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Cubic Relations of Autonomous and Controlled Motivation to Achievement: A Cross-National Validation of Self-Determination Theory Using Response Surface Analysis

Fernando Núñez-Regueiro¹

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Abstract

Self-determination theory (SDT) proposes to explain the relations between motivational states and human development. In education, a central tenet of the theory is that experiencing autonomous motivation in school activities (i.e., genuine pleasure and enjoyment) fosters optimal learning processes, whereas experiencing controlled motivation (i.e., pressure from social or instrumental incentives) undermines them. Although the theory is well established empirically, little is known about how these motivations combine in their effects on achievement at school (interactions), and whether their effects depend on the intensity of motivations (nonlinearities) or on the context of study (national differences). Applying cubic response surface analysis to the TIMSS 2019 dataset on mathematics ($N=152,825$ 8th grade students from 37 countries), as well as replication data ($N=169,269$ 8th grade students from TIMSS 2015, $N=270$ college students from SDT data), this study uncovers the existence of various kinds of nonlinear-interactive motivational processes in achievement, three of which systematically account for cross-national differences. In substance, these findings demonstrate that predictions based on SDT are close to universally true (93% of students), although they may not generalize well to extreme states of autonomous or controlled motivation (nonlinear and interactive processes). Implications for research and interventions on motivational processes are discussed.

Keywords Self-determination theory · Cross-national validation · Invariance testing · Response surface analysis · Cubic polynomials

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Introduction

A core tenet of self-determination theory (SDT; Ryan & Deci, 2017) is that healthy development is most sustained when individuals feel autonomously motivated. Autonomous motivation relates to behaviors that the individual views as reflecting personal values and interests essential to the self, to which he or she fully consents to as an “autonomous” being. To the contrary, actions undertaken under pressure of one’s authoritative self will induce a state of motivation characterized by external regulation or “control” over the self, leading to poor social adjustment and health. In educational contexts, autonomous motivation (vs. controlled motivation) is associated with higher levels of learning and achievement (Ryan & Deci, 2020).

One issue that remains poorly understood is whether autonomous and controlled motivations combine in their effects on achievement, and whether their effects are nonlinear. At present, studies have shown not only the positive effect of autonomous motivation on achievement but also the positive, negative, or null effects of controlled motivation (J. L. Howard et al., 2021; Taylor et al., 2014). The latter mitigated results might hide unknown higher-order effects, such as extreme degrees of controlled motivation above which a negative effect becomes positive (nonlinearity), or whose impact depends on autonomous motivation levels (interaction). Shedding light on nonlinear or interactive processes is important to understand whether and when autonomous or controlling appeals made by educators can be beneficial to learning, depending on the student being taught. For example, some analyses of student profiles found that high levels of controlled motivation (vs. low levels) were more positively related to achievement when autonomous motivation was low, suggesting a possible compensatory mechanism (Gillet et al., 2017). If this is the case, then using controlling motivational appeals might prove an adaptive—and a better tailored—teaching practice toward low-motivated students. Pioneering studies (Brunet et al., 2015; Mouratidis et al., 2021) more directly tested for nonlinear and interactive effects but reported contradictory findings, leaving the debate open on this matter. More research is therefore needed to clarify whether there are optimal degrees of autonomous or controlled motivation, and whether there exist interactive effects on achievement—including negative or positive synergies—between both kinds of motivations.

Another issue is whether SDT’s claims about relations between motivational processes and achievement generalize to diverse educational contexts from across the globe, confirming self-determined motivation as a panhuman phenomenon with general applicability. Although SDT’s universality is generally assumed, the bulk of the data draws mainly from English-speaking (i.e., Australia, Canada, UK, and USA) or Scandinavian countries (J. L. Howard et al., 2021; Taylor et al., 2014). The largest meta-analysis on the topic even found that some motivational processes might differ between Western and non-Western countries, but the evidence from non-Western countries (mostly Asian countries) was too limited to reach definitive conclusions (J. L. Howard et al., 2021). Additionally,

no study formally tested SDT's universality using dedicated techniques, that is, cross-national analyses of parametric invariance using exactly the same measures of motivation across multiple countries from around the globe. Evidence from parametric invariance tests provides a stronger basis for inferences, by gauging whether the effects of autonomous and controlled motivation on achievement occur in the same way and with the same intensity across countries (strong or "accessibility" universality), in the same way but at varying intensities (moderate or "functional" universality), or whether they differ in a nontrivial way depending on countries (weak or "existential" universality; Norenzayan & Heine, 2005). There is therefore a research need to characterize SDT's degree of universality more precisely and, by implication, to understand whether its motivational principles can be successfully applied everywhere or, on the contrary, need to be adjusted to specific cultures.

