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FABIO ACERBI<sup>a</sup>

## Byzantine Geometric Metrology An Overview with an Edition of the *Compendium*\*

ABSTRACT: The article presents an overview of Byzantine geometric metrology and an edition of a complete yet anonymous set of metrological prescriptions, here called the *Compendium*.

KEYWORDS: Byzantine mathematics, metrology, stylistic codes, manuscript tradition

### INTRODUCTION

Geometric metrology is a body of prescriptions for measuring the space content (area, volume) of geometric figures and their most relevant linear or planar components (side, perimeter, height, diameter, diagonal, surface, etc.). These prescriptions are formulated by means of dedicated stylistic codes. If we exclude Hero of Alexandria's *Metrica*, which the Greek and the Latin traditions unanimously regard as the founding treatise of the entire discipline, none of these prescriptions is backed up—I say “validated”—by any proof in Euclidean style. Consequently, geometric metrology is not a deductive discipline but a set of collections of mathematical recipes aimed at preserving any bit of a specific technical lore: alternative algorithms for computing the same quantity are compiled as a matter of course, even if they frequently differ by trifling details only; within a single collection, a specific algorithm can be exemplified more than once, by simply changing the numerical values of the input data. This helps explain why geometric metrological writings suffer from inflation and thereby make up a huge corpus<sup>1</sup>. The size of the corpus is further increased by the fact that geometric metrology bears obvious and necessary connections with two related yet distinct disciplines: metrology, which provides lists of units of measurement and the conversions among them, and fiscal geometry (or “geodesy”, for it was intended to assist land-surveyors), which does not deal with geometric objects as such but with land portions *qua* geometric objects and employs in a systematic way “wrong” prescriptions to measure most of these objects. Even within geometric metrology proper, much of stereometry sets out to measure material objects *qua* compounded and schematic geometric objects, thereby resorting by default to approximations.

Very much as with any other scientific discipline, the Byzantine scholars handed down the Greek heritage of geometric metrology and enriched it by their editorial activities and by composing new works. The latter are usually second-order compilations, possibly simplified as for contents and reformulated and homogenised as for style. This is exactly the case with the text that is published in the present paper. This text is anonymous and without a title: I shall call it “the *Com-*

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\* Reproductions of most manuscripts mentioned in this article can be found through the website <https://pinakes.irht.cnrs.fr/>, which also provides additional bibliography. I had access to some of the manuscripts thanks to the digital repository of the project Sin-aps (Alexander von Humboldt-Professorship, FAU Erlangen-Nürnberg). Ramon Masià and Bernard Vitrac helped me to improve this study.

<sup>1</sup> The estimate in F. ACERBI – B. VITRAC, Héron d'Alexandrie, *Metrica* (*Mathematica Graeca Antiqua* 4). Pisa–Roma 2014 (Hero, *Metrica* henceforth), 443–445 gives a total of about 570 problem-tokens in the several collections of the corpus, which amount to 370–420 distinct problems, that is, problems such that either the object, or the numerical input data, or the feature to be computed, or the algorithm used to compute it are different.

*pendium*”. With one single—and inexplicable—exception, the *Compendium* does not set out its algorithms by means of paradigmatic examples, but by formulating general procedures. This stylistic choice is without parallels in the entire geometric metrological corpus<sup>2</sup> and makes the *Compendium* a most interesting piece of mathematical writing.

The present edition of the *Compendium* describes all its manuscript witnesses and establishes their genealogical relations, which are summarised in a stemma. This is preceded by an overview of Byzantine geometric metrology and geodesy, by a summary of the contents of the *Compendium* by some speculations about its possible author, and followed by an exposition of the structure and of the style of the *Compendium*. The edition is accompanied by a translation and by an annotated paraphrase; the latter features symbolic transcriptions of all metrological algorithms contained in the text. An Appendix sets out the original structure and some codicological data of the prototype of the manuscript tradition of the *Compendium*. Within the Bibliography, the section on “Manuscript Sources” lists about 330 Greek manuscripts that contain metrological (in a broad sense) and geometric metrological items, fiscal geometry included<sup>3</sup>.

### BYZANTINE GEOMETRIC METROLOGY: AN OVERVIEW

Geometric metrological texts are fairly frequent among scientific papyri. They usually contain a few items and characteristically put emphasis on measuring “real” objects. The most substantial collections are included in compilations of metrological problems accompanied by computational tools obviously relevant to solving these problems, such as resolutions of common fractions into unit fractions, and by problems traditionally categorised as “recreational mathematics”. Listing the contents of P.Math (4<sup>th</sup> century) will suffice to characterise such *Ur-Rechenbücher*<sup>4</sup>: 3 model contracts (2 of them for loan of money); 5 lists of units of measurement of length, surface, volume, or liquid capacity, 32 problems of measurement of plane (15 items) and solid (17 items) geometric figures or of objects regarded as such; 6 problems of resolution of common fractions into unit fractions; 3 problems of proportional partition (shipload of wheat; distribution of amount of wheat; the three *thēsauroi* problem); 1 problem of pursuit; 1 application of the rule of three (pay); 1 problem is too fragmentary to allow any safe reconstruction. Important geometric metrological sources are also P.Vindob. gr. inv. 19996 (MPER N.S. I.1; 1<sup>st</sup> century), and the Akhmīn mathematical papyrus (P.Cair. cat. 10758; 7<sup>th</sup> century); these two papyri contain 30 and 3 problems of measurement of solids, respectively<sup>5</sup>.

<sup>2</sup> A very compact parallel featuring only generic algorithms is edited as *Geom.* 22, 3–24. On the difference between “procedures” and “algorithms” as stylistic resources see the Section on structure and style of the *Compendium*. With the exception of this Section and of the Preliminaries to the edition, the mathematical sense of “algorithm” is used throughout.

<sup>3</sup> The sense is “broad” because, for both simplicity’s and reference’s sake, the list includes metrological items that do not necessarily pertain to the measurement of lengths and surfaces only. The survey has also revealed a further witness of the arithmetical problem noted A<sub>3</sub> in F. ACERBI, I problemi aritmetici attribuiti a Demetrio Cidone e Isacco Argiro. *Estudios Bizantinos* 5 (2017) 131–206: it is Holkham gr. 106 (15<sup>th</sup> century; *Diktyon* 48174), f. 89r.

<sup>4</sup> For *Rechenbücher* in Greek language see F. ACERBI, Byzantine *Rechenbücher*: An Overview with an Edition of *Anonymi L* and J. *JOeB* 69 (2019) 1–57.

<sup>5</sup> A survey of geometric metrological papyri can be found in Hero, *Metrica* 557–569. Editions of the three papyri just mentioned are in R. S. BAGNALL – A. JONES, Mathematics, Metrology, and Model Contracts. A Codex from Late Antique Business Education (*P.Math.*). New York 2019 (a survey of geometric metrological papyri, only overlapping with the one just cited, can be read on 163–177), whose approach should be modulated according to the review in *Plekos* 23 (2021) 21–27, and the complement in R. S. BAGNALL – A. JONES, P.Math. Leaf A Verso, Mathematical Problem a3 Revisited: A New Algorithm in Greek Mensurational Mathematics. *Pylon* 2 (2022); H. GERSTINGER – K. VOGEL, Eine stereometrische Aufgabensammlung im Papyrus Graecus Vindobonensis 19996. *Mitteilungen aus der Nationalbibliothek in Wien. Papyrus Erzherzog Reiner. Neue Serie I.* Wien 1932; J. BAILLET, Le papyrus mathématique d’Akhmīm. *Mémoires publiés par les membres de la Mission Archéologique Française au Caire* 9,1 (1892) 1–89, respectively.

As for the evidence in Byzantine manuscripts, geometric metrological texts were transmitted as collections of paradigmatic problems. These were usually graded by the complexity of the figure involved, prefaced by or interspersed with generalities about measuring geometric figures and with lists of relevant shapes (both with an unmistakable Neoplatonic tinge) and of units of measurement and their conversions. The main collections are contained in six independent manuscripts that pre-date the mid 14<sup>th</sup> century<sup>6</sup>. These are Seragl. G.Ī.1 (the *codex unicus* of Hero's *Metrica* too; ca. 960; *Diktyon* 33946), Vat. gr. 215 (*Liber geeponicus* and Cassianus Bassus' *Geoponica*; 11<sup>th</sup> century; *Diktyon* 66846), Par. gr. 1670 (late 12<sup>th</sup> century; *Diktyon* 51293), Vat. gr. 1038 (mainly Euclid and Ptolemy, plus the *De mensuris*; late 13<sup>th</sup> century; *Diktyon* 67669), Par. suppl. gr. 387 (early 14<sup>th</sup> century; *Diktyon* 53135), and Par. gr. 2448 (a scientific miscellany with Euclid and treatises of the "little astronomy", plus Diophanes and the texts edited as *Geom. 22*; Cyprus; early 14<sup>th</sup> century; *Diktyon* 52080). Minor, and frequently exiguous, metrological and geometric metrological collections are dispersed among many more manuscripts (about 330 tokens are listed among the "Manuscript Sources" below). These texts may simply comprise short lists of units of measurement and their conversions. Many of these texts are fillers or sections of personal notebooks and anthologies, others are integral to the textual constellation in which they are included, as is the case with fiscal (and more generally juridical) reference books or with the selection of imperial decrees called *Synopsis basilicorum major*. As a matter of fact, the manuscripts containing the geometric metrological collections usually feature other textual units, whose separation from the metrological items may be questionable. Cases in point are the pseudo-Heronian *Definitiones* and *Rechenbuch*-like problems. For instance, a florilegium contained in Seragl. G.Ī.1, edited partly as *Geom. 24* and partly as *Stereom. I*, 68–97, mainly comprises "separation" problems (see sect. 13 of the *Compendium*), which can be read (and in fact were solved) as problems of Diophantine analysis in fictitious geometric metrological guise (no land-surveyor will ever be confronted with such problems).

The most important witnesses of the geometric metrological corpus are Seragl. G.Ī.1, Par. gr. 1670, and Par. suppl. gr. 387. If we take into account the articulation of texts exhibited by titles, decoration, and of course by homogeneity of contents and of style<sup>7</sup>, we come to realise that these manuscripts are in fact collections of collections, some of which are ascribed to Hero while others are anonymous. Exactly of this kind is the magnificent Seragl. G.Ī.1, a pure geometric metrological collection and the only witness to Hero of Alexandria's *Metrica*. More varied, penned by two main hands, and possibly an assembly of texts copied for personal use by at least two scholars even if owned by George Choumnos (*PLP* 30945), *megas stratopedarchēs* in AD 1341–42, is Par. suppl. gr. 387, which compiles computistical items, metrological and geometric metrological collections edited as *Geometrica* and *Stereometrica I–II*, the texts edited as the Heronian *Definitiones*, at least two independent *Rechenbücher*, along with a galaxy of very short notes of logistic, astronomical, and astrological content. Par. gr. 1670 deserves a less sketchy description. This manuscript is a computational primer resulting from a conscious selection of texts, entrusted to an excellent copyist, and intended for conservation purposes. The manuscript was designed to carry a complete technical record, both as regards the proposed texts and on account of the possibility of a double level of use. Linguistic excellence, an inflexible formulaic rigidity, and the solutions of layout adopted in Par. gr. 1670 marked a turning point in the development of high-brow technical literature in Byzan-

<sup>6</sup> On the structure of the collections and on the manuscripts carrying them see Hero, *Opera omnia* IV IV–XXVI; V I–LXV; Hero, *Metrica* 88–90 and 429–588, with detailed descriptions of their contents and a bibliography, to which I. PÉREZ MARTÍN, Enseignement et service impérial à l'époque paléologue, in M.-H. Blanchet, R. Estangüi Gómez (eds.), *Le monde byzantin du XIII<sup>e</sup> au XV<sup>e</sup> siècle. Anciennes ou nouvelles formes d'impérialité*. Paris 2021 (*TM* 25/1), 451–502: 496–498 (on Par. suppl. gr. 387), must be added. For the contents of the collection in Par. gr. 1670 see also F. ACERBI, *Struttura e concezione del vademecum computazionale Par. gr. 1670*. *Segno e Testo* 19 (2021) 167–255: 169–173, summarised below.

<sup>7</sup> See Hero, *Metrica* 441–443, where 14 distinct collections are singled out.

tium. This manuscript is the only independent witness of the *Palatia Logarikē* (ff. 3r–13r) and of the *Nea Logarikē* (13r–21v), the treatises of fiscal accounting composed shortly after the death of Alexios I Komnenos in AD 1118. After them, one finds: 21v–33v and 33v–34v, multiples and submultiples of monetary units; 35r–44v, a collection of algorithms for dividing numbers  $1 \dots n$  by  $n$ , with  $n = 5 \dots 12$ , followed (44v–46v) by a list of the results of the same divisions, with  $n$  ranging from 5 to 20; 46v–61v, an Easter Computus, which assumes AM 6691 [= AD 1183] as the current year<sup>8</sup>; 61v (the only filler of the entire manuscript, yet penned by the main hand), measure of a stone parallelepiped (edited in Hero, *Opera omnia* IV XVII), in fact an exercise in computing with fractions; 62r–131v a metrological collection edited as *Geom.* 2–23, whose contents are as follows. The definitions of Book I of the *Elements*. On the origin of geometry as land measurement (*Geom.* 2). Sections on the subject-matter of geometric metrology, on units of measurement and their conversions, and on the standard productivity of soil (*Geom.* 3–4) prelude the problems (*Geom.* 5–21). These measure space content and relevant components of: squares (8 items); rectangles (5 items); right-angled triangles (17 items, plus the rules traditionally ascribed to Pythagoras and Plato for providing the sides of numerical right-angled triangles: 5 + 5 items); equilateral (13 items), isosceles (12 items), scalene (28 items), generic (10 items, Heronian algorithm) triangles; isosceles triangles with an inscribed square (29 items, plus 2 items on the species of triangles and on the incommensurability of side and diagonal of a square); rhombi (6 items); rectangles (23 items); rhomboids (21 items); trapezia (45 items); circles (36 items); semicircles (14 items); segments less than a semicircle (4 items) and greater than a semicircle (14 items, followed by 13 items of problems about circles); regular  $n$ -gons, with  $n = 5 \dots 12$  (10 items). After this, one finds a general prescription to the effect of dividing complex shapes into triangles (1 item), the area of the circle according to Archimedes (1 item), and Patrikios' addition (2 items, see below). Further texts include a succinct compendium featuring only generic algorithms (*Geom.* 22, 3–24) and an earlier version of the preliminary sections (*Geom.* 23, 1–22).

As repeatedly intimated above, the standard edition of the geometric metrological corpus, namely, volumes IV and V of the Heronian *Opera omnia*<sup>9</sup>, presents these collections dismembered and reassembled to form fictitious treatises called *Geometrica* and *Stereometrica* I–II<sup>10</sup>. Partial exceptions to this picture of patchwork transmission and of invasive modern editorial practices are as follows. The pseudo-Heronian *De mensuris* was transmitted via the Archimedean tradition and Vat. gr. 1038; it is edited as a separate work in vol. V of the Heronian *Opera omnia*<sup>11</sup>. The short treatises of Diophanes and Didymos are also contained in some of the manuscripts that carry the main geometric metrological compilations; owing to the fact that they exhibit an author name that is not Hero's, they were edited as self-contained works by J. L. Heiberg at the end of his life, in the slim volume *Mathematici graeci minores*.

After a very short preface on the species of measurement and before two final sections on units of measurement, the *De mensuris* is a badly organised compilation of problems of measurement of material (sects. 1–19, 22, 24–26, 38, 49) and geometric (sects. 28–37, 39–48, 50–59) objects, plane (14 items) and solid (38 items), interspersed with the standard problem of the filling of a tank (sects. 20–21) and with conversions of units of measurement (sects. 23, 27, 60–61). Didymos' very short tract is metrologically and practically oriented, as is already suggested by its title *On the measurement of all kinds of timber*. A long opening on units of measurement is followed by 14

<sup>8</sup> The Computus is edited in F. ACERBI, *Byzantine Easter Computi (StT 562)*. Città del Vaticano 2024, 507–520.

<sup>9</sup> This was preceded by the landmark edition F. HULTSCH, *Heronis Alexandrini Geometricorum et Stereometricorum reliquiae*. Berlin 1864.

<sup>10</sup> In particular, Heiberg dismembered the *Liber geeponicus* ascribed to Hero, whose main witness is Vat. gr. 215, ff. 1r–24r.

<sup>11</sup> Edition [Hero of Alexandria], *De mensuris*, manuscript tradition studied in Hero, *Opera omnia* V XXXIV–XXXVII. Discussion and summary of its contents in Hero, *Metrica* 488–491 and 577–579.

problems of measurement of plane (2 items, a square and a rectangle) and solid (12 items) chunks of timber shaped as geometric figures, mostly rectangular parallelepipeds and pyramidal wedges. Diophanes' text is a reasonably well-conceived compilation of geometric metrological problems<sup>12</sup>. The initial sections on circle measurement (sects. 1–7 and 9) has been intercalated by a section that describes a trapezium whose sides realise an harmonic mean (8); two sequences on the measurement of the equilateral triangle (20) and of the  $n$ -gons, with  $n = 5 \dots 12$  (10–17, 18 octagon only, 19, 23) have been intercalated by problems on the spherical segment and on the cone (21–22); sect. 24 contains a series of definitions of the sphere and its components, of the cube, and of a species of rectangular parallelepiped; the final two sections (25–26) expound measurements of the sphere and its parts and components and of the volume of the cylinder and of the cone, featuring quotations of the basic results proved by Archimedes about these figures.

Most of these texts come from late Greek antiquity, but the highly sectional nature of the corpus encouraged increase by accretion. Cases in point are a measurement problem conceived by the Praetorian prefect Modestus (*ca.* 372) and edited as *Stereom.* II, 54, the Byzantine additions that some “very illustrious” Patrikios and Makarios made to the corpus and that are edited as *Geom.* 21, 26–27 and 28–30, and the editorial work Patrikios made on *Stereom.* I, 21<sup>13</sup>.

Let us now come to Byzantine elaborations. I begin with the geometric metrological items included in *Rechenbücher*. After the texts summarily described above, Par. suppl. gr. 387 contains, on ff. 118v–140v and 148r–161v, two *Rechenbücher*, the former of which has been edited<sup>14</sup>. Geometric metrological problems can be found as nos. 22 (cube), 92 (apparently, a spherical body which is transformed into a cylinder), 115 (square), 117–118 (parallelepiped and cylinder, respectively) of the former and on ff. 151r (parallelepiped), 153v marg. (equilateral triangle), 158r–v (cylindrical tank), 159v–160r (= no. 92 just above) of the latter. Vindob. phil. gr. 65 (*ca.* 1436; *Diktyon* 71179), ff. 11r–126r and 126v–140r, also contains two *Rechenbücher*<sup>15</sup>. Nos. 51 (rectangular parcel), 87–88 (volume of a tower, both) of the second *Rechenbuch* are geometric metrological problems. The first *Rechenbuch* is a huge compilation that includes 60 geometric metrological problems (nos. 166–188, 201–233, and 235–238; nos. 18[4]–20[1] are missing because of the loss of two bifolia but their enunciations are given in the initial index). After a short section of preliminaries, these problems are distributed as follows: 167–168, find the diameter of a circle from the circumference and vice versa; 169, how to determine the diameter and the circumference of a circle if only a part of it is accessible; 170–171, inscribe a square into a circle and into a semicircle; 172–173, inscribe a circle into a square and into a rhombus; 174, inscribe an equilateral triangle into a circle; 175, find the height of an equilateral triangle from its side; 176–177, inscribe a circle and a square into an equilateral triangle; 178, find the distance from the centre to a vertex of an equilateral triangle; 179, find a side of a right angled-triangle from the other two sides, and inscribe a circle into the triangle; 180, inscribe a square into a right angled-triangle; 181, find the diagonal of a square and of a rectangle from its sides, and vice versa; 182–184, find the height of a scalene triangle; 184, find the area of a square from its side and the area of a circle from its circumference; 185–186, inscribe a square and a circle into a scalene triangle; 187, find the distance from the centre to a vertex of a scalene triangle; 188, find the circumcentre of a scalene triangle; find the area of: 201–

<sup>12</sup> See the discussion and the synopsis in Hero, *Metrica* 481–488.

<sup>13</sup> See the discussions in Hero, *Metrica* 450–455, 491–492, and 507–509, and ACERBI, *Struttura* 180. A patrikios Nikephoros was appointed professor of geometry by Constantine VII (r. 913–959); P. LEMERLE, *Le premier humanisme byzantin (Bibliothèque Byzantine. Études 6)*. Paris 1971, 264–265. However, “Patrikios” is here indisputably a name.

<sup>14</sup> K. VOGEL, *Ein byzantinisches Rechenbuch des frühen 14. Jahrhunderts (WBS 6)*. Wien 1968.

<sup>15</sup> The first is edited in M. D. CHALKOU, *The Mathematical Content of the Codex Vindobonensis Phil. Graecus 65 (ff. 11–126). Introduction, Edition and Comments (Byzantina Keimena kai Meletai 41)*. Thessalonikē 2006, the second in H. HUNGER – K. VOGEL, *Ein byzantinisches Rechenbuch des 15. Jahrhunderts (Österreichische Akademie der Wissenschaften, Philosophisch-historische Klasse, Denkschriften 78.2)*. Vienna 1963.

212, a circle and of regular polygons of 40, 30,  $2k$  with  $k = 10 \dots 3$ , and 5 sides; 213–216, rectangles and parallelograms, trapezia, rhombi, and a rhomboid; 217–219, equilateral, acute- and obtuse-angled isosceles, and scalene triangles; 220, arrow-shaped quadrilaterals; 221, the star-shaped figures resulting from the superposition of two equilateral triangles and of two arrow-shaped quadrilaterals; 222, compounded figures; 223, area of isoperimetric figures: the same sequence as in nos. 201–212, plus a square, rectangles, an equilateral triangle, a rectangle and a compounded figure; 224, transition to *geōdotikai paradoseis modēsmon*, with applications to measuring the area of: 224–225, circular, square, and rectangular parcels; 226, towns of several shapes; 227–230, pillars, a sphere, a tent; find the volume of: 231, cylindrical bags; 232, silos; 233, a tank minus a column plunged into it; 235–237, amphorae of several shapes; 238, how to square a circle.

Some byzantine compilations are markedly authorial products. A *Geodaesia*, whose anonymous author is traditionally called “Hero of Byzantium”, was written *ca.* 938<sup>16</sup>. After a preface and a lacuna involving at least one folio of the prototype of the tradition, Vat. gr. 1605 (mid 11<sup>th</sup> century; *Diktyon* 68236), ff. 42r–57v<sup>17</sup>, the extant text can be divided into eleven sections. Of these, sections 2–6 and 10–11 are reworked excerpts from Hero of Alexandria’s *Dioptra*: sect. 2, height of a distant wall (= *Dioptra* 12); 3–4, distance of inaccessible points (= *Dioptra* 10); 5, position of the straight line joining inaccessible points (= *Dioptra* 10); 6, surface of some plane figures by decomposition into triangles, maybe incomplete (see *Dioptra* 23–24); 10, flow of a source (= *Dioptra* 31); 11, angular distance between stars (= *Dioptra* 32); in this section, the author refers by title to his own writing *Positioning of Sundials*; the last two sections contain extensive verbatim excerpts. Sections 7–9 compile geometric metrological items from a source not yet identified and carry out computations of: circumference and area of a circle (7); surface, volume, and centre of gravity of cube, cylinder, sphere, cone, rectangular parallelepiped, and pyramid (8); volume of a water tank (9). In the preface, the author stresses the importance of the dioptra for siegecraft and asserts that he will compose a synopsis of previous writings and make abstract theories accessible to the layman. He mentions Euclid, Archimedes, and Hero; he quotes definitions 25, 18, 19, 20, 12, 13 of Book XI of the *Elements* and the enunciation of Archimedes’ *Measurement of a Circle* 1.

Ioannes Peditasimos († 1310–14; *PLP* 22235) wrote an *Epitome on land-measuring and land-partitioning*, usually referred to as *Geometria*, dealing with geometric metrological matters. This is to a large extent made up of extracts from the main geometric metrological collections, in particular from manuscripts of compilations edited as parts of the *Geometrica*. Peditasimos starts by drawing a distinction between geometry and geodesy, the latter being the discipline concerned with partitioning parcels (sects. 1–2). He then expounds a classification of reference directions and milestones (see below), lines, figures, and angles relevant to land measurement (3–7). “Postulates” and “lemmas” are then stated; these are metrological conversions, and a mix of arithmetical and geometric assumptions, respectively (8–9). Prescriptions for measuring lines and figures are expounded, according to the following sequence<sup>18</sup>: area of a square (10–14); side of a square of given area (16–17; only a *post factum* check); diagonal of a square, with a digression on the several categories of ratios (18); rectangles, which Peditasimos calls “parallelograms” (19–25); equilateral triangle, with a comparison between the methods of Nikomachos (applied to a triangular number), Hero, and “fiscal geometry” (26): the latter is deemed inadequate, the former two are declared to be compatible, but the final prescription amounts to drawing a height and calculating its length (in such a way

<sup>16</sup> Edition Hero of Byzantium 114–150. The date is argued in Th-H. MARTIN, *Recherches sur la vie et les ouvrages d’Héron d’Alexandrie*. Paris 1854, 267–275.

<sup>17</sup> The *Geodaesia* is preceded by a poliorcetic compilation traditionally ascribed to the same author, edited in Hero of Byzantium 26–112. See A. DAIN, *La tradition du texte d’Héron de Byzance*. Paris 1933.

<sup>18</sup> For most geometric species, seven examples are given, depending on whether the relevant lengths are expressed in one of the three units of measurement assumed as simple or in the four possible combinations of them (see sect. 15).

that the two residual methods, too, are compatible); right-angled (27), isosceles (28–30), scalene (31), generic (32; Heronian algorithm) triangles; rhombi (33), rhomboids (34); right-angled (35), isosceles (36), scalene (37–38) trapezia; circles (39), semicircles (40), segments less than a semicircle (41) and greater than a semicircle (42). Compounded figures are then discussed (43). Finally, geodesy is treated, with prescriptions for partitioning (not necessarily complex) figures into simpler ones; the sequence of figures is the same as above (44–59, plus two species of *tymbē*).

The so-called *Geodaesia* is a Byzantine collection of excerpts from earlier geometric metrological texts<sup>19</sup>. It deals only with triangles and quadrangles. Sections on the subject-matter of geometric metrology and on units of measurement prelude the problems; these measure space content and relevant components of: squares (sect. 7; 9 items); rectangular parallelograms (sect. 8; 4 items); right-angled triangles (sect. 9; 18 items, including the rules traditionally ascribed to Pythagoras and Plato for providing the sides of numerical right-angled triangles); equilateral triangles (sect. 10; 11 items); scalene triangles (sect. 11; 8 items); generic triangles by means of the Heronian algorithm (sect. 12; 2 items). The style adopts the procedural code more often than in the standard compilations. The result, while anonymous and made of extracts, bears a clear authorial stamp. Given the date of the manuscript witnesses to the *Geodaesia*, I am inclined to date it (and the *Compendium*) to the first half of the 14<sup>th</sup> century.

A short text by Isaak Argyros († ca. 1380; *PLP* 1285)<sup>20</sup>, sometimes transmitted in the form of a *Letter to Kolybas* (this slightly enriched version is almost certainly an authorial recension), deals with the problem of how to measure non-rectangular figures. Argyros first provides a definition of a right angle; he shows then, by comparing a square and a rhombus with equal sides, that two equilateral quadrangles with equal sides need not have equal areas. Triangles are measured by drawing and measuring a height. The final section, about measuring a generic quadrilateral by partition in a central rectangle and a number of peripheral triangles, is not satisfactorily argued, and ends with resorting to a prescription typical of fiscal geometry (see just below). In several manuscripts, Argyros' text is followed by a short note on Bryson's circle quadrature and by a capsule geometric metrological collection (20 items: units of measurements, problems that measure area and relevant components of: square; rectangles; quadrilaterals; right-angled, scalene, equilateral, acute- and obtuse-angled triangles; circle), entitled *From Hero's Geodesy* and comprising the algorithms edited as *Geom.* 6, 1–2; 7, 1–3, 5–6; 11, 1–2, 24, 31–36; 17, 4–6, 8, 7. Heiberg shows that these extracts come from the *Liber geeponicus* in Vat. gr. 215; he ascribes their compilation to Argyros himself<sup>21</sup>. Assemblies like this are easily found, as we see turning our attention to fiscal geometry.

A tradition of fiscal geometry or “geodesy” in the Greek language surfaces in Byzantine times. The latter parameter was taken into account, in a particular system of imposition, by making some basic units of measurement a function of the quality of the soil. We may draw a difference between geometric metrology and geodesy because, in principle, the former employs “wrong” prescriptions only when they are approximations—for instance, of the area of a circle—the latter programmatically applies in a systematic way “wrong” prescriptions<sup>22</sup>. Parcels were imposed depending on their

<sup>19</sup> For the edition, the identification of the sources, and a discussion of the manuscript tradition, see Hero, *Opera omnia* V LXVI–XCVII, and XCVIII–CXI, CXVIII–CXXIII, 222–232 for a synopsis and editions of other Byzantine texts and scholia.

<sup>20</sup> Critical edition in Argyros, *Epistola*. An edition based on Par. gr. 2419 (15<sup>th</sup> century; *Diktyon* 52051), ff. 197v–198r, a highly incorrect copy that also carries a heavy recension, and Par. gr. 2013 (16<sup>th</sup> century; *Diktyon* 51640), ff. 151v–152v, can be read in J. LEFORT – R.-C. BONDOUX – J.-C. CHEYNET – J.-P. GRELOIS – V. KRAVARI – J.-M. MARTIN, *Géométries du fisc byzantin (Réalités byzantines 4)*. Paris 1991, 154–158. The manuscript tradition of Argyros' short text greatly overlaps with the ones of the *Geodaesia* and of the *Compendium*; the prototypes of the two stemmatic branches are Vat. gr. 1411 (end of the 14<sup>th</sup> century; *Diktyon* 68042) and Marc. gr. Z. 323 (coll. 639; here ca. 1428; *Diktyon* 69794).

<sup>21</sup> Hero, *Opera omnia* V IC.

<sup>22</sup> Justificatory discourse about geometric metrology not coming from fiscal milieu suggests that it was identified with geometry and thereby kept distinct from geodesy: read the preface of Pediasimos' *Geometria*, at 7, 4–9 Friedlein.



area and on the quality of their soil. In fiscal geometry, for practical reasons, the area of a parcel was estimated by applying a reduced set of standard prescriptions involving relevant linear elements of the parcel. As a matter of fact, all parcels, even triangular ones, were treated as more or less “degenerate” quadrilaterals. Almost all such estimates lead to an overestimation of the area of a parcel, that increases with the irregularity of its contour but that, however, seldom amounts to more than some ten per cent. This overestimate was partly compensated for by the application of a standard reduction rate, usually ten per cent. There were three basic prescriptions for estimating the area of a region: (1) for irregular parcels, multiply one-fourth of the perimeter by itself; (2) for semi-regular parcels of approximately constant length and appreciably variable width, multiply the former by the arithmetic mean of a number of measures of the latter (this is Patrikios’ rule); (3) for quadrilaterals (and hence triangles), take the product of the arithmetic mean of the opposite sides. The last prescription can be further simplified for isosceles trapezia and for rectangles: replace one or both arithmetic means by one of the equal sides. The elementary arithmetical operations involved in the prescriptions never go beyond multiplication and halving.

The texts of fiscal geometry are organised exactly as geometric metrological collections are: possibly an introduction dealing with generalities about land-surveying and units of measurement, then certainly a series of problems serving as paradigmatic examples, usually graded by the complexity of the figure involved. The corpus of such writings has no real cohesion: they are scattered in a number of (frequently juridical) manuscripts<sup>23</sup>. These texts are all anonymous, with two notable exceptions: a short treatise of someone called “George” and a poem ascribed to Michael Psellos (b. 1018)<sup>24</sup>. In particular, George’s work—which displays the verbosity that characterises so many Byzantine scientific writings—expounds the subject-matter of geometric metrology (sects. 205–217), the units of measurement and their conversions, along with pieces of information on the quality of soil (218–223), and basics about the practice of land-surveying (225); problems that measure the area of a series of figures follow: square (224); rectangle (226); generic quadrilaterals (227–228); trapezium with a triangle on its top (229); equilateral (230), isosceles acutangle (231), scalene (232) triangles; circles (233–234); a segment of a circle (235). The poem ascribed to Psellos expounds units of measurement and their conversions (284–286); reference directions and milestones (287); problems that measure the area of the following figures: square (288); trapezium (289); equilateral triangle (290); rectangle (291); circle (292); gnomon (293); irregular shapes (294–295); a spherical cap (296); a slim trapezium (297); a cross (298). The rest of the poem (299–308) gives prescriptions about the best practice of land-surveying (299, 301, and 304–308), the relationships between some unit of measurement and the quality of the soil (300), and how to measure at sea (302) and on ragged terrain (303).

The manuscripts that contain the most important compilations of fiscal geometry are Bucur. BAR, gr. 493 (17<sup>th</sup> century; Litzica 624; *Diktyon* 10566), ff. 125v–133r; Par. gr. 1043 (15<sup>th</sup> century; *Diktyon* 50636), ff. 141r–144r; Par. suppl. gr. 676 (14<sup>th</sup> century; *Diktyon* 53411), ff. 89r–92v; Vin-

<sup>23</sup> The sources are edited in F. HULTSCH, *Metrologorum scriptorum reliquiae*. I–II. Lipsiae 1864–66 I, in E. SCHILBACH, *Byzantinische metrologische Quellen (Byzantinā Keimena kai Meletai* 19). Thessalonikē 1982, and in the excellent LEFORT *et al.*, *Géométries* (the latter with a rich discussion; the unity of some compilations is better highlighted in this book than in Schilbach’s, who sometimes explodes a single collection into independent units); analysis of the units of measurement in F. HULTSCH, *Griechische und römische Metrologie*. Berlin 1882, E. SCHILBACH, *Byzantinische Metrologie (Handbuch der Altertumswissenschaft* XII,4). München 1970. See also J. LEFORT, *Le cadastre de Radolibos (1103), les géomètres et leurs mathématiques*. *TM* 8 (1981) 269–313.

<sup>24</sup> Editions in Hero, *Opera omnia* V cv–cvii (only George, and partial but with a complete list of parallel passages in pseudo-Heronian compilations); SCHILBACH, *Quellen* 86–92 (only George, and partial) and 116–125; LEFORT *et al.*, *Géométries* 136–152 (George, complete and with identification of the sources) and 184–200 (Psellos), to whose sections I refer; Psellos, *Poems* 415–425. On Psellos’ poem see also P. MOORE, *Iter Psellianum*, A detailed list of manuscript sources for all works attributed to Michael Psellos, including a comprehensive bibliography (*Subsidia Mediaevalia* 26). Toronto 2005 POE.58.

dob. jur. gr. 1 (11<sup>th</sup> century; *Diktyon* 71008), ff. 345r–346r; Vindob. jur. gr. 2 (1<sup>st</sup> half of the 14<sup>th</sup> century; *Diktyon* 71009), f. 380r–v; Vindob. jur. gr. 10 (2<sup>nd</sup> half of the 13<sup>th</sup> century; *Diktyon* 71017), ff. 85v–88v; Zaborda, Nikanor 121 (12<sup>th</sup> century; *Diktyon* 72403), ff. 226r–229v; the filler in Laur. Plut. 74.5 (bound in disorder; 2<sup>nd</sup> half of the 12<sup>th</sup> century; one of the products of Ioannikios’ scriptorium; *Diktyon* 16660), ff. 182r–186r, which is followed by extracts from John of Damascus and by first-rate computistical prescriptions. The important, multi-hand anthology in Laur. Plut. 58.24 (12<sup>th</sup> century; *Diktyon* 16660) contains on f. 124r, after two *Rechenbuch*-style problems, two short problems of measurement of plane parcels (1 rectangle and 1 sail-like trapezium) and a conversion between units of measurement<sup>25</sup>. Fiscal geometry also features in Vat. Pal. gr. 367 (Cyprus; 1317–20; *Diktyon* 66099), ff. 88v–91r: capacity of ships and measurement of quantities of specific goods like oil, wine, and salt; and 94r–97v: title *archē syn theō tēs geōmetrias*, 13 problems of measurement of plane parcels (4 triangles, 1 square, 3 rectangles, 2 trapezia, 1 of which is said to be sail-like, 1 generic quadrilateral, 2 threshing-floors shaped as a circle) which are interspersed by sequences on units of measurement. These texts close a fully-fledged *Rechenbuch* that also contains an Easter Computus<sup>26</sup>. Personal anthologies as the one in Par. gr. 2419 (the notebook of George Meidiates), ff. 195v–198v, may assemble items like George, Argyros’ *Letter to Kolybas*, and an additional set of prescriptions for quadrilaterals, triangles, and circles. Structured texts like these usually feature, among the introductory considerations, extracts from geometric metrological compilations; George has long extracts, and even mentions Hero of Alexandria.

