p)_j^- - \frac{\delta_0 \Delta z_j}{2} (w_{j+1/2}^p + w_{j-1/2}^p) \right] \end{aligned}$$

679 where $(\delta_z w^p)_j^+ = \max(w_{j+1/2}^p - w_{j-1/2}^p, 0)$ and $(\delta_z w^p)_j^- = \min(w_{j+1/2}^p - w_{j-1/2}^p, 0)$.
 680 Integrating from $z_{j-1/2}$ to $z_{j+1/2}$ the continuity equation and ϕ_p equations we obtain

$$\begin{aligned} (a^p w^p)_{j+1/2} - (a^p w^p)_{j-1/2} &= \Delta z_j (E_j - D_j) \\ (a^p w^p \phi^p)_{j+1/2} - (a^p w^p \phi^p)_{j-1/2} &= \Delta z_j E_j \bar{\phi}_j - (\Delta z_j D_j / 2) \left(\phi_{j+1/2}^p + \phi_{j-1/2}^p \right) \end{aligned}$$

681 which can also be extended to the horizontal momentum equation formulated using U_p .
 682 Since at this stage $w_{j+1/2}^p$ and $w_{j-1/2}^p$ are known, the continuity equation is used to com-
 683 pute $a_{j-1/2}^p$ through

$$\begin{aligned} a_{j-1/2}^p &= a_{j+1/2}^p \left\{ \frac{2w_{j+1/2}^p - \text{Em}D_j}{2w_{j-1/2}^p + \text{Em}D_j} \right\} \\ \text{Em}D_j &= \beta_1 (\delta_z w^p)_j^+ + \beta_2 (\delta_z w^p)_j^- + \min \left\{ \frac{\delta_0 \Delta z_j}{2} (w_{j+1/2}^p + w_{j-1/2}^p), -2(w_{\min}^p) \right\} \quad (\text{C9}) \end{aligned}$$

684 Note that a_p is subject to a boundedness requirement as $0 \leq a_p \leq 1$. Assuming $0 \leq$
 685 $a_{j+1/2}^p \leq 1$, sufficient conditions to guarantee that $a_{j-1/2}^p \leq 1$ are $\beta_1 \leq 1$ and $\beta_2 \geq$
 686 1 and a sufficient condition to guarantee that $a_{j-1/2}^p \geq 0$ is $\beta_2 < 2$. Moreover a con-
 687 straint is added on the background detrainment δ_0 in (C9) to guarantee that $a_{j-1/2}^p =$
 688 0 as soon as $w_{j+1/2}^p = w_{j-1/2}^p = -w_{\min}^p$ which occurs once outside the plume.

689 Once $a_{j-1/2}^p$ is known, it is possible to compute $\phi_{j-1/2}^p$ (as well as $U_{j-1/2}^p$). The
 690 proposed discretization ensures that the compatibility between the continuity and the
 691 tracer equations is maintained at the discrete level (*i.e.* we recover the continuity equa-
 692 tion for $\phi_{j+1/2}^p = \phi_{j-1/2}^p = 1$ and $\bar{\phi}_j = 1$).

693 The same reasoning can be applied to solve the k_p equation, which presents no ad-
 694 ditional difficulties as all necessary quantities $w_{j\pm 1/2}^p$, $a_{j\pm 1/2}^p$ and $u_{j\pm 1/2}^p$ are known.

695 In summary, the proposed discretization guarantees that w_p is strictly negative, that
 696 a_p is bounded between 0 and 1, and that the continuity and tracer equations are com-
 697 patible, without the need for an iterative solution procedure.

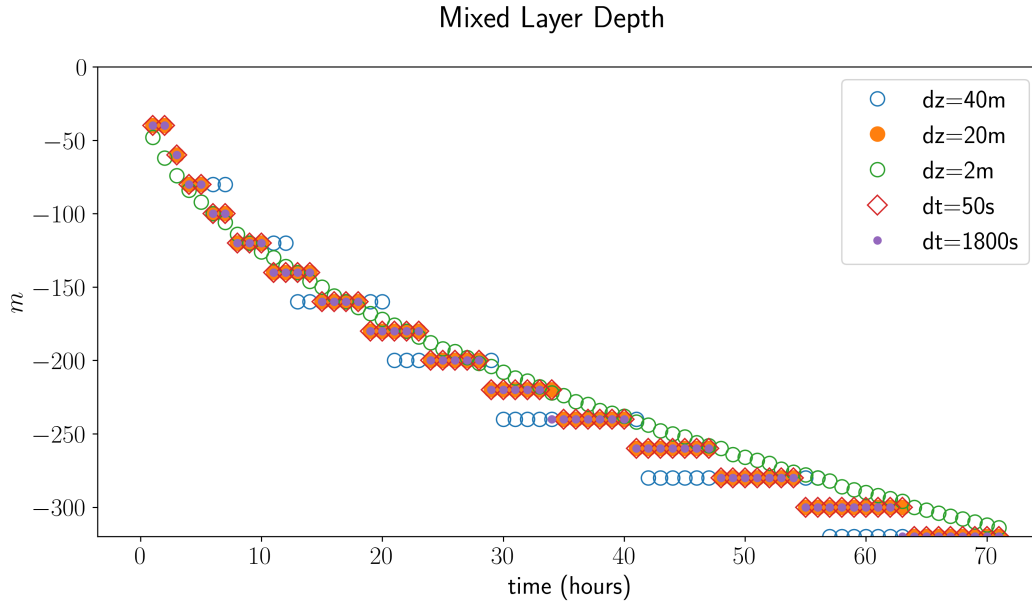


Figure D1: Mixed layer depth (MLD) computed as the minimum of temperature flux for different time steps (dt) and number of levels (nz), LES data and analytical expression.

698 Appendix D Time step and vertical resolution sensitivity

699 On figure D1 we expose mixed layer depth (MLD) sensitivity to time step and ver-
 700 tical resolution for the case W005_C500. During deep convection, the mixed layer can
 701 reach depths at which vertical grid spacing is important, justifying the relevance of large
 702 vertical resolution such as $dz = 40m$.

703 Open Research

704 Data Availability Statement

705 Data from the Lion mooring (located in the Gulf of Lion; Mediterranean sea) are
 706 freely accessible from Bosse et al. (2023). The output from LES simulations and the ini-
 707 tial and surface boundary conditions for the Hymex/ASICS-MED experiments are avail-
 708 able at the Zenodo archive <https://zenodo.org/records/10619442>.

709 Software Availability Statement

710 The LES model Méso-NH can be downloaded at <http://mesonh.aero.obs-mip.fr/mesonh57>.
 711 All the SCM codes used in this study have been made available and can be found at the
 712 Zenodo archive <https://zenodo.org/records/10619442>. It includes the single-column model
 713 with Eddy-Diffusivity Mass-Flux turbulent closure developed from scratch. The latter
 714 consists of low-level code written in Fortran interfaced with Python using F2PY (Peterson,
 715 2009). The single-column simulations analyzed in this study can be executed from a high-
 716 level Python driver code without any intervention on the Fortran code. The high-level
 717 Python driver code and scripts to reproduce the figures are available in the Zenodo archive.
 718 The Fortran code contains inline documentation following the FORD (Fortran Documenter)
 719 format.

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