The present study aims to address these two issues simultaneously, by investigating the linear and nonlinear-interactive relations of autonomy and control on mathematics achievement worldwide, using cubic response surface analysis (RSA) and the database TIMSS (Trends in International Mathematics and Science Study). Cubic RSA allows for the investigation of complex relations by accounting for various kinds of interactive and nonlinear effects of autonomous and controlled motivation on achievement. In this study, we integrate cubic RSA with invariance testing to examine the generality of these effects across countries.

Defining the Motivational Spectrum at School: a Bi-Dimensional Projection of Self-Determination Theory's Taxonomy

Autonomous Versus Controlled Motivational Processes at School

As an organismic theory of human motivation, SDT views the self as the process that organizes individual functioning. This self-organizing process integrates and synthesizes the full scope of human functioning, including physiological, neurological, phenomenological, and behavioral experiences (Di Domenico & Ryan, 2017). From this perspective, autonomous motivation is a motivational state in which behavior is appraised as reflecting self-organization and, thus, as aligning with one's volition. In such instances, people engage with full congruence by internalizing these behaviors as means of personal empowerment and accomplishment. SDT emphasizes that autonomous motivation is inherent to human beings, meaning it is a necessary condition for optimal functioning (Deci & Ryan, 1985; Ryan & Deci, 2017).

To measure autonomous motivation, SDT focuses on people's perceptions about the cause of their own behavior. A first step is to consider whether the cause of behavior is attributed to the person (personal causation) or, on the contrary, by external influences independent from the person's agency (impersonal causation; Heider, 1958). In the latter case, autonomous motivation is excluded. A second operational step is to refine the notion of personal causation by its perceived locus of causality (PLOC; De Charms, 1968). The PLOC clarifies that some intentional actions originate in the person's volition and are therefore internally regulated (I-PLOC)

and fully autonomous, whereas others are externally regulated by demands or constraints that contradict the self-as-process and control it (E-PLOC). Altogether, these attributional theories converge in equating autonomous motivation with behavior that is perceived as regulated by the person's volition (i.e., I-PLOC), which can be measured in the form of interest, curiosity, or enjoyment in conducting the behavior. By contrast, controlled motivation can be measured in the form of rewards, punishments, or other forms of social or instrumental incentives that regulate the self externally, moving it as a pawn (E-PLOC; Ryan & Deci, 2017). For simplicity, the terms "autonomous motivation" (AM) and "controlled motivation" (CM) will henceforth be used for motivational states reflecting, respectively, internal and external PLOC.

In the present research, the above operationalization is used to characterize student motivation in mathematics as a function of indicators of genuine interest denoting autonomous motivation (i.e., I-PLOC) or external incentives denoting controlled motivation (i.e., E-PLOC; see "Method" section). An important assumption of the study is that autonomous and controlled motivational processes can combine to different degrees, in the sense that students may experience congruent or incongruent levels of genuine interest (autonomous motivation) or external pressures (controlled motivation) to learn mathematics. As explained next, this bidimensional approach also assumes that SDT's taxonomy of prototypical student motivational regulations (Fig. 1A) can be expressed as a function of combined levels of autonomous or controlled motivations (Fig. 1B).

Student Motivational Regulations as a Function of Autonomous and Controlled Motivation: Toward a Bidimensional Approach

SDT's taxonomy recognizes six major kinds of motivational regulations, which are defined based on the motivational orientation they appeal to in a given discipline, e.g., in mathematics (Ryan & Connell, 1989; Ryan & Deci, 2020; Vansteenkiste et al., 2018). On one extreme, *internal regulation* characterizes behavior driven by the pleasure, joy, or curiosity taken in doing mathematics. In this case, the orientation is fully autonomous (high AM), not being pressured by external incentives at all (low CM). On the other extreme, *external regulation* characterizes motivation that is void of genuine interest (low AM) and only views mathematics as a means to comply with social incentives, such as being rewarded with a better social position or avoiding negative consequences associated with not learning mathematics (high CM).