A relevant portion of the corpus of fiscal geometry is contained in manuscripts earlier than the 12<sup>th</sup> century; some texts must predate a reform, amounting to a redefinition of a basic unit of measurement, introduced by one of the Byzantine emperors of the 11<sup>th</sup> century called Michael. All in all, about one-third of the corpus certainly predates 1204.

## CONTENTS AND AUTHORSHIP OF THE *COMPENDIUM*

The contents of the *Compendium* can be outlined as follows<sup>27</sup>. As is customary, the text opens with definitions concerning the subject-matter of geometric metrology (sects. 1 and 23), with list of units of measurement (2 and 23), and with conversions between some units of measurement (3). The bulk of the work expounds how to measure the space content and relevant components of: squares (4); rectangles (5); equilateral triangles (6); right-angled triangles (7); obtuse- and acute-angled triangles (9); circles inscribed in and circumscribed to triangles (10); rhombi (11); rhomboids (12); the circle (13); semicircles (14); segments of a circle other than a semicircle (17); regions enclosed between mutually tangent circles, and squares inscribed in and circumscribed to circles (18); trapezia (19); regular  $n$ -gons, with  $n = 5 \dots 12$  (20 and 22); irregular figures (21); the sphere (24); cones (25); obelisks (26), cylinders (27); cubes (28); wedges (29); *meiouroi*, namely, prisms with triangular or quadrangular base (30); columns (31); bricks (32); pyramids (33). One is also explained how

<sup>25</sup> The texts in Laur. Plut. 74.5 are edited in SCHILBACH, Quellen II, 18, 21, 24, 6, 15, 6, 22, and LEFORT *et al.*, Géométries 78–103. The text in Laur. Plut. 58.24 is as follows: ὀφείλει μετρεῖσθαι τὸ τετράγωνον χωρίον μέσον τὸ πλάτος καὶ τὸ μῆκος. καὶ ἐὰν εὐρεθῇ μῆκος  $\eta$  καὶ πλάτος  $\epsilon$ , ψηφίξει οὕτως: πέντα  $\eta$ ,  $\mu$ . καὶ ἔστιν [οὕτως] † $\eta$ , ἥτοι σι† μῶδιοι. εἰ δὲ ἔνι ἀρμενοειδές, μετρεῖσθαι ὀφείλει τὸ πλάτος εἰς δύο, ἥτοι ἐπὶ τὴν στενὴν καὶ ἐπὶ τὴν πλατεῖαν, καὶ ὀφείλεσθαι τὰ ἡμίση τοῦ ποσοῦ τῶν ὀργῶν, καὶ τὰ ἡμίση ψηφίξεσθαι μετὰ τοῦ μήκους, καὶ ἀναβιβάζειν. ἔστι δὲ ὁ μῶδιος  $\sigma$  ὀργῶν, ἢ δὲ λίτρα ὀργῶν  $\epsilon$ . The Computi in these two manuscripts are edited in F. ACERBI, Byzantine Easter Computi 978–982 and 975–978, respectively.

<sup>26</sup> These problems, as well as short texts on ff. 69v, 73v, 76v, 79r–v, 80r–v, 83v, 84r, 98r, are edited, sometimes broken into smaller units and in perturbed order, in SCHILBACH, Quellen I, 5cd; II, 4, 14, 16, 18; III, 1, 2ek; IV, 4d, 8bf, and, more satisfactorily, in LEFORT *et al.*, Géométries 48–59. The Computus is edited in F. ACERBI, Byzantine Easter Computi 534–541.

<sup>27</sup> See the Annotated Paraphrase below for details. A summary of the *Compendium*, with indication of the initial borrowings from the *Geometrica*, was already given in Hero, *Opera omnia* V IC–CII.

to construct numerical right-angled triangles by using Pythagoras' and Plato's prescriptions (8), a criterion for establishing the species of a circular segment (15), and how to compute the diameter of the circle equal to a given rectilinear region (16). Sections 4–7 and 11 also provide constructions of the objects at issue.

The authorship of the *Compendium* can only be a matter of speculation. Its tradition strongly suggests that this work is a 14<sup>th</sup>-century elaboration. The presence of Isaak Argyros' and Nicholas Rhabdas' works in the prototype of the tradition encourage making a case for identifying either of them as the author. The subject-matter appears to favour Rhabdas (*fl.* 1321–40; *PLP* 1437), who was a land-surveyor, but this is counterbalanced by the fact that Argyros wrote a metrological text. As we shall see, the state of the text of the *Compendium* points to a damaged model, possibly a draft waiting for being edited (even the title is missing). There are several mathematical mistakes, not all of which can be justified by material damages or copying mistakes. This does not decide the issue, for both Argyros and Rhabdas were prone to commit trivial mistakes<sup>28</sup>. The question of authorship must be left open.

### THE MANUSCRIPT WITNESSES OF THE *COMPENDIUM*

Only seven manuscript witnesses of the *Compendium* are known to me<sup>29</sup>. Six of them are more or less obvious systematic apographs of the only independent witness, namely, Vat. gr. 1411. The stemmatic relations in which Vat. gr. 1411 is involved have been established in a series of studies. I shall briefly expound these relations at the beginning of the next Section.

B. Berlin, Staatsbibliothek (Preußischer Kulturbesitz), Philipps 1555 (*Diktyon* 9456) + Oxford, Bodleian Library, Auct. T.I.22 (Misc. 200; *Diktyon* 47148); mm 324×240; a set of quires copied in Bartolomeo Zanetti's atelier and now bound as two separate codices; 1539–42; copyists <George> (ff. 104r–106v + 2r–17v) and <Nicholas Kokolos> (1r–72v + 65r–78v), <Bartolomeo Zanetti> (// + 41r–64v) plus hands A (73v–103v + 18r–24v) and B (// + 25r–40v)<sup>30</sup>. The *Compendium* is on ff. 102v–103v *des. γεγονότος ἀριθμοῦ* + 18r–24v *inc. ἄφελε τὸν τῆς βάρσεως*. Other works in the manuscripts: Philipps 1555: ff. 1r–54v Aristides Quintilianus, *De musica*; 54v–63r *Anonymus Bellermann*; 64v–69v Bacchius Senex; 69v–72r Dionysius [Pseudo-Bacchius], *De arte musica*; 72r *Bacchii epigramma*; 72r–v *Mesomedis hymni tres*; 73v–91r Ioannes Pediasimos, *scholia in Cleomedis Caelestia*; 91r–92r [Ioannes Pediasimos], two astronomical texts excerpted from Cassius Dio, with a diagram<sup>31</sup>; 93r–100v [Hero], *Geodaesia*; 100v–102r Isaak Argyros, *Epistola ad Kolybam*; 104r–106r *Introductio in Euclidis Elementa, inc. mut. ἔ]χει*; 106r Neophytos Prodromenos, *De numeris indicis*. Bodl. Auct. T.I.22: ff. 2r–5v Theodoros Prodromos, *De magno et parvo*; 6r–17v *Introductio in Euclidis Elementa, des. mut. ἔ]χει*; 25r–40v Eusebius Caesariensis, *Contra Hieroclem*; 41r–64v *Catena in Canticum Cantorum*; 65r–78v [Plutarch], *De musica*.

<sup>28</sup> See F. ACERBI, *Riscrivere Tolomeo a Bisanzio. Concezione ed appropriazione delle Tabulae Novae di Isacco Argiro*. *Bollettino dei Classici* 44 (2023) 1–112 and F. ACERBI, *A New Logistic Text of Nicholas Rhabdas*. *Byz* 92 (2022) 17–45 (which also see for Rhabdas being a land-surveyor), respectively.

<sup>29</sup> The editions of all works contained in the manuscripts are listed in the “Printed Sources” section of the Bibliography.

<sup>30</sup> W. STUEMUND – L. COHN, *Verzeichniss der griechischen Handschriften der Königlichen Bibliothek zu Berlin*. I. Codices ex bibliotheca Meermanniana Phillipici Graeci nunc Berolinenses. Berlin 1890, 65; T. J. MATHIESEN, *Ancient Greek Music Theory. A Catalogue Raisonné of Manuscripts (International Inventory of Musical Sources Bxi)*. München 1988, 79–82; A. CATALDI PALAU, *A Catalogue of Greek Manuscripts from the Meerman Collection in the Bodleian Library*. Oxford 2011, 180–186. The first two copyists of the Bodleian manuscript are identified in *RGK* I 65 and 310, respectively. See also *RGK* II 84 and 429, III 107.

<sup>31</sup> For these excerpts see also R. B. TODD, *The Manuscripts of John Pediasimus' Quotations from Dio Cassius*. *Byz* 56 (1986) 275–284.

These two manuscript collect quires produced in the copying campaign sponsored by Guillaume Pellicier, who was the French ambassador in Venice from June 1539 to October 1542<sup>32</sup>. Problems arose when these quires were bound in two separated codices. The *Compendium*—and, more generally, the geometric metrological items, copied by hand A along with Pediasimos' scholia—was split as indicated above; Pseudo-Plutarch *De musica* was separated from the other treatises of harmonic theory (this production unit was copied by Nicholas Kokolos). It is also obvious that ff. 6r–17v of Bodl. Auct. T.I.22 must be followed by ff. 104r–106r of Phillipps 1555, and this makes the entire production unit copied by George Kokolos. For these isagogic texts see F. ACERBI, I codici matematici di Bessarione, in A. Rigo, N. Zorzi (eds.), I libri di Bessarione. Studi sui manoscritti del Cardinale a Venezia e in Europa (*Bibliologia* 59). Turnhout 2021, 107–218: 131–138 and 143. The model of this isagogic compilation is Par. gr. 1928 (2<sup>nd</sup> quarter of the 14<sup>th</sup> century; copyist mostly Neophytos Prodromenos, but here Matthew Chortatzes [RGK II 367]; *Diktyon* 51555), ff. 6r–15r. With the exception of the *Geodaesia* and of Argyros' short treatise, the quires penned by hand A are a copy of Vat. gr. 1411 (V), as follows (the original folio numbers of V are given, see the Appendix):

B	73v–92v	102v–103v + 18r–24v
V	328r–335v	140r–149r

- D. El Escorial, Real Biblioteca del Monasterio de S. Lorenzo, Φ.I.10 (gr. 188; *Diktyon* 15142); mm 333×215; ca. 1542; copyists Nicholas Murmuris (ff. 1r–127v and 144r–231v, subscription on f. 209v, dated April 29, 1542), <Ioannes Mauromates> (128r–143r, 7), Petrus <Carnabacas> (143r, 8–143v)<sup>33</sup>. The *Compendium* is on ff. 100r–108r. Other works in the manuscript: ff. 1r–50r Hero, *Pneumatica* I–II; 50v–70r Hero, *Automata*; 73r–77r Manuel Moschopoulos, *De numeris quadratis*; 77v–83v Maximus Planudes, *Psephophoria secundum Indos – recensio* Rhabdas, des. 61, 8 εἴρηται; 84r–88v Nicholas Rhabdas, *Epistola ad Khatzykem*; 89r–90v arithmetical tables; 91r–v arithmetical problems C<sub>1</sub>, A<sub>1</sub>, A<sub>2</sub>; 91v–98v [Hero], *Geodaesia*; 98v two magic squares 6<sup>2</sup> and 10<sup>2</sup>; 99r–100r Isaak Argyros, *Epistola ad Kolybam*; 108v–124r Nicholas Rhabdas, *Epistola ad Tsavoukhem*; 124v–126r Isaak Argyros, On the first stereographic projection used in Ptolemy's *Geography*; 126r–v [Michael Psellos], *De duodecim ventis*<sup>34</sup>; 126v–127r prose text on the winds; 127r–v three short geographical texts; 128r–143v George Pachymeres, *Paraphrasis in Aristotelis mechanica*<sup>35</sup>; 144r–159v Ioannes Pediasimos, *scholia in Cleomedis Caelestia*; 159v–160v [Ioannes Pediasimos], two astronomical texts excerpted from Cassius Dio, with a diagram; 161r–164r Nikephoros Gregoras, *protheōria in*

<sup>32</sup> See A. CATALDI PALAU, Les vicissitudes de la collection de manuscrits grecs de Guillaume Pellicier. *Script* 40 (1986) 32–53. After Pellicier's death, parts of his collection followed the trajectory of ownership Claude Naulot → Jesuits of Clermont → Gerard Meerman (and hence the Bodleian Library) → Thomas Phillipps (and hence the Preußische Staatsbibliothek).

<sup>33</sup> See G. DE ANDRÉS, Catálogo de los Códices Griegos de la Real Biblioteca de El Escorial. II. Códices 179–420. Madrid 1965, 15–17, and *CCAG* XI.2 19–24. For the distribution of the hands see D. HARLFINGER, Die Textgeschichte der Pseudoaristotelischen Schrift PERI ATOMÓN GRAMMÓN. Amsterdam 1971, 196. For Murmuris see *RGK* I 314bis, II 434, III 507 (copies dated 1540–43); for Mauromates see *RGK* I 171, II 229, III 283 (copies dated 1545–65), for Carnabacas see *RGK* I 346–347; II 474–475; III 551 (copies dated 1542–46). See also A. BRAVO GARCÍA, Dos copistas griegos de Asulanus y de Hurtado de Mendoza. *Faventia* 3 (1981) 233–239; O. L. SMITH, On Some Manuscripts of Heron, *Pneumatica*. *Script* 27 (1973) 96–101, 98–101. These two studies are also relevant to Par. gr. 2428.

<sup>34</sup> For the five Psellian treatises see in the order in which they are copied in this manuscript, the items POE.59, PHI.111, PHI.112, THE.165, THE.168 in MOORE, *Iter*. The two theological texts are argued to be spurious in P. GAUTIER, *Le De daemonibus* du Pseudo-Psellos. *REB* 38 (1980) 105–194 and P. GAUTIER, Pseudo-Psellos: *Graecorum opiniones de daemonibus*. *REB* 46 (1988) 85–107, the short poem is deemed inauthentic in Psellos, *Poems* XXIX.

<sup>35</sup> This paraphrasis, which is part of an Aristotelian compendium in twelve books and still lies unedited, is almost identical to the original. This fact has given rise to a series of annotations on ff. VIr and 128r: see P. MORAUX – D. HARLFINGER – D. REINSCH – J. WIESNER, *Aristoteles Graecus. Die griechischen Manuskripte des Aristoteles. Erster Band. Alexandrien – London. Berlin – New York* 1976, 164; *CCAG* XI.2 22.

*Synesii De insomniis*; 164r–187v, Synesios, *De insomniis cum Gregorae scholiis*<sup>36</sup>; 189r–v [Manuel Moschopoulos], *De insomniis*; 189v–195r *excerpta e Meletii tractatu De natura hominis*; 195r–196r *excerpta de medicina*<sup>37</sup>; 196v Ippocrates, *Elementa*<sup>38</sup>; 197r–198v Diocles of Carystus, *Ad Antigonem regem de tuenda ualetudine*; 199r–200v *collectio remediorum*<sup>39</sup>; 200v–205v *excerpta de medicina*; 206r Manuel Holobolos, *In mortem Andronici Tornicis*; 206v–209v Demetrios Triklinios, *Excogitatum de lunae figuris*; 210r–217v Michael Psellos, in *Oracula Chaldaica expositio*; 217v–218v Michael Psellos, *Summaria Chaldaicorum dogmatum expositio*; 219r–221r [Michael Psellos], *Graecorum opiniones de daemonibus*; 221v–231v [Michael Psellos], *De operatione daemonum*.

A set of quires realised by copyists who worked for Diego Hurtado de Mendoza, the Spanish ambassador in Venice from May 1539 to October 1547. After the two Heronian treatises, and excluding George Pachymeres' *Paraphrasis* (penned by different hands), Scorial. Φ.I.10 is a partial yet systematic copy of Vat. gr. 1411 (V), as follows (lost portions of V are within square brackets):

D	73r–124r	124v–127v	144r–160v	161r–196v	197r–205v	206r–209v	210r–231v
V	128r–162v	59r–60v	328r–335v	[198r–234v]	[251r–258v]	[336–337]	163r–179v

- K. Istanbul, Topkapı Sarayı, G.İ.19 (*Diktyon* 33965); mm 135×90, 1<sup>st</sup> half of the 15<sup>th</sup> century; copyist <Isidoros of Kiev><sup>40</sup>. The *Compendium* is on ff. 90v–105v. Other works in the manuscript: ff. 4r–50v Aristoteles, *De generatione et corruptione*; 51r–69v Proclus, *Elementatio physica*; 76r–90r [Heraclitus], *Geodaesia*; 106r–107v arithmetical problems A<sub>3</sub>, C<sub>2</sub>, C<sub>1</sub>, A<sub>1</sub>, A<sub>2</sub>; 108r–109r *astrologica et meteorologica quaedam*<sup>41</sup>; 110r–115v *Anonymus*, προγνωστικὸν ἀπὸ τῶν ἐν τῇ παλάμῃ γραμμῶν; 116r–151r <Aelianus>, *Theoria tactica*; 151v *Anonymus*, σύνταξις ὀπλιτῶν τετράγωνος ἔχουσα ἐντὸς καβαλλάριον; 152r representation of the παράταξις τετράγωνος; 153r–155v Leo VI the Wise, *excerptum tacticum*<sup>42</sup>; 156r–238v Hero, *Pneumatica* I–II; 246r–247v arithmetical problems; 247v–

<sup>36</sup> The manuscript tradition of Gregoras' commentary is studied in P. PIETROSANTI, Per un contributo all'edizione critica del *Commento* di Niceforo Gregora al *de insomniis* di Sinesio di Cirene. *Acme* 49 (1996) 157–175. For the ascription of the subsequent item to Moschopoulos see G. MERCATI, I codici Vaticani latino 3122 e greco 1411, *Archivio Storico Italiano* 78 (1920) 269–282, reprinted in ID., *Opere minori*. Volume IV (1917–1936) (*StT* 79). Città del Vaticano 1937, 154–168, 279 n. 1.

<sup>37</sup> See *CCAG* XI.2 23, to be checked with R. FÖRSTER, Eine Handschrift des Serail. *Philologus* 42 (1884) 167–170, 169 (ff. 248r–250v [= 239r–241v Förster] of Seragl. G.İ.19), MERCATI, *Codici* 279, and E. MARTINI, *Catalogo dei manoscritti greci esistenti nelle biblioteche italiane*. Milano 1896, 402. Meletius can be read in *PG* LXIV 1075–1310.

<sup>38</sup> For this and the following item see H. DIELS, *Die Handschriften der antiken Ärzte*. I–II. Berlin 1905–06 I, 46, II 27–28, respectively.

<sup>39</sup> For this and the following item see *CCAG* XI.2 23–24; MERCATI, *Codici* 279–280; MARTINI, *Catalogo* 404–405.

<sup>40</sup> Descriptions in FÖRSTER, *Handschrift* and in MORAUX – HARLFINGER – REINSCH – WIESNER, *Aristoteles Graecus* 373–375. See also M. RASHED, *Die Überlieferungsgeschichte der aristotelischen Schrift De generatione et corruptione (Serta Graeca 12)*. Wiesbaden 2001, 21, 250, 273–274 and Abb. 8. The copyist was first singled out as *Anonymus* 8 in HARLFINGER, *Textgeschichte* 418, then identified with Isidoros of Kiev in D. HARLFINGER, *Griechische Handschriften und Aldinen: Eine Ausstellung anlässlich der XV. Tagung der Mommsen-Gesellschaft in der Herzog-August-Bibliothek Wolfenbüttel*. Herzog-August-Bibliothek Wolfenbüttel, 16. Mai bis 29. Juni 1978 (*Ausstellungskataloge der Herzog-August-Bibliothek* 24). Braunschweig 1978, 51. This manuscript is not included in M. MANFREDINI, *Inventario dei codici scritti da Isidoro di Kiev*. *Studi Classici e Orientali* 46 (1998) 611–624. On Isidoros see *RGK* I 155, II 205, III 258; *PLP* 8306.

<sup>41</sup> Two texts: the former (f. 108r) *tit.* περὶ ὧν σημαίνει τῷ κόσμῳ ὁ ἐν τῇ ἀνάστρω σφαίρα ὄφισ κινούμενος, *inc.* τινὲς τῶν φιλοσόφων εἶπον εἶναι ἀναστρων σφαίραν, ἄνω δηλονότι τῆς ἡ<sup>15</sup> τῶν ἀπλανῶν σφαίρας, ἦντινα καὶ θ<sup>9ν</sup> λέγουσι σφαίραν *des.* τὰ δὲ τῆς πενίας ζῳδιὰ εἰσιν ἡ Παρθένος καὶ ὁ Κύων; the latter (ff. 108v–109r) *tit.* περὶ νεφέλης, χιόνος, χαλάζης, βροχῆς, ὀμίχλης, πάχνης, κρυστάλου καὶ δροσίας, *inc.* ἡ νεφέλη ἀτμός ἐστι καὶ ἀναθυμίασις τοῦ κύκλω τῆς γῆς ὠκεανοῦ *des.* καὶ οὕτως εὐρίσκονται καὶ κάτω τὰ τέσσαρα ταῦτα, ἡ ὀμίχλη, ἡ πάχνη, ἡ δροσία καὶ ὁ κρύσταλος, κατὰ μίμησιν τῶν ἐπάνω τεσσάρων.

<sup>42</sup> *Tactica, constitutio* IV.6 *segg., inc.* πρώτη κεφαλὴ ὁ στρατηγός, καὶ μετ' αὐτὸν οἱ μεράρχαι (*PG* 107, 701A6) *des.* πρὸς τοὺς ἐν τῇ πόλει ἐκ ταύτης μάχονται.

248r *collectio remediorum*<sup>43</sup>; 248r–250v *excerpta de medicina*; 251r–273r Adamantios, *Physiognomonica* I–II, *des.* 2, 36, 1 τὰ πλείω; 273r–276v παρασημειώσεις προγνωστικαὶ περὶ τῆς μελλούσης τοῦ ἀέρος καταστάσεως; 276v–277v παρασημειώσεις ἀπὸ τῆς τῶν ἀλόγων ζώων διαθέσεως; 278r–281r Ioannes Pediasimos, two astronomical texts excerpted from Cassius Dio, with three diagrams; 282r–291v [Ammonius] <Nikephoros Gregoras>, *De constructione astrolabii*; 292r–316r Ioannes Philoponos, *De usu astrolabii; nota de astrolabio*; 317r–319v George Kodinos, *De officiis* XX; 319v–330r *Donatio Constantini*; 330v–332r Michael Psellos, *De bellica dispositione*<sup>44</sup>.

Seragl. G.Ī.19 is a partial yet systematic copy of Vat. gr. 1411 (V), as follows (the original foliation of Seragl. G.Ī.19 is in the row above):

K	76r–90r	90v–105v	106r–107v	246r–247v	248r–250v	251r–273r	273r–277v	278r–279r
K	73r–87r	87v–95v	96r–97v	237r–238v	239r–241v	242r–264r	264r–268v	269r–270r
V	140r–143v	144v–149r	124r, 140r	[162]	[233]	259r–267v	299r–301v	335r–v

M. München, Bayerische Staatsbibliothek, Cod. graec. 100 (*Diktyon* 44544); mm 330×230; composite; ca. 1550 and 1551; copyists Main 5 Mon. 27 (ff. 1r–347r) and Ioannes Murmuris (348r–458r, subscription on f. 458r, dated 1551), plus several annotators<sup>45</sup>. The *Compendium* is on ff. 273v–284r. Other works in the manuscript: ff. 1r–47r, 90r–103r, 137r–142v, 328v–347r Damascius, in *Phaedonem*; 48r–89v Proclus, in *Cratylum*; 103r–137r Damascius, in *Philebum*; 142v–152r Theophylact Simocatta, *Quaestiones physicae*; 152r–170v Nemesius, *De natura hominis*<sup>46</sup>, from chapt. 35; 170v–185v Demetrios Kydones, *De contemnenda morte, des.* 36, 13 εἰρήσθω; 186r–209r Ioannes Pediasimos, *scholia in Cleomedis Caelestia*; 209r–v [Ioannes Pediasimos], two astronomical texts excerpted from Cassius Dio, with a diagram; 210r–220r Michael Psellos, in *Oracula Chaldaica expositio*; 220r–221r Michael Psellos, *Summaria Chaldaicorum dogmatum expositio*; 221r–224r [Michael Psellos], *Graecorum opiniones de daemonibus*; 224r–237r [Michael Psellos], *De operatione daemonum*; 238r–243v Manuel Moschopoulos, *De numeris quadratis*; 244r–252v Maximus Planudes, *Psephophoria secundum Indos – recensio Rhabdas, des.* 61, 8 εἴρηται; 253r–v Maximus Planudes, *Verse for Ptolemy's Geography*<sup>47</sup>; 254r–266r Isaak Argyros, *De cyclis solis et lunae ad Andronicum*; 267r–270v Isaak Argyros, *Tabulae novae (Almagest)*; 272r–273v Isaak Argyros, *Epistola ad Kolybam*; 284v–287v Nicholas Rhabdas, *Epistola ad Tsavoukhem, des.* 126, 26; 288r–328v Olympiodoros, in *Phaedonem*; 348r–457v Theodoros Metochites, *Stoicheiōsis Astronomikē* I.1–31, 34.

The two sets of copies that make up this manuscript were collected, and very likely commissioned, by Joannes Jakob Fugger (first trace in a 1557 inventory); his collection was acquired in 1571 by the ducal court library in München. Damascius' and Olympiodoros' commentaries stem from Marc. gr. Z. 196 (coll. 743; middle of the 9<sup>th</sup> century; *Diktyon* 69667) and from Ricc. 37 (*olim* K.II.31; 15<sup>th</sup> century; *Diktyon* 17037), respectively. The

<sup>43</sup> *Tit. περί τινων ὑγιεινῶν, inc.* Γαλενοῦ ὡς ἤκουσα *des.* ὕδατι ζέσας καὶ ψυχράνας, πότιζε.

<sup>44</sup> See MOORE, *Iter*, item CET.DISC.4.

<sup>45</sup> Description in M. MOLIN PRADEL, *Katalog der griechischen Handschriften der Bayerischen Staatsbibliothek München. Band 2. Codices graeci Monacenses 56-109*. Wiesbaden 2013, 270–279. Identification of the copyists in B. MONDRAIN, *Copistes et collectionneurs de manuscrits grecs au milieu du XVI<sup>e</sup> siècle: le cas de Johann Jakob Fugger d'Augsbourg*. *BZ* 84–85 (1992) 354–390; For Murmuris see *RGK* I 172, II 230. On Metochites' treatise see B. BYDÉN, *Theodore Metochites' Stoicheiōsis Astronomikē and the Study of Natural Philosophy and Mathematics in Early Palaiologan Byzantium (Studia Graeca et Latina Gothoburgensia 66)*. Göteborg 2003.

<sup>46</sup> A study of the textual tradition of this text is in M. MORANI, *La tradizione manoscritta del De natura hominis di Nemesio (Scienze Filologiche e Letteratura 18)*. Milano 1981.

<sup>47</sup> The poem is followed by a 3-verse epigram. On the former see also F. PONTANI, *The World on a Fingernail: An Unknown Byzantine Map, Planudes, and Ptolemy. Traditio* 65 (2010) 177–200, and C. M. MAZZUCCHI, *Il Tolomeo Ambr. D 527 inf. e i versi di Massimo Planude sulle carte della Geografia (Ambr. A 199 sup.)*, in: *Miscellanea Graecolatina I*, a cura di F. Gallo. Roma 2013, 259–266.

model for Nemesius is Ambr. D 338 inf. (gr. 967; end of the 15<sup>th</sup> century; *Diktyon* 42627). Theodoros Metochites' *Stoicheiōsis Astronomikē* was copied from Marc. gr. Z. 329 (coll. 734; middle of the 14<sup>th</sup> century; *Diktyon* 69800). The rest of the scientific section is a partial yet systematic copy of Vat. gr. 1411 (V), as follows:

M	186r–209r	209r–v	210r–237r	238r–252v	253r–v	254r–267r	267r–271v	272r–287v
V	108r–115r	115r–v	26r–42v	118r–126v	127r–v	167r–174v	160v–164r	17r–24v

- p. Paris, Bibliothèque Nationale de France, Parisinus gr. 2428 (*Diktyon* 52060); composite; ca. 1542; two anonymous copyists, one responsible for ff. 73r–115v, the other for the rest<sup>48</sup>. The *Compendium* is on ff. 214v–225r. Other works in the manuscript: ff. 1r–51v Hero, *Pneumatica* I–II; 52r–71v Hero, *Automata*; 73r–115v Theon of Smyrna, *Expositio rerum mathematicarum ad legendum Platonem utilium*; 116r–178v Nemesius, *De natura hominis*; 181r–185v Manuel Moschopoulos, *De numeris quadratis*; 186r–193v Maximus Planudes, *Psephophoria secundum Indos – recensio* Rhabdas, des. 61, 8 εἴρηται; 194r–200v Nicholas Rhabdas, *Epistola ad Khatzykem*; 201r–202v arithmetical tables; 203r–v arithmetical problems C<sub>1</sub>, A<sub>1</sub>, A<sub>2</sub>; 203v–212v [Hero], *Geodaesia*; 212v two magic squares 6<sup>2</sup> and 10<sup>2</sup>; 213r–214v Isaak Argyros, *Epistola ad Kolybam*; 225r–245v Nicholas Rhabdas, *Epistola ad Tsavoukhem*; 246r–248r Isaak Argyros, On the first stereographic projection used in Ptolemy's *Geography*; 248v–249r [Michael Psellos], *De duodecim ventis*; 249r–v prose text on the winds; 249v–250r three short geographical texts.

If we exclude Theon of Smyrna (penned by a different hand) and Nemesius, Par. gr. 2428 is a partial yet systematic copy of Scorial. Φ.I.10 (D), and hence of Vat. gr. 1411 (V), as follows:

p	1r–71v	181r–245v	246r–250r
D	1r–70r	73r–124r	124v–127v
V	/	128r–162v	59r–60v

The two apographs were almost certainly produced in strict succession, within the atelier that in Venice worked for Diego Hurtado de Mendoza; see SMITH, *Some Manuscripts* 100: “In all probability, Par. 2428 was copied while Φ. I. 10 was still in the scriptorium”.

- R. Vatican City, Biblioteca Apostolica Vaticana, Rossianus 986 (Gollob 16; *olim* XI.136; *Diktyon* 66453); mm 230×150; 2<sup>nd</sup> quarter of the 15<sup>th</sup> century; some Demetrius (subscription on f. 389r) and two different hands (ff. 97v–104v and 150v–151v)<sup>49</sup>. The *Compendium* is on ff. 114v–122v. Other works in the manuscript: \*\*\*; f. 97v *nota astronomica*; 98r–104v Table of Noteworthy Cities, extracted from Ptolemy's *Geography*; 105r–112v [Hero], *Geodaesia*; 113r–114v Isaak Argyros, *Epistola ad Kolybam*; 123r–141v Nicholas Rhabdas, *Epistola ad Tsavoukhem*, des. 186, 18; 142r–146v Nicholas Rhabdas, *Epistola ad Khatzykem*; 147r–148v arithmetical tables; 148v *definitiones metricae*; 149r–150r extracts from Maximus Planudes, *Psephophoria secundum Indos*, and from Nicholas Rhabdas, *Epistola ad Khatzykem*<sup>50</sup>; 150v–151v Hermes Trismegistos, *De terrae motu*; 153r–210r Cleomedes, *Caelestia* I–II; 212r–230v Ioannes Pediasimos, *scholia in Cleomedis Caelestia*; 230v–231v Ioannes Pediasimos, two astronomical texts excerpted from Cassius Dio, with two diagrams;

<sup>48</sup> See BRAVO GARCÍA, *Dos copistas*; O. L. SMITH, *Some Manuscripts* 99–101; Ch. ASTRUC, *Sur quelques manuscrits de Némésius d'Émèse*. *Script* 19 (1965) 288–293, 290–291.

<sup>49</sup> A detailed description of the manuscript is in E. GOLLOB, *Die griechische Literatur in den Handschriften der Rossiana in Wien*. I. Teil. *Sitzungsberichte der Kaiserliche Akademie der Wissenschaften in Wien. Philosophisch-Historische Klasse* 164(3) (1910) 1–116, 43–65, and see also CCAG V.4 107–109. I list only the scientific items; the omitted works are marked by \*\*\*.

<sup>50</sup> The four extracts from Planudes' treatise are as in Planudes, *Psephophoria* 27, 1–29, 14; 31, 18–23; 33, 2–10; 35, 1–2; the one from Rhabdas' is edited in Rhabdas, *Epistola ad Khatzykem* 96, 13–98, 22 τὴν μονάδα.

232r–v criticism of the second diagram<sup>51</sup>; 233r description of the lunar phases; 233v–234r [Michael Psellos], *De duodecim ventis*; 234r–v prose text on the winds; 234v–235r three short geographical texts; 235v astronomical text; 236r–v arithmetical text allegedly excerpted from Diophantos' *Arithmetica*; 236v–237v arithmetical problems; 237v–238v arithmetical problems A<sub>3</sub>, C<sub>2</sub>, C<sub>1</sub>, A<sub>1</sub>, A<sub>2</sub>; 238v–239v *notae et diagrammata de harmonia*<sup>52</sup>; \*\*\*.

Vat. Ross. 986 is an early systematic copy of Vat. gr. 1411 (V), as follows:

R	97v–104v	105r–141v	142r–148v	148v–151v	153r–210r	212r–232v	233r
V	/	140r–[162]	137r–140r	/	1r–33v	328r–335v	33v–34r
R	233v–235r	235v	236r–v	236v–237v	237v–238v	238v–239v	
V	59v–60v	34v	/	[162]	124r, 140r	/	

The text on f. 236r–v of Vat. Ross. 986 was very likely contained in Vat. gr. 1411 too.

V. Vatican City, Biblioteca Apostolica Vaticana, Vat. gr. 1411 (*Diktyon* 68042); mm 220×144; unitary but incomplete and bound in disorder, the first four quires of the original codex are ff. 6–38 of Ambr. A 92 sup. (gr. 23; *Diktyon* 42205); end of the 14<sup>th</sup> century, watermark range 1370–1414; a single copyist, in a typical *Eugenikosschrift*<sup>53</sup>. The *Compendium* is on ff. 17v–23r. Other works in the manuscript: ff. 1r–9v Adamantios, *Physiognomonica* I–II, des. 2, 36, 1 τὰ πλείω; 10r–11v Nicholas Rhabdas, *Epistola ad Khatzykem*; 12r–13r arithmetical tables; 13r arithmetical problems C<sub>1</sub>, A<sub>1</sub>, A<sub>2</sub>; 13r–16v [Hero], *Geodaesia*; 16v two magic squares 6<sup>2</sup> and 10<sup>2</sup>; 17r–v Isaak Argyros, *Epistola ad Kolybam*; 23r–25v Nicholas Rhabdas, *Epistola ad Tsavoukhem*, des. mut. 132, 31 ἐστιν ὁ κε; 26r–32r Michael Psellos, in *Oracula Chaldaica expositio*; 32r–33r Michael Psellos, *Summaria Chaldaicorum dogmatum expositio*; 33r–34v [Michael Psellos], *Graecorum opiniones de daemonibus*; 34v–42v [Michael Psellos], *De operatione daemonum*; 43r–60r Ioannes Philoponos, in *Nicomachi Introductionem arithmetica I – recensio* II<sup>54</sup>; 61r–92v Nikomachos, *Introductio arithmetica* I–II; 93r–106r Ioannes Philoponos, in *Nicomachi Introductionem arithmetica II – recensio* II; 106r arithmetical problems A<sub>3</sub>, C<sub>2</sub>; 108r–115r Ioannes Pediasimos, *scholia in Cleomedis Caelestia*; 115r–v [Ioannes Pediasimos], two astronomical texts excerpted from Cassius Dio, with a diagram; 118r–121r Manuel Moschopoulos, *De numeris quadratis*; 122r–126v Maximus Planudes, *Psephophoria secundum Indos – recensio* Rhabdas, des. 61, 8 εἶρηται; 127r–v Maximus Planudes, *Verse for Ptolemy's Geography*; 128r–150v Ptolemy, *Geographia* I.1–II.1.11, VII.5.1–16, VIII.1.1–2.3; 151r–v Isaak Argyros, On the first stereographic projection used in Ptolemy's *Geography*; 151v–152r [Michael Psellos], *De duodecim ventis*; 152r–v prose text on the winds; 152v three short geographical texts; 153r–160r Isaak Argyros, *De constructione astrolabii*; 160v–163v Isaak Argyros, *Tabulae novae (Almagest)* plus a scholium; 164r incomplete table of the length of daylight, as a function of the solar longitude in degrees of each sign, for the latitude of 42°; 167r–174v Isaak Argyros, *De cyclis solis et lunae ad Andronicum*; 174v–179r *varia astronomica, geographica, astrologica*.

The original contents and ordering of Vat. gr. 1411 can be reconstructed from the very detailed index on ff. 1r–8v of Vat. lat. 3122, a manuscript that contains only translations of works contained in Vat. gr. 1411 (MER-

<sup>51</sup> This and the following item are edited in ACERBI, Problemi, Texts 2 and 3, and see also notes 16 and 17. For the former, see also Triklinios, *Excogitatum de lunae figuris* 167.

<sup>52</sup> See ACERBI, Problemi, 137–138 n. 21 and Texts 5 and 6.

<sup>53</sup> A detailed description and a reconstruction of the original codex—completed in the Appendix below—and a bibliography can be found in ACERBI, Problemi 138–143, which also see for the texts on ff. 174v–179r. For the copyist see C. GIACOMELLI – D. SPERANZI, *Dispersi e ritrovati. Gli oracoli caldaici, Marsilio Ficino e Gregorio (iero)monaco. Scripta* 12 (2019) 113–142, 130 n. 5.

<sup>54</sup> On the several recensions of Philoponos' commentary, see most recently F. ACERBI, *La tradition manuscrite de la « Recension IV » du commentaire à l'Introductio arithmetica de Nicomaque. Revue d'Histoire des Textes* 18 (2023) 35–96.



CATI, Codici, but see already H. Stevenson in R. FÖRSTER, *De Adamantii Physiognomonicis recensendis. Philologus* 46 (1888) 250–275, 259–262 and n. 12). This index allows us to identify ff. 6–38 of Ambr. A 92 sup. (gr. 23; *Diktyon* 42205) as the first four quires of the original Vat. gr. 1411. The reconstruction is confirmed by the fact that the manuscript Napoli, Biblioteca dei Girolamini, C.F. 2.11 (*olim* XXII.1; *Diktyon* 45944; see MARTINI, *Catalogo* 397–415) is a partial yet systematic copy of some of the lost quires of Vat. gr. 1411, as follows:

N	112r–185v	186r–193v
V	163r–179v, [180–267]	[305–322]

Vat. gr. 1411 belonged, still intact, to the library of Domenico Grimani (†1523): A. DILLER – H. D. SAFFREY – L. G. WESTERINK, *Bibliotheca Graeca Manuscripta Cardinalis Dominici Grimani* (1461-1523). Venezia 2003, 108–109. We find it already dismembered in the library of Fulvio Orsini (†1600) (inscription *ex libris Fulvij Orsini n. 134* on f. IIr, and see P. DE NOLHAC, *La bibliothèque de Fulvio Orsini*. Paris 1887, 184 n. 5 and 347–348: “Libro di varii autori d’Astronomia, di Geometria et di Mathematica con sue figure, scritto in papiro in-4°, coperto di carta pecora”). The same conclusion we reach from the inventory made by Domenico Ranaldi (†1606), where we find the codex “già mutilo e stravolto come ora” (MERCATI, *Codici* 271 n. 2). See the Appendix for the original contents and ordering of Vat. gr. 1411.

#### THE RELATIONSHIPS BETWEEN THE MANUSCRIPT WITNESSES OF THE *COMPENDIUM*

Before discussing the variant readings, I list the studies that confirm the stemma to be presented below. These studies are critical editions of works other than the *Compendium* contained in the manuscript witnesses described above. These works are Argyros, *Epistola ad Kolybam* (ACERBI, *Epistola*, stemma at the end of sect. 3.2); Adamantios (FÖRSTER, *Scriptores*, stemma at CXVIII); Argyros’ tract on the stereographic projection (TSIOTRAS, *Exēgētikē paradosē*, 194–214 and stemma at 215); Argyros, *Tabulae novae* (ACERBI, *Riscrivere*, stemma at 41); arithmetical problems marked “C” and “A” (ACERBI, *Problemi*, stemma at 163); Hero, *Pneumatica* (SMITH, *Some Manuscripts* 98–101); [Hero], *Geodaesia (Opera omnia V LXVI–XCVII)*, stemma at XCVII); Pediasimos, *Scholia* (CABALLERO SÁNCHEZ, *El Comentario*, stemma at 139); Planudes, *Psephophoria* (ALLARD, *Maxime Planude*, stemma at 14); Rhabdas, both letters (Heiberg replaced Tannery’s original readings of Par. gr. 2428 with those of Vat. gr. 1411!); Psellos (GAUTIER, *De daemonibus* and *Pseudo-Psellos*, stemmas at 124 and 91, respectively, O’ MEARA, *Philosophica Minora*, stemma at XXIV).

In Vat. gr. 1411, the *Compendium* begins, without a title, after Argyros’ *Epistola ad Kolybam* and three blank lines. Six blank lines also separate the exposition of stereometry from what precedes. Majuscule or minuscule rubricated letters are very frequent; they mark the beginning of every self-contained metrological unit. Marginal titles to the sections abound. Abbreviated words, standard compendia, and numerals are used throughout. A long scholium that refers to “the author” (*ho technikos*) of the text, two short marginal notes, five signs *nota bene*, a few outlines of geometric shapes given in the margins, and two interlinear glosses point to an annotated model. Twelve blank spaces of several letters are included in the text (lines 19, 39, 40, 42, 55, 104, 286, 291, 302, 303, 342), marking corruptions or damages. Most of these features will be retained in the apographs. The quirks listed above point to a damaged, at times corrupted (see lines 64–65, 298–299, 309–310, and compare the errors or omissions at lines 24, 111, 141, 152, 159, 164, 182, 202, 211, 216, 235, 314, 316, 327, 356), yet annotated model. A possibility is that an uncorrected draft had been slightly annotated and then used as the blueprint for the copy in Vat. gr. 1411.

The relations between the manuscript witnesses of the *Compendium* are discussed below. The variant readings of any witness are listed by taking the text of its model as a reference. The variant readings I call “characteristic” are the *Leitfehler* and are not shared by other manuscript witnesses.

The minor variant readings are listed in reduced font size; these variant readings are categorised by the kind of innovation. The following features are not recorded among the variant readings: presence or absence of elision and of movable *ny* and *sigma*; use of standard symbols; differences in accent and punctuation; writing numerals by means of numeral letters or by spelling them in full. All variant readings are identified by the number of the line(s) on which they occur.

The copy in Berol. Phillipps 1555 + Bodl. Auct. T.I.22 exhibits five characteristic omissions: 62 τούτου — ἀναμφιβόλως, 90–91 τετραπλασίασον — βάσιν, 236 ἀριθμοῦ, 325 τοῦ κόνου, and 336 ἐφ' ἑαυτό, — ὡσαύτως; and a mistake: 230–231 ἔτι γενομένου] ἐπιζητουμένου. Minor variant readings are as follows (I just give the list of the spelling mistakes that involve double consonants: σκόπελλον, τέσσαρσι, βούλλει, περισείας, τεσάρων, τέσσαρα, ἄλην, ἐλάσονος, ἄλως, μέλεις, τρισάκις, ἄλαι):

*Omissions.* 197 τὸ 343 αὐτὸ *Mistakes.* 65 ἰσόπλευρων 74–75 πολλαπλασίασον 88 καθέτου 119 πλευρῶν 130 ἐκατέραν 137 πολλὰς 179 τὸν 210 δὲ ἢ 230–231 ἐπιζητουμένου 233 ποίει 269 χωρία 271 ὅσα πλευροῖς 279 ἑδεκαγώνια 285 χωρὸς 331 Κολούρα 344 Κύβου 371 κώβου 374 εὐρὸν 378 ἐφ' ἄφ' *Inversions.* 104–105 τὸν ἀριθμὸν τῆς βάσεως *Variants.* 265 ἐφ' εἰς *Spellings.* 308 εικοσάκης

The copy in Scorial. Φ.I.10 exhibits the following characteristic innovations: the mistakes 2 ἦδη 261 τῶν πλευρῶν] πλευρὰν and the omissions 99 πλευρᾶς 162 τε. The copyist systematically misread the compendium for -εῖν as the one for -ὄν. The minor variant readings are as follows:

*Mistakes.* 19 ὁ] ἢ 57, 89 θέλεις 65 ἐγκαρδίως 73, 106, 202, 222, 232, 235, 241, 293, 296 345 (sed corr.) εὐρὸν 91 λήψει 106 καλειφθέντος 115 τριγώνου] δ<sup>ο</sup> 117 εὐρεθὴ 157 τοιού 208 τυγχάνει 210 οἶονδηποτοῦν 211 λαβῶν 252 γινόμενον 252 δωδεκαπλάσιον 269 πολλαπλασιασμὸν 275 κύκλον 353 τετραγωνικὴν 369 σύνθεσ sed καὶ συνθεῖς marg.

Par. gr. 2428 is a copy of Scorial. Φ.I.10, whose variant readings it adopts. Characteristic innovations are the omissions 129 δύο 187–188 ποίει οὕτως and the mistake 259, 292 ε<sup>ο</sup>. The minor variant readings are as follows:

*Restorations.* 157 τοιούτω *Omissions.* 38 τὸ 80 τὴν<sup>2</sup> 81 τὸ<sup>2</sup> 221 τῆ 255 τὸ<sup>1</sup> 352 κύβου *Mistakes.* 4 ἐμπηγνύμονται 43–44 ἀναπολειφθέντον 62 ἀναμφιβάλως 65 ἐγγάφης 69 μέσιν 71 βάσις 102 κατὰ] κα- 118 θέλεις 202 πρόσθεν 229 ἀπομένων 245 πάντος 268 εἰ 332 διαμέτρεις 338 πλευρῶν 355 σφινίσκου 369 σύνθεσ sed συνθεῖς marg. 370 ψήφησον *Additions.* 335 ἔχοντος τοῦ *Dittographies.* 39–40 τὴν κάθετον — λάβε τὸ  $\sphericalangle$  255–256 μίαν τῶν πλευρῶν

The very correct copy in Seragl. G.I.19 exhibits the characteristic omissions 247–249 τὰς δὲ τούτων — εἰρήκαμεν and 309–310 δίπλωσον — ἐπιφάνειαν. The minor variant readings are as follows:

*Restorations.* 210 εὐθυγράμμου *Omissions.* 6 καὶ<sup>2</sup> 8 καὶ 320 τὸ 325 ὁ *Mistakes.* 50 ζητῆς 55 ἔξει 66 διάξεις 72 μέθοδον 73 ὅποιαν ἐθέλεις] εἰ θέλεις 118 βούλη 146 γινομένου λάβε 166 γενομένου 235 θέλης 266 ἀριθμὸν *Additions.* 66 γοῦν καὶ *Inversions.* 299 τὴν ὄλην

The very incorrect copy in Monac. gr. 100 exhibits the following characteristic innovations: omissions 16–17 βῆμα — σχοῖνος 64 ἐπὶ γωνίαν 113–114 μέρισον — ἀριθμὸς, 146 τῆς περιμέτρου — λάμβανε τὸ  $\sphericalangle$ , 298 Ἔτι καὶ ἄλλως τὴν ἐπιφάνειαν 347–350 Ἔτι καὶ ἄλλως — τὸ στερεόν; mistakes 1 Ἐπεὶ δὴ γεωμετρία συνέστηκε ἐπίπεδος 65 ἐγκαρσίεως γράφης, 187–188 ἀπὸ μόνης τῆς βάσεως. ποίει οὕτως. πολλαπλασίασον] ποίησον, 256 τὸ γενομένου ἀρίθμησον.

The copyist also omitted all *marginalia*. The minor variant readings are as follows:

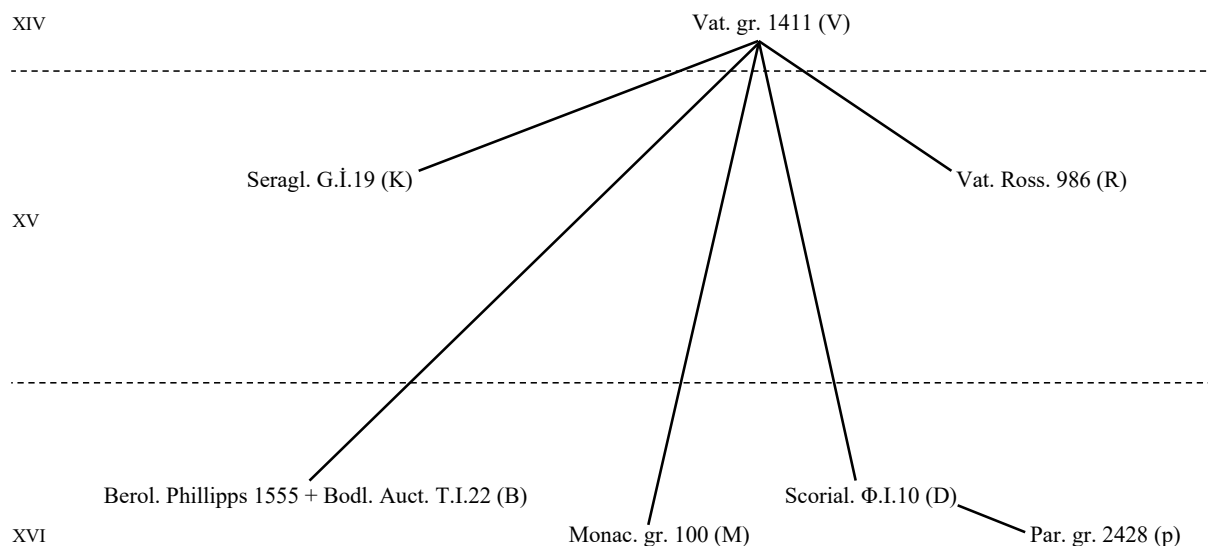
*Omissions.* 49 δὲ 74 τοῦ 117 ὁ 154 τὸ 162 τε 187 τὸ 191 δὲ 231 οἱ 255 εὐρήσεις τὸ τοῦ 267 τῶν 268 καὶ διαιρούμενα 325 δὲ<sup>1</sup> 327 βάσιν 362 εἶτα 366 καὶ τῆς ἐφένδρας *Mistakes.* 11 ἰσοπλευρές 19 ὁ] ἢ 24 ποιήσης 27 διαπλασίασον 45 βούλοι 51, 53 γενομένου 64 τεμῆς 76 πολλαπλασιασμὸν 77 τετραγωνὴ 80 γνώμην 81 τρίπλωσιν 89 θέλης]

ἀφέλης 94 τῆς<sup>1</sup>] τοῦ 94 ὑποθεινούσης 97 οὕτω] τούτω 102 τὸν<sup>1</sup>] τὸ 105 γενομένου 106 πλευρὰν 106 γενομένου 111 θελήση 117 μερίσοντα 117 εὐρέσθαι 118 θέλεις 133 τοιοῦτο 141 θέλεις 147 ἔσται 148–149 τρίπλωσιν 150 ἔτι] εἰ δὲ βούλει 153, 154, 156 κα<sup>ov</sup> 155 Ἔστι 157 τὴν] τὸν 160 πλευρὰν ἴδιον αὐτῆς κη<sup>ov</sup> 167 θέλεις 174 προσηθεῖσι καθολῶς 175 γενομένου 176 καθολῶς 177 διαπλασίασον 184 λέγεται] γίνεται 186 ἡμικυκλίου 190 δ<sup>ov</sup> 195 τῆς<sup>2</sup>] τοῦ 200 ἀπὸ] ἐπὶ 201 τὸ ἐμβαδὸν] τὴν τοῦ ἐμβαδοῦ 202 τρίπλωσιν 204 καθολῶς 205 φανερωῶς 207 ἔλατον 211 τεσσαρασκαιδεκάκις 215 ἡμικύκλον 217 ὁ ἀπὸ τοῦ 218 τούτου 218 θέλεις 221 τοῦ] αὐτοῦ 225 ταῦτα ἐφ’] ταῦθα 239 γνωρισμός 258 ἀριθμὸν 259 Ἐνεαγωνίου 264 Δωδεκαγωνίου 269 πολλαπλασιασμὸν 275 ιδ<sup>ov</sup>] ιδίων 281 μηχανηκώτατος 284 πειραμὶς 289 χωρήσομεν 294 τοῦ γενομένου ἀριθμοῦ 305 κβ<sup>ov</sup>] κμ<sup>ov</sup> 307 ὦν τὸ μβ<sup>ov</sup>] ὦ τὸ κμ<sup>ov</sup> 313 τὸν] τὸ 314, 317 κβ<sup>ov</sup>] κη<sup>ov</sup> 323 ἄρον] εὔρον 325 ἔδειξεν 327 ἴσα] ια 330 τοιοῦτου] τούτου 330 τοῦ κώνου τὸ] τὸ κώνου τούτου 331 ἀτέλους 334 τὸ<sup>2</sup>] τὸν 335 Ὀβελίσκος δέ. ὀβελίσκου 336 ἑαυτῷ 336–337 μίαν τῶν πλευρῶν τοῦ ὀβελίσκου ἐφ’ ἑαυτήν 350 Εἰς 351 ἑαυτήν 358 πολλαπλασιάζων *Additions*. 134 ρόμβου τὸ ἐμβαδὸν 137–138 τὴν δὴ διάμετρον εἰς ἑαυτήν ἐφ’ ὅλην 138 μέρος δύο 152 ἀριθμοῦ κα<sup>ov</sup> καὶ τοῦ καταλειφθέντος 154 Ἔτι δὲ 189 εἰς ἑαυτήν, τὸν γενόμενον ἀριθμὸν ποιήσον ἑνδεκάκις 200 τὸ δὲ δ<sup>ov</sup> 205 τοῦ κύκλου 241 Τραπεζίου δὲ 255 τὴν μίαν 281 ἐπὶ τὸν τρόπον καὶ λόγον 293 Ἄλλως δὲ 360 τίτ. Ἔτι καὶ ἄλλως, μέτρησον τὸ μείζον πλασιασμὸν πολλαπλασιάζων τὸ πάχος ἐπὶ τὸ πλάτος *Inversions*. 254 τῶν πλευρῶν μίαν 259 τὸ ἐμβαδὸν τοῦ [[δεκαγωνίου]] marg. ὀκταγωνίου 263 τῶν πλευρῶν μίαν *Dittographies*. 78 τὴν βᾶσιν καὶ τὴν κάθετον 224 αὐτοῦ 259 ἐμβαδὸν *Variants*. 244 οὕτως

The copy in Vat. Ross. 986 exhibits the following characteristic innovations: title Ἡρώως εἰσαγωγή τῶν γεωμετρούμενων χωρίων; omissions 30–32 ἐπὶ τὴν ἑτέραν — μίαν πλευρὰν, 115–118 Ἐὰν δὲ ἐκτός — ἔσται διάμετρος, 150–152 ἔτι καὶ ἄλλως — ἐμβαδὸν, 203–204 καὶ ἕξεις — τὸ ἴδιον κα<sup>ov</sup>, 250 τὸ ἐμβαδὸν εὐρεῖν. ποίει οὕτως, 256–257 ποίει οὕτως 273 εἰ δὲ ε, τὸ ε<sup>ov</sup>; mistake 162 ιδεῖν. The copyist also omitted most *marginalia*. The minor variant readings are as follows:

*Restorations*. 210 εὐθυγράμμου *Omissions*. 104, 243 δὲ 276 δὲ<sup>2</sup> *Mistakes*. 32 ἀμφοτέρων 57 θέλεις 80 πολλαπλασίασον 85 = μονάδα] ἡμίσει α 165 βούλει 264 Δωδεκαγωνίου *Inversions*. 268 ἀσφαλῶς καὶ

The above discussion allows to organise the following stemma of the textual tradition of the *Compendium*.



STRUCTURE AND STYLE OF THE *COMPENDIUM*

The *Compendium* adopts a mix of the procedural and of the algorithmic stylistic codes<sup>55</sup>, the former being clearly predominant in our text and the latter—bar one single exception—exhibiting no numerical examples. These two codes are stylistic resources that formulate chains of operations on mathematical objects *qua* numerically quantified, and such that the output of an operation is taken as the input of the operation next in order. In particular, the procedural code was used to express in words operational sequences that we would summarise in a “static” equality.

The procedural code formulates its prescriptions as a sequence of coordinated principal clauses with the verb in the imperative or in the first person plural, present or future; to each principal clause are subordinated one or more participial clauses coordinated with each other; the participle is a satellite and performs the function of modifier of the operating subject. There is an initialising clause, which feeds the initial input into the procedure, and an end clause, which identifies the result of the chain of operations as the quantity to be calculated. This quantity is usually declared in a clause that precedes the entire procedure. This code is used to formulate operatory prescriptions in the most general way (but it is also used to process numerical examples); the verb forms—either finite or participial forms—represent the operations; each verb form corresponds to one operation; the involved mathematical objects, the “operands”, are the complements of the verb forms and are designated by (sometimes long) definite descriptions. The operations may be unary or binary. In Byzantine sources, procedures are prominently featured in Easter Computi, in the 11<sup>th</sup>-century manual best witnessed in Par. gr. 2425 (*Diktyon* 52057)<sup>56</sup>, in Theodoros Metochites’ *Stoicheiōsis Astronomikē* (ca. 1316), in George Chrysokokkes’ *Syntaxis Persica* (1346), in Theodoros Meliteniotes’ *Tribiblos astronomikē* (1352), and in the anonymous *Paradosis in Tabulas Persicas* (1352).

The algorithmic code processes *only* paradigmatic examples featuring specific numerical values. After the initialising clause, the algorithms are expressed as a sequence of principal clauses coordinated by asyndeton; each clause formulates exactly one step of the algorithm and comprises a verb form in the imperative and a system of one or two objects<sup>57</sup>—a direct and an indirect object—in the form of demonstrative or (cor)relative pronouns or of numerals. The operation is often expressed by means of the preposition that introduces the indirect object, without any verb form: “these by 11” instead of “multiply these by 11”. The result of each operation is identified as such in a dedicated clause, with the verb in the present indicative (forms of *ginomai* “to yield”, “to result”, the latter for participial forms only), sometimes replaced by an adjective in predicative position (mainly *loipos* “as a remainder” after a subtraction); both syntactic structures are equivalent to our equality sign. An end clause identifies the result of the chain of operations as the quantity to be calculated. This quantity was usually declared in a clause that precedes the entire algorithm. The main feature of an algorithm is the systematic use of parataxis by asyndeton: no coordinants, (almost) no connectors, no subordination. The algorithmic flow is usually one-step: any step (1) accepts a number that is the output of the immediately preceding step as input and (2) inserts new data by means of the second operand. Operations in which neither operand is the output of the immediately preceding step are less frequent. Such operations induce a hiatus in the algorithmic flow; the hiatus is often syntactically marked by the presence of particles or of specific verb forms. The algorithmic code is used exclusively (with the exception of the *Compendium*) in the geometric metrological

<sup>55</sup> The stylistic codes of Greek mathematics are described in F. ACERBI, *The Logical Syntax of Greek Mathematics (Sources and Studies in the History of Mathematics and the Physical Sciences)*, Heidelberg – New York 2021, sect. 1, 1–3.

<sup>56</sup> See the edition in A. JONES, *An Eleventh-Century Manual Of Arabo-Byzantine Astronomy (Corpus des Astronomes Byzantins 3)*, Amsterdam 1987. A preliminary study is in O. NEUGEBAUER, *Commentary on the Astronomical Treatise Par. gr. 2425 (Académie royale de Belgique. Classe des Lettres et des Sciences morales et politiques. Mémoires 59.4)*, Bruxelles 1969.

<sup>57</sup> Accordingly, the operation is unary or binary, respectively.

*corpus*. In Meliteniotes' *Tribiblos astronomikē*, the prescriptions for using the tables are frequently carried out three times: by means of a general procedure (called *methodos*), by means of a procedure featuring actual numerical values (*hypodeigma* "example"), and finally by means of an algorithm in the sense just explained, often organised as a tabular set-up (*psēphophoria* "computation" or *ekthesis tōn arithmōn* "setting-out of the numbers").

The procedural and the algorithmic codes can be formalised by the kind of symbolic transcriptions I shall use in my paraphrases. These transcriptions are more faithful both to the syntactic structure and to the "mathematical content" of the original algorithms than the oft-used "static" formulas that summarise an entire algorithm in one single equality, thereby erasing the algorithm's operational structure. Let us read a procedure taken from section 17 of the *Compendium* as an example. The independent procedural steps are distributed in different rows.

Ἐὰν δὲ θέλῃς καὶ τὴν περίμετρον ἦτοι τὴν περιφέρειαν τοῦ τοιούτου τμήματος εὐρεῖν, ἀρίθμησον τὴν βᾶσιν εἰς ἑαυτήν.	Whenever you also wish to find the circumference, viz. the perimeter, of such a segment, count the base into itself.
ὁμοίως καὶ τὴν κάθετον·	Similarly the height too;
εἶτα τὸν ἀπὸ τοῦ πολλαπλασιασμοῦ τῆς καθέτου ἀριθμὸν τετραπλασίασον,	afterwards, quadruple the number from the multiplication of the height,
καὶ πρόσθεε τῷ τῆς βάσεως πολλαπλασιασμῷ,	and add <it> to the multiplication of the base,
καὶ τοῦ γενομένου λάβε πλευρὰν τετραγωνικὴν,	and take the square root of the result,
καὶ τῇ εὐρεθείσῃ πλευρᾷ τὸ τῆς καθέτου προσθεῖς δ'οῦν ἕξεις τὴν περιφέρειαν.	and adding $\frac{1}{4}$ of the height to the root that has been found you will get the circumference.

In the symbolic transcriptions, the assignments are enclosed in parentheses; self-contained procedural steps—namely, the ones singled out by commas or by upper points in the Greek text—are separated by an arrow  $\rightarrow$ ; procedural hiatuses are marked by a lower point; the final equality identifies the magnitude to be computed. Accordingly, the symbolic transcription of the procedure just read is as follows.

$$(b, h) \rightarrow b^2 \cdot h^2 \rightarrow 4h^2 \rightarrow 4h^2 + b^2 \rightarrow \sqrt{(4h^2 + b^2)} \rightarrow \sqrt{(4h^2 + b^2)} + \frac{1}{4}(h) = P_T.$$

Details about the lexical content of the *Compendium* are given in the next Section.

### THE LEXICAL CONTENT OF THE *COMPENDIUM*

With the help of Ramon Masià, I have studied the lexical content of the *Compendium*<sup>58</sup>. This approach is interesting because, with Ptolemy's *Psephophoria*, the *Compendium* is the longest self-contained text of Greek and Byzantine mathematics programmatically written in procedural language. Long treatises such as Ptolemy's *Almagest*<sup>59</sup>, Theon of Alexandria's primer on Ptolemy's *Handy Tables*, and Theodoros Meliteniotes' *Tribiblos astronomikē* do contain a greater overall amount of text written in procedural language, but this is only a part of the entire text. This part can be a large one: 83.6% of Book III of Meliteniotes' treatise (20210 words out of 24155) is written in procedural language; the widely-circulating anonymous abridgement of Book III called *Paradosis in Tabulas Persicas* radicalises this approach, for 96.3% of it (15219 words out of 15799) is written in procedural language. The importance and the diffusion of primers on the *Tabulae Persicae* and

<sup>58</sup> Compare the similar discussions in Hero, *Metrica* 59–74 and ACERBI, Logical Syntax 28–33.

<sup>59</sup> See *Alm.* II, 9; III, 8; III, 9; V, 9; V, 19; VI, 9–10; XI, 12; XIII, 6. In all these treatises, the procedures describe how to use the tables for computing specific quantities.

on the *Handy Tables* in the mid 14<sup>th</sup> century may have prompted authors to write texts that adopt the procedural code exclusively and do not work out numerical examples: the two cases I know of are the *Compendium* and the anonymous primer on the *Tabulae Persicae* witnessed in Vat. gr. 210 (1350–60; *Diktyon* 66841), ff. 40r–42v. This trend can be compared with the aforementioned (and very short: 531 words) geometric metrological compendium edited as *Geom.* 22, 3–24, a late antique or early Byzantine elaboration featuring only generic algorithms, and with the algorithmic code that predominates throughout the late-12<sup>th</sup> century computational primer Par. gr. 1670.

Before setting out the results of the lexical analysis, a couple of definitions are in order. Every independent sequence of signs in a text is an “occurrence”; the identical occurrences are a “form”; the set of forms referred to one and the same term is a “lemma”. The *Compendium* contains 5008 occurrences, which is a bit less than a quarter of the word content of Hero’s *Metrica*. As the *Compendium* merges the lexica of several disciplines, it is lexically rich: there are 444 lemmas, which is 8.87% of the number of occurrences, to be compared, for instance, with 8.80% of Ptolemy’s *Psephophoria* (445 lemmas and 5060 words), 0.32% of the *Elements* (524 and 163267), and 3.00% of the *Metrica* (612 and 20407). The bias of short writings towards raising the relative lexical content also plays a role here. The following table sets out the relative frequency of the main morpho-syntactic categories the lemmas belong to in the four works mentioned above.

cat.	<i>Compendium</i>		<i>Psephophoria</i>		<i>Metrica</i>		<i>Elements</i>	
	occ.	%	occ.	%	occ.	%	occ.	%
adjective	644	12.86	575	11.36	2106	10.32	17635	10.80
adverb	227	4.53	109	2.15	423	2.07	3560	2.18
article	1026	20.49	1267	25.04	4026	19.74	35740	21.89
denot. lett.	0	0.00	0	0.00	2257	11.06	30723	18.82
noun	979	19.55	1083	21.40	2285	11.20	15482	9.48
numeral	60	1.20	49	0.97	1227	6.02	0	0.00
particle	624	12.46	504	9.96	2887	14.15	23179	14.20
preposition	289	5.77	635	12.55	1395	6.84	13371	8.19
pronoun	237	4.73	195	3.85	879	4.31	4771	2.92
verb	922	18.41	643	12.71	2914	14.29	18772	11.50

Because of the large spectrum of mathematical objects to be measured and because of the presence of lists of notions and objects related to land-surveying, nouns and noun phrases are well represented in the *Compendium*. This, however, is common feature of procedural texts, as the data from the *Psephophoria* shows: in texts that adopt the demonstrative code, most of the nominal content is carried by the denotative letters (and, consequently, by the articles that make the letters a noun phrase), as the data from the *Elements* and, less markedly, from the *Metrica* confirm.

The number of verb lemmas that naturally feature in geometric metrological algorithms is increased in the *Compendium* by the several verbs that denote the geometric operations and the spatial relations that are necessary to describe some of the geometric configurations to be measured (29 lemmas), by the synonyms used to name the four arithmetical operations (4 for addition, 7 for subtraction plus 2 verbs that identify the result, 7 for multiplication), by the spectrum of different verbs that denote the taking of multiples (10 lemmas, among which 2 pairs of synonyms), and by several yet poorly occurring verbs among the metadiscursive items (12 lemmas). Likewise, there are 3 prepositions that denote multiplication. Conversely, there are almost no synonyms among the names of geometric objects and of their components. As the *Compendium* does not include deductions, the connectors that characterise Euclidean-style Greek mathematics are almost absent: 3 occurrences of *oun*, 2 of *dē*, 1 of *hōste*, none of the standard deductive connector *ara* “therefore”. Conversely, owing to the characterising feature of the procedural code, the conjunction *kai* is over-

represented: 7.07% of the overall occurrences against 4.11% of the *Elements* and 4.75% of the *Metrica*. Likewise, as squaring is formulated by multiplying an item by itself, the reflexive pronoun *heautou* (54 occurrences) is over-represented. Nearly the same number of occurrences (49, of which 47 are in the form of the standard sign) has the numeral adjective *hēmisis*; more generally, numeral adjectives (ordinals = unit fractions) and their signs are abundant. Adverbs also abound; their number is increased by the several multiplicative adverbs terminating in *-akis*. Generality is expressed by adverbs like *aei*, *diapantos*, and *katholikōs*, by adjectives like *katholikos* and *pas*, and by the standard participial modifier *tychōn*. The style is plain (only two verb forms in the optative, plus 1 occurrence of *mellō* + infinitive), the oscillation between indicative and subjunctive after *ean* is exactly what we should expect in a Byzantine text. The main areas of subordination are the participial clauses that characterise the procedural code and the fact that many problems are introduced by a conditional clause (the protasis is introduced more frequently by *ean* than by *ei*). The oscillation between present imperative and aorist imperative is exactly what we should expect; as is also to be expected, imperatives are over-abundant. The perfect tense is seldom used; it normally has a confective-stative value that is obviously related to expressing the state of affairs that such-and-such a numerical value for a quantity to be measured is the result of a sequence of operations (see, however, the standard exception of the perfect of *bainō*).

The following thematic word index includes my integrations (most of which are nearly certain) but not my corrections. The Greek lemmas are followed by their translation(s) and by the numbers of the sections of the *Compendium* in which they appear.

### *Metrological lexicon*

#### *Surveying*

ἀνατολή: rising (1); ἄρκτος: Ursa (1); γεωμετρία: geometry (1); δύσις: setting (1); ἐμπήγνυμι: to stick (1); ἐπίπεδος: plane (1); κλίμα: reference direction (1); μεσημβρία: midday (1); ὀξύς: pointed (1), εἰς ὀξύ: in a point (33); σημεῖον: point (1); σκόπελον: milestone (1).

#### *Measuring and units of measurement*

μέτρον: unit of measurement (2, 23).

μέτρησις: measurement (1, 23, 33).

καταμετρέω: to measure out (21); μετρέω: to measure (13, 16, 17, 28).

ἐμβαδομετρικός: planimetric (1, 23); εὐθυμετρικός: rectilinear (1); στερεομετρία: stereometry (24); στερεομετρικός: volumetric (1).

ἄκενα: akena (2, 3); βῆμα: step (2); δάκτυλος: digit (2); δίαυλος: double-stadium (2, 3); διχάς: forefinger span (2, 3); ιούγερον: jugerum (2, 3); κόνδυλος: knuckle (2); μίλιον: mile (2, 3); μόδιος: modius (17); οὐργυία: fathom (2); παλαιστή(ς): palm (2, 23); παρασάγγελος: parasang (2, 3); πήχυς: cubit (2); πλέθρον: plethron (2, 3); πούς: foot (2); σπιθαμή: span (2); στάδιον: stadium (2, 3); σχοίνιον: schoinion (17); σχοῖνος: schoinos (2, 3); σωκάριον: cable (2, 3).

ἑξάγων: hexagon (23); λίτρα: pound (23).

καθαρός σίτος: winnowed grain (23); κέγγρος: millet (23); κριθή: barley (23).

### *Geometric lexicon*

#### *Objects and their properties*

γένος: genus (1); εἶδος: species (1, 23); θεώρημα ultimate species (1, 4, 23); σχῆμα: figure (12, 18, 20, 28).

ἄκρον: extremity (1, 7, 12); κέντρον: centre (4, 6, 13, 15, 24); κορυφή: apex (25), vertex (1).

ἄξων: axis (24); ἀποτομή: projection (9); βάθος: depth (28); βάσις: base (1, 6–9, 14, 15, 17, 19, 24–26); γραμμή: line (1, 4); διαγώνιος: diagonal (1, 4, 5, 11, 12, 28); διάμετρος: diameter (1, 7, 10, 12–14, 16–18, 24, 25, 27, 28, 31); διάστασις: dimension (28); εὐθεῖα: straight line (1, 6, 7, 25);

κάθετος: height (1, 6–10, 14, 17, 19, 24–26, 33); κλίμα: oblique side, slope (25, 33); κορυφή: arch (15), upper side (19); μήκος: length (5, 12, 21, 25, 27–32); παράλληλος: parallel (1); πάχος: thickness (23, 29, 30); περίμετρος: circumference (1, 13, 14, 16, 17, 25); περιφέρεια: circumference (13, 14, 24, 25, 27, 31), perimeter (17); πλάτος: width (5, 12, 21, 28–30); πλευρά: side (4–7, 9–12, 18, 20, 25, 26, 28, 33); <ή> πρὸς ὀρθάς: perpendicular (1, 7, 15, 19); σκέλος: side (1); ὑποτείνουσα: hypotenuse (1, 7–10, 19); ὕψος: height (24, 25, 28, 29).