SDT also defines intermediate kinds of motivational regulations reflecting mixed levels of AM or CM. Thus, *introjected regulation* occurs if mathematics partly contribute to define one's identity in relation to others' approval or to the self (e.g., getting good grades to look smart or to avoid guilt from parents; medium AM), but the associated learning goals are not endorsed and contradict one's volition (e.g., disliking mathematics as a discipline; high CM). By contrast, *identified regulation* occurs when the activities imposed by the learning context are endorsed for their high relevance to important personal goals, thereby making the activities both externally and internally regulated. For example, being required to complete home assignments in

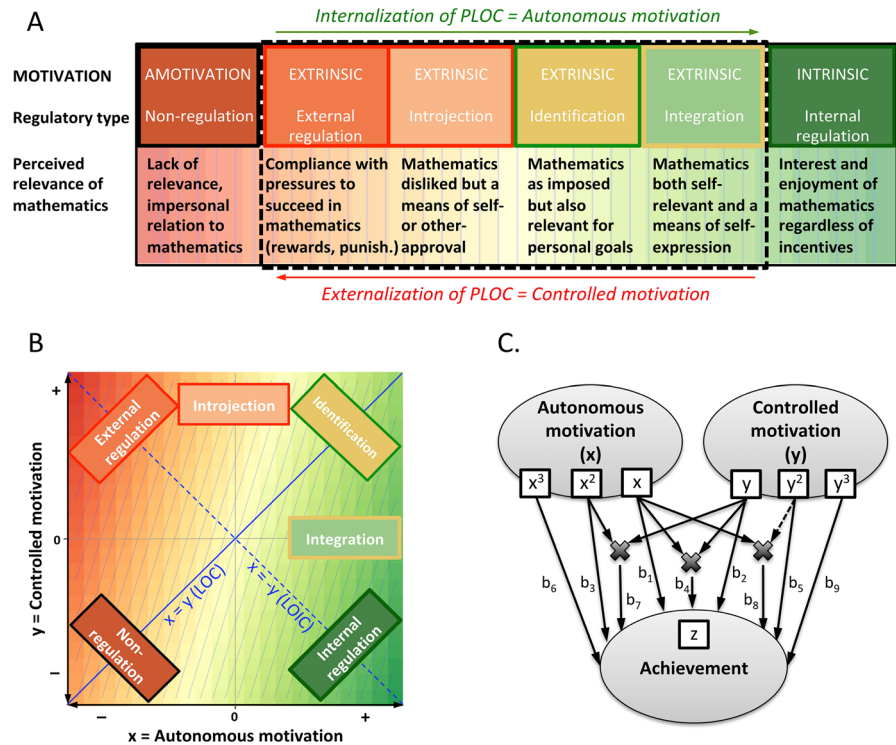


Fig. 1 Student motivational regulations (**A**, **B**) and achievement (**C**) as a function of autonomous and controlled motivational processes. Note. PLOC=perceived locus of control; LOC and LOIC=lines of congruence and incongruence (see Appendix). The diversity of regulatory types (Ryan & Connell, 1989; Ryan & Deci, 2020) is projected in the form of a bidimensional autonomous-controlled motivational spectrum (**B**) (Brunet et al., 2015; Mouratidis et al., 2021; Vansteenkiste et al., 2009), and related to achievement using cubic polynomial regression modeling (**C**) (Núñez-Regueiro & Juhel, 2022). Autonomous and controlled motivation scales relate to levels in the population: “-,” low; “0,” average; “+,” high

mathematics may be conducive to enthusiasm if it is perceived as a major means to enter a career requiring mathematics skills. In this case, both interest and incentive may intervene simultaneously (high AM, high CM). A fuller form of internalization is obtained through *integrated regulation*, notably when mathematics activities at school coincide with genuine interests and personal goals of self-realization. This may occur if the assigned task concerns a domain that one takes pleasure indulging in and mastering (e.g., algebra). In this case, the heightened state of personal interest and ownership (high AM) is expected to diminish the appraisal of school tasks as being imposed externally (medium CM). Finally, a sixth state of *non-regulation* may occur if mathematics are not related to any kind of motivation (low AM, low CM), and are therefore viewed as irrelevant to the self-organizational process and to external pressures to do well in mathematics.

While SDT’s taxonomy enables pinpointing the qualitative differences associated with distinct categories of motivation at school, researchers have long recognized that motivational regulations reflect more general dimensions of relative

autonomous or controlled motivation. However, two distinct approaches on dimensionality have been proposed.

Unidimensional Approach (Simplex)

According to the unidimensional or “simplex” approach, regulations can be aligned along a continuum ranging from least (external regulation, introjection) to most autonomous (identification, integration; Ryan & Connell, 1989; Ryan & Deci, 2020). More recent versions also include non-regulation and internal regulation as minimal and maximal regulatory styles of autonomous motivation, respectively (J. L. Howard et al., 2021). Here, increases in autonomous motivation imply decreases in controlled motivation (negative correlation is assumed), as denoted by the red and green arrows of external and internal PLOC running in opposite directions in Fig. 1A. Despite its popularity, the unidimensional approach has some complications. Empirically, unidimensional models do not fit the data for specific disciplines (e.g., in mathematics, English, French; Guay & Bureau, 2018) or activities (e.g., physical activity; Gunnell & Gaudreau, 2015), and they appear not to be able to account for variations in intrinsic motivation (Bureau et al., 2023). The unidimensionality assumption also contradicts findings that indicators of autonomous and controlled motivation correlate positively or not significantly, instead of negatively (Li et al., 2018; Mouratidis et al., 2021; Taylor et al., 2014; Vansteenkiste et al., 2009), and that their effects on achievement are complementary rather than substitutive (Brunet et al., 2015; Li et al., 2018; Mouratidis et al., 2021).