ἀμβλεῖα: obtuse (1); γωνία: angle (1, 6, 7); ὀξεῖα acute (1); ὀρθή: right (1).

ἔδρα: floor (31); ἐφέδρα: ceiling (31); ῥομβοειδής: rhomboid (1); ῥόμβος: rhombus (1, 6, 11, 12); τετράγωνον: quadrangle (1, 4–7, 12, 13, 18, 19, 22, 23), quadrangular (28); τραπέζιον: trapezium (1, 19); τρίγωνον: triangle (1, 6–8, 11, 12, 21, 22).

ἀψίς: arch (1); ἡμικύκλιον: semicircle (1, 14, 15, 17); κύκλος: circle (1, 4, 6, 7, 10, 12–18, 22, 24–27, 29, 31, 33); τμήμα: segment (1, 14, 15, 17, 24).

δεκάγωνον: decagon (20, 22); δωδεκάγωνον: dodecagon (20, 22); ἑνδεκαγώνιον: hendecagon (20, 22); ἑνεαγώνιον: enneagon (20, 22); ἑξάγωνον: hexagon (20, 22); ἑπτάγωνον: heptagon (20, 22); ὀκτάγωνον: octagon (20, 22); πεντάγωνον: pentagon (20, 22).

ἐμβαδόν: area (4–7, 9–14, 16–25, 27, 31), content (24, 25); ἐπιφάνεια: surface (24, 27, 31); χωρησις: span (4, 6, 24); χωρίον: region (13, 16, 21).

ἡμισφαίριον: hemisphere (24); κίων: column (23, 31); κύβος: cube (23, 28, 32); κύλινδρος: cylinder (23, 25, 27); κώνος: cone (25, 26); μείουρος: meiouros (23, 30); ὀβελίσκος: obelisk (23, 26, 33); πλινθίς: brick (23, 32); πυραμῖς: pyramid (23, 33); σφαῖρα: sphere (23, 24, 25, 28); σφηνίσκος: wedge (23, 29); τεταρτημόριον: quadrant (24).

στερεόν: solid (23, 24, 28), volume (24–33).

ἀτέλεστος: without the end (25); ἀμβλογώνιος: obtuse-angled (1, 7–9); ἑτερομήκης: oblong (5, 12, 32); εὐθύγραμμος: rectilinear (16); ἰσογώνιος: equiangular (21); ἰσόπλευρος: equilateral (1, 4, 6, 7, 11, 18, 21, 22); ἰσοσκελής: isosceles (1, 6, 7); κόλουργος: truncated (25, 33); κωνοειδής: cone-like (33); ὀξυγώνιος: acute-angled (1, 7–9); ὀρθογώνιος: right-angled (1, 4, 7, 8, 12, 19); παραλληλόγραμμος: parallelogrammic (32); πολύπλευρος: polylateral (1, 20, 21, 33); πολυγώνιος: polygon (1, 20, 21, 33); σκαληνός: scalene (1, 6, 7, 10, 12); τραπεζοειδής: trapezium-like (33).

### *Geometric operations and spatial relations*

ἄγω: to draw (4, 15); ἀποτελέω: to complete (7); ἄπτομαι: to touch (4, 18); βαίνω: to stand (3); διαγράφω: to trace out (4, 6, 10, 12); διάγω: to draw through (7); διαιρέω: to partition (12, 17, 21, 24); δίδωμι: to give (8, 13, 16, 18, 21); διήκω: to reach (25); ἐγγράφω: to inscribe (7); ἐκτείνω: to extend (21); ἐμβάλλω: to insert (28); ἐπισυνάπτω: to join up (12); ἐφάπτομαι: to be tangent (6, 18); κατασκευάζω: to construct (4); κατατέμνω: to cut out (21); λήγω: to end (33); περιγράφω: to circumscribe (10, 18); περιλαμβάνω: to comprehend (25); ποιέω: to make (4–8, 16, 22, 31); προσαναπληρώω: to complete out (17); στηρίζω: to set firmly (15); συμπίπτω: to meet (17); συνάπτω: to join (11); συνίστημι: to construct (6–8); τέμνω: to cut (5–7, 19, 30, 31); τίθημι: to set (7); ὑπόκειμαι: to be supposed (15); ὑποστόρνυμι: to spread out (15).

γένεσις: generation (8).

### *Relations*

ἀναλογέω: to be associated (31); ἄνισος: unequal (1, 21); δίχα: in half (7, 24); ἑκκεντρος: eccentric (13); ἐκτός: outside (10, 18); ἐλάττων / -σσων: less(er) (7, 8, 13, 15, 17, 25, 30); ἐντός: inside (6, 7, 10, 12, 18); ἥττων: smaller (1, 9, 10, 12, 15, 24, 31); ἴσος: equal (7, 8, 15, 16, 18, 25); μέγιστος: great (17, 24); μείζων: greater (1, 7–10, 12, 13, 15, 17, 25, 30, 31); μέσον: in the middle (5, 6, 17, 19, 30, 31); μέσος: middle (7, 18); μικρός: small (12); πλείων: more (15, 21); πλήρης: full (15).



### Operational lexicon

#### Investigation

βούλωμαι: to want (4–8, 10, 13, 18, 28); γινώσκω: to know (7, 10, 18, 28); διαστέλλω: to separate (13); ἐθέλω: to wish (7); εὔρεσις: finding (6); εὐρίσκω: to find (4–15, 17–21, 24–33); ζητέω: to seek (5–7); θέλω: to wish (5–10, 12–14, 17, 18, 25, 28); μανθάνω: to learn (6).

#### Initialising a procedure

εὐρήσεις / εὔρεῖν / εὔρεθήσεται οὕτως: you will find / will be found as follows (6, 7, 15, 24, 31, 32); κρατέω: to keep (9); λαμβάνω: to take care (1, 8, 9, 19, 25, 27, 31); ποίει / ποιήσον / ποιῶ οὕτως: do / I do as follows (10, 11, 13, 13, 14, 17–20, 18, 24–26, 24, 28, 28–31, 33).

#### Counting and reckoning

ἰδίᾳ: separately (13).

#### Identification of the output

ἀποφαίνομαι: to declare (16); διαγινώσκω: to determine (11, 21); εἰμι: to be (5–14, 20, 21, 24, 25, 28, 29); ἔχω: to get (4–9, 13, 14, 17–20, 24–31, 33); νοέω: to realise (13); οἶδα: to know (13); περίστημι: to turn out as (19); ὑπάρχω: to happen to be (8, 13).

### Numerical objects

#### Unknown quantities

ἀριθμός: number (4–11, 13, 14, 17–20, 24–26, 30); ὄρος: term (8); πλῆθος: multiplicity (8); ποσόν: quantity (4, 6); ποσότης: quantity (13).

#### Numerals

δέκα: ten (23); δύο: two (5, 7, 9, 11, 12, 13, 22, 25); εἷς: one (4–13, 18–20, 24–26, 28, 33); ἕξ: six (1, 22); τέσσαρες: four (4, 7, 16–18); τρεῖς: three (1, 6, 10, 12, 18, 21, 28).

ωμα: 841 (13); ρηδ: 154 (13); ξς: 66 (20, 22); ξδ: 64 (3); να: 51 (20, 22); με: 45 (20, 22); μδ: 44 (24); μγ: 43 (20, 22); λη: 38 (22, 23); λς: 36 (17); λ: 30 (22); κθ: 29 (13, 20, 22); η: 18 (1); ις: 16 (3); ιε: 15 (20, 22); ιδ: 14 (22); ιγ: 13 (17, 22); ιβ: 12 (17, 22); ια: 11 (22, 24); ι: 10 (1, 22); θ: 9 (17); η: 8 (3); ζ: 7 (22); ς: 6 (17); ε: 5 (1, 22); δ: 4 (1, 3, 17, 21, 22); γ: 3 (3, 22); β: 2 (1, 3, 31) α: 1 (3, 23).

ἄρτιος: even (8); περιττός: odd (8).

#### Numerical sets

μονάς: unit (8); δυάς: dyad (8).

#### Parts

δέκατον: a tenth (13); ἑβδομον: a seventh (13, 14); εἰκοστέμπετον: a twenty-fifth (11); ἥμισος: half (6, 9); μέρος: part (6, 12–16, 20, 21, 24, 25, 29); τεσσαρακοστοτέταρτον: a forty-fourth (14); τέταρτον: a fourth, a quarter (6, 29); τρισκαιδέκατον: a thirteenth (6); τρίτον: a third (6, 25).

ω:  $\frac{2}{3}$  (3, 23, 25); υ:  $\frac{1}{2}$  (3, 6–9, 11, 13–15, 17–20, 24–26, 28–31); γ<sup>ov</sup>:  $\frac{1}{3}$  (3, 13, 20, 21, 24, 25, 33); δ<sup>ov</sup>:  $\frac{1}{4}$  (13, 14, 17, 20, 21, 33); ε<sup>ov</sup>:  $\frac{1}{5}$  (3, 7, 21); ς<sup>ov</sup>:  $\frac{1}{6}$  (20, 24); ζ<sup>ov</sup>:  $\frac{1}{7}$  (13, 14, 20, 27); η<sup>ov</sup>:  $\frac{1}{8}$  (20); ι<sup>ov</sup>:  $\frac{1}{10}$  (6, 13, 20); ια<sup>ov</sup>:  $\frac{1}{11}$  (13, 16); ιβ<sup>ov</sup>:  $\frac{1}{12}$  (20); ιδ<sup>ov</sup>:  $\frac{1}{14}$  (13, 14, 17, 18, 24); ιε<sup>ov</sup>:  $\frac{1}{15}$  (3); ια<sup>ov</sup>:  $\frac{1}{21}$  (13, 14, 24); κβ<sup>ov</sup>:  $\frac{1}{22}$  (13, 24); κη<sup>ov</sup>:  $\frac{1}{28}$  (13, 14); κθ<sup>ov</sup>:  $\frac{1}{29}$  (13); λ<sup>ov</sup>:  $\frac{1}{30}$  (6); μβ<sup>ov</sup>:  $\frac{1}{42}$  (24); ξ<sup>ov</sup>:  $\frac{1}{60}$  (28).

### Operations

#### Addition

ένωω: to unite (7); προστίθημι: to add (8, 13, 14, 17, 23); συνάγω: to gather (6, 10, 13, 14, 19, 26); συντίθημι: to compose (5, 6, 9, 10, 14, 17, 19, 21, 24, 25, 29–31).

σύνθεσις: composition (6, 10, 21).

ἀμφοτέρω: both (5, 7, 19); συναμφοτέροι: both of them together (6, 13).

ὁμοῦ: together (6, 30, 31).

#### *Subtraction*

αἶρω: to take up (6, 9, 25); ἀφαιρέω: to remove (6–9, 12–14, 17, 24, 26, 33); ἐάω: to leave alone (13); ἐκβάλλω: to throw away (13); ἐξαιρέω: to remove out (6, 19); κουφίζω: to raise (6, 9, 11, 13, 14); ὑφαιρέω: to remove (6).

#### *Multiplication*

ἀριθμέω: to count (14, 17, 18, 20, 21, 24, 30, 33); ἐρωτάω: to ask (14); λαμβάνω: to take (9, 10, 13, 18, 20, 24, 25, 28); μετρέω: to measure (14, 20, 24, 30); ποιέω: to do (6, 11, 13, 14, 16–18, 20, 24, 25, 28, 30, 33); πολλαπλασιάζω: to multiply (4–14, 19, 20, 24, 27–31, 33); ψηφίζω: to compute (31).

εἰς: into (5, 8, 9, 11, 13, 14, 17, 18, 20, 24, 30); ἐπί: by (4–15, 17, 19–21, 24–31, 33); μετά: with (6).

πολλαπλασιασμός: multiplication (5, 7–9, 17).

κυβίζω: cube (24); λαμβάνω κυβικῶς: to take cubically (24).

#### *Taking multiples*

διπλασιάζω: to double (13, 18, 24, 28); διπλόω: to double (24, 33); δωδεκαπλασιάζω: to duodecuple (20); ἑνδεκαπλασιάζω: to undecuple (13, 24); ἑξαπλασιάζω: to sextuple (20); ἑπταπλασιάζω: to septuple (13); πενταπλασιάζω: to quintuple (8); τετραπλασιάζω: to quadruple (7, 8, 10, 17); τριπλασιάζω: to triple (8, 18); τριπλόω: to triple (7, 13, 14).

διπλασιασμός: doubling (24); τετραπλασιασμός: quadruplication (10).

τετραπλασίων: four times (24); τριπλασίων: three times (27).

ἅπαξ: once (6); δεκάκις καὶ ἅπαξ: ten times and once (24); δίς: twice (24, 28); εἰκοσάκις: twenty times (13); εἰκοσάκις καὶ δίς: twenty times and twice (13, 24); ἑνδεκάκις: eleven times (14, 24); ἑξηκοντάκις: sixty times (28); ἑπτάκις: seven times (13, 14); ὀγδοηκοντάκις καὶ ὀκτάκις: eighty-eight times (13); τεσσαρεσκαίδεκάκις: fourteen times (13, 16); τετράκις: four times (24); τριακοντάκις: thirty times (6); τρισσάκις: three times (24).

ια<sup>15</sup>: 11 times (14, 24).

#### *Division*

μερίζω: to divide (6, 9, 10, 17, 29).

παρά: by (6, 9, 10, 17).

μερισμός: division (9, 10).

#### *Taking parts*

λαμβάνω: to take (6–8, 11, 13, 14, 16, 18, 20, 21, 24–31, 33).

#### *Not a part but as a whole*

ὅλος: whole (6, 7, 11, 13, 14, 17, 24, 33).

#### *Taking a square root*

λαμβάνω πλευρὰν τετραγωνικήν: to take the square root (4–7, 9, 13, 16–18, 25, 26, 28, 33).

#### *Identification of the result of an operation*

ἀποβαίνω: to step off (6, 9, 13); γίνομαι: to result (participle only, 6–11, 13, 14, 16–20, 24, 25, 28–30, 33), to yield (17), to come about (6, 8, 19), to come to be (4, 12, 23); ἐναπολείπομαι: to be

left out (6, 13, 25); καταλείπομαι: to be left over (8, 9, 11, 13, 14, 19, 26); λοιπός: remainder, re-maining (6–8, 14, 24, 33, 7, 17); περισσεία: excess (6).

#### *Connectors, adverbs, prepositions, and particles*

ἀλλά: but (7, 21, 24); ἅμα: together (13); ἀνάπαλιν: inverse (7); ἀπό: from (4–17, 19, 21, 24–27, 31); ἀῤῥθις: anew (6, 7, 17, 19, 24, 28, 29, 33); γάρ: for, in fact (6, 8, 9, 17–19, 19, 24, 32, 33); γοῦν: then ... really (7, 9, 23); δέ (1, 2, 4–29, 31, 33); δῆ: indeed (13, 27); ἐάν: whenever (4, 6, 7, 8, 11–13, 15–19); εἰ: if (4, 5, 7–9, 12–14, 19, 21, 25, 28); εἶτα: afterwards (5–7, 9, 10, 13, 14, 17, 24, 25, 30, 33); εἴτε ... εἴτε: whether ... or (7, 21); εἴτουν: or simply (1); ἐκ, ἐξ: from (5, 7, 9, 9, 10, 11, 19, 25, 26, 33); ἔκτοτε: thence (6); ἐν: in, inside, for, (4, 6, 6, 7, 13, 19, 19, 23, 23, 24, 24, 26); ἐξῆς: in succession, next (4, 21, 25); ἔτι: further (6, 13, 14, 18, 20, 24, 25, 28–30); ἕως οὗ: until (17); ἢ: or (1, 6, 7, 12, 13, 15, 24, 28); ἥγουν: namely (7, 9, 12, 13, 19, 21, 25, 28); ἥτοι: or, viz. (1, 17, 24, 31); καί: and, also (1–33); κατά: according to, in, (8, 9, 9, 11, 13, 19, 21, 24, 26); μέν (1, 6, 7, 9, 17, 19, 21, 23, 33); μεταξύ: between (13); μέχρι: as far as (25); μήτε ... μήτε: neither ... nor (8); ὅπως: how (4); ὅσον ἐπί: as for (23); ὅτι: that (7, 16, 22, 24, 25); οὐκ, οὐχ: not (7, 21); οὖν: then (1, 23, 25); οὕτω: thus, in such a way (4, 32); πάλιν: again (7, 8, 22); παρά: by, according to, other than (1, 6, 9, 10, 13, 17); -περ: exactly (20, 21, 27, 32); πότερον ... ἢ: whether ... or (15); τε: and (1, 7, 9, 13, 33); ὡς: as (25); ὥστε: so that (17).

#### *Generality*

ἀεὶ: always (13, 20); ἅπας: all and every (6), any (1); διαπαντός: altogether (8, 17); καθολικός: generic, general (6, 6); καθολικῶς: generally (6, 13, 14); καθόλου: in general (8, 19); πάντως: in every instance (19); πᾶς: all, every (19, 25), everyone (12, 19); τυχόν: at random (8, 19).

#### *Metadiscourse*

αἰτία: reason (25); ἀκριβέστατα: most exactly (7); ἀκριβέστερον: more exactly (22); ἀκριβής: exact (11); ἄλλως: otherwise (6, 11–14, 22, 24, 28–30); ἀναμφιβόλως: unambiguously (6); ἄνωθεν: above (6, 8); ἀνωτέρω: above (25); ἀποδείκνυμι: to prove (22, 24, 25); ἀρχή: beginning (23); ἀσφαλῶς: faultlessly (21); αὐτόθεν: immediately (18); γεωμετρικός: geometric (8); γνώριμος: recognised (18); γνώρισμα σαφές: feature that makes \*\* plainly recognised (8); δηλαδή: clearly (1, 7); δῆλος: clear (7, 12, 19); δηλώω: to highlight (13, 23); διάφοροι: manifold (33); δίκη: like (33); ἐγκαρσίως: athwart (7); εἶσω: inside (8); ἐκτίθημι: to set out (7, 33); ἐκτός: outside (8); ἐν μέρει ... ἐν μέρει: respectively (19); ἐξαίρετος: specific (23); ἐπιστημονικώτερος: more scientific (19); ἐτέρως: in another way (11); ἔφοδος: algorithm (24, 26); ἰστέον: one must know (7, 16, 22); καλέω: to call (1, 15); κανών: rule (6, 8); κάτωθεν: below (8); λέγω: to call, to mean, to say (1, 4, 8, 14, 25, 32, 33); λόγος: rule, account (21, 33); μέθοδος: procedure (6, 7, 9, 11, 19); μηχανικώτατος: the most-clever (22); μόνος: only (6, 7, 13, 14); οἶον τί: what do [I mean] by this (8); ὅμοιος: similar (9); ὁμοίως: similarly (6, 17, 18, 21, 25, 30); ὀνομάζω: to name (1); ὄρος: term (1); ποιέω: to do (19); πολλοί: many (13); προγράφω: to write above (11, 24); προλαμβάνω: to be taken above (9, 19); προρρηθείς: aforementioned (17); προσηκόντως: accurately (33); πρότερον: beforehand (13); πρότερος: previous (24); συνεστηκέναι: to be composed of (1); τεχνικός: author (7); τουτέστι: that is (4, 6, 17, 24, 25, 28, 29, 31); τρόπος: way (9, 13, 19); ὕστερον: finally (6, 28); φανερός: evident (15); χράομαι: to use (23); χωρέω: to proceed (4, 24); ὡσαύτως: likewise (24, 26).

#### *Modalities and imperatives*

relative pronoun + ἄν + optative: whichever (6, 14); δεῖ, δέη: one must (9, 16); μέλλω + infinitive: should (23).

### Pronouns

ἄλλος: other (7, 11, 33); ἀλλήλων: each other (4, 6, 18); ἑαυτοῦ: itself (4–11, 13, 14, 17, 18, 20, 24–26, 28, 30, 33); ἕκαστος: each (4, 6, 9, 13, 18, 23, 28, 33); ἕτερος: another (5, 11, 19, 33); ὅδε: this here (18, 24); οἰοσθηποτοῦν: whatsoever (10, 16); ὁποῖος: whichever (7, 13); ὅς: this, which, whose (6, 13, 13–15, 17, 19, 20, 24, 24, 26, 28, 33, 33); ὅσος: as much, as many, *all those* (12, 18, 21, 21); ὅστις: this very, which, whose (13, 14, 29); οὗτος: this (1, 2, 4–14, 16–25, 27, 28, 31–33); πόσος: how much (6, 10, 18, 28); τις: any, someone (16, 19); τοιοῦτος: such (4, 6, 8, 12, 13, 17, 25, 31); τοσοῦτος: so much (12, 16, 18, 21, 28).

### Particulars

ἔλκω: to drag (23); ἡμέτερος: our (23); ἴδιος: its own (6, 11, 13, 14); τυγχάνω: to happen to be (15); φύω + infinitive: naturally (6); χεῖρ: hand (23).

### Authors

Ἀρχιμήδης: Archimedes (22, 24, 25); Εὐκλείδης: Euclid (13); Πατρίκιος: Patrikios (21); Πλάτων: Plato (8); Πυθαγόρας: Pythagoras (8).

## PRELIMINARIES TO THE EDITION

*Edition.* I have corrected the text of Vat. gr. 1411 whenever mathematical or syntactical consistency required to do it; some lacunas marked in Vat. gr. 1411 could not be integrated with a reasonable degree of certainty and I refrained from doing it. There is no critical apparatus; the proposed corrections are inserted in the text, which includes within braces all marginal and interlinear notes of the manuscript. Some features of the layout of Vat. gr. 1411 are also marked. I have retained the original accents of proclitics and enclitics; otherwise, the accents are normalised to the conventions presently in use. All letters in majuscule are in majuscule in the text too; several other rubricated initial letters are in minuscule. I have introduced section numbers and paragraphs. I have punctuated the text anew, following the rules I use in editing Greek and Byzantine procedural texts<sup>60</sup>. In particular, these rules for the procedural code prescribe that a comma separates principal clauses from one another; no comma separates the conjoined participial clauses subordinated to each principal clause. As for the algorithmic code, an upper point separates both the consecutive steps of a continuous algorithmic flow and the operations from their result. A full stop is inserted when a hiatus occurs in the algorithm, and before the identification of the final result. I have added parentheses and N-dashes to single out incidental clauses and identifications of objects (namely, clauses introduced by *toutesti* “that is”).

*Translation.* The translated text includes my corrections. The paragraph structure of the translation is the same as the one of the Greek text. With a few exceptions, different Greek terms are translated with different English terms; the converse is less frequently the case. The translation is literal and sometimes obviously non-idiomatic; the lexical choices conform to the thematic word index set out above. I have tried to render the several nuances of the Greek perfect tense required by the context. Sequences that I have integrated in the Greek text whenever Vat. gr. 1411 marked a lacuna are put between square brackets in the translation too. Integrations not marked by a lacuna but required either because the Greek text is corrupt or for a better understanding of the English translation are put between angular brackets.

<sup>60</sup> These rules are expounded in ACERBI, Logical Syntax, sect. 1, 4.

EDITION OF THE *COMPENDIUM*

|<sub>17v13</sub> <1> {*marg.* ὄροι} Ἐπίπεδος γεωμετρία συνέστηκεν ἕκ τε κλιμάτων καὶ σκοπέλων καὶ γραμμῶν καὶ γωνιῶν, ἐπιδέχεται δὲ γένη, εἶδη καὶ θεωρήματα. κλίματα μὲν οὖν εἰσὶ δ' ἀνατολή, δύσις, ἄρκτος καὶ μεσημβρία. σκόπελον δὲ ἐστὶ τὸ λαμβανόμενον ἅπαν σημεῖον ἢ λίθος τις – ὄξυς δηλαδὴ – ἢ ξύλον τῷ ἄκρῳ τοῦ κλίματος ἐμπηγνύμενον. γραμμαὶ δὲ εἰσὶ ι' εὐθεῖα, παράλληλος, βάσις, κορυφή, σκέλη, διαγώνιος, κάθετος ἢ καὶ πρὸς ὀρθὰς καλουμένη, ὑποτείνουσα, περίμετρος καὶ διάμετρος. Γωνία δὲ εἰσὶ τρεῖς· ὀρθή, ἀμβλεῖα καὶ ὀξεῖα. γένη τῆς μετρήσεως εἰσὶ τρία· εὐθυμετρικόν, ἐμβαδομετρικόν καὶ στερεομετρικόν. εἶδη δὲ τῆς μετρήσεως εἰσὶ ε' τετράγωνα, τρίγωνα, ρόμβοι, τραπέζια καὶ κύκλοι. ἔχουσι δὲ θεωρήματα ιη, ταῦτα· τετραγώνων θεωρήματα β, τετράγωνον ἰσόπλευρον καὶ ὀρθογώνιον καὶ τετράγωνον ἑτερομήκες· τριγώνων θεωρήματα ἕξ, τρίγωνον ἰσόπλευρον, τρίγωνον ἰσοσκελές, τρίγωνον ὀρθογώνιον, τρίγωνον ἀμβλυγώνιον, τρίγωνον ὀξυγώνιον καὶ τρίγωνον σκαληνόν· τραπέζιων θεωρήματα δ, τραπέζιον ἰσοσκελές, τραπέζιον ὀρθογώνιον, ἀμβλυγώνιον καὶ ὀξυγώνιον, ὃ καὶ σκαληνὸν λέγεται εἴτουν ἄνισον· ρόμβων θεωρήματα β, ρόμβος καὶ ῥομβοειδές. Τὰ δὲ παρὰ ταῦτα πολυπλευρα καὶ πολυγώνια καὶ λέγονται καὶ ὀνομάζονται. Κύκλων θεωρήματα δ, κύκλος, ἡμικύκλιον ἢτοι ἀψίς, τμήμα μείζον ἡμικυκλίου καὶ τμήμα ἥττον ἡμικυκλίου.

<2> Μέτρα δὲ ἐστὶ ταῦτα· δάκτυλος, κόνδυλος, παλαιστή, διχάς, σπιθαμή, πούς, πήχυς, βῆμα, οὐργυρία, σωκάριον, πλέθρον, ἰούγερον, δίαυλος, στάδιον, ἄκενα, μίλιον, σχοῖνος καὶ παρασάγγης.

<3> Τὸ πλέθρον ἔχει σχοινία σωκάρια α ω ιε<sup>ov</sup>, τὸ ἰούγερον ἔχει σωκάρια γ γ<sup>ov</sup>, ὁ δίαυλος στάδια β, τὸ στάδιον {*sp.* 5 *litt.*} η μ, ἢ ἄκενα σπιθαμὰς ις, τὸ μίλιον {*sp.* 3 *litt.*} ξδ ω ε<sup>ov</sup>, ὁ σχοῖνος μίλια δ καὶ ὁ παρασάγγης δ.

<4> Τούτων οὕτω λεχθέντων |<sub>18r</sub> ἐξῆς ἐπὶ τὸ ἐμβαδὸν τῶν θεωρημάτων χωρήσωμεν καὶ ὅπως τούτων ἕκαστον κατασκευάζεται. {*marg.* sign of a square} Τὸ ἰσόπλευρον τετράγωνον οὕτω γίνεται. ἐὰν δ<sup>as</sup> κύκλους διαγράψῃς ἀπτομένους ἀλλήλων καὶ ἀπὸ κέντρου πρὸς κέντρον ἄξις γραμμὴν {*fort. lege* γραμμὰς} ἐν τοῖς τέσσαρσι, ποιήσεις τετράγωνον ἰσόπλευρον καὶ ὀρθογώνιον. τούτου τὸ ἐμβαδόν – τουτέστι τὸ ποσὸν τῆς χωρήσεως – εἰ βούλει εὐρεῖν, πολλαπλασίασον μίαν τῶν πλευρῶν ἐφ' ἑαυτήν, καὶ διὰ τοῦ γινομένου ἀριθμοῦ ἕξεις τὸ ἐμβαδόν. τὴν δὲ διαγώνιον τούτου εἰ βούλει εὐρεῖν, διπλασίασον τὸ ἐμβαδόν, καὶ τοῦ τοιοῦτου ἀριθμοῦ λαβὼν πλευρὰν τετραγωνικὴν ἕξεις τὴν διαγώνιον.

<5> Τμηθέντος δὲ μέσον τοῦ ἰσοπλεύρου δύο ποιήσεις ἑτερομήκη τετράγωνα, οὗ τὸ ἐμβαδὸν εἰ θέλεις εὐρεῖν, πολλαπλασίασον τὴν τοῦ μήκους μίαν πλευρὰν ἐπὶ τὴν ἑτέραν τοῦ πλάτους, καὶ ἕξεις ἀπὸ τούτων τὸ ἐμβαδόν. τὴν δὲ διαγώνιον τούτου εἰ βούλει εὐρεῖν, πολλαπλασίασον τὴν τοῦ μήκους μίαν πλευρὰν εἰς ἑαυτήν, εἶτα καὶ τὴν τοῦ πλάτους, καὶ συνθεῖς ἀμφοτέρους τοὺς ἐκ τοῦ πολλαπλασιασμοῦ ἀριθμοὺς λάβε τούτων πλευρὰν τετραγωνικὴν, καὶ αὕτη ἔσται ἡ ζητούμενη τοῦ ἑτερομήκους τετραγώνου διαγώνιος.

<6> {*marg.* sign of a triangle} Τὸ δὲ ἰσόπλευρον τρίγωνον οὕτω συνίστασθαι πέφυκε. τριῶν κύκλων ἀλλήλοις ἐφαπτομένων ἐὰν ἀπὸ κέντρου πρὸς κέντρον διαγράψῃς εὐθείας ἐν τοῖς τρισὶ, τρίγωνον ποιήσεις ἰσόπλευρον. τούτου τὸ ἐμβαδόν εὐρήσεις οὕτως. πολλαπλασίασον μίαν τῶν πλευρῶν ἐφ' ἑαυτήν, καὶ τοῦ γινομένου ἀριθμοῦ μέρος λαβὼν τρίτον καὶ ι<sup>ov</sup> ἕξεις τὸ ἐμβαδόν. {*sp.* 12 *litt.*} [Τοῦ καθολικοῦ τριγώνου τὸ ἐμβαδόν εὐρήσεις οὕτως. ποιήσον] τὴν κάθετον ἐπὶ μίαν τῶν πλευρῶν, καὶ τούτου λάβε τὸ μ, καὶ ἕξεις τὸ ἐμβαδόν. ἔτι καὶ ἄλλως. πολλαπλασίασον {*sp.* 10 *litt.*} τὸ μ μᾶς τῶν πλευρῶν ἐφ' ὅλην τὴν κάθετον[, καὶ ἕξεις τὸ ἐμβαδόν. Ἐὰν δὲ ἀπὸ τῶν πλευρῶν μόνων ζητεῖς μαθεῖν καθολικῶς τὸ ἐμβαδόν τοῦ τριγώνου,] {*sp.* 15 *litt.*} σύνθεσ ὁμοῦ καὶ τὰς τρεῖς πλευράς, καὶ τὸ μ τοῦ ἀπὸ τῆς τοιαύτης συνθέσεως ἀριθμοῦ ὑφελὼν ἀπὸ τοῦ ἐναπολειφθέντος μ ἄφελε ἅπαξ ἐκάστην τῶν πλευρῶν, καὶ τὰς εὐρεθείσας περισσείας ἀφ' ἐκάστης αὐτῶν μίαν μὲν, ἢν ἂν βούλοιο, πολλαπλασίασον μετὰ τοῦ ἡμίσεος τοῦ ἀποβάντος ἀριθμοῦ ἀπὸ τῆς συνθέσεως τῶν αὐτῶν πλευρῶν, τὸν δὲ γενόμενον ἀριθμὸν ποιήσον ὁμοίως μετὰ τῆς β<sup>as</sup> περισσείας, καὶ τὸν ἔτι γενόμενον μετὰ τῆς τρίτης, καὶ τοῦ συναχθέντος ὕστερον ἀπὸ τῶν τριῶν ἀριθμοῦ ἄρον πλευρὰν τετραγωνικὴν, καὶ ἕξεις τὸ ἐμβαδόν. {*marg.* ση(μείωσαι)} οὗτος καθολικὸς κανὼν

TRANSLATION OF THE *COMPENDIUM*

1. Terms. Plane geometry is composed of reference directions, milestones, lines, and angles; it admits of genera, species, and ultimate species. Then, the reference directions are 4: rising, setting, Ursa, and midday. A milestone is any point that is taken, or some stone—pointed, clearly—or log stuck into the extremity of a reference direction. The lines are 10: straight line, parallel, base, vertex, sides, diagonal, height, also called perpendicular, hypotenuse, circumference, and diameter. The angles are three; right, obtuse, and acute. The genera of measurement are three: rectilinear, planimetric, and volumetric. The species of measurement are 5: quadrangles, triangles, rhombi, trapezia, and circles. They have 18 ultimate species, as follows: 2 ultimate species of quadrangle: equilateral and right-angled quadrangle and oblong quadrangle; six ultimate species of triangle: equilateral triangle, isosceles triangle, right-angled triangle, obtuse-angled triangle, acute-angled triangle, and scalene triangle; 4 ultimate species of trapezium: isosceles trapezium, right-angled, obtuse-angled, and acute-angled trapezium, which is also called scalene or simply unequal; 2 ultimate species of rhombus: rhombus and rhomboid. The items other than these are both called and named polylaterals and polygons. 4 ultimate species of circle: circle, semicircle or arch, segment greater than a semicircle, and segment smaller than a semicircle.

2. The units of measurement are as follows: digit, knuckle, palm, forefinger span, span, foot, cubit, step, fathom, cable, plethron, jugerum, double-stadium, stadium, akena, mile, schoinos, and parasang.

3. The plethron has  $1\frac{2}{3}\frac{1}{15}$  cable schoinia, the jugerum has  $3\frac{1}{3}$  cables, the double-stadium 2 stadia, the stadium [...]  $8\frac{1}{2}$ , the akena 16 spans, the mile [...]  $64\frac{2}{3}\frac{1}{15}$ , the schoinos 4 miles, and the parasang 4.

4. This being thus said, let us next proceed to the area of the ultimate species and to how can each of these be constructed. The equilateral quadrangle comes to be as follows. Whenever you trace out 4 circles touching each other and inside the four you draw lines from centre to centre, you will make an equilateral and right-angled quadrangle. If you want to find the area of this—that is, the quantity of its span—multiply one of the sides by itself, and you will get the area by means of the resulting number. If you want to find the diagonal of this, double the area, and taking the square root of such a number you will get the diagonal.

5. Once an equilateral <quadrangle> has been cut in the middle, you will make two oblong quadrangles; if you wish to find the area of these, multiply the one side of the length by the other of the width, and you will get the area from them. If you want to find the diagonal of this, multiply the one side of the length into itself, afterwards the one of the width too, and composing both numbers <resulting> from the multiplication take the square root of these, and this will be the sought diagonal of the oblong quadrangle.

6. The equilateral triangle can naturally be constructed as follows. Three circles being tangent to each other, whenever inside the three you trace out a line from centre to centre, you will make an equilateral triangle. You will find the area of this as follows. Multiply one of the sides by itself, and taking a third and  $\frac{1}{10}$  part of the resulting number you will get the area. [You will find the area of a generic triangle as follows. Do] the height by one of the sides, and take  $\frac{1}{2}$  of this[, and you will get the area. Further, otherwise too. Multiply]  $\frac{1}{2}$  one of the sides by the whole height[, and you will get the area. Whenever you seek to learn generally the area of a triangle from its sides only,] compose together the three sides, and removing  $\frac{1}{2}$  the number <resulting> from such a composition remove each of the sides once from what has been left out, and the excesses found from each of them, multiply one of them, whichever you want, with half the number stepping off from the composition of the same sides, similarly do the resulting number with the 2<sup>nd</sup> excess, and the further resulting <number> with the third, and take up the square root of the number finally gathered from the three, and you will get the area.

50 εἰς τὴν τῶν ἀπάντων τριγώνων τοῦ ποσοῦ τῆς χωρήσεως εὑρεσιν. Ἐὰν δὲ ἀπὸ μόνου τοῦ ἔμβαδου  
 ζητεῖς μαθεῖν τὴν τοῦ ἰσοπλεύρου τριγώνου πλευρὰν, λάβε τριακοντάκις τὸ ἔμβαδόν, καὶ τοῦ  
 γινομένου αὐθις λάβε τὸ  $19^{\text{ov}}$ , καὶ ἔτι τοῦ τρισκαιδεκάτου πλευρὰν τετραγωνικὴν, καὶ ἕξεις τὴν  
 πλευρὰν. τὴν δὲ κάθετον εὐρήσεις οὕτως. πολλαπλασίασον μίαν τῶν πλευρῶν ἐφ' ἑαυτήν, καὶ τοῦ  
 γινομένου ἀριθμοῦ ἔξελε τὸ ἴδιον τέταρτον – ἢ τὸ  $\frac{1}{4}$  μιᾶς πλευρᾶς πολλαπλασιασθὲν ἐφ' ἑαυτὸ  
 55 ἄφελε ἀπὸ τοῦ τοιοῦτου ἀριθμοῦ· ταῦτόν γάρ ἐστι  $\frac{1}{4}$ , καὶ τοῦ ἐναπολειφθέντος ἔκτοτε ἀριθμοῦ  
 λάβε πλευρὰν τετραγωνικὴν, καὶ ἕξεις τὴν κάθετον. {*sp.* 12 *litt.*} [ἔτι καὶ ἄλλως.] κούφισον μιᾶς  
 τῶν πλευρῶν τὸ ἴδιον  $1^{\text{ov}}$  καὶ τὸ  $10^{\text{ov}}$ , καὶ τὸ λοιπὸν ἔσται ἡ κάθετος. Ἐὰν δὲ ἐντὸς τριγώνου ἰσο-  
 πλεύρου βούλη διαγράψαι τετράγωνον ἰσόπλευρον καὶ θέλης μαθεῖν πόσου ἔσται <sup>|18v</sup> ἐκάστη πλευ-  
 ρά, σύνθεθι βάσιν καὶ κάθετον. εἴθ' ὁμοίως πολλαπλασίασον τὴν βάσιν μετὰ τῆς καθέτου, καὶ μέρι-  
 σον παρὰ τὰ συναμφοτέρα – τουτέστι παρὰ τὸν ἀπὸ τῆς καθέτου καὶ τῆς βάσεως συντεθέντα ἀριθ-  
 60 μόν  $\frac{1}{4}$ , καὶ ἕξεις τὴν τοῦ τετραγώνου πλευρὰν. Καὶ ἐν τοῖς σκαληνοῖς τριγώνοις οὕτω γίνεται. Τὸ δὲ  
 ἰσοσκελὲς οὕτω συνίσταται, ἐὰν ῥόμβον ἰσόπλευρον μέσον τέμῃς – τουτέστιν ἀπὸ γωνίας εἰς γωνί-  
 αν. τούτου δὲ καὶ τὸ ἔμβαδόν καὶ τὴν κάθετον διὰ τῶν ἄνωθεν μεθόδων εὐρήσεις ἀναμφιβόλως.

<7> Τὸ δὲ ὀρθογώνιον τρίγωνον οὕτω συνίσταται ἀκριβέστατα, ἐὰν τέμῃς δίχα τὸ ἰσοσκελὲς. τὸ  
 ἰσόπλευρον τετράγωνον εἰ δίχα τεμεῖς, ἀπὸ γωνίας ἐπὶ γωνίαν δηλαδή, †[...]. καὶ ἴσον ἑτερόμηκες  
 65 καὶ ἰσόπλευρον τρίγωνον†. καὶ ἐὰν ἐντὸς κύκλου δύο διαμέτρους ἐγκαρσίως ἐγγράψῃς εἴτ' ἐκ τῶν  
 τεσσάρων ἄκρων αὐτῶν διάξῃς εὐθείας, τέσσαρα ἀποτελέσεις ὀρθογώνια τρίγωνα. Εἰ γοῦν τὸ  
 ἔμβαδόν αὐτοῦ θέλεις εὐρεῖν, πολλαπλασίασον ἢ τὴν βάσιν ὅλην ἐφ' ὅλην τὴν πρὸς ὀρθάς, καὶ τοῦ  
 γινομένου λάβε τὸ  $\frac{1}{4}$ , ἢ τὸ  $\frac{1}{4}$  τῆς βάσεως ἐφ' ὅλην τὴν πρὸς ὀρθάς. {*marg.* ἰστέον ὅτι ὁ τεχνικός ἐν  
 μὲν τοῖς ὀρθογωνίοις τριγώνοις τὴν μέσην τῶν πλευρῶν τίθεται βάσιν τὴν δὲ μείζω καὶ ἐλάσσω  
 70 ταύτης ποιεῖ ὑποτείνουσας, ἐν δὲ τοῖς σκαληνοῖς τριγώνοις, εἴτε ὀξυγώνια εἰσι ταῦτα εἴτε ἀμβλυ-  
 γώνια, οὐχ οὕτως, ἀλλὰ τὴν μὲν μείζονα τῶν ἄλλων τίθεται βάσιν τὰς δὲ λοιπὰς δύο τὰς ἐλάττονας  
 ταύτης ποιεῖ ὑποτείνουσας, καὶ δῆλον ἐξ ὧν αὐτὸς περὶ τούτων τῶν μεθόδων ἐξέθετο.} μίαν δὲ τῶν  
 τούτου πλευρῶν ὁποῖαν ἐθέλεις εὐρεῖν, εὐρήσεις οὕτως. εἰ μὲν τὴν πρὸς ὀρθάς, πολλαπλασίασον  
 τὴν ὑποτείνουσαν ἐφ' ἑαυτήν, καὶ ἀπὸ τοῦ γεγονότος ἀριθμοῦ ἄφελε τὸν τῆς βάσεως πολλαπλα-  
 75 σιασμόν, καὶ τοῦ λοιποῦ λάβε πλευρὰν τετραγωνικὴν, καὶ ἕξεις τὴν κάθετον. Εἰ δὲ τὴν βάσιν  
 βούλει εὐρεῖν, ποίησον τὸ ἀνάπαλιν (ἡγουν ἄφελε τὸν τῆς καθέτου πολλαπλασιασμόν ἀπὸ <τοῦ>  
 τῆς ὑποτείνουσης), καὶ τοῦ λοιποῦ τετραγωνικὴ πλευρὰ ἔσται ἡ βάσις. Εἰ δὲ τὴν ὑποτείνουσαν ζη-  
 τεῖς, πολλαπλασίασον τὴν βάσιν καὶ τὴν κάθετον ἐφ' ἑαυτάς· εἶτα ἐνώσας ἀμφοτέρους τοὺς ἀριθ-  
 80 μούς τούτων λάβε πλευρὰν τετραγωνικὴν, καὶ ἕξεις τὴν ὑποτείνουσαν. Ἐὰν δὲ ἀπὸ μόνης τῆς ὑπο-  
 τείνουσης ζητῆς γνῶναι τὴν τε βάσιν καὶ τὴν κάθετον, τετραπλασίασον τὴν ὑποτείνουσαν, καὶ  
 ταύτης λάβε τὸ  $9^{\text{ov}}$ , καὶ ἕξεις τὴν βάσιν· τρίπλωσον αὐθις τὴν ὑποτείνουσαν, καὶ πάλιν λάβε τὸ  $9^{\text{ov}}$ ,  
 καὶ ἕξεις τὴν κάθετον.

<8> Ἐὰν δὲ ἀπὸ πλήθους περιτοῦ τρίγωνον ὀρθογώνιον βούλει συστήσασθαι κατὰ Πυθαγό-  
 ραν, πολλαπλασίασον τὴν κάθετον ἐφ' ἑαυτήν, καὶ ἀπὸ τοῦ γενομένου ἀριθμοῦ ἄφελε μονάδα  
 85 μίαν, καὶ τοῦ καταλειφθέντος τὸ  $\frac{1}{4}$  ἔσται ἡ βάσις· πρόσθεθι τῷ τοιοῦτῳ  $\frac{1}{4}$  μονάδα, καὶ ἕξεις τὴν  
 ὑποτείνουσαν. εἰ δ' ἀπὸ πλήθους ἀρτίου θέλεις πάλιν τρίγωνον ὀρθογώνιον συστήσασθαι κατὰ  
 Πλάτωνα, λάβε τὸ  $\frac{1}{4}$  τῆς βάσεως εἰς ἑαυτό, καὶ τοῦ γινομένου μονάδα ἀφελὼν τὸ λοιπὸν ἔσται ἡ  
 κάθετος· πρόσθεθι τῇ καθέτῳ δυάδα, καὶ ἕξεις τὴν ὑποτείνουσαν. καθόλου δὲ ἡ τῶν ὀρθογωνίων  
 τριγώνων γένεσις οὕτω γίνεται. Ἐὰν ἀπὸ τοῦ τυχόντος ἀριθμοῦ θέλης τρίγωνον ὀρθογώνιον ποιῆ-  
 90 σαι, τριπλασίασον τὸν δοθέντα ἀριθμόν, καὶ ἕξεις τὴν κάθετον· τετραπλασίασον αὐτόν, καὶ ἕξεις  
 τὴν βάσιν· πενταπλασίασον αὐτόν, καὶ λήψῃ τὴν ὑποτείνουσαν. γνώρισμα δὲ σαφὲς τοῦ ὀρθογω-  
 νίου τριγώνου τὸ μήτε εἶσω {*s.l.* κάτωθεν} τοῦ γεωμετρικοῦ κανόνος τοὺς ὄρους τῶν ἀριθμῶν  
 εὐρίσκεισθαι μήτε ἐκτός {*s.l.* ἄνωθεν}· οἷον τί λέγω, {*marg.* οὗτος ἐστὶν ὁ γεωμετρικὸς κανὼν} τὸν  
 95 πολλαπλασιασμόν τῆς καθέτου καὶ τῆς βάσεως ἴσον εἶναι διαπαντὸς τῷ τῆς ὑποτείνουσης· εἰ γὰρ  
 |19r μείζων ἐστίν, ὀξυγώνιον ὑπάρχει· εἰ δὲ ἐλάττων, ἀμβλυγώνιον.

This is the general rule for finding the quantity of the span of all and every triangle. Whenever you seek to learn the side of an equilateral triangle from its area only, take thirty times the area, and take  $\frac{1}{13}$  of the result anew, and further the square root of the thirteenth, and you will get the side. You will find the height as follows. Multiply one of the sides by itself, and out of the resulting number remove its own quarter—or, remove  $\frac{1}{2}$  one side multiplied by itself from such a number, for it amounts to the same—and take the square root of the number thence left out, and you will get the height. [Further, otherwise too.] From one of the sides raise its own  $\frac{1}{10}$  and  $\frac{1}{30}$ , and the remainder will be the height. Whenever you want to trace out an equilateral quadrangle inside an equilateral triangle and wish to learn how much each side will be, compose the base and the height. Afterwards, similarly, multiply the base with the height, and divide by both of them together—that is, by the number composed out of the base and the height—and you will get the side of the quadrangle. And for the scalene triangles it will come about as follows. The isosceles one can be constructed as follows, whenever you cut an equilateral rhombus in the middle—that is, from angle to angle. You will also unambiguously find both the area and the height of this by means of the above procedures.

7. The right-angled triangle can be constructed most exactly as follows, whenever you cut an isosceles in half. If you will cut an equilateral quadrangle in half, clearly from angle to angle, [...] and an equal oblong and an equilateral triangle. And whenever you inscribe two diameters athwart inside a circle, <and> afterwards you draw through straight lines from the four extremities, you will complete four right-angled triangles. Then, if you really wish to find its area, multiply either the whole base by the whole perpendicular, and take  $\frac{1}{2}$  the result, or  $\frac{1}{2}$  the base by the whole perpendicular. {*marg.* One must know that, for right-angled triangles, the author sets the middle side as base and makes hypotenuses the one greater and the one lesser than this; for scalene triangles, whether they are acute-angled or obtuse-angled, not in this way, but he sets the <side> greater than the others as base and makes hypotenuses the remaining two, the ones lesser than this, and this is clear from what he himself set out about these procedures.} You will find one, whichever you wish to find, of the sides of this as follows. If the perpendicular, multiply the hypotenuse by itself, and remove the multiplication of the base from the number that turns out to result, and take the square root of the remainder, and you will get the height. If you want to find the base, do the inverse (namely, remove the multiplication of the height from the <multiplication of the> hypotenuse), and the square root of the remainder will be the base. If you seek the hypotenuse, multiply the base and the height by themselves; afterwards, uniting both numbers take the square root of these, and you will get the hypotenuse. Whenever you seek to know both the base and the height from the hypotenuse only, quadruple the hypotenuse, and take  $\frac{1}{5}$  of this, and you will get the base. Triple the hypotenuse anew, and again take  $\frac{1}{5}$ , and you will get the height.

8. Whenever you want to construct a right-angled triangle from an odd multiplicity according to Pythagoras, multiply the height by itself, and remove one unit from the resulting number, and  $\frac{1}{2}$  what has been left over will be the base; add a unit to such a  $\frac{1}{2}$ , and you will get the hypotenuse. Again, if you wish to construct a right-angled triangle from an even multiplicity according to Plato, take  $\frac{1}{2}$  the base into itself, and removing a unit from the result the remainder will be the height; add a dyad to the height, and you will get the hypotenuse. In general, the generation of the right-angled triangles comes about as follows. Whenever you wish to make a right-angled triangle from a number taken at random, triple the given number, and you will get the height; quadruple it, and you will get the base; quintuple it, and you will take the hypotenuse for yourself. A feature that makes a right-angled triangle plainly recognised is that the terms of the numbers are found neither inside {*s.l.* below} nor outside {*s.l.* above} the geometric rule. What do I mean by this? {*marg.* This is the geometric rule} I mean that the multiplication of the base and of the height is altogether equal to the one of the hypotenuse, for if it is greater, <the triangle> happens to be acute-angled, if it is less, obtuse-angled.



<9> τοῦ δὲ ἀμβλυγωνίου καὶ ὀξυγωνίου τὸ ἐμβαδὸν κατὰ τὰς προλαβούσας μεθόδους εὐρίσκεται. ἡ δὲ κάθετος αὐτῶν εὐρίσκεται οὕτω. πολλαπλασίασον ἐκάστην τῶν πλευρῶν ἐφ' ἑαυτήν· εἶτα σύνθεσ τῶν δύο πλευρῶν τὸν πολλαπλασιασμόν (ἦγουν τῆς τε βάσεως καὶ τῆς μείζονος ὑποτείνουσῆς), καὶ ἄρον ἐκ τούτων τὸν τῆς ἥττονος πλευρᾶς πολλαπλασιασμόν, καὶ τοῦ καταλειφθέντος ἀριθμοῦ τὸ μὲν ἦμισυ κράτησον τὸ δὲ  $\sphericalangle$  ἕασον· εἶτα μέρισον τὸ κρατηθὲν παρὰ τὸν τῆς βάσεως ἀριθμόν, καὶ τὸ ἐκ τοῦ μερισμοῦ ἀποβὰν ἔσται ἡ μείζων ἀποτομή. Καὶ τὴν ἥττονα δὲ ἀποτομὴν κατὰ τὸν ὅμοιον εὐρήσεις τρόπον· συνθεῖς γὰρ τὸν τῆς ἥττονος πλευρᾶς πολλαπλασιασμόν καὶ τὸν τῆς βάσεως καὶ ἐξ αὐτῶν ἀφελὼν τὸν τῆς μείζονος πλευρᾶς πολλαπλασιασμόν τοῦ καταλειφθέντος ἀριθμοῦ τὸ μὲν  $\sphericalangle$  {*sp. 7 litt.*} [ἔασας] τὸ δὲ  $\sphericalangle$  κρατήσας καὶ παρὰ τὸν τῆς βάσεως ἀριθμόν μερίσας τὸ ἀπὸ τοῦ μερισμοῦ γενόμενον ἢ ἥττων ἔσται ἀποτομή. Εἶτα εἰ θέλεις τὴν τούτου κάθετον εὐρεῖν, λάβε μίαν τῶν ὑποτείνουσῶν πλευρῶν εἰς ἑαυτήν, καὶ ἐκ τοῦ γινομένου ἀριθμοῦ κούφισον τὸν ἀπὸ τῆς ἀποτομῆς γεγονότα ἀριθμόν, καὶ τοῦ καταλειφθέντος λαβὼν πλευρὰν τετραγωνικὴν ἕξεις τὴν κάθετον. Εἰ γοῦν τὸν τῆς μείζονος ὑποτείνουσῆς πλευρᾶς λάβῃς ἀριθμόν, ἄφελε ἐξ αὐτοῦ καὶ τὸν τῆς μείζονος ἀποτομῆς πολλαπλασιασμόν· εἰ δὲ τὸν τῆς ἥττονος λάβῃς, ἄφελειν ἐξ αὐτῆς δεῖ καὶ τὸν τῆς ἥττονος ἀποτομῆς ἀριθμόν.

<10> Ἐὰν ἐντὸς τοῦ οἰο<υ>δητοῦν τριγώνου θελήσῃς κύκλον διαγράψαι καὶ εὐρεῖν τὴν τούτου διάμετρον, ποιεῖ οὕτως. τετραπλασίασον τὸ τοῦ τριγώνου ἐμβαδόν· εἶτα συνθεῖς τὰς τρεῖς τούτου πλευρὰς παρὰ τὸν ἀπὸ τῆς συνθέσεως γεγονότα ἀριθμόν μέρισον τὸν ἐκ τοῦ τετραπλασιασμοῦ τοῦ ἐμβαδοῦ συναχθέντα ἀριθμόν, καὶ ὁ ἀπὸ τοῦ μερισμοῦ εὐρεθεῖς ἀριθμὸς ἢ τοῦ κύκλου ἔσται διάμετρος. Ἐὰν δὲ ἐκτὸς ἰσοπλεύρου τριγώνου θελήσῃς κύκλον περιγράψαι καὶ βούλῃ γινῶναι πόσου ἔστιν ἢ τούτου διάμετρος, λάβε μίαν τῶν πλευρῶν ἐφ' ἑαυτήν, καὶ τὸν γενόμενον ἀριθμόν μέρισον παρὰ τὴν κάθετον αὐτοῦ, καὶ ὁ ἀπὸ τοῦ μερισμοῦ εὐρεθεῖς ἀριθμὸς ἢ τοῦ κύκλου ἔσται διάμετρος. Ἐὰν δὲ ἐκτὸς τριγώνου σκαληνοῦ βούλῃ περιγράψαι κύκλον καὶ θέλῃς εὐρεῖν τὴν τούτου διάμετρον, πολλαπλασίασον τὸν ἀριθμόν τῆς ἥττονος ὑποτείνουσῆς πλευρᾶς ἐπὶ τὸν τῆς μείζονος, καὶ τὸν ἀπὸ τούτων γενόμενον ἀριθμόν μέρισον παρὰ τὴν κάθετον, καὶ ὁ ἀπὸ τοῦ μερισμοῦ γενόμενος ἀριθμὸς ἢ τοῦ κύκλου ἔσται διάμετρος.

<11> {*marg. diagram*} Ῥόμβος ἀκριβῆς διαγινώσκειται, ἐὰν δύο συνάψῃς ἰσόπλευρα τρίγωνα. {*marg. diagram*} Ῥόμβου δὲ τὸ ἐμβαδὸν εὐρεῖν. ποιεῖ οὕτως. τὸ  $\sphericalangle$  τῆς μιᾶς τῶν διαγωνίων πολλαπλασίασον ἐπὶ τὴν ἐτέραν ὄλην, καὶ ὁ γενόμενος ἐξ αὐτῶν ἀριθμὸς ἔσται τοῦ ῥόμβου τὸ ἐμβαδόν. καὶ ἄλλως. ποιήσον τὴν μίαν ἐπὶ τὴν ἄλλην, καὶ τοῦ γενομένου λάβε τὸ  $\sphericalangle$ . καὶ ἐτέρως. πολλαπλασίασον μίαν τῶν πλευρῶν εἰς ἑαυτήν, καὶ ἀπὸ τοῦ γενομένου κούφισον τὸ ἴδιον εἰκοστόπεμπτον, καὶ ὁ καταλειφθεὶς ἀριθμὸς τὸ τοῦ ῥόμβου ἔστιν ἐμβαδόν. τὰς δὲ διαγωνίους τούτου καὶ τὰς πλευρὰς εὐρήσεις κατὰ τὰς προγραφείσας μεθόδους τοῦ ἰσοπλεύρου τριγώνου.

<12> {*marg. inf. two diagrams*} Ῥομβοειδὲς δὲ γίνεται, ἐὰν δύο ἐπισυνάψῃς τρίγωνα σκαληνὰ ἢ ἐὰν ἀπὸ τετραγώνου |<sub>19v</sub> ἑτερομήκους ἀφ' ἑκατέρων τῶν ἄκρων δύο ἀφέλῃς ὀρθογώνια τρίγωνα. τὸ δὲ ἐμβαδὸν αὐτοῦ εἰ θέλεις εὐρεῖν, πολλαπλασίασον τῶν τοῦ μήκους πλευρῶν μίαν ἐπὶ τὴν ἥττονα διαγώνιον ἢ μίαν τῶν τοῦ πλάτους ἐπὶ τὴν μείζονα διαγώνιον, καὶ εὐρήσεις τὸ ἐμβαδόν. καὶ ἄλλως. δῖελε τὸ τοιοῦτον σχῆμα εἰς τρία μέρη (ἦγουν εἰς ἓν τετράγωνον καὶ εἰς ὀρθογώνια δύο τρίγωνα), καὶ πᾶσιν ἐσεῖται δῆλον ἀπὸ τούτου τὸ ἐμβαδόν. εἰ δὲ κύκλον ἐντὸς ῥόμβου διαγράψῃς, τοσοῦτου ἔσται καὶ ἡ διάμετρος ὅσου καὶ ἡ μικρὰ τῶν διαγωνίων.

{*marg. sign like a rotated S and sign of a circle*}

<13> Κύκλου τὸ ἐμβαδὸν εὐρήσεις κατὰ πολλοὺς τρόπους. πολλαπλασίασον τὴν διάμετρον ἐφ' ὄλην τὴν περίμετρον, καὶ τοῦ γινομένου ἀριθμοῦ λάβε μέρος δ<sup>ov</sup>, καὶ ἕξεις τούτου τὸ ἐμβαδόν. Ἐὰν δὲ θέλῃς ἀπὸ μόνῃς τῆς περιμέτρου τὸ ἐμβαδὸν εὐρεῖν, ποιεῖ οὕτως. πολλαπλασίασον τὴν περίμετρον εἰς ἑαυτήν· εἶτα τὸν ἀπὸ τούτου γινόμενον ἀριθμόν ἐπταπλασίασον, καὶ τοῦ γενομένου λάβε μέρος κη<sup>ov</sup> {*lege* πη<sup>ov</sup>}, καὶ ἕξεις τὸ ἐμβαδόν. Ἐὰν δὲ θέλῃς ἀπὸ μόνῃς τῆς διαμέτρου τὸ ἐμβαδὸν εὐρεῖν, ποιήσον ταύτην ἐφ' ἑαυτήν, καὶ τὸν γενόμενον ἀριθμόν ἐνδεκαπλασίασον, καὶ τοῦ ἔτι γενομένου λάβε τὸ ιδ<sup>ov</sup>, καὶ ἕξεις τὸ ἐμβαδόν. Ἔτι καὶ ἄλλως παρ' Εὐκλείδη τὸ ἐμβαδὸν εὐρεῖν. ποιήσον τὴν διάμετρον ἐφ' ἑαυτήν, καὶ ἀπὸ τοῦ γενομένου ἀριθμοῦ ἄφελε τὸ ἴδιον ζ<sup>ov</sup> καὶ ιδ<sup>ov</sup>, καὶ τὸ

9. The area of the obtuse-angled and acute-angled <triangles> can be found according to the procedures taken above. Their height can be found as follows. Multiply each of the sides by itself; afterwards, compose the multiplication of the two sides (namely, both of the base and of the greater hypotenuse), and take up the multiplication of the smaller side from these, and keep half the number left over and leave  $\frac{1}{2}$  alone afterwards, divide what has been kept by the number of the base, and what has stepped off from the division will be the greater projection. You will also find the smaller projection in a similar way; for composing the multiplication of the smaller side and the one of the base and removing from them the multiplication of the greater side, [leaving]  $\frac{1}{2}$  [alone] the number left over and keeping <its>  $\frac{1}{2}$  and dividing by the number of the base what results from the division will be the smaller projection. Afterwards, if you wish to find the height of this, take one of the hypotenuse sides into itself, and raise the number that turns out to result from the projection from the result, and taking the square root of what has been left over you will get the height. Then, if you really take the number of the greater hypotenuse side, remove the multiplication of the greater projection too from it; if you take the <number> of the smaller one, one must remove the number of the smaller projection too from it.

10. Whenever you wish to trace out a circle inside a triangle whatsoever and to find the diameter of this, do as follows. Quadruple the area of the triangle. Afterwards, composing the three sides of this divide the number gathered from the quadruplication of the area by the number that turns out to result from the composition, and the number found from the division will be the diameter of the circle. Whenever you wish to circumscribe a circle outside an equilateral triangle and want to know how much the diameter of this is, take one of the sides by itself, and divide the resulting number by its height, and the number found from the division will be the diameter of the circle. Whenever you want to circumscribe a circle outside a scalene triangle and wish to know the diameter of this, multiply the number of the smaller hypotenuse side by the one of the greater one, and divide the number resulting from them by the height, and the number resulting from the division will be the diameter of the circle.

11. An exact rhombus is determined whenever you join two equilateral triangles. Find the area of a rhombus. Do as follows. Multiply  $\frac{1}{2}$  one of the diagonals for the whole other, and the number resulting from them will be the area of the rhombus. And in another way. Do the one by the other, and take  $\frac{1}{2}$  the result. And otherwise. Multiply one of the sides into itself, and from what results raise its own twenty-fifth, and the number left over is the area of the rhombus. You will find the diagonals and the sides of this according to the procedures for the equilateral triangle written above.

12. A rhomboid comes to be whenever you join up two scalene triangles, or whenever you remove two right-angled triangles from both extremities of an oblong quadrangle. If you wish to find its area, multiply one of the sides of the length by the smaller diagonal or one of those of the width by the greater diagonal, and you will find the area. And otherwise. Partition such a figure into three parts (namely, into one quadrangle and two right-angled triangles), and from this the area will be clear to everyone. If you will trace out a circle inside a rhombus, as much is also the small diagonal, so much will also be the diameter.

13. You will find the area of a circle in many ways. Multiply the diameter by the whole circumference, and take a  $\frac{1}{4}$  part of the resulting number, and you will get the area of this. Whenever you wish to find the area from the circumference only, do as follows. Multiply the circumference into itself; afterwards, septuple the number resulting from this, and take a  $\frac{1}{88}$  part of the result, and you will get the area. Whenever you wish to find the area from the diameter only, do this by itself, and undecuple the resulting number, and take a  $\frac{1}{14}$  part of what further results, and you will get the area. Further, find the area otherwise too, according to Euclid. Do the diameter by itself, and from the resulting number remove its own  $\frac{1}{7}$  and  $\frac{1}{14}$ , and what has been left over happens to be the area.

145 καταλειφθὲν ὑπάρχει τὸ ἐμβαδόν. Ἔτι καὶ ἄλλως, τὸ  $\omega$  τῆς διαμέτρου ἐφ' ὅλην τὴν περίμετρον ἢ τὸ  $\omega$  τῆς περιμέτρου ἐφ' ὅλην τὴν διάμετρον, καὶ τοῦ γινομένου λάμβανε τὸ  $\omega$ , καὶ ἔξεις τὸ ἐμβαδόν· ἢ τὸ  $\omega$  τῆς περιμέτρου ἐπὶ τὸ  $\omega$  τῆς διαμέτρου, καὶ τὸ γενόμενον ἐστὶ τὸ ἐμβαδόν. Ἔτι καὶ ἄλλως, τὸ  $\omega$  τῆς διαμέτρου ἐφ' ἑαυτό, καὶ τοῦ γενομένου λάβε μέρος ζ<sup>ov</sup>, καὶ ἔχε τοῦτο ἰδίᾳ· εἶτα τρίπλωσον τὸν τοιοῦτον ἀριθμὸν προσθεὶς αὐτῷ καὶ τὸ δηλωθὲν ζ<sup>ov</sup>, καὶ τὸν γενόμενον ἀριθμὸν ἴσθι

150 εἶναι τοῦ κύκλου τὸ ἐμβαδόν. ἔτι καὶ ἄλλως ἀπὸ μόνης τῆς διαμέτρου τὸ ἐμβαδὸν εὐρεῖν. ποιήσον τὴν διάμετρον ἐφ' ἑαυτήν, καὶ τοῦ γενομένου ἀριθμοῦ ἄφελε τὸ ἴδιον δ<sup>ov</sup>, τῷ δὲ ἐναπολειφθέντι πρόσθεσ τῷ {lege τὸ} τοῦ {ὄλου} ἀριθμοῦ κα<sup>ov</sup>, καὶ ἔξεις τὸ ἐμβαδόν. εἰ δὲ βούλει τὴν διάμετρον εὐρεῖν, ἄφελε ἀπὸ τῆς περιμέτρου τὸ ἴδιον κβ<sup>ov</sup>, καὶ τοῦ καταλειφθέντος λάβε τὸ γ<sup>ov</sup>, καὶ ἔξεις τὴν διάμετρον. Ἔτι καὶ ἄλλως τὴν διάμετρον εὐρεῖν. λάβε τῆς περιμέτρου τὸ κβ<sup>ov</sup>, καὶ ποιήσον ἐπτάκις, καὶ ἔξεις τὴν διάμετρον. Ἔτι καὶ ἄλλως τὴν διάμετρον εὐρεῖν. ἐπταπλασίασον τὴν περίμετρον, καὶ τοῦ γενομένου λάβε μέρος κβ<sup>ov</sup>, καὶ ἔξεις τὴν διάμετρον. Ἐὰν δὲ θέλῃς ἀπὸ τῆς διαμέτρου τὴν

155 περίμετρον εὐρεῖν, τρίπλωσον τὴν διάμετρον, καὶ πρόσθεσ τῷ τοιοῦτῳ ἀριθμῷ καὶ τὸ τῆς διαμέτρου ζ<sup>ov</sup>, καὶ τὸν ἀποβαίοντα ἀριθμὸν τὴν τοῦ κύκλου νόει περίμετρον. Ἔτι καὶ ἄλλως τὴν περίμετρον {lege διάμετρον} εὐρεῖν. πολλαπλασίασον τὴν περίμετρον εἰς ἑαυτήν, καὶ τῷ γενομένῳ ἀριθμῷ πρόσθεσ τὸ ταύτης γ<sup>ov</sup> ἐκβαλὼν πρότερον τὸ ἴδιον αὐτῆς κβ<sup>ov</sup>, καὶ τοῦ συναχθέντος ἀριθμοῦ λάβε τὸ ι<sup>ov</sup>. τοῦ δὲ τοιοῦτου δεκάτου λαβὼν πλευρὰν τετραγωνικὴν ἔξεις τὴν διάμετρον. Ἐὰν δὲ ἀφ' ἑνὸς καὶ μόνου ἀριθμοῦ θέλῃς διαστεῖλαι καὶ εὐρεῖν τὴν τε περίμετρον καὶ τὴν διάμετρον, πολλαπλασίασον ἐπτάκις τὸν τοιοῦτον ἀριθμὸν, καὶ τοῦ γενομένου λάβε μέρος κθ<sup>ov</sup>, καὶ ἔξεις τὴν

160 διάμετρον, ὄντινα δὴ ἀριθμὸν ἀπὸ τοῦ καταλειφθέντος {lege δοθέντος} ἀφελὼν ἀριθμοῦ |<sub>20r</sub> τὸν καταλειφθέντα νόει εἶναι περίμετρον. Ἐὰν δὲ βούλη πρότερον τὴν περίμετρον εὐρεῖν, ποίει τὸν δοθέντα ἀριθμὸν δις καὶ εἰκοσάκις, καὶ τοῦ γινομένου μέρος κθ<sup>ov</sup> λαβὼν ἔξεις τὴν περίμετρον, ἢ ἀφελὼν ἀπὸ τοῦ δοθέντος ἀριθμοῦ τὸ καταλειφθὲν ἔσται ἡ διάμετρος. Ἐὰν δὲ θέλῃς ἀπὸ μόνου τοῦ ἐμβαδοῦ τὴν τε περίμετρον καὶ τὴν διάμετρον εὐρεῖν, ποιήσον οὕτως. τὴν τοῦ ἐμβαδοῦ ποσότητα λάβε τεσσαρεσκαδεκάκις, καὶ τοῦ γενομένου λάβε μέρος ια<sup>ov</sup>, οὗ πλευρὰ τετραγωνικὴ ἔσται ἡ

170 διάμετρος. τὴν δὲ περίμετρον εὐρεῖν. ποιήσον τὴν τοῦ ἐμβαδοῦ ποσότητα ὀγδοηκοντάκις καὶ ὀκτάκις, καὶ τοῦ γενομένου λάβε μέρος ἑβδομον, οὗ πλευρὰ τετραγωνικὴ ἔσται ἡ περίμετρος.

Δοθέντων δὲ συναμφοτέρων τῶν ἀριθμῶν (ἤγουν τῆς διαμέτρου, τῆς περιμέτρου καὶ τοῦ ἐμβαδοῦ τοῦ κύκλου ἐν ἀριθμῷ ἐνί) διαστεῖλαι καὶ εὐρεῖν ἕκαστον ἀριθμὸν. ποίει οὕτως. τὸν δοθέντα ἀριθμὸν ὅποιος ἐστὶν αἰεὶ πολλαπλασίαζε ἐπὶ τὸν ρνδ· τοῦτοις δὲ προστίθει καθολικῶς ὦμα, καὶ τοῦ

175 γινομένου λάμβανε πλευρὰν τετραγωνικὴν· ἀπὸ δὲ τοῦ τοιοῦτου ἀριθμοῦ (ἤγουν τοῦ τετραγώνου) κούφιζε καθολικῶς κθ, καὶ τοῦ καταλειφθέντος μέρος λαβὼν ια<sup>ov</sup> ἔξεις τὴν διάμετρον. τὴν δὲ περίμετρον θέλων εὐρεῖν τὸν ἐναπολειφθέντα ἀριθμὸν διπλασίασον, καὶ τοῦ γενομένου μέρος λαβὼν ζ<sup>ov</sup> ἔξεις τὴν περίμετρον. τὸ δὲ ἐμβαδὸν εὐρεῖν. ποιήσον τὸ  $\omega$  τῆς διαμέτρου ἐπὶ τὸ  $\omega$  τῆς περιμέτρου, καὶ ἔξεις τὸ ἐμβαδόν.

180 Δύο δὲ κύκλων περὶ τὸ αὐτὸ κέντρον ὄντων ἢ καὶ ἐκκέντρων ὑπαρχόντων τὸ μεταξὺ τῶν περιφερειῶν αὐτῶν χωρίον εὐρεῖν. μέτρησον ἅμα ἐκάτερον τῶν κύκλων, καὶ ἄφελε τὸν μείζονα ἀπὸ τοῦ ἐλάττονος {lege τὸν ἐλάττονα ἀπὸ τοῦ μείζονος}, καὶ ἐκάστου εὐρήσεις ἰδίᾳ τὸ ἐμβαδόν.

<14> Τμήματος δὲ κύκλου ἡμικυκλίου τὸ ἐμβαδὸν εὐρεῖν. ποίει οὕτως. πολλαπλασίασον τὴν διάμετρον τοῦ ἡμικυκλίου, ἥτις καὶ βᾶσις λέγεται, ἐπὶ τὴν περίμετρον, καὶ τοῦ γενομένου λάβε τὸ

185 δ<sup>ov</sup>, καὶ ἔξεις τὸ ἐμβαδόν. εἰ δὲ θέλεις ἀπὸ τῆς καθέτου καὶ τῆς περιμέτρου τὸ ἐμβαδὸν εὐρεῖν, ἐρωτησάτω ἡ κάθετος τὴν περίμετρον, καὶ ὄν ἂν εὔροις ἀριθμὸν, τοῦτο ἐστὶ τὸ τοῦ ἡμικυκλίου ἐμβαδόν. {marg. ση(μειῶσαι)} Ἔτι καὶ ἄλλως τὸ ἐμβαδὸν εὐρεῖν ἀπὸ μόνης τῆς βάσεως. ποίει οὕτως. πολλαπλασίασον τὴν βᾶσιν εἰς ἑαυτήν, καὶ τὸν γενόμενον ἀριθμὸν ποιήσον ια<sup>15</sup>, οὗ μέρος κη<sup>ov</sup> ἔσται τὸ ἐμβαδὸν τοῦ ἡμικυκλίου. ἄλλως τὸ ἐμβαδὸν εὐρεῖν. ποιήσον τὴν βᾶσιν εἰς ἑαυτήν, καὶ

190 ἀπὸ τοῦ γενομένου ἄφελε τὸ ἴδιον ζ<sup>ov</sup> καὶ ιδ<sup>ov</sup>, καὶ τοῦ καταλειφθέντος ἀριθμοῦ τὸ  $\omega$  λαβὼν ἔξεις τὸ ἐμβαδόν. ἀπὸ δὲ τῆς καθέτου μόνης τὸ ἐμβαδὸν εὐρεῖν. ποίει οὕτως. ἀριθμήσον τὴν κάθετον εἰς ἑαυτήν, καὶ τὸν γενόμενον ἀριθμὸν ποιήσον ἑνδεκάκις· τούτου δὲ λαβὼν τὸ ζ<sup>ov</sup> ἔξεις τὸ ἐμβαδόν.

Further, otherwise too.  $\frac{1}{2}$  the diameter by the whole circumference or  $\frac{1}{2}$  the circumference by the whole diameter, and take  $\frac{1}{2}$  the result, and you will get the area; or  $\frac{1}{2}$  the circumference by  $\frac{1}{2}$  the diameter, and the result is the area. Further, otherwise too.  $\frac{1}{2}$  the diameter by itself, and take a  $\frac{1}{7}$  part of the result, and get this separately; afterwards, triple such a number adding the highlighted  $\frac{1}{7}$  too to it, and know that the resulting number is the area of the circle. Further, otherwise too, find the area from the diameter only. Do the diameter by itself, and from the resulting number remove its own  $\frac{1}{4}$ , and add  $\frac{1}{21}$  of the number to what has been left out, and you will get the area. If you want to find the diameter, from the circumference remove its own  $\frac{1}{22}$ , and take  $\frac{1}{3}$  of what has been left over, and you will get the diameter. Further, find the diameter otherwise too. Take  $\frac{1}{22}$  of the circumference, and do seven times, and you will get the diameter. Further, find the diameter otherwise too. Septuple the circumference, and take  $\frac{1}{22}$  of the result, and you will get the diameter. Whenever you wish to find the circumference from the diameter, triple the diameter, and add  $\frac{1}{7}$  of the diameter too to such a number, and realise that the number stepping off is the circumference of the circle. Further, find the diameter otherwise too. Multiply the circumference into itself, and add  $\frac{1}{3}$  of this to the resulting number throwing away beforehand its own  $\frac{1}{22}$ , and take  $\frac{1}{10}$  of the gathered number; taking the square root of such a tenth you will get the diameter. Whenever you wish to separate and find both the circumference and the diameter from one and only one number, multiply seven times such a number, and take a  $\frac{1}{29}$  part of the result, and you will get the diameter; removing indeed this very number from the given number realise that the <number> left over is the circumference. Whenever you want to find the circumference beforehand, do the given number twice and twenty times, and taking a  $\frac{1}{29}$  part of the result you will get the diameter; removing this from the given number what has been left over will be the diameter. Whenever you wish to find both the circumference and the diameter from the area only, do as follows. Take the quantity of the area fourteen times, and take a  $\frac{1}{11}$  part of the result, the square root of which will be the diameter. Find the circumference. Do the quantity of the area eighty-eight times, and take a seventh part of the result, the square root of which will be the circumference.

Both numbers together (namely, the diameter, the circumference, and the area of the circle in one single number) being given, separate and find each number. Do as follows. Always multiply the given number, whichever it is, by 154; add generally 841 to these, and take the square root of the result; raise generally 29 from such a number (namely, from the square <root>), and taking a  $\frac{1}{11}$  part of what has been left over you will get the diameter. Wishing to find the circumference double the number left out, and taking a  $\frac{1}{7}$  part of the result you will get the circumference. Find the area. Do  $\frac{1}{2}$  the diameter by  $\frac{1}{2}$  the circumference, and you will get the area.

Two circles being around the same centre or even happening to be eccentric, find the region between their circumferences. Measure together both circles, and remove the lesser from the greater, and you will find the area of each separately.

14. Find the area of a semicircular segment of a circle. Do as follows. Multiply the diameter of the semicircle, which is also called base, by the circumference, and take  $\frac{1}{4}$  of the result, and you will get the area. If you wish to find the area from the height and the circumference, let the height ask the circumference, and whichever number you find, this is the area of the semicircle. Further, find the area otherwise too, from the base only. Do as follows. Multiply the base into itself, and do the resulting number 11 times, whose  $\frac{1}{28}$  part will be the area of the semicircle. Find the area otherwise. Do the base into itself, and from the result remove its own  $\frac{1}{7}$  and  $\frac{1}{14}$ , and taking  $\frac{1}{2}$  the number left over you will get the area. Find the area from the height only. Do as follows. Count the height into itself, and do the resulting number eleven times; taking  $\frac{1}{7}$  of this you will get the area.

ἀπὸ δὲ μόνης τῆς περιφερείας τὸ ἐμβαδὸν εὐρεῖν. ποίει οὕτως. μέτρησον τὴν περιφέρειαν εἰς ἑαυ-  
 τήν· εἶτα καὶ ἐπτάκις· τούτων δὲ μέρος τεσσαρακοστοτέταρτον λαβὼν ἕξεις τὸ ἐμβαδόν. ἀπὸ δὲ  
 195 τῆς βάσεως καὶ τῆς καθέτου τὸ ἐμβαδὸν εὐρεῖν. ποίει οὕτως. ἀρίθμησον τὴν βάσιν ἐπὶ τὴν κάθετον,  
 καὶ τοῦ συναχθέντος κούφισον τὸ ζ<sup>ov</sup> καὶ ιδ<sup>ov</sup>, καὶ τὸ λοιπὸν ἔστι τὸ ἐμβαδόν. ἀπὸ δὲ τῆς καθέτου  
 καὶ τῆς περιφερείας τὸ ἐμβαδὸν εὐρεῖν. ποιήσον τὴν κάθετον ἐπὶ τὴν περιφέρειαν, καὶ τὸ τούτου ἔ-  
 λαβὼν ἕξεις τὸ ἐμβαδόν. τὸ ἔτι τῆς καθέτου ποιήσον ἐφ' ὅλην τὴν περιφέρειαν, καὶ ἕξεις τὸ τοῦ ἡμι-  
 κυκλίου ἐμβαδόν. |<sub>20v</sub> τὴν βάσιν ὅλην ἐφ' ὅλην τὴν περιφέρειαν, καὶ τοῦ γινομένου τὸ δ<sup>ov</sup> ἔστι τὸ  
 200 ἐμβαδόν. τὸ ἔτι τῆς βάσεως ἐπὶ τὸ ἔτι τῆς περιφερείας, καὶ τὸ ἀπὸ τούτων ἔστι τὸ ἐμβαδόν. τὸ δ<sup>ov</sup> τῆς  
 περιφερείας ἐπὶ τὴν βάσιν, καὶ ἕξεις τὸ ἐμβαδόν ἀπὸ τούτου. τὴν δὲ περιφέρειαν τοῦ ἡμικυκλίου  
 ἀπὸ τῆς βάσεως {lege καθέτου} εὐρεῖν. ποίει οὕτως. τρίπλωσον τὴν κάθετον, καὶ πρόσθεσθε καὶ τὸ  
 τῆς καθέτου ἕβδομον, καὶ ἕξεις τὴν περιφέρειαν. Ἔτι καὶ ἄλλως. σύνθεσθε βάσιν καὶ κάθετον, καὶ τῶ  
 γινομένῳ ἀριθμῷ προστίθει καθολικῶς τὸ ἴδιον κα<sup>ov</sup>, καὶ ἕξεις τὴν περιφέρειαν.

205 <15> Τμήματος δὲ κύκλου ὑποκειμένου καὶ τῆς βάσεως ὑπεστρωμένης καὶ φανεραῦς οὔσης τῆς  
 καθέτου, ἥτις καὶ πρὸς ὀρθᾶς καλεῖται, ἀπὸ τοῦ κέντρου τῆς κορυφῆς ἐπὶ τὴν βάσιν ἀχθείσης καὶ  
 ἐστηριγμένης εὐρεῖν πότερον ἡμικύκλιόν ἐστιν ἢ ἕλαττον ἢ μείζον τοῦ ἡμικυκλίου. εὐρήσεις οὐ-  
 τως. ἐὰν ἢ πρὸς ὀρθᾶς ἴση τῶ ἔτι μέρει τῆς βάσεως τυγχάνῃ, ἡμικύκλιόν ἐστι πληρὲς· ἐὰν δὲ πλεῖον,  
 τοῦ ἡμικυκλίου μείζον· ἐὰν δὲ ἥττον, τοῦ ἡμικυκλίου ἕλασσον.

210 <16> Ἐὰν δὲ ἡ χωρίον τινὸς δοθέντος εὐθυγράμ<μ>ου οἰουδηποτοῦν τούτῳ ἴσον κύκλον ποιῆ-  
 σαι, δεῖ λαβεῖν τὸ ια<sup>ov</sup> μέρος τοῦ ἐμβαδοῦ καὶ τοῦτο ποιῆσαι τεσσαρεσκαίδεκάκις, καὶ τοῦ γενομέ-  
 νου πλευρὰν λαβὼν τετραγωνικὴν τοσοῦτου ἀποφαίνου τὴν τοῦ κύκλου διάμετρον.

Ἰστέον δὲ ὅτι τὸ ἐμβαδὸν τὸ ἀπὸ τῆς διαμέτρου καὶ τῆς περιμέτρου τοῦ κύκλου μετρούμενον  
 ἴσον ἐστὶν ἐμβαδοῖς κύκλων τεσσάρων.

215 <17> Τμήματος κύκλου ἕλαττονος ἡμικυκλίου τὸ ἐμβαδὸν εὐρεῖν. ποίει οὕτως. σύνθεσθε βάσιν  
 καὶ κάθετον· τούτων δὲ τὸ ἔτι ποιήσον ἐπὶ τὴν κάθετον. εἶτα ποιήσον αὐτὴς τὸ ἔτι <τῆς βάσεως> ἐφ'  
 ἑαυτῶ· τούτου δὲ λαβὼν τὸ ιδ<sup>ov</sup> σύνθεσθε τῶ προρρηθέντι ἀριθμῷ, καὶ ὁ ἀπὸ τούτου γινόμενος ἀριθ-  
 μός ἐστι τὸ ἐμβαδὸν τοῦ τοιοῦτου τμήματος. Ἐὰν δὲ θέλῃς καὶ τὴν περίμετρον ἦτοι τὴν περιφέρει-  
 αν τοῦ τοιοῦτου τμήματος εὐρεῖν, ἀρίθμησον τὴν βάσιν εἰς ἑαυτήν. ὁμοίως καὶ τὴν κάθετον· εἶτα  
 220 τὸν ἀπὸ τοῦ πολλαπλασιασμοῦ τῆς καθέτου ἀριθμὸν τετραπλασίασον, καὶ πρόσθεσθε τῶ τῆς βάσεως  
 πολλαπλασιασμῷ, καὶ τοῦ γενομένου λάβε πλευρὰν τετραγωνικὴν, καὶ τῆ εὐρεθείσῃ πλευρᾷ τὸ τῆς  
 καθέτου προσθείς δ<sup>ov</sup> ἕξεις τὴν περιφέρειαν. τμήματος δὲ μείζονος ἡμικυκλίου τὸ ἐμβαδὸν εὐρεῖν.  
 ποίει οὕτως. ἔστω τμήμα μείζον ἡμικυκλίου, οὗ ἢ μὲν βάσις σχοινίων ιβ ἢ δὲ κάθετος θ. εὐρεῖν τὸ  
 225 ἐμβαδὸν αὐτοῦ. ποίει οὕτως. προσαναπληρούσθω διαπαντὸς ἢ κάθετος ἕως οὗ συμπέσῃ τῶ κύκλῳ,  
 καὶ διαιρείτω τὰ τῆς βάσεως σχοινία μέσον· γίνεται ς· ταῦτα ἐφ' ἑαυτὰ· γίνεται λς· ταῦτα μέριξε  
 παρὰ τὴν κάθετον – τουτέστι παρὰ τὸν θ – γίνεται δ· ὥστε ἢ διάμετρος τοῦ ὅλου κύκλου ἐστὶ  
 σχοινίων ιγ· τοῦ γὰρ ἕλασσονος μοδίων ἐστὶ τεσσάρων. Ἐὰν δὲ ἀφέλωμεν ἀπὸ τοῦ κύκλου τὸ  
 ἕλαττον τμήμα, ἕξομεν καὶ τὸ λοιπὸν μέγιστον τμήμα τοῦ κύκλου μεμετρημένον.

230 <18> Τριῶν κύκλων ἀπτομένων ἀλλήλων εὐρεῖν τοῦ μέσου σχήματος τὸ ἐμβαδόν. ποίει οὕτως.  
 μίαν τῶν διαμέτρων λάβε εἰς ἑαυτήν, καὶ τὸν γινόμενον ἀριθμὸν διπλασίασον, καὶ τοῦ ἔτι γενο-  
 μένου λάβε τὸ ιδ<sup>ov</sup>, καὶ ἕξεις τὸ ἐμβαδόν. ἔστωσαν δὲ οἱ κύκλοι ἴσοι.

Τεσσάρων δὲ ἴσων ὁμοίως κύκλων ἀλλήλοις ἐφαπτομένων εὐρεῖν τοῦ μέσου σχήματος τὸ  
 ἐμβαδόν. ποιήσον οὕτως. μίαν τῶν διαμέτρων ἀρίθμησον εἰς ἑαυτήν, καὶ τὸν γινόμενον ἀριθμὸν  
 235 τριπλασίασον, καὶ τοῦ γενομένου λάβε τὸ ιδ<sup>ov</sup>, καὶ ἕξεις τὸ ἐμβαδόν. Ἐὰν ἐντὸς κύκλου βούλη  
 περιγράψαι {lege ἐγγράψαι} τετράγωνον ἰσόπλευρον |<sub>21r</sub> καὶ θέλῃς εὐρεῖν πόσον ἐσεῖται ἐκάστη  
 πλευρά, ποιήσον τὴν διάμετρον τοῦ δοθέντος κύκλου εἰς ἑαυτήν, καὶ τοῦ γενομένου ἀριθμοῦ λάβε  
 τὸ ἔτι, καὶ τὴν τοῦδε λαβὼν πλευρὰν τετραγωνικὴν ἕξεις τὴν τοῦ τετραγώνου πλευρὰν. Ἐὰν δὲ  
 ἐκτὸς κύκλου περιγράψῃς τετράγωνον καὶ βούλη γῶναι τὴν τούτου πλευρὰν, αὐτόθεν ἐσεῖται  
 240 γνῶριμος· ὅση γὰρ ἐστὶν ἢ τοῦ δοθέντος κύκλου διάμετρος, τοσαύτη ἐστὶ καὶ ἢ τοῦ τετραγώνου  
 πλευρά.

Find the area from the circumference only. Do as follows. Measure the circumference into itself; afterwards, seven times too; taking a forty-fourth part of these you will get the area. Find the area from the base and the height. Do as follows. Count the base by the height, and raise  $\frac{1}{7}$  and  $\frac{1}{14}$  of the gathered number, and the remainder is the area. Find the area from the height and the circumference. Do the height by the circumference, and taking  $\frac{1}{2}$  this you will get the area. Do  $\frac{1}{2}$  the height by the whole circumference, and you will get the area of the semicircle. The whole base by the whole circumference, and  $\frac{1}{4}$  of the result is the area.  $\frac{1}{2}$  the base by  $\frac{1}{2}$  the circumference, and what <results> from these is the area.  $\frac{1}{4}$  of the circumference by the base, and you will get the area from this. Find the circumference of the semicircle from the height. Do as follows. Triple the height, and add a seventh of the height too, and you will get the circumference. Further, otherwise too. Compose the base and the height, and to the resulting number add generally its own  $\frac{1}{21}$ , and you will get the circumference.

15. A segment of a circle being supposed, and the base being spread out and evident, the height, which is also called perpendicular, being drawn and firmly set from the centre of the arch to the base, find whether it is a semicircle, or less than, or greater than a semicircle. You will find as follows. Whenever the perpendicular happens to be equal to the  $\frac{1}{2}$  part of the base, it is a full semicircle; whenever it is more, it is greater than a semicircle; whenever it is smaller, it is less than a semicircle.

16. Whenever, any rectilinear region whatsoever being given, one must make a circle equal to this, one must take the  $\frac{1}{11}$  part of the area and do this fourteen times, and taking the square root of the result declare that the diameter of the circle is so much.

One must know that the area measured from the diameter and the circumference of a circle is equal to the areas of four circles.

17. Find the area of a segment of a circle less than a semicircle. Do as follows. Compose the base and the height; do  $\frac{1}{2}$  these by the height. Afterwards, do  $\frac{1}{2}$  <the base> by itself anew; taking  $\frac{1}{14}$  of this compose with the aforementioned number, and the number resulting from this is the area of such a segment. Whenever you also wish to find the circumference, viz. the perimeter, of such a segment, count the base into itself. Similarly the height too; afterwards, quadruple the number from the multiplication of the height, and add <it> to the multiplication of the base, and take the square root of the result, and adding  $\frac{1}{4}$  of the height to the root that has been found you will get the circumference. Find the area of a segment greater than a semicircle. Do as follows. Let there be a segment greater than a semicircle, whose base is 12 schoinia and the height 9. Find its area. Do as follows. Let the height be completed out altogether until it meets the circle, and let it partition the schoinia of the base in the middle; it yields 6; these by themselves; it yields 36; divide these by the height—that is, by 9—; it yields 4; so that the diameter of the whole circle is of 13 schoinia; for <the height> of the lesser <segment> is of four modii. Whenever we remove the lesser segment from the circle, we shall also get that the remaining great segment of the circle turns out to be measured.

18. Three circles touching each other, find the area of the middle figure. Do as follows. Take one of the diameters into itself, and double the resulting number, and take  $\frac{1}{14}$  of what further results, and you will get the area. Let the circles be equal.

Similarly, four equal circles being tangent to each other, find the area of the middle figure. Do as follows. Count one of the diameters into itself, and triple the resulting number, and take  $\frac{1}{14}$  of the result, and you will get the area. Whenever you want to inscribe an equilateral quadrangle inside a circle and wish to find how much each side will be, do the diameter of the given circle into itself, and take  $\frac{1}{2}$  the resulting number, and taking the square root of this here you will get the side of the quadrangle. Whenever you circumscribe an equilateral quadrangle outside a circle and want to know its side, this will be immediately recognised: for as much is the diameter of the given circle, so much is also the side of the quadrangle.

<19> Τραπεζίου ὀρθογωνίου τὸ ἐμβαδὸν εὐρεῖν. ποίει οὕτως. σύνθεσ κορυφήν καὶ βάσιν, καὶ  
 τεμῶν τὸν συντεθέντα ἀριθμὸν μέσον τὸ  $\omega$  τούτου πολλαπλασίασον ἐπὶ τὴν κάθετον (ἤγουν τὴν  
 πρὸς ὀρθάς), καὶ ἕξεις τὸ ἐμβαδόν. καὶ καθόλου δὲ ἐν πᾶσι τοῖς τραπεζίοις ἡ αὐτὴ ἔσται μέθοδος.  
 Καὶ καθ' ἕτερον δὲ τρόπον ἐπιστημονικώτερον καὶ οὕτω ποιήσει τις. ἐὰν γὰρ ἀπὸ τοῦ τυχόντος  
 245 τραπεζίου ἐξέλῃς τετράγωνον, τὸ καταλειφθὲν πάντως εἰς ἓν τῶν τριγῶνων περισταθήσεται, οὗ  
 γενομένου δῆλον ἐσεῖται πᾶσι τὸ ἐμβαδόν· εἰ γὰρ ἐν μέρει μὲν λάβῃς τοῦ τετραγώνου ἐν μέρει δὲ  
 αὐθις τοῦ τριγώνου, τὸ ἐξ ἀμφοτέρων συναγόμενον ἐσεῖται τοῦ τραπεζίου τὸ ἐμβαδόν. τὰς δὲ  
 τούτων καθέτους καὶ τὰς ὑποτείνουσας εὐρήσεις ὡς ἐν ταῖς προλαβούσαις μεθόδοις τῶν τριγῶνων  
 εἰρήκαμεν.

250 <20> πολυπλεύρων δὲ καὶ πολυγωνίων τὸ ἐμβαδὸν εὐρεῖν. ποίει οὕτως.

Πενταγώνου ἰσοπλεύρου τὸ ἐμβαδὸν εὐρεῖν. ποίει οὕτως. πολλαπλασίασον μίαν τῶν πλευρῶν  
 εἰς ἑαυτήν, καὶ τὸν γενόμενον ἀριθμὸν δωδεκαπλασίασον, καὶ τοῦ γενομένου μέρος λαβὼν ζ<sup>ov</sup>  
 ἕξεις τοῦ πενταγώνου τὸ ἐμβαδόν. Ἐξαγώνου ἰσοπλεύρου τὸ ἐμβαδὸν εὐρεῖν. ποίει οὕτως.  
 μέτρησον μίαν τῶν πλευρῶν εἰς ἑαυτήν, καὶ τοῦ γενομένου λάβε τὸ γ<sup>ov</sup> καὶ τὸ ι<sup>ov</sup>, ὅπερ ἐξαπλα-  
 255 σιάσας εὐρήσεις τὸ τοῦ ἑξαγώνου ἐμβαδόν. Ἑπταγώνου ἰσοπλεύρου τὸ ἐμβαδὸν εὐρεῖν. μίαν τῶν  
 πλευρῶν ἀρίθμησον εἰς ἑαυτήν, καὶ τὸν γινόμενον ἀεὶ ἀρίθμει ἐπὶ τὸν μγ, καὶ τοῦ γενομένου λαβὼν  
 μέρος ιβ<sup>ov</sup> ἕξεις τοῦ ἑπταγώνου τὸ ἐμβαδόν. Ὀκταγώνου ἰσοπλεύρου τὸ ἐμβαδὸν εὐρεῖν. ποίει  
 οὕτως. πολλαπλασίασον μίαν τῶν πλευρῶν εἰς ἑαυτήν, καὶ τὸν γενόμενον ἀρίθμησον ἐπὶ τὸν κθ·  
 τούτου δὲ τὸ ζ<sup>ov</sup> λαβὼν ἕξεις τοῦ ὀκταγώνου τὸ ἐμβαδόν. Ἐννεαγωνίου ἰσοπλεύρου τὸ ἐμβαδὸν  
 260 εὐρεῖν. ποιήσον μίαν τῶν πλευρῶν εἰς ἑαυτήν, καὶ τὸν γενόμενον μέτρησον ἐπὶ τὸν να, ὧν μέρος  
 η<sup>ov</sup> ἔστι τὸ ἐμβαδόν. Δεκαγωνίου ἰσοπλεύρου τὸ ἐμβαδὸν εὐρεῖν. λάβε μίαν τῶν πλευρῶν εἰς ἑαυ-  
 τήν, καὶ τὸν γενόμενον ἀρίθμησον ἐπὶ τὸν ιε, καὶ τοῦ ἔτι γενομένου τὸ  $\omega$  λαβὼν ἕξεις τὸ ἐμβαδόν  
 τοῦ δεκαγωνίου. Ἐνδεκαγωνίου ἰσοπλεύρου τὸ ἐμβαδὸν εὐρεῖν. ἀρίθμησον μίαν τῶν πλευρῶν εἰς  
 ἑαυτήν, καὶ τὸν γενόμενον ποιήσον ἐπὶ τὸν ζς· τούτου δὲ τὸ ζ<sup>ov</sup> ἔσται τὸ ἐμβαδόν. Δωδεκαγώνου  
 265 ἰσοπλεύρου τὸ ἐμβαδὸν εὐρεῖν. πολλαπλασίασον μίαν τῶν πλευρῶν ἐφ' ἑαυτήν, καὶ τὸν γενόμενον  
 ἐπὶ τὸν με· τούτων δὲ λαβὼν τὸ δ<sup>ov</sup> ἕξεις τὸ ἐμβαδόν.

<21> Ὅσα δὲ τῶν πολυπλεύρων καὶ πολυγωνίων σχημάτων οὐκ εἰσὶν ἰσόπλευρα καὶ ἰσογώνια  
 ἀλλὰ ἄνισα, ταῦτα εἰς τρίγωνα κατατεμνόμενα καὶ διαιρούμενα καταμετρεῖται, καὶ ἀσφαλῶς τὸ  
 τούτων διαγινώσκεται ἐμβαδόν. Δοθέντος χωρίου ἄνισα πλάτη ἔχοντος καὶ εἰς πολλαπλάσιον μῆ-  
 270 κος ἔκτεινομένου εὐρεῖν τούτου τὸ ἐμβαδόν κατὰ Πατρίκιον. {*marg.* ση(μείωσαι)} σύνθεσ τὰ πλά-  
 τη ὅσαπερ εὔροις, εἴτε τρία εἰσὶν εἴτε δ' εἴτε πλείονα, καὶ τοσοῦτον μέρος ἀπὸ τούτων λαβὼν κατὰ  
 τὸν τῆς συνθέσεως λόγον ἀρίθμει |<sub>21v</sub> τούτου ἐπὶ τὸ μῆκος, καὶ τὸ γινόμενον ἔσται τοῦ χωρίου τὸ  
 ἐμβαδόν· ἤγουν εἰ μὲν τρία συνθήσεις, λάβε τὸ γ<sup>ov</sup>, εἰ δὲ δ', τὸ δ<sup>ov</sup>, εἰ δὲ ε', τὸ ε<sup>ov</sup>, καὶ ἐξῆς ὁμοίως  
 κατὰ τὸν αὐτὸν λόγον.

275 <22> Ἰστέον δὲ ὅτι τῶν ια τετραγῶνων ἰσοπλεύρων τὸ ἐμβαδὸν ιδ<sup>ovv</sup> ποιεῖ κύκλων ἐμβαδόν· τὰ  
 ιγ ἰσόπλευρα τετράγωνα λ τρίγωνα ποιοῦσιν ἰσόπλευρα· τὰ δὲ ε τετράγωνα, γ πεντάγωνα· τὰ δὲ ιγ  
 τετράγωνα, ε ἑξάγωνα· τὰ μγ τετράγωνα, ιβ ἑπτάγωνα· τὰ κθ τετράγωνα, ἕξ ὀκτάγωνα· τὰ να τε-  
 τράγωνα, ι ἑννεαγώνια· τὰ ιε τετράγωνα, δύο δεκάγωνα, καὶ ἄλλως δὲ πάλιν ἀκριβέστερον· τὰ λη  
 τετράγωνα, ε δεκάγωνα· τὰ ζς τετράγωνα, ζ ἑνδεκαγώνια· τὰ δὲ με τετράγωνα, δ δωδεκάγωνα.  
 280 ταῦτα Ἀρχιμήδης ἀπέδειξεν ὁ μηχανικώτατος.

<23> Ταῦτα μὲν οὖν τὰ εἶδη καὶ τὰ θεωρήματα ὅσον ἐπὶ τῶν ἐμβαδομετρικῶν ἐπιπέδων· ἐπὶ δὲ  
 τῶν στερεῶν προστιθεμένου ἐκάστη μετρήσει καὶ τοῦ πάχους ἐξαίρετα γίνονται θεωρήματα. Εἰσὶ  
 δὲ στερεῶν εἶδη δέκα· σφαῖρα, κῶνος, ὀβελίσκος, κύλινδρος, κύβος, σφηνίσκος, μείουρος, κίων,  
 285 πλινθὶς καὶ πυραμὶς. τὰ δὲ μέτρα κᾶν τοῖς στερεοῖς τὰ αὐτὰ μέλλεις χρῆσθαι ἃ καὶ ἐν τῇ τῶν ἐπι-  
 πέδων ἀρχῇ ἐδηλώσαμεν. ὁ γοῦν διὰ τῆς ἡμετέρας χειρὸς ἐν τετραγώνῳ στερεὸς παλαιστῆς ἔλκει  
 σίτου καθαροῦ λίτραν α καὶ ω, κριθῆς δὲ λίτραν α καὶ {*sp.* 2 *litt.*}, καὶ κέγγρου λίτραν α καὶ ἐξά-  
 για λη. {6 *versus vacui*}

19. Find the area of a right-angled trapezium. Do as follows. Compose the upper side and the base, and cutting the composed number in the middle multiply  $\frac{1}{2}$  this by the height (namely, the perpendicular), and you will get the area. And in general the procedure will be the same for all trapezia. Someone will also do in another, more scientific, way as follows. In fact, whenever you remove a quadrangle out of a trapezium taken at random, what is left over will in every instance turn out as one of the triangles; this coming about the area will be clear to everyone; for, if you take care respectively of the quadrangle and of the triangle anew, what is gathered from both will be the area of the trapezium. You will find the heights and the hypotenuses of these as we have said in the procedures for the triangles taken above.

20. Find the area of polylaterals and polygons. Do as follows.

Find the area of an equilateral pentagon. Do as follows. Multiply one of the sides into itself, and duodecuple the resulting number, and taking a  $\frac{1}{7}$  part of the result you will get the area of the pentagon. Find the area of an equilateral hexagon. Do as follows. Measure one of the sides into itself, and take  $\frac{1}{3}$  and  $\frac{1}{10}$  of the result; sextupling exactly this you will get the area of the hexagon. Find the area of an equilateral heptagon. Count one of the sides into itself, and always count the result by 43, and taking a  $\frac{1}{12}$  part of the result you will get the area of the heptagon. Find the area of an equilateral octagon. Do as follows. Multiply one of the sides into itself, and count the result by 29; taking  $\frac{1}{6}$  of this you will get the area of the octagon. Find the area of an equilateral enneagon. Do one of the sides into itself, and measure the result by 51, a  $\frac{1}{8}$  part of which is the area. Find the area of an equilateral decagon. Take one of the sides into itself, and count the result by 15, and taking  $\frac{1}{2}$  what further results you will get the area of the decagon. Find the area of an equilateral hendecagon. Count one of the sides into itself, and do the result by 66;  $\frac{1}{7}$  of this will be the area. Find the area of an equilateral dodecagon. Multiply one of the sides into itself, and the result by 45; taking  $\frac{1}{4}$  of these you will get the area.

21. All those of the polylateral and polyangular figures that are not equilateral and equiangular but unequal, these will be measured out once cut out and partitioned into triangles, and the area of these will be faultlessly determined. A region that has unequal widths and extending into a multiple length being given, find the area of this according to Patrikios. Compose the widths, exactly as many as you find, whether they are three or 4 or more, and taking so much a part from these as according to the rule of composition count this by the length, and the result will be the area of the region; namely, if you will compose three, take  $\frac{1}{3}$ , if 4,  $\frac{1}{4}$ , if 5,  $\frac{1}{5}$ , and similarly in succession according to the same rule.

22. One must know that the area of 11 equilateral quadrangles makes the area of 14 circles; 13 equilateral quadrangles make 30 equilateral triangles; 5 quadrangles, 3 pentagons; 13 quadrangles, 5 hexagons; 43 quadrangles, 12 heptagons; 29 quadrangles, six octagons; 51 quadrangles, 10 enneagons; 15 quadrangles, 2 decagons, and again, otherwise, more exactly: 38 quadrangles, 5 decagons; 66 quadrangles, 3 hendecagons; 45 quadrangles, 4 dodecagons. Archimedes the most-clever proved these.

23. Then, these are the species and the ultimate species as for the planimetric surfaces; as for the solids, once the thickness is also added to each measurement, there comes to be the specific ultimate species. There are ten species of solids: sphere, cone, obelisk, cylinder, cube, wedge, meiuoros, column, brick, and pyramid. In the case of solids too you should use the same units of measurement as those we also highlighted at the beginning for the planes. Then, the solid palm in a quadrangle through our hand really drags 1 and  $\frac{2}{3}$  pound of winnowed grain, 1 and [...] pound of barley, and 1 pound and 38 hexagia of millet.



## &lt;24&gt; περι στερεομετρίας

{*marg.* περι σφαίρας} Ἄλλ' ἐπὶ τὸ ἐμβαδὸν τῶν στερεῶν χωρήσωμεν. σφαίρας τὸ ἐμβαδὸν  
 290 εὐρεῖν. ποιεῖ οὕτως. τὴν διάμετρον ἐφ' ἑαυτήν· καὶ τὸν γενόμενον αὐθις ἀριθμὸν ἐπὶ τὸν ια· εἶτα  
 τοῦ γενομένου λάβε τὸ ιδ<sup>ov</sup>, καὶ πολλαπλασίασον αὐτὸ ἐπὶ τὸ ὕψος τῆς σφαίρας· {*sp.* 15 *litt.*}  
 {*marg.* ση(μείωσαι)} τούτων δὲ ἄ και ζ<sup>ov</sup> μέρος λαβὼν ἕξεις τῆς σφαίρας τὸ στερεόν, ἦτοι τὸ  
 ἐμβαδὸν τῆς χωρήσεως. Ἄλλως εὐρεῖν τὸ στερεόν. ποιήσον οὕτως. λάβε κυβικῶς τὸν τῆς σφαίρας  
 ἄξωνα – τουτέστι τὴν διάμετρον –, καὶ τὸν ἀπὸ τούτου γενόμενον ἀριθμὸν ποιήσον ια<sup>ic</sup>, ὧν μέρος  
 295 λαβὼν κα<sup>ov</sup> ἕξεις τῆς σφαίρας τὸ στερεόν. τὴν δὲ {*marg.* τουτέστι τὸ ἐμβαδὸν τῆς ὅλης ἐπιφανείας}  
 ἐπιφάνειαν εὐρεῖν. ποιήσον οὕτως. ἀρίθμησον εἰς ἑαυτὴν τὴν διάμετρον, καὶ τὸν γενόμενον ἀριθ-  
 μὸν ποιήσον τετράκις, καὶ τὸν ἔτι γενόμενον ποιήσον ἑνδεκάκις· τούτων δὲ μέρος λαβὼν ιδ<sup>ov</sup> ἕξεις  
 τὴν ἐπιφάνειαν. Ἔτι καὶ ἄλλως τὴν ἐπιφάνειαν εὐρεῖν. †διπλασίασον τὴν διάμετρον, καὶ τὸν γενό-  
 μενον λάβε δεκάκις καὶ ἄπαξ, † καὶ ἕξεις τὸ τῆς |<sub>22r</sub> ἐπιφανείας ἐμβαδὸν – τουτέστιν ὅλην τὴν ἐπι-  
 300 φάνειαν. {*marg.* ση(μείωσαι)} Ἔτι καὶ ἄλλως τὴν ἐπιφάνειαν εὐρεῖν. ποιήσον τὴν διάμετρον ἐφ'  
 ἑαυτήν, καὶ τὸν γενόμενον αὐθις ἐπὶ τὸν μδ, ὧν μέρος ιδ<sup>ov</sup> ἔσται ἡ ἐπιφάνεια. ἀπέδειξε δὲ ὁ Ἀρχι-  
 μῆδης ὅτι ἡ ἐπιφάνεια τῆς σφαίρας {*sp.* 8 *litt.*} [τετρα]πλασίον ἔστιν ἑνὸς τῶν μεγίστων κύκλων  
 τοῦ διαιροῦντος {*sp.* 7 *litt.*} [δίχα] τὴν σφαῖραν. μέγιστος δὲ κύκλος ἔστιν ὁ τὸ αὐτὸ κέντρον ἔχων  
 τῆς σφαίρας. ἡ δὲ διάμετρος τῆς σφαίρας εὐρεθήσεται κατὰ τὴν προγραφείσαν ἐν τοῖς κύκλοις  
 305 ἔφοδον· ἑνὸς γὰρ τῶν ἐν τῇ σφαίρα μεγίστων κύκλων τῆς περιφερείας τὸ κβ<sup>ov</sup> ἀφελὼν τοῦ λοιποῦ  
 λάβε τὸ γ<sup>ov</sup>, καὶ ἕξεις τὴν διάμετρον. Ἡμισφαιρίου τὸ στερεὸν εὐρεῖν. ποιεῖ οὕτως. κύβισον τὴν  
 διάμετρον· καὶ τὸν γενόμενον ια<sup>ic</sup>, ὧν τὸ μβ<sup>ov</sup> ἔστι τοῦ ἡμισφαιρίου τὸ στερεόν. τὴν δὲ ἐπιφάνειαν  
 εὐρεῖν οὕτως. τὴν διάμετρον ἐφ' ἑαυτήν· καὶ τὸν γενόμενον εικοσάκις καὶ δὶς· τούτων δὲ λαβὼν  
 μέρος ιδ<sup>ov</sup> ἕξεις τὴν ἐπιφάνειαν. Ἔτι καὶ ἄλλως. †δίπλωσον τὴν κάθετον, καὶ τὸν ἀπὸ τοῦ διπλα-  
 310 σιασμοῦ γενόμενον ἑνδεκάκις λαβὼν ἕξεις τὴν ἐπιφάνειαν. † τμήματος δὲ ἠττονος ἡμισφαιρίου τὸ  
 στερεὸν εὐρεῖν. ποιεῖ οὕτως. τὸ ἄ τῆς βάσεως μέτρησον ἐφ' ἑαυτό· καὶ τὸν γενόμενον τρισσάκις.  
 εἶτα τὴν κάθετον ἐφ' ἑαυτήν· καὶ τὸν γενόμενον σύνθεσ τῷ προτέρῳ ἀριθμῷ· εἶτα τὸν ὅλον  
 ἀριθμὸν μετρήσας ἐπὶ τὴν κάθετον αὐθις τὸν ἀπὸ τοῦδε γενόμενον ἑνδεκαπλασίασον, καὶ τοῦ  
 γενομένου μέρος λαβὼν κβ<sup>ov</sup> {*lege* κα<sup>ov</sup>} ἕξεις τὸ τοῦ τμήματος στερεόν. τεταρτημορίου δὲ  
 315 σφαίρας τὸ στερεὸν εὐρεῖν οὕτως. κύβισον τὴν βάσιν ἢ τὴν κάθετον τοῦ τεταρτημορίου – ταῦτόν  
 γὰρ ἔστι – καὶ τὰ γενόμενα δὶς· εἶτα ἑνδεκάκις· τούτων δὲ μέρος κβ<sup>ov</sup> {*lege* κα<sup>ov</sup>} ἔστι τοῦ  
 τεταρτημορίου τὸ στερεόν.

<25> {*marg.* περι κώνου} Κώνου τὸ στερεὸν εὐρεῖν. ποιεῖ οὕτως. λάβε τὴν διάμετρον τοῦ κώ-  
 νου καὶ τὴν περίμετρον, καὶ εὐρὲ τοῦ κύκλου τὸ ἐμβαδόν· εἶτα τὸ γ<sup>ov</sup> τῆς καθέτου τοῦ κώνου λα-  
 320 βὼν (ἡγουν τὴν ἀπὸ τῆς κορυφῆς τοῦ κώνου μέχρι τῆς περιφερείας διήκουσαν εὐθεῖαν) καὶ ἐπὶ τὸ  
 τοῦ κύκλου ποιήσας ἐμβαδὸν τὸ στερεὸν εὐρήσεις τοῦ κώνου. Ἔτι κώνου τὴν κάθετον καὶ τὸ στε-  
 ρεὸν εὐρεῖν. λάβε τῆς διαμέτρου τὸ ἄ ἐφ' ἑαυτό. καὶ μίαν ἀπὸ τῶν κλιμάτων πλευρὰν τοῦ κώνου  
 ὡσαύτως ἐφ' ἑαυτήν· καὶ ἀπὸ τοῦ γενομένου ἐκ ταύτης ἀριθμοῦ ἄρον τὸν εὐρεθέντα ἀριθμὸν ἀπὸ  
 τοῦ ἄ τῆς βάσεως· τῶν δὲ ἑναπολειφθέντων πλευρὰν λαβὼν τετραγωνικὴν ἕξεις τὴν κάθετον τοῦ  
 325 κώνου. τὸ δὲ ἐμβαδὸν τοῦ κώνου εὐρήσεις ὡς ἀνωτέρω εἴρηται. ἀπέδειξε δὲ ὁ Ἀρχιμήδης ὅτι πᾶσα  
 σφαῖρα ω μέρος ἔστι τοῦ περιλαμβάνοντος αὐτὴν κυλίνδρου – τουτέστι τοῦ τὴν αὐτὴν βάσιν καὶ  
 τὸ αὐτὸ ὕψος ἔχοντος {ἴσα}. Καὶ πᾶς κῶνος τρίτον μέρος ἔστι κυλίνδρου τοῦ τὴν αὐτὴν βάσιν  
 ἔχοντος αὐτῷ καὶ τὸ ὕψος ἴσον. Εἰ οὖν θέλεις σφαίρας εὐρεῖν τὸ στερεὸν τῆς ἐχούσης, ὡς εἴρηται,  
 τὴν βάσιν καὶ τὸ ὕψος ἴσον τῷ κυλίνδρῳ, λάβε τὸ ω τοῦ στερεοῦ τοῦ κυλίνδρου, καὶ ἕξεις τῆς  
 330 σφαίρας τὸ στερεόν. τὸ δὲ γ<sup>ov</sup> ὁμοίως τοῦ τοιοῦτου κυλίνδρου λαβὼν εὐρήσεις τοῦ κώνου τὸ στε-  
 ρεὸν διὰ τὴν εἰρημένην αἰτίαν. Κολούρου δὲ κώνου – τουτέστιν ἀτελέστου – τὸ στερεὸν εὐρεῖν.  
 ποιεῖ οὕτως. σύνθεσ τὰς δύο διαμέτρους – τουτέστι τὴν |<sub>22v</sub> μείζονα καὶ τὴν ἐλάσσονα – τούτου δὲ  
 τὸ ἄ ἔσται ἡ τοῦ περιλαμβάνοντος αὐτὴν κύκλου διάμετρος· ἐξῆς εὐρίσκων τοῦ κύκλου τὸ  
 ἐμβαδὸν ποιήσον αὐτὸ ἐπὶ τὸ μῆκος, καὶ ἕξεις τοῦ κώνου τὸ στερεόν.

## 24. On stereometry

On the sphere. But let us proceed to the content of the solids. Find the content of the sphere. Do as follows. The diameter by itself; and the newly resulting number by 11; afterwards, take  $\frac{1}{14}$  of the result, and multiply it by the height of the sphere; taking the  $\frac{1}{2}$  and  $\frac{1}{6}$  part of these you will get the volume of the sphere, viz. the content of the span. Find the volume otherwise. Take cubically the axis of the sphere—that is, the diameter—and do the number resulting from this 11 times; taking a  $\frac{1}{21}$  part of this you will get the volume of the sphere. Find the surface {*marg.* that is, the area of the whole surface}. Do as follows. Count the diameter into itself, and do the resulting number four times, and the further resulting number eleven times; taking the  $\frac{1}{14}$  part of these you will get the surface. Further, find the surface otherwise too. Double the diameter, <and do the result into itself,> and take the result ten times and once, and <taking the  $\frac{1}{14}$  part of these> you will get the area of the surface—that is, the whole surface. Further, find the surface otherwise too. Do the diameter by itself, and the result by 44 anew, a  $\frac{1}{14}$  part of which will be the surface. Archimedes proved that the surface of the sphere is [four] times one of the great circles which partition the sphere [in half]. A great circle is the one having the same centre as the sphere. The diameter of the sphere will be found according to the algorithm written above for the circles, for removing  $\frac{1}{22}$  of the circumference of one of the great circles in the sphere take  $\frac{1}{3}$  of the remainder, and you will get the diameter. Find the volume of a hemisphere. Do as follows. Cube the diameter; and the result 11 times,  $\frac{1}{42}$  of which is the volume of the hemisphere. Find the surface as follows. The diameter by itself; and the result twenty times and twice; taking a  $\frac{1}{14}$  part of these you will get the surface. Further, otherwise too. Double the height, <and do the result into itself,> and taking eleven times the <number> resulting from doubling <and further taking the  $\frac{1}{28}$  part of these> you will get the surface. Find the volume of a segment smaller than a hemisphere. Do as follows. Measure  $\frac{1}{2}$  the base by itself; and the result three times. Afterwards, the height by itself; and compose the result with the previous number; afterwards, measuring anew the whole number by the height undecuple the <number> resulting from this here, and taking a  $\frac{1}{21}$  part of the result you will get the volume of the segment. Find the volume of a quadrant of a sphere as follows. Cube the base or the height of the quadrant—for it amounts to the same—; and what results twice; afterwards, eleven times; a  $\frac{1}{21}$  part of these is the volume of the quadrant.

25. On the cone. Find the volume of a cone. Do as follows. Take the diameter and the circumference of the cone, and find the area of the circle; afterwards, taking a  $\frac{1}{3}$  of the height of the cone (namely, the straight line reaching from the apex of the cone as far as the circumference) and doing <it> by the area of the circle you will get the volume of the cone. Further, find the height and the volume of the cone. Take  $\frac{1}{2}$  the diameter by itself. And likewise, one side on the slope of the cone by itself; and, from the number resulting from this, take up the number found from  $\frac{1}{2}$  the base; taking the square root of what has been left out you will get the height of the cone. You will find the content of the cone as said above. Archimedes proved that every sphere is  $\frac{2}{3}$  part of the cylinder that comprehends it—that is, the one having the same base and the same height. And every cone is a third part of the cylinder that has the same base as it and an equal height. Then, if you wish to find the volume of a sphere that has, as said, the base and the height equal to <the ones of> a cylinder, take  $\frac{2}{3}$  of the volume of the cylinder, and you will get the volume of the sphere. Similarly, taking  $\frac{1}{3}$  of such a cylinder you will find the volume of the cone because of the said reason. Find the volume of a truncated—that is, without the end—cone. Do as follows. Compose the two diameters—that is, the greater and the lesser—;  $\frac{1}{2}$  this will be the diameter of the circle that comprehends it; next, finding the area of the circle do it by the length, and you will get the volume of the cone.

- 335 <26> {*marg.* περί ὀβελίσκου} Ὀβελίσκου ἔχοντος ἐν τῇ βάσει κύκλον εὐρεῖν αὐτοῦ τὴν κάθετον. ποίει οὕτως. λάβε τὸ  $\sphericalangle$  τῆς βάσεως ἐφ' ἑαυτό, καὶ μίαν τῶν τοῦ ὀβελίσκου πλευρῶν ὡσαύτως ἐφ' ἑαυτήν, ἐξ ὧν ἀφελε τὸν συναχθέντα ἀριθμὸν ἀπὸ τοῦ  $\sphericalangle$  τῆς βάσεως, καὶ τοῦ καταλειφθέντος πλευρᾶν λαβὼν τετραγωνικὴν ἕξεις τὴν τοῦ ὀβελίσκου κάθετον. τὸ δὲ στερεὸν εὐρήσεις κατὰ τὴν τοῦ κώνου ἔφοδον.
- 340 <27> {*marg.* περί κυλίνδρου} Κυλίνδρου τὸ στερεὸν εὐρεῖν. λάβε ἀπὸ τῆς περιφερείας, ὡσπερ δὴ καὶ ἀπὸ τοῦ κύκλου, τὸ ἐμβαδὸν, καὶ τοῦτο πολλαπλασίασον ἐπὶ τὸ μῆκος, καὶ ἕξεις τοῦ κυλίνδρου τὸ στερεόν. τὴν δὲ ἐπιφάνειαν εὐρεῖν. λάβε τῆς διαμέτρου τοῦ κυλίνδρου τὸ {*sp.* 2 *litt.*} [τριπλάσιον] καὶ ζ<sup>ov</sup>, καὶ πολλαπλασίασον αὐτὸ ἐπὶ τὸ μῆκος αὐτοῦ, καὶ ἕξεις τὴν ἐπιφάνειαν.
- 345 <28> {*marg.* περί κύβου} Κύβον μετρήσαι – τουτέστι σχῆμα τετράγωνον στερεὸν ὑπὸ τριῶν διαστάσεων περιεχόμενον (ἦγουν μήκους, πλάτους καὶ ὕψους ἢ βάθους). εὐρεῖν αὐτοῦ τὸ στερεόν. πολλαπλασίασον τὸ μῆκος ἐπὶ τὸ πλάτος, καὶ τὸν γενόμενον αὐθις ἐπὶ τὸ ὕψος ἢ ἐπὶ τὸ βάθος, καὶ τοσοῦτου ἔσται τὸ τοῦ κύβου στερεόν. Ἔτι καὶ ἄλλως εὐρεῖν τὸ τοῦ κύβου στερεόν. ποίει οὕτως, μίαν τῶν πλευρῶν ποιήσον ἐξηκοντάκις· καὶ τὸν ἔτι γενόμενον ἐπὶ τὸ πλάτος· καὶ τὸν αὐθις γενόμενον ἐπὶ τὸ ὕψος ἢ ἐπὶ τὸ βάθος· καὶ τοῦ γενομένου ὕστερον λαβὼν τὸ ζ<sup>ov</sup> ἕξεις τοῦ κύβου τὸ στερεόν. Εἰ εἰς σφαιραν θελήσω κύβον τετράγωνον ἐμβαλεῖν, καὶ βουλευθῶ γνῶναι πόση ἔσται ἐκάστη τοῦ κύβου πλευρά, ποιῶ οὕτως. τὸ  $\sphericalangle$  τῆς διαμέτρου τῆς σφαίρας ποιῶ ἐφ' ἑαυτό, καὶ τὸν γενόμενον διπλασιάζω, οὗ πλευρὰ τετραγωνικὴ ἔσται ἢ τοῦ κύβου πλευρά. τὴν δὲ διαγώνιον τούτου εὐρεῖν. λάβε μίαν τῶν πλευρῶν ἐφ' ἑαυτήν, καὶ τὸν γενόμενον ποιήσον δῖς, οὗ πλευρὰ τετραγωνικὴ ἔσται ἢ τοῦ κύβου διαγώνιος καὶ τῆς σφαίρας διάμετρος.
- 355 <29> {*marg.* περί σφηνίσκου} Σφηνίσκου τὸ στερεὸν εὐρεῖν. ποίει οὕτως. σύνθεσ πλάτος καὶ πάχος, καὶ τὸν γενόμενον μέρισον {*lege* ποιήσον} ἐπὶ τὸ πάχος, καὶ τὸν αὐθις γενόμενον ἐπὶ τὸ μῆκος – τουτέστιν ἐπὶ τὸ ὕψος – τοῦ δὲ γενομένου τέταρτον μέρος λαβὼν ἕξεις τοῦ σφηνίσκου τὸ στερεόν. Ἔτι καὶ ἄλλως. πολλαπλασίασον τὸ πάχος ἐπὶ τὸ πλάτος, καὶ τὸν γενόμενον αὐθις ἐπὶ τὸ μῆκος, οὗτινος τὸ  $\sphericalangle$  ἔσται τὸ στερεὸν τοῦ σφηνίσκου.
- 360 <30> {*marg.* περί μειούρου} Μειούρου τὸ στερεὸν εὐρεῖν. ποίει οὕτως. ἀρίθμησον τὸ πλάτος ἐπὶ τὸ πάχος, καὶ τοῦ γενομένου λαβὼν τὸ  $\sphericalangle$  ποιήσον ἐπὶ τὸ μῆκος, καὶ ἕξεις τὸ στερεόν. Ἔτι καὶ ἄλλως. μέτρησον τὸ μείζον πλάτος εἰς ἑαυτό. καὶ τὸ ἔλασσον ὁμοίως εἰς ἑαυτό· εἶτα σύνθεσ ὁμοῦ τοὺς ἀριθμοὺς, καὶ τεμῶν αὐτοὺς μέσον πολλαπλασίασον τὸ  $\sphericalangle$  ἐπὶ τὸ μῆκος, καὶ εὐρήσεις τὸ στερεόν.
- 365 <31> {*marg.* περί κίονος} Κίονος εὐρεῖν τὸ στερεόν. ποίει οὕτως. σύνθεσ τὰς β διαμέτρους (ἦτοι τῆς ἑδρας καὶ τῆς ἐφέδρας), καὶ τεμῶν μέσον εὐρὲ τὴν ἀναλογοῦσαν τῇ τοιαύτῃ διαμέτρῳ περιφέρειαν, καὶ ποιήσας κύκλου ἐμβαδὸν ἀπὸ τούτων πολλαπλασίασον τὸ τοιοῦτον ἐμβαδὸν ἐπὶ τὸ μῆκος τοῦ κίονος, καὶ ἕξεις τὸ τούτου στερεόν. |<sub>23r</sub> τὴν δὲ ἐπιφάνειαν εὐρεῖν οὕτως. λάβε ἑδρας καὶ ἐφέδρας τοὺς κύκλους – τουτέστι τῆς μείζονος καὶ τῆς ἥττονος περιφερείας –, καὶ συνθεῖς ταύτας ὁμοῦ λάβε τούτων τὸ  $\sphericalangle$ , ὃ καὶ ψήφισον ἐπὶ τὸ μῆκος, καὶ ἕξεις τὴν τοῦ κίονος ἐπιφάνειαν.
- 370 <32> {*marg.* περί πλινθίδος} Πλινθίδος τὸ στερεὸν οὕτως εὐρεθήσεται ὡσπερ ἐπὶ τοῦ κύβου· καὶ τοῦτο γὰρ κύβος παραλληλόγραμμος καὶ ἑτερομήκης λέγεται.
- 375 <33> {*marg.* περί πυραμίδος} Πυραμίδος τὴν τε κάθετον καὶ τὸ στερεὸν εὐρεῖν. ποίει οὕτως. καὶ τὴν μὲν κάθετον εὐρεῖν. πολλαπλασίασον μίαν τῶν πλευρῶν ἐφ' ἑαυτήν, καὶ τὸν γενόμενον διπλώσον· εἶτα τοῦ γενομένου λάβε τὸ δ<sup>ov</sup>. καὶ αὐθις ἀρίθμησον ἐν τῶν κλιμάτων ἐφ' ἑαυτό, καὶ ἐκ τοῦ γενομένου τὸ ῥηθὲν δ<sup>ov</sup> ἀφελὼν τοῦ λοιποῦ λάβε πλευρᾶν τετραγωνικὴν, καὶ ἕξεις τὴν κάθετον. τὸ δὲ στερεὸν εὐρεῖν. ποιήσον τὴν πλευρᾶν ἐφ' ἑαυτήν, καὶ τοῦ γενομένου τὸ γ<sup>ov</sup> λαβὼν ἀρίθμησον ἐφ' ὅλην τὴν κάθετον, καὶ ἕξεις τῆς πυραμίδος τὸ στερεόν.
- 380 Τῶν δὲ πυραμίδων διαφόρων οὐσῶν διάφοροι καὶ αἱ τούτων μετρήσεις εἰσίν. αἱ μὲν γὰρ αὐτῶν ἐπὶ τετραγώνου εἰσι βεβηκυῖαι, αἱ δὲ ἐπὶ τριγώνου, ἕτεραι δὲ ἐπὶ κύκλου, καὶ ἄλλαι ἐπὶ πολυγωνίων καὶ πολυπλεύρων· καὶ τούτων αὐθις αἱ μὲν εἰς ὄξυ λήγουσιν ὀβελίσκου δίκην, αἱ εἰσι κωνοειδεῖς, ἄλλαι τραπεζοειδεῖς, καὶ ἕτεραι κόλουροι, ὧν ἐκάστης λόγον προσηκόντως ἐκθήσομεν.

26. On the obelisk. An obelisk having a circle in its base, find its height. Do as follows. Take  $\frac{1}{2}$  the base by itself, and likewise one of the sides of the obelisk by itself, from which remove the number gathered from  $\frac{1}{2}$  the base; and taking the square root of what has been left over you will get the height of the obelisk. You will find the volume according to the algorithm of the cone.

27. On the cylinder. Find the volume of a cylinder. Take the area from the circumference, indeed exactly as from the circle too, and multiply this by the length, and you will get the volume of the cylinder. Find the surface. Take [three times] and  $\frac{1}{7}$  the diameter of the cylinder, and multiply it by its length, and you will get the surface.

28. On the cube. Measure a cube—that is, a quadrangular solid figure contained by three dimensions (namely, length, width, and height or depth). Find its volume. Multiply the length by the width, and the result by the height or by the depth anew, and so much will be the volume of the cube. Further, find the volume of the cube otherwise too. Do as follows. Do sixty times one of the sides; and the further resulting <number> by the width; and the new result by the height or by the depth, and taking finally  $\frac{1}{60}$  of the result you will get the volume of the cube. If I will wish to insert a quadrangular cube into a sphere, and wanted to know how much will be each side of the cube, I do as follows. I do  $\frac{1}{2}$  the diameter of the sphere by itself, and double the result, the square root of which will be the side of the cube. Find the diagonal of this. Take one of the sides by itself, and do the result twice, the square root of which will be the diagonal of the cube and the diameter of the sphere.

29. On the wedge. Find the volume of a wedge. Do as follows. Compose width and thickness, and do the result by the thickness, and the new result by the length—that is, by the height—; taking the fourth part of the result you will get the volume of the wedge. Further, otherwise too. Multiply the thickness by the width, and the result by the length anew, whose  $\frac{1}{2}$  will be the volume of the wedge.

30. On the meiouros. Find the volume of a meiouros. Do as follows. Count the width by the thickness, and taking  $\frac{1}{2}$  the result do <it> by the length, and you will get the volume. Further, otherwise too. Measure the greater width by itself. And similarly, the lesser by itself; afterwards, compose the numbers together, and cutting them in the middle multiply their  $\frac{1}{2}$  by the length, and you will get the volume.

31. On the column. Find the volume of a column. Do as follows. Compose the 2 diameters (viz. of the floor and of the ceiling), and cutting <them> in the middle find the circumference associated with such a diameter, and making the area of a circle from these multiply such an area by the length of the column, and you will get the volume of this. Find the surface as follows. Take the circles of the floor and of the ceiling—that is, the circumferences of the greater and of the smaller <circles>—, and composing these together take  $\frac{1}{2}$  these, which compute by the length too, and you will get the surface of the column.

32. On the brick. The volume of a brick will be found exactly in such a way as for the cube; for this is also called a parallelogrammic and oblong cube.

33. On the pyramid. Find both the height and the volume of a pyramid. Do as follows. And find the height. Multiply one of the sides by itself, and double the result; afterwards, take  $\frac{1}{4}$  of the result. And count one of the oblique sides by itself anew, and removing the said  $\frac{1}{4}$  from the result take the square root of the remainder, and you will get the height. Find the volume. Do the side by itself, and taking  $\frac{1}{3}$  of the result count by the whole height, and you will get the volume of the pyramid.

As the pyramids are manifold, the measures of these are manifold too. In fact, some of them happen to stand on a quadrangle, some on a triangle, others on a circle, and others on polygons and polylaterals; and anew, some of these, which are cone-like, end in a point like an obelisk, others are trapezium-like, and others truncated; we shall set out accurately an account of each of these.

AN ANNOTATED PARAPHRASE OF THE *COMPENDIUM*

For every section of the *Compendium*, I shall provide a summary of the contents and point to parallel procedures in Hero's *Metrica* and in other geometric metrological collections; mathematical details are given in the footnotes. The procedures are formalised by means of algorithmic transcriptions, with the conventions listed above. In the algorithms<sup>61</sup>, I kept the distinction made in the text between taking a part of a quantity (which I formalise as a preposed unit fraction) and dividing a quantity by a number (which I formalise as a quotient sign followed by the divisor); the latter is employed whenever the divisor is the value of a geometric object and not a pure number.

As for notation,  $s$  denotes the side of an equilateral and equiangular figure;  $s$  with a subscript numeral is a side of a different ultimate species;  $l, w$  are the two dimensions of a rectangle;  $a, b, c$  the sides of a triangle, and  $h_a$  the height dropped to side  $a$ ;  $b, h, u$  are the base, height, and hypotenuse of a right-angled triangle;  $P_x$  is the projection of side  $x$  of a triangle on a reference side;  $d$  is a diagonal or a diameter;  $d_i$  and  $d_e$  are the diameters of the circle inscribed in and circumscribed to a triangle, respectively;  $p$  is the circumference of a circle;  $h$  is the height of a figure;  $b$  is the base of a segment of a circle or of a sphere;  $k$  is the oblique side of a cone;  $l, w, h$  are the three dimensions of a rectangular parallelepiped;  $A$  is the area of a figure;  $A_x$ , with  $x = 3 \dots 12$ , is the area of an equilateral and equiangular figure;  $V$  is the volume of a figure; subscript  $R$  = rectangle,  $Tr$  = triangle;  $R_h$  = rhombus or rhomboid;  $C$  = circle;  $H$  = semicircle or hemisphere;  $T_i$  and  $T_e$  = squares inscribed in or circumscribed to a circle;  $Tra$  = trapezium;  $S$  = sphere;  $Qu$  = quadrant;  $Co$  = cone;  $Cy$  = cylinder;  $O$  = obelisk;  $K$  = cube;  $Sf$  = wedge;  $M$  = meiouros, that is, a tapering parallelepiped;  $Ki$  = column;  $P$  = pyramid.

1. A series of definitions: reference direction, milestone, lines, angles, genera of measurement, species of measurement (that is, classes of figures: quadrangles, triangles, rhombi, trapezia, and circles), ultimate species (*theōrēmata*, a standard designation in geometric metrology) of figures: two ultimate species of quadrangles, six of triangles, two rhombi, four trapezia, four circles.

This section coincides with *Geom.* 3, 1–4 + 3, 15 + 3, 18 + 3, 22–23 = *Geod.* 3, 1–4 + 3, 15 + 3, 18 + 3, 22–23.

2. List of units of measurement.

This section almost coincides with *Geom.* 23, 4.

3. Conversions between some units of measurement.

This section coincides with *Geod.* 4.

4. Construction of a square by joining the centres of four mutually tangent<sup>62</sup> circles.

Algorithm for finding the area of a square:  $(s) \rightarrow s^2 = A_4$ .

Algorithm for finding the diagonal of a square:  $(A_4) \rightarrow 2A_4 \rightarrow \sqrt{(2A_4)} = d$ .

This basic algorithm for the area is employed throughout *Geom.* 5; the algorithm for the diagonal is as in *Geom.* 5, 3 SV. For the area see also *Geod.* 7.

5. Construction of a rectangle by cutting a square in half.

Algorithm for finding the area of a rectangle:  $(l, w) \rightarrow lw = A_R$ .

Algorithm for finding the diagonal of a rectangle:  $(l, w) \rightarrow l^2 \cdot w^2 \rightarrow \sqrt{(l^2 + w^2)} = d$ .

<sup>61</sup> In this section, the mathematical meaning of “algorithm” is intended throughout.

<sup>62</sup> Strictly speaking, this is false: no four equal circles can be mutually tangent.

The basic algorithm for the area is employed throughout *Geom.* 6 (but not in 6, 2 SV, where we find the surveyor's formula) and 14, 1–2, 7, 10; the algorithm for the diagonal is as in *Geom.* 6, 1 SV. For the area see also *Geod.* 8.

6. Construction of an equilateral triangle by joining the centres of three mutually tangent circles.

Algorithm for finding the area of an equilateral triangle<sup>63</sup>:  $(s) \rightarrow s^2 \rightarrow (\frac{1}{3} + \frac{1}{10})s^2 = A_3$ .

Algorithm for finding the area of a generic triangle:  $(a, h_a) \rightarrow ah_a \rightarrow \frac{1}{2}(ah_a) = A_{Tr}$ .

Algorithm for finding the area of a generic triangle:  $(a, h_a) \rightarrow h_a[\frac{1}{2}(a)] = A_{Tr}$ .

(Heronia) algorithm for finding the area of a generic triangle from its sides only:

$(a, b, c) \rightarrow (a + b + c) \rightarrow \frac{1}{2}(a + b + c) - a. \frac{1}{2}(a + b + c) - b. \frac{1}{2}(a + b + c) - c \rightarrow$   
 $[\frac{1}{2}(a + b + c)][\frac{1}{2}(a + b + c) - a] \rightarrow [\frac{1}{2}(a + b + c)][\frac{1}{2}(a + b + c) - a][\frac{1}{2}(a + b + c) - b] \rightarrow$   
 $\rightarrow [\frac{1}{2}(a + b + c)][\frac{1}{2}(a + b + c) - a][\frac{1}{2}(a + b + c) - b][\frac{1}{2}(a + b + c) - c] \rightarrow$   
 $\rightarrow \sqrt{\{[\frac{1}{2}(a + b + c)][\frac{1}{2}(a + b + c) - a][\frac{1}{2}(a + b + c) - b][\frac{1}{2}(a + b + c) - c]\}} = A_{Tr}$ .

Algorithm for finding the side of an equilateral triangle from its area:

$(A_3) \rightarrow 30A_3 \rightarrow \frac{1}{13}(30A_3) \rightarrow \sqrt{[\frac{1}{13}(30A_3)]} = s$ .

Algorithm for finding the height of an equilateral triangle:  $(s) \rightarrow s^2 \rightarrow s^2 - \frac{1}{4}(s^2) \rightarrow \sqrt{[s^2 - \frac{1}{4}(s^2)]} = h$ .

Algorithm for finding the height of an equilateral triangle:  $(s) \rightarrow s^2 - [\frac{1}{2}(s)]^2 \rightarrow \sqrt{\{s^2 - [\frac{1}{2}(s)]^2\}} = h$ .

Algorithm for finding the height of an equilateral triangle:  $(s) \rightarrow s - (\frac{1}{10} + \frac{1}{30})s = h$ .

Algorithm for inscribing a square in an equilateral triangle<sup>64</sup>:  $(s, h) \rightarrow s + h. sh \rightarrow (sh)/(s + h) = s_4$ .

Construction of an isosceles triangle by cutting a rhombus in half.

The algorithm for finding the area of an equilateral triangle is as in *Geom.* 10, 1–2, 6, 9, *Dioph.* 20, 1, *Geod.* 10, 1–2, 6, 8–9. The algorithms for the area of a generic triangle—or slight variations on them—are employed throughout *Geom.* 11–12 and *Geod.* 10. The Heronian algorithm is validated and formulated in *Metr.* I, 8; it is applied in *Geom.* 12, 30–32 and *Geod.* 12. The inverse of the side-area formula is in *Geom.* 10, 11 and 22, 3, *Dioph.* 20, 2. The three algorithms for finding the height of an equilateral triangle are as in *Geom.* 10, 12, *Dioph.* 20, 3, *Geod.* 10, 11; *Geom.* 10, 13, *Dioph.* 20, 4; *Geom.* 10, 3, 4, *Geod.* 10, 3, 7, respectively. For the last algorithm see *Geom.* 12, 43 and 24, 24–25.

7. Constructions of a right-angled triangle: cut an isosceles triangle in half; cut a square along a diagonal; trace two random diameters in a circle and join their extremities: four right-angled triangles result. Marginal note: denominations of the sides of triangles according to their intended graphical representation in the *Compendium*: in right-angled triangles, the greater leg is set as the base; in scalene triangles, the greater side is set as the base.

Algorithm for finding the area of a right-angled triangle:  $(b, h) \rightarrow bh \rightarrow \frac{1}{2}(bh) \rightarrow A$ .

Algorithm for finding the area of a right-angled triangle:  $(b, h) \rightarrow \frac{1}{2}(b)h \rightarrow A$ .

Algorithm for finding the height of a right-angled triangle:  $(b, u) \rightarrow u^2 \rightarrow u^2 - b^2 \rightarrow \sqrt{(u^2 - b^2)} = h$ .

Algorithm for finding the base of a right-angled triangle:  $(h, u) \rightarrow u^2 - h^2 \rightarrow \sqrt{(u^2 - h^2)} = b$ .

Algorithm for finding the hypotenuse of a right-angled triangle:  $(b, h) \rightarrow b^2. h^2 \rightarrow \sqrt{(b^2 + h^2)} = h$ .

Algorithm for finding the base of a right-angled triangle from the hypotenuse only<sup>65</sup>:

$(u) \rightarrow 4u \rightarrow \frac{1}{5}(4u) = b$ .

Algorithm for finding the height of a right-angled triangle from the hypotenuse only:

$(u) \rightarrow 3u \rightarrow \frac{1}{5}(3u) = h$ .

The basic algorithms for right-angled triangles are employed throughout *Geom.* 7 and *Geod.* 9.

<sup>63</sup> The standard approximation  $\frac{26}{15}$  for  $\sqrt{3}$  is used in this section.

<sup>64</sup> This algorithm applies in general to isosceles triangles.

<sup>65</sup> Despite being formulated as referring to a generic right-angle triangle, this and the subsequent algorithm apply to the  $(3n, 4n, 5n)$  right-angled triangle only.

8. Algorithms for providing the sides of numerical right-angled triangles, according to Pythagoras and to Plato, respectively:

$$(n = 2m + 1 = h) \rightarrow n^2 \rightarrow n^2 - 1 \rightarrow \frac{1}{2}(n^2 - 1) = b. \frac{1}{2}(n^2 - 1) + 1 = u.$$

$$(n = 2m = b) \rightarrow [\frac{1}{2}(n)]^2 \rightarrow [\frac{1}{2}(n)]^2 - 1 = h \rightarrow h + 2 = u.$$

Algorithm for providing the sides of a numerical right-angled triangle from a given number:  $(n) \rightarrow$   
 $\rightarrow 3n = h. 4n = b. 5n = u.$

Criterion for establishing the species of a triangle: if  $b^2 + h^2 = u^2$ , the triangle is right-angled; if  $b^2 + h^2 > u^2$ , it is acute-angled, if  $b^2 + h^2 < u^2$ , it is obtuse-angled.

For the formulas traditionally ascribed to Pythagoras and to Plato see *Geom.* 8, 1 and 9, 1, and *Geod.* 9, 14 and 15, respectively.

9. Obtuse- and acute-angled triangles. Their area is taken by means of the procedures already expounded. Algorithms for finding the greater and the smaller projections on the base of the sides of obtuse- and acute-angled triangles:

$$(a, b, c) \rightarrow a^2 \cdot b^2 \cdot c^2 \rightarrow a^2 + b^2 \rightarrow (a^2 + b^2) - c^2 \rightarrow \frac{1}{2}[(a^2 + b^2) - c^2] \rightarrow \{\frac{1}{2}[(a^2 + b^2) - c^2]\}/a = P_b.$$

$$(a, b, c) \rightarrow \{\frac{1}{2}[(a^2 + c^2) - b^2]\}/a = P_c.$$

Algorithms for finding the height associated with the base of obtuse- and acute-angled triangles:

$$(a, b, c) \rightarrow b^2 \rightarrow b^2 - (P_b)^2 \rightarrow \sqrt{[b^2 - (P_b)^2]} = h_a.$$

$$(a, b, c) \rightarrow c^2 \rightarrow c^2 - (P_c)^2 \rightarrow \sqrt{[c^2 - (P_c)^2]} = h_a.$$

This section is possibly a synthesis of *Geod.* 11, 2–4 (scalene triangle). The same algorithms can be found in *Geom.* 12, 1, 4–6, 9–12, 16, 19–20, and 23–24 (acute-angled).

10. Inscribing and circumscribing circles to triangles.

Algorithm for finding the diameter of the circle inscribed in a triangle<sup>66</sup>:

$$(a, b, c) \rightarrow 4A_{Tr}. a + b + c \rightarrow (4A_{Tr})/(a + b + c) = d_i.$$

Algorithm for finding the diameter of the circle circumscribed to an equilateral triangle:

$$(s, h) \rightarrow s^2 \rightarrow (s^2)/h = d_e.$$

Algorithm for finding the diameter of the circle circumscribed to a triangle<sup>67</sup>:

$$(b, c, h) \rightarrow bc \rightarrow (bc)/h = d_e.$$

The first algorithm is as in *Geom.* 24, 27, 29, 31, 33, 35, and 37; the second, as in *Geom.* 24, 32; the third, as in *Geom.* 24, 34, 36 and 38.

11. Construction of a rhombus by joining two equilateral triangles.

Algorithm for finding the area of a rhombus:  $(d_1, d_2) \rightarrow d_1[\frac{1}{2}(d_2)] = A_{Rh}.$

Algorithm for finding the area of a rhombus:  $(d_1, d_2) \rightarrow d_1 d_2 \rightarrow \frac{1}{2}(d_1 d_2) = A_{Rh}.$

Algorithm for finding the area of a rhombus<sup>68</sup>:  $(s) \rightarrow s^2 \rightarrow s^2 - \frac{1}{25}(s^2) = A_{Rh}.$

The diagonals and the side of the specific rhombus considered here can be found by applying the algorithms expounded for the equilateral triangle.

The first algorithm can be found in *Geom.* 13, 1–3, the second in *Geom.* 13, 2.

<sup>66</sup> This relation is a by-product of the proof that in *Metr.* I, 8 validates the Heronian formula for finding the area of a generic triangle from its.

<sup>67</sup> The algorithm for the equilateral triangle is a particular case of this algorithm, whose relation is easily proved by using similar triangles.

<sup>68</sup> This is inaccurate or corrupt: given the standard formula for the area of an equilateral triangle,  $\frac{1}{25}$  should be replaced by  $\frac{2}{15}$ .

12. Construction of a rhomboid by joining two scalene triangles, or by removing two (equal) right-angled triangles from opposite corners of a rectangle.

Algorithm for finding the area of a rhomboid<sup>69</sup>:  $(s_1, s_2, d_1, d_2) \rightarrow s_1 d_2 = A_{Rh}$ .

Algorithm for finding the area of a rhomboid:  $(s_1, s_2, d_1, d_2) \rightarrow s_2 d_1 = A_{Rh}$ .

The area of a rhomboid can also be found by dividing it into one rectangle and two right-angled triangles, and by measuring each of these.

The diameter of a circle inscribed in a rhombus is equal to the shorter diagonal.

13. The circle.

(a) Algorithm for finding the area of a circle:  $(d, p) \rightarrow dp \rightarrow \frac{1}{4}(dp) = A_C$ .

(b) Algorithm for finding the area of a circle<sup>70</sup>:  $(p) \rightarrow p^2 \rightarrow 7p^2 \rightarrow \frac{1}{88}(7p^2) = A_C$ .

(c) Algorithm for finding the area of a circle:  $(d) \rightarrow d^2 \rightarrow 11d^2 \rightarrow \frac{1}{14}(11d^2) = A_C$ .

(d) Algorithm for finding the area of a circle:  $(d) \rightarrow d^2 \rightarrow d^2 - (\frac{1}{7} + \frac{1}{14})d^2 = A_C$ .

(e) Algorithm for finding the area of a circle:  $(d, p) \rightarrow \frac{1}{2}(d)p \rightarrow \frac{1}{2}[\frac{1}{2}(d)p] = A_C$ .

(f) Algorithm for finding the area of a circle:  $(d, p) \rightarrow \frac{1}{2}(p)d \rightarrow \frac{1}{2}[\frac{1}{2}(p)d] = A_C$ .

(g) Algorithm for finding the area of a circle:  $(d, p) \rightarrow \frac{1}{2}(d)\frac{1}{2}(p) = A_C$ .

(h) Algorithm for finding the area of a circle:  $(d) \rightarrow [\frac{1}{2}(d)]^2 \rightarrow \frac{1}{7}[\frac{1}{2}(d)]^2 \rightarrow 3[\frac{1}{2}(d)]^2 + \frac{1}{7}[\frac{1}{2}(d)]^2 = A_C$ .

(i) Algorithm for finding the area of a circle:  $(d) \rightarrow d^2 \rightarrow d^2 - \frac{1}{4}(d^2) \rightarrow [d^2 - \frac{1}{4}(d^2)] + \frac{1}{21}[d^2 - \frac{1}{4}(d^2)] = A_C$ .

(p) Algorithm for finding the diameter of a circle:  $(p) \rightarrow p - \frac{1}{22}(p) \rightarrow \frac{1}{3}[p - \frac{1}{22}(p)] = d$ .

(q) Algorithm for finding the diameter of a circle:  $(p) \rightarrow \frac{1}{22}(p) \rightarrow 7[\frac{1}{22}(p)] = d$ .

(r) Algorithm for finding the diameter of a circle:  $(p) \rightarrow 7p \rightarrow \frac{1}{22}(7p) = d$ .

(x) Algorithm for finding the circumference of a circle:  $(d) \rightarrow 3d \rightarrow 3d + \frac{1}{7}(d) \rightarrow p$ .

(s) Algorithm for finding the diameter of a circle<sup>71</sup>:

$(p) \rightarrow p^2 \rightarrow p^2 - \frac{1}{22}(p^2) + \frac{1}{3}(p^2) \rightarrow \frac{1}{10}[p^2 - \frac{1}{22}(p^2) + \frac{1}{3}(p^2)] \rightarrow \sqrt{\frac{1}{10}[p^2 - \frac{1}{22}(p^2) + \frac{1}{3}(p^2)]} = d$ .

Algorithms for finding the circumference and the diameter of a circle from their sum<sup>72</sup>:

$(n = d + p) \rightarrow 7n \rightarrow \frac{1}{29}(7n) = d \rightarrow n - d = p$ .

$(n = d + p) \rightarrow 22n \rightarrow \frac{1}{29}(22n) = p \rightarrow n - p = d$ .

(t) Algorithm for finding the diameter of a circle:  $(A) \rightarrow 14A \rightarrow \frac{1}{11}(14A_C) \rightarrow \sqrt{[\frac{1}{11}(14A)]} = d$ .

(y) Algorithm for finding the circumference of a circle:  $(A) \rightarrow 88A \rightarrow \frac{1}{7}(88A_C) \rightarrow \sqrt{[\frac{1}{7}(88A)]} = p$ .

Algorithm for finding the diameter, the circumference, and the area of a circle from their sum<sup>73</sup>:

$(n = d + p + A) \rightarrow 154n \rightarrow 154n + 841 \rightarrow \sqrt{(154n + 841)} \cdot \sqrt{(154n + 841) - 29} \rightarrow \frac{1}{11}[\sqrt{(154n + 841) - 29}] = d$ .

$2[\sqrt{(154n + 841) - 29}] \rightarrow \frac{1}{7}\{2[\sqrt{(154n + 841) - 29}]\} = p \rightarrow \frac{1}{2}(d)\frac{1}{2}(p) = A_C$ .

If a circle is included in another circle, the area between the two is found by the lesser from the greater.

Algorithm (a) can be found in *Geom.* 17, 1, 18; 21, 5; 22, 18; Dioph. 2; (b) in *Geom.* 17, 3, 13, 27, 32–33; 22, 17; (c) in *Geom.* 17, 4, 15, 25–26, 30–31; 21, 25; 22, 15, Dioph. 1, 2; (d) in *Geom.* 17, 5, 14; 24, 40; (e) in *Geom.* 21, 5 C and 6; (g) in *Geom.* 17, 2, 19; 21, 6; 22, 18, Dioph. 2; a slight variant of (h) in *Geom.* 17, 16; (i) in *Geom.* 17, 17; (p) in *Geom.* 17, 11; (q) in *Geom.* 17, 7; (r) in *Geom.* 17, 6, 12; 21, 4, Dioph. 3; (x) in *Geom.* 17, 8, 10, 23, 24, 29; 22, 16; 24, 45, Dioph. 1, 1; a slight variant of (t) in *Geom.* 17, 22 and 21, 8; (y) in *Geom.* 17, 22 and 24, 42. Algorithms (c) and (x) can also be found in *De mens.* 35. The first separation algorithm is as in

<sup>69</sup> This is true only if the diagonal and side are both the shorter ones and if the shorter diagonal is perpendicular to the shorter side; the subsequent algorithm and the final relation are false.

<sup>70</sup> The standard approximation  $\frac{22}{7}$  for the ratio between the circumference and the diameter of a circle is used in this section.

<sup>71</sup> The algorithm is corrupt, and the present wording ambiguous. The fraction of  $p^2$  to be added to  $p^2$  is  $\frac{1}{22}(\frac{3}{11})$ .

<sup>72</sup> There are three separation problems in the *Compendium*. The following two problems are solved by using the additional relation  $p = \frac{22}{7}(d)$ .

<sup>73</sup> This problem of separation amounts to solving for  $d$  the quadratic equation  $d + \frac{22}{7}(d) + (\frac{1}{14})d^2 = n$ , that is, the equation  $11d^2 + 2 \times 29d - 14n = 0$ . The algorithm yields exactly the standard quadratic formula applied to this specific equation.



*Geom.* 22, 18 and 24, 43. The third separation algorithm is a synthesis of *Geom.* 21, 9–10; see also *Geom.* 24, 46, and the exposition in Hero, *Metrica* 495–496. For the last statement compare *Geom.* 21, 1, at Hero, *Opera omnia* IV 374, 3–6. See also the synopsis in Hero, *Metrica* 585–588.

#### 14. The semicircle.

- (a) Algorithm for finding the area of a semicircle:  $(b,p) \rightarrow pb \rightarrow \frac{1}{4}(pb) = A_H$ .  
 (b) Algorithm for finding the area of a semicircle<sup>74</sup>:  $(h,p) \rightarrow hp = A_H$ .  
 (c) Algorithm for finding the area of a semicircle:  $(b) \rightarrow b^2 \rightarrow 11b^2 \rightarrow \frac{1}{28}(11b^2) = A_H$ .  
 (d) Algorithm for finding the area of a semicircle:  $(b) \rightarrow b^2 \rightarrow b^2 - (\frac{1}{7} + \frac{1}{14})b^2 \rightarrow \frac{1}{2}[b^2 - (\frac{1}{7} + \frac{1}{14})b^2] = A_H$ .  
 (e) Algorithm for finding the area of a semicircle:  $(h) \rightarrow h^2 \rightarrow 11h^2 \rightarrow \frac{1}{7}(11h^2) = A_H$ .  
 (f) Algorithm for finding the area of a semicircle:  $(p) \rightarrow p^2 \rightarrow 7p^2 \rightarrow \frac{1}{44}(7p^2) = A_H$ .  
 (g) Algorithm for finding the area of a semicircle:  $(b,h) \rightarrow bh \rightarrow bh - (\frac{1}{7} + \frac{1}{14})b^2 = A_H$ .  
 (h) Algorithm for finding the area of a semicircle:  $(h,p) \rightarrow hp \rightarrow \frac{1}{2}(hp) = A_H$ .  
 (i) Algorithm for finding the area of a semicircle:  $(h,p) \rightarrow [\frac{1}{2}(h)]p = A_H$ .  
 (j) Algorithm for finding the area of a semicircle:  $(b,p) \rightarrow bp \rightarrow \frac{1}{4}(bp) = A_H$ .  
 (k) Algorithm for finding the area of a semicircle:  $(b,p) \rightarrow \frac{1}{2}(b)\frac{1}{2}(p) = A_H$ .  
 (l) Algorithm for finding the area of a semicircle:  $(b,p) \rightarrow \frac{1}{4}(p)(b) = A_H$ .  
 (x) Algorithm for finding the circumference of a semicircle:  $(h) \rightarrow 3h \rightarrow 3h + \frac{1}{7}(h) = p_H$ .  
 (y) Algorithm for finding the circumference of a semicircle:  $(b,h) \rightarrow b + h \rightarrow b + h + \frac{1}{21}(b + h) = p_H$ .

This section follows the plan of *Geom.* 18: algorithm (a) can be found in *Geom.* 18, 1; (c) in *Geom.* 18, 4 and 16; (d) in *Geom.* 18, 5; (e) in *Geom.* 18, 6; (f) in *Geom.* 18, 7 and 22, 22; (g) in *Geom.* 18, 8; (h) in *Geom.* 18, 10; (i) in *Geom.* 18, 11; (j) in *Geom.* 18, 12 and 15; (k) in *Geom.* 18, 13; (l) in *Geom.* 18, 14; (x) in *Geom.* 18, 2; (y) in *Geom.* 18, 3.

15. Criterion for establishing the species of a circular segment: if  $h = \frac{1}{2}(b)$ , it is a semicircle; if  $h > \frac{1}{2}(b)$ , it is greater than a semicircle; if  $h < \frac{1}{2}(b)$ , it is less than a semicircle.

This section coincides with *Geom.* 20, 14.

#### 16. Algorithm for finding the diameter of the circle equal to a given rectilinear region:

$$(A) \rightarrow \frac{1}{11}(A) \rightarrow 14[\frac{1}{11}(A)] \rightarrow \sqrt{\{14[\frac{1}{11}(A)]\}} = d.$$

Relation between the area of the rectangle contained by the diameter and the circumference of a circle and the area of the circle:  $(d,p) \rightarrow dp = 4A$ .

The algorithm coincides with *Geom.* 21, 8, at Hero, *Opera omnia* IV 380, 4–9. For the relation compare *Geom.* 21, 5 A.

#### 17. Segments of a circle other than a semicircle.

Algorithm for finding the area of a segment of a circle less than a semicircle:

$$(b,h) \rightarrow b + h \rightarrow h[\frac{1}{2}(b + h)]. [\frac{1}{2}(b)]^2 \rightarrow \frac{1}{14}[\frac{1}{2}(b)]^2 + h[\frac{1}{2}(b + h)] = A_T.$$

Algorithm for finding the circumference of a segment of a circle less than a semicircle:

$$(b,h) \rightarrow b^2. h^2 \rightarrow 4h^2 \rightarrow 4h^2 + b^2 \rightarrow \sqrt{(4h^2 + b^2)} \rightarrow \sqrt{(4h^2 + b^2)} + \frac{1}{4}(h) = P_T.$$

Algorithm for finding the area of a segment of a circle greater than a semicircle (in fact, only the diameter of the circle is computed):

$$(b,h) \rightarrow \frac{1}{2}(b) \rightarrow [\frac{1}{2}(b)]^2 \rightarrow \{[\frac{1}{2}(b)]^2\}/h \rightarrow h + \{[\frac{1}{2}(b)]^2\}/h = d.$$

A lesser segment of a circle being measured, the complementary segment turns out to be measured too.

<sup>74</sup> This is incorrect: compare (h). The verb form ἐρωτησάτω suggests that a corruption has occurred.

The first algorithm is as in *Geom.* 19, 1, 3, 5, and 8; 20, 6, 11, the second, as in *Geom.* 19, 2 and 4; 20, 13. The third algorithm is verbatim the same as in *Geom.* 20, 1, at Hero, *Opera omnia* IV 362, 9–22; the same algorithm is used in *Geom.* 20, 4 AC; 20, 8, *De mens.* 32.

### 18. Miscellanea about circles.

Algorithm for finding the area of the region contained by three mutually tangent circles<sup>75</sup>:

$$(d) \rightarrow d^2 \rightarrow 2d^2 \rightarrow \frac{1}{14}(2d^2) = A_{3C}.$$

Algorithm for finding the area of the region contained by four mutually tangent circles:

$$(d) \rightarrow d^2 \rightarrow 3d^2 \rightarrow \frac{1}{14}(3d^2) = A_{3C}.$$

Algorithm for finding the side of a square inscribed in a circle:  $(d) \rightarrow d^2 \rightarrow \frac{1}{2}(d^2) \rightarrow \sqrt{[\frac{1}{2}(d^2)]} = s_{Ti}$ .

Algorithm for finding the side of a square circumscribed to a circle:  $(d) \rightarrow d = s_{Te}$ .

The first two algorithms can also be found in Dioph. 4–5. The inverse of the last two algorithms is in *Geom.* 24, 17–18.

### 19. Algorithm for finding the area of a (right-angled) trapezium:

$$(s_1, s_2, h) \rightarrow (s_1 + s_2) \rightarrow h[\frac{1}{2}(s_1 + s_2)] = A_{Tra}.$$

The area of a right-angled trapezium can also be found by dividing it into one rectangle and one (right-angled) triangle, and by measuring each of these according to the already expounded procedures.

For this general algorithm and for the subsequent argument see *Geom.* 16, 1–2, 11, 14; most of sect. 16 expounds algorithms in which the trapezium is divided into a quadrangle and one or more triangles, or into triangles only.

### 20. Algorithms for finding the area of a regular $n$ -gon ( $n = 5 \dots 12$ ):

$$(s) \rightarrow s^2 \rightarrow 12s^2 \rightarrow \frac{1}{7}(12s^2) = A_5.$$

$$(s) \rightarrow s^2 \rightarrow (\frac{1}{3} + \frac{1}{10})s^2 \rightarrow 6(\frac{1}{3} + \frac{1}{10})s^2 = A_6.$$

$$(s) \rightarrow s^2 \rightarrow 43s^2 \rightarrow \frac{1}{12}(43s^2) = A_7.$$

$$(s) \rightarrow s^2 \rightarrow 29s^2 \rightarrow \frac{1}{6}(29s^2) = A_8.$$

$$(s) \rightarrow s^2 \rightarrow 51s^2 \rightarrow \frac{1}{8}(51s^2) = A_9.$$

$$(s) \rightarrow s^2 \rightarrow 15s^2 \rightarrow \frac{1}{2}(15s^2) = A_{10}.$$

$$(s) \rightarrow s^2 \rightarrow 66s^2 \rightarrow \frac{1}{7}(66s^2) = A_{11}.$$

$$(s) \rightarrow s^2 \rightarrow 45s^2 \rightarrow \frac{1}{4}(45s^2) = A_{12}.$$

Compare this section with *Metr.* I, 18–25, *Geom.* 21, 14 and 17–23; 22, 7–14, Dioph. 10–17, and *De mens.* 53 (octagon). See also the synoptic exposition in Hero, *Metrica* 484–488.

### 21. Irregular rectilinear figures are measured by dividing them into triangles.

Algorithm for finding the area of an irregular region by taking the mean of a number of measures of its width, according to Patrikios:  $(a, b_i) \rightarrow \sum_i^n b_i \rightarrow a[\frac{1}{n}(\sum_i^n b_i)] = A$ .

The first statement is an inflation of *Geom.* 21, 24, at Hero, *Opera omnia* IV 386, 11–13. Patrikios' algorithm as given here is a rewriting in procedural form of *Geom.* 21, 26–27.

### 22. Relations between the area of a square and the area of a series of regular figures that have the same side (resp. diameter in the case of a circle) as the square.

$$11A_4 = 14A_C.$$

<sup>75</sup> This is greatly inaccurate: given the standard formula for the area of an equilateral triangle,  $\frac{2}{14}$  should be replaced by  $(\frac{17}{30})(\frac{1}{14})$ .

$$13A_4 = 30A_3.$$

$$5A_4 = 3A_5.$$

$$13A_4 = 5A_6.$$

$$43A_4 = 12A_7.$$

$$29A_4 = 6A_8.$$

$$51A_4 = 10A_9.$$

$$15A_4 = 2A_{10}.$$

$$38A_4 = 5A_{10}.$$

$$66A_4 = 7A_{11}.$$

$$45A_4 = 4A_{12}.$$

This section appears to be a slight rewriting of Dioph. 19.

### 23. Species of solids. The units of measurement for solids are the same as for plane figures.

The part on the species of solid coincides with *Geom.* 3, 24 and *Geod.* 3, 24. See the discussion in Hero, *Metrica* 465–466.

### 24. The sphere.

Algorithm for finding the volume of a sphere:

$$(d) \rightarrow d^2 \rightarrow 11d^2 \rightarrow \frac{1}{14}(11d^2) \rightarrow h[\frac{1}{14}(11d^2)] \rightarrow (\frac{1}{2} + \frac{1}{6})h[\frac{1}{14}(11d^2)] = V_S.$$

$$\text{Algorithm for finding the volume of a sphere: } (d) \rightarrow d^3 \rightarrow 11d^3 \rightarrow \frac{1}{21}(11d^3) = V_S.$$

Algorithm for finding the area of the surface of a sphere:

$$(d) \rightarrow d^2 \rightarrow 4d^2 \rightarrow 11 \times 4d^2 \rightarrow \frac{1}{14}(11 \times 4d^2) = A_S.$$

Algorithm for finding the area of the surface of a sphere<sup>76</sup>:

$$(d) \rightarrow 2d \leftrightarrow (2d)^2 \rightarrow 11 \times (2d)^2 \leftrightarrow \frac{1}{14}[11 \times (2d)^2] = A_S.$$

$$\text{Algorithm for finding the area of the surface of a sphere: } (d) \rightarrow d^2 \rightarrow 44d^2 \rightarrow \frac{1}{14}(44d^2) = A_S.$$

Result established by Archimedes: the surface of the sphere is four times one of its great circles.

$$\text{Algorithm for finding the diameter of a sphere: } (p) \rightarrow \frac{1}{3}[p - \frac{1}{22}(p)] = d.$$

$$\text{Algorithm for finding the volume of a hemisphere: } (d) \rightarrow d^3 \rightarrow 11d^3 \rightarrow \frac{1}{42}(11d^3) = V_H.$$

$$\text{Algorithm for finding the area of the surface of a hemisphere: } (d) \rightarrow d^2 \rightarrow 22d^2 \rightarrow \frac{1}{14}(22d^2) = A_H.$$

Algorithm for finding the area of the surface of a hemisphere<sup>77</sup>:

$$(h) \rightarrow 2h \leftrightarrow (2h)^2 \rightarrow 11(2h)^2 \leftrightarrow \frac{1}{28}[11(2h)^2] = A_H.$$

Algorithm for finding the volume of a segment smaller than a hemisphere:

$$(b, h) \rightarrow [\frac{1}{2}(b)]^2 \rightarrow 3[\frac{1}{2}(b)]^2. h^2 \rightarrow h^2 + 3[\frac{1}{2}(b)]^2 \rightarrow 11h[h^2 + 3[\frac{1}{2}(b)]^2] \rightarrow \frac{1}{21}\{11h[h^2 + 3[\frac{1}{2}(b)]^2]\} = V_T.$$

Algorithm for finding the volume of a quadrant of a sphere:

$$(b, h) \rightarrow b^3 \rightarrow 2b^3 \rightarrow 11 \times 2b^3 \rightarrow \frac{1}{21}(11 \times 2b^3) \rightarrow V_{Qu}.$$

The first algorithm for the volume of a sphere is as in *Stereom.* I, 1. The second algorithm for the volume of a sphere and the first for its surface are as in *Stereom.* I, 2–3, 7, 65, 68 and 72, Dioph. 7 (a slight variant) and 25, 7–8, *De mens.* 36. The second and the third algorithm for its surface are as in *Stereom.* I, 5 and 6, respectively. The algorithm for the volume of a hemisphere is as in *Stereom.* I, 56, 66, and 69, Dioph. 25, 9. As for its surface, compare the second algorithm with *Stereom.* I, 73 (Dioph. 25, 10 is incomplete). The algorithm for the volume of a segment less than a hemisphere is as in *Stereom.* I, 71, Dioph. 21, 1; a slightly different formulation is in *De mens.* 47. The algorithm for the quadrant is as in Dioph. 26, 4.

### 25. The cone.

$$\text{Algorithm for finding the volume of a cone: } (d, p, h) \rightarrow (d, p) \rightarrow A_C \rightarrow [\frac{1}{3}(h)]A_C = V_{Co}.$$

<sup>76</sup> The text of the algorithm is incomplete.

<sup>77</sup> The text of the algorithm is incomplete.

Algorithm for finding the height of a cone:  $(d,k) \rightarrow [\frac{1}{2}(d)]^2 \cdot k^2 \rightarrow k^2 - [\frac{1}{2}(d)]^2 \rightarrow \sqrt{\{k^2 - [\frac{1}{2}(d)]^2\}} = h$ .  
 Results established by Archimedes (the cylinder contains the sphere and the cone):  $\frac{2}{3}V_{Cy} = V_S$ ;  
 $\frac{1}{3}V_{Cy} = V_{Co}$ .

Algorithm for finding the volume of a truncated cone:  $(d_1,d_2,h) \rightarrow \frac{1}{2}(d_1 + d_2) = d \rightarrow hA_C = V_F$ .

The first two algorithms are as in *Stereom.* I, 12 and 14, and as in I, 15 for the truncated cone.

## 26. The obelisk (a cone).

Algorithm for finding the height of an obelisk:

$$(d,k) \rightarrow [\frac{1}{2}(d)]^2 \cdot k^2 \rightarrow k^2 - [\frac{1}{2}(d)]^2 \rightarrow \sqrt{\{k^2 - [\frac{1}{2}(d)]^2\}} = h.$$

The volume of an obelisk is found according to the algorithm for the cone.

The algorithm is as in *Stereom.* I, 18. For the final sentence see *Stereom.* I, 18, at Hero, *Opera omnia* V 18, 10–15 CM.

## 27. The cylinder.

Algorithm for finding the volume of a cylinder:  $(p,h) \rightarrow A_C \rightarrow hA_C = V_{Cy}$ .

Algorithm for finding the area of a cylindrical surface:  $(d,h) \rightarrow (3 + \frac{1}{7})d \rightarrow h(3 + \frac{1}{7})d = A_{Cy}$ .

The two algorithms are as in *Stereom.* I, 19–20. The reference to the circle has a parallel in *Stereom.* I, 19, at Hero, *Opera omnia* V 20, 2–4.

## 28. Definition of the cube.

Algorithm for finding the volume of a cube:  $(l,w,h) \rightarrow lw \rightarrow hlw = V_K$ .

Algorithm for finding the volume of a cube:  $(l,w,h) \rightarrow 60l \rightarrow w60l \rightarrow lw60h \rightarrow \frac{1}{60}(lw60h) = V_K$ .

Algorithm for finding the side of a cube inscribed in a sphere<sup>78</sup>:

$$(d) \rightarrow [\frac{1}{2}(d)]^2 \rightarrow 2[\frac{1}{2}(d)]^2 \rightarrow \sqrt{\{2[\frac{1}{2}(d)]^2\}} = s.$$

Algorithm for finding the diagonal of a cube:  $(s) \rightarrow s^2 \rightarrow 2s^2 \rightarrow \sqrt{(2s^2)} = d$ .

The first algorithm is as in *Stereom.* I, 22–24, with an obvious parallel in *Stereom.* I, 22, at Hero, *Opera omnia* V 22, 8–10.

## 29. The wedge (a truncated pyramid with a quadrangular base).

Algorithm for finding the volume of a wedge<sup>79</sup>:

$$(l,w,h) \rightarrow (l + w) \rightarrow w(l + w) \rightarrow hw(l + w) \rightarrow \frac{1}{4}[hw(l + w)] = V_{Sf}.$$

Algorithm for finding the volume of a wedge<sup>80</sup>:  $(l,w,h) \rightarrow wl \rightarrow hwl \rightarrow \frac{1}{2}(hwl) = V_{Sf}$ .

The first algorithm is as in *Stereom.* I, 26, the second as in I, 27. See also *Stereom.* I, 25.

## 30. The meiouros (a prism with triangular or quadrangular base).

Algorithm for finding the volume of a meiouros<sup>81</sup>:  $(l,w,h) \rightarrow wl \rightarrow h[\frac{1}{2}(wl)] = V_M$ .

Algorithm for finding the volume of a meiouros<sup>82</sup>:

$$(w_1,w_2,h) \rightarrow w_1^2 \cdot w_2^2 \rightarrow w_1^2 + w_2^2 \rightarrow h[\frac{1}{2}(w_1^2 + w_2^2)] = V_M.$$

The first algorithm is as in *Stereom.* I, 28; for the second, *Stereom.* II, 17; II, 59, 4.

<sup>78</sup> This and the subsequent algorithm are incorrect, as they apply to the case of a square inscribed in a circle.

<sup>79</sup> The algorithm uses the area of a rectangle whose dimensions are the arithmetic means of the dimensions of the two bases instead of using the arithmetic mean of the areas of the bases, namely,  $\frac{1}{2}(w_1 + w_2)\frac{1}{2}(l_1 + l_2)$  instead of  $\frac{1}{2}(w_1l_1 + w_2l_2)$ . In the present case, the latter is equal to  $\frac{1}{2}(w_1^2 + w_2^2)$ . The formulation is careless and very likely corrupt.

<sup>80</sup> This solid is a rectangular parallelepiped cut along a diagonal plane, that is, a prism with a triangular base.

<sup>81</sup> This solid is in fact a prism with a triangular base.

<sup>82</sup> The bases are two squares.

## 31. The column (a truncated cone).

Algorithm for finding the volume of a column:  $(d_1, d_2, h) \rightarrow \frac{1}{2}(d_1 + d_2) \rightarrow p \rightarrow hA_C = V_{Ki}$ .

Algorithm for finding the area of a column:  $(p_1, p_2, h) \rightarrow \frac{1}{2}(p_1 + p_2) \rightarrow h[\frac{1}{2}(p_1 + p_2)] = A_{Ki}$ .

The two algorithms are as in *Stereom.* I, 21, whose final sentence (I, 21, 3) states that “the exposition of the column is a correction of the same Patrikios, for the ancients did not mix the two diameters”. “To mix” means here “to add”. See also *Stereom.* II, 10–12.

## 32. The brick.

The algorithm for finding the volume of a brick is the same as for the cube.

Compare *Stereom.* I, 29.

## 33. The pyramid.

Algorithm for finding the height of a pyramid<sup>83</sup>:  $(s, k) \rightarrow s^2 \rightarrow 2s^2 \rightarrow \frac{1}{4}(2s^2). k^2 \rightarrow \sqrt{[k^2 - \frac{1}{4}(2s^2)]} = h$ .

Algorithm for finding the volume of a pyramid:  $(s, h) \rightarrow s^2 \rightarrow h^{\frac{1}{3}}(s^2) = V_P$ .

The two algorithms are as in *Stereom.* I, 31. A slightly different formulation of the first algorithm—it computes  $\frac{1}{2}(s^2)$  instead of  $\frac{1}{4}(2s^2)$ —is in *Stereom.* II, 55–56, *De mens.* 39. See also *De mens.* 41.

## APPENDIX. THE ORIGINAL STRUCTURE OF VAT. GR. 1411

The subsequent two tables set out the original structure of Vat. gr. 1411 and some of its codicological features. In both tables, the double frames separate independent production units, possibly unified under a prim(ary) quire number sequence according to the subject-matter (quires 5–7, geography; 8–16, arithmetic; 17–19, metrology and logistic). The original quires were further unified under the final numbering sequence. The lines per page are 29 (44–46 signs per line), 31 (50–52 signs), or 43 (68–70 signs). The script fr(ame) A is mm 212 × 153 = 20 [169] 23 × 15 [113] 25; B is mm 220 × 144 = 22 [153] 45 × 19 [91] 34.

There is an obvious correlation between the dimension of the script frame, the lines per page, and the nature of the copied work. Anc(ient) texts and the fill(ers) associated with them are assigned 29 lines and frame B (the 27 lines for Ptolemy’s *Geography* are forced by the abundant inter-linear annotation); Byz(antine) texts are assigned 31 lines and frame A; com(mentaries) and the fill(ers) associated with them get 43 lines and frame A. Psellos’ poem is laid out on two columns. In the second table, the prim(ary) quire numbers highlight the sectional character of the process of copying.

quires	orig. fol.	pres. fol.	text	nature	no. lines	fr.
1–4	[1r]–33v	6r–37v	Cleomedes, <i>Caelestia</i> I–II	anc.	29	B
	33v–34r	37v–38r	περί ὧν ποιεῖται σχημάτων ἢ σελήνη πρὸς τὸν ἥλιον	fill.	29	B
	34v	38v	περὶ τοῦ ἡλίου πότε εἰσέρχεται ἐν τῷ Κριῶ καὶ διὰ πόσων ἡμερῶν διέρχεται τοῦτόν τε καὶ ἕκαστον τῶν λοιπῶν ζῳδίων	fill.	29	B
	34v	38v	πότε ἀφ’ ἑκάστου εἰς ἕκαστον τῶν ζῳδίων μεταβαίνει	fill.	29	B
5–7	35r–v	127r–v	Planudes, <i>Verse</i>	fill.	27–26	[B]
	36r–58v	128r–150v	Ptolemy, <i>Geogr.</i> I.1–II.1.11, VII.5.1–16, VIII.1.1–2.3	anc.	27	B
	59r–v	151r–v	Argyros, On the first projection	com.	43	A
	59v–60r	151v–152r	[Psellos], <i>De duodecim ventis</i>	byz.	33	A
	60r–v	152r–v	prose text on the winds	fill.	31	A
	60v	152v	three short geographical texts	fill.	31	A

<sup>83</sup> The base is a square.

8–13	61r–78r	43r–60r	Philoponos, <i>in Ar. I – recensio II</i>	com.	43	A
	78v	60v	blank		blank	A
	79r–110v	61r–92v	Nikomachos, <i>Ar. I–II</i>	anc.	29	B
14–15	111r–124r	93r–106r	Philoponos, <i>in Ar. II – recensio II</i>	com.	43	A
	124r	106r	problems A <sub>3</sub> , C <sub>2</sub>	fill.	43	A
	124v–125v	106v–107v	blank		blank	A
	126	116	blank		blank	A
16	127	117	blank		blank	A
	128r–131r	118r–121r	Moschopoulos, <i>De numeris quadratis</i>	byz.	31	A
	131v	121v	blank		blank	A
	132r–136v	122r–126v	Planudes, <i>Psephophoria</i>	byz.	31	A
17–18	137r–138v	10r–11v	Rhabdas, <i>Epistola ad Khatzycem</i>	fill.	43	A
	139r–140r	12r–13r	arithmetical tables	fill.	63	A
	140r	13r	problems C <sub>1</sub> , A <sub>1</sub> , A <sub>2</sub>	fill.	43	A
	140r–143v	13r–16v	[Hero], <i>Geodaesia</i>	byz.	43	A
	143v	16v	magic squares 6 <sup>2</sup> and 10 <sup>2</sup>	fill.	/	A
	144r–v	17r–v	Argyros, <i>Epistola ad Kolybam</i>	fill.	43	A
	144v–149r	17v–23r	<i>metrologica quaedam</i>	byz.	31	A
	150r–152v	23r–25v	Rhabdas, <i>Epistola ad Tsavoukhem</i>	byz.	31	A
19	153–162	[Rhabdas, <i>Epistola ad Tsavoukhem</i> ]	byz.	lost	A	
20–21 +	163r–169r	26r–32r	Psellos, <i>in Oracula Chaldaica</i>	byz.	31	A
	169r–170r	32r–33r	Psellos, <i>Summaria expositio</i>	byz.	31	A
	170r–171v	32r–34v	[Psellos], <i>Opiniones de demonibus</i>	byz.	31	A
	171v–179v	34v–42v	[Psellos], <i>De operatione daemonum</i>	byz.	31	A
22–31	180–258	[ <i>Cebetis tabula</i> ; Epictetos, <i>Encheiridion</i> (extr.); epistolographers; Synesios, <i>De insomniis</i> , Gregoras' <i>protheōria</i> , <i>De somniis</i> ascribed to Moschopoulos; medical excerpts]		lost	A	
32–33	259r–267v	1r–9v	Adamantios, <i>Physiognomonica I–II</i>	byz.	31	A
	268–274	III–IX	blank		blank	A
34–35	275r–282r	153r–160r	Argyros, <i>De constr. astrolabii</i>	byz.	31	A
	282v–288r	160v–164r	Argyros, <i>Tabulae novae</i>	byz.	31	A
	286–287				lost	A
	288v–290v	164v–166v	blank		blank	A
36	291r–298v	167r–174v	Argyros, <i>De cyclis</i>	byz.	31	A
	298v	174v	μέθοδος δι' ἧς εὐρίσκονται αἱ καθ' ἐκάστην νύκτα ὑπὸ τῆς σελήνης φωτιζόμεναι τῆς νυκτὸς ὥραι	fill.	31	A
37	299r	175r	περὶ διαφορᾶς ἀνέμων καὶ ὅτι καθολικοὶ ἄνεμοι τέσσαρες εἰσιν	fill.	31	A
	299r–300v	175r–176v	παρασημειώσεις προγνωστικαὶ περὶ τῆς μελλούσης τοῦ ἀέρος καταστάσεως	fill.	31	A
	300v–301v	176v–177v	παρασημειώσεις ἀπὸ τῆς τῶν ἀλόγων ζώων διαθέσεως	fill.	31	A
	302r–303r	178r–179r	περὶ τῶν ἐξ ἐκατέρου τῶν ἀπὸ τῆς γῆς ἀναφερομένων ἀναθυμιάσεων ἀποτελουμένων, ἧγουν νεφέλης καὶ ὄμβρου βροντῆς τε καὶ ἀστραπῆς καὶ τῶν ἄλλων τῶν τοιούτων	fill.	31	A
	303v–304v	179v–180v	blank		blank	A
38–40	305–327		[Gregoras, <i>Solutiones quaestionum</i> ; <i>De hebdomadario</i> ; extracts from Synesios, <i>De dono</i> , and John, <i>Apocalypsis</i> ]		lost	A
41	328r–335r	108r–115r	Pediasimos, <i>scholia in Cleomedem</i>	com.	43	A
	335r–v	115r–v	[Pediasimos], two astronomical texts	com.	43	A
	336–337		[Holobolos, <i>In mortem</i> ; Demetrios Triklinios, <i>Excogitatum</i> ]		lost	A

The quire structure of the original Vat. gr. 1411 is as in the following page.

#	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
type	IV	IV	IV	V	IV	IV	V	IV	IV	IV	IV	IV	V	IV	IV	V	IV	IV	V
last folio	8	16	24	34	42	50	60	68	76	84	92	100	110	118	126	136	144	152	162
quire no.	α	β	γ	δ	ε	ς	ζ	η	θ	ι	ια	ιβ	ιγ	ιδ	ιε	ις	ιζ	ιη	
prim. quire no.					α	β	γ									θ		β	

#	20	21	(22–31]	32	33	34	35	36	37	[38–40]	41
type	IV	IV		IV	IV	IV	IV	IV	III		V
last folio	170	178	258	266	274	282	290	298	304	327	337
quire no.	κ	κα	κβ–	λβ	λγ	λδ	λε	λς	λζ		
prim. quire no.		β	γ–								

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