Bidimensional Approach (Circumplex)

Alternatively, according to the bidimensional or “circumplex” approach, autonomous and controlled motivations may co-exist or exist independently from one another (Brunet et al., 2015; Mouratidis et al., 2021). Various SDT studies thus report that student profiles exist where both kinds of motivations coincide at high or moderate levels or, on the contrary, oppose each other (Gillet et al., 2017; Hayenga & Corpus, 2010; J. L. Howard et al., 2016; Ratelle et al., 2007; Vansteenkiste et al., 2009). In other words, students may feel both autonomous and controlled in their relation to schoolwork, or experience only one form of motivation. In addition, from a theoretical perspective, the state of non-regulation assumed in SDT concerns students who are regulated neither internally (low AM) nor externally (low CM) in their schoolwork (Ryan & Deci, 2020). The very existence of this category of regulation implies that the two dimensions of autonomous and controlled motivation are not two opposite sides of the same coin but, instead, cumulative systems of motivation that may shut down at the same time, creating amotivation.

In summary, the bidimensional approach to motivational regulations appears empirically and theoretically sound from an SDT perspective, as it enables integrating all types of student regulations (Fig. 1B). What remains to be understood is whether and how autonomous and controlled motivation combine in their effects on achievement.

How Autonomous and Controlled Motivation Relate to School Achievement: Hypothesizing Interactive and Nonlinear Processes

The added value of the motivational spectrum is to acknowledge that autonomous and controlled motivations may combine in their effects on achievement. This approach complements research based on SDT's regulatory styles (Fig. 1A), by quantifying to what degree (i.e., intensity) and under which conditions (i.e., within which areas of the spectrum) the combination of autonomous and controlled motivations are associated with increased or decreased levels of achievement at school. As we explain next, this implies identifying interactive or nonlinear processes, notably by estimating a model including main effects (b_1, b_2), quadratic effects (b_3, b_5), interactive effects (b_4), cubic effects (b_6, b_9), and interactive-quadratic effects (b_7, b_8) of autonomy and control gradients on achievement (see Fig. 1C). Such gains in flexibility and precision nevertheless increase the complexity of the analysis. Clear hypotheses based on preliminary evidence are therefore needed to guide the analysis (Fig. 2).

Preliminary Evidence from Variable-Centered and Person-Centered Studies

Variable-centered studies aim to identify the effect of one variable (e.g., autonomous motivation) on another (e.g., achievement), while assuming that the effect is homogenous in the population and, in general, does not vary as a function of other defining characteristics (e.g., controlled motivation; M. C. Howard & Hoffman, 2018). This approach has the benefit of providing direct evidence about the educational effects of one motivational construct, but it is limited by a lack of investigation of interactive processes, e.g., between autonomous and controlled motivation. Accordingly, variable-centered studies in SDT have shed light on the extent to which autonomous and controlled motivations independently support or undermine school performance, while also revealing inconsistencies concerning the impact of controlled motivation. Using gradient measures based on the combination of autonomy-oriented regulatory styles (i.e., identification, integration, internal regulation), studies in the school or university contexts consistently demonstrated that higher levels of autonomy are associated with higher levels of exam performance or achievement in a foreign language, reading and mathematics (Becker et al., 2010; J. L. Howard et al., 2021; León et al., 2015; Li et al., 2018; Taylor et al., 2014; Vansteenkiste et al., 2005). By contrast, analyses of control-oriented regulatory styles (i.e., introjection and external regulation) or their combination have resulted in contradictory findings, some supporting its positive effect on math achievement ($\beta_{\text{external}} = .22$; Taylor et al., 2014), others its negative effect on reading ($\beta_{\text{controlled}} = -.56$; Becker et al., 2010) or grade point average ($\beta_{\text{controlled}} = -.24$; Li et al., 2018), or its non-significant effect on foreign language acquisition ($\beta_{\text{controlled}} = -.04$; Vansteenkiste et al., 2005). Similarly, meta-analyses across multiple measures of achievement concluded either to the negative effect ($\beta_{\text{external}} = -.22, \beta_{\text{introjected}} = -.12$; Taylor et al., 2014) or nonsignificant effects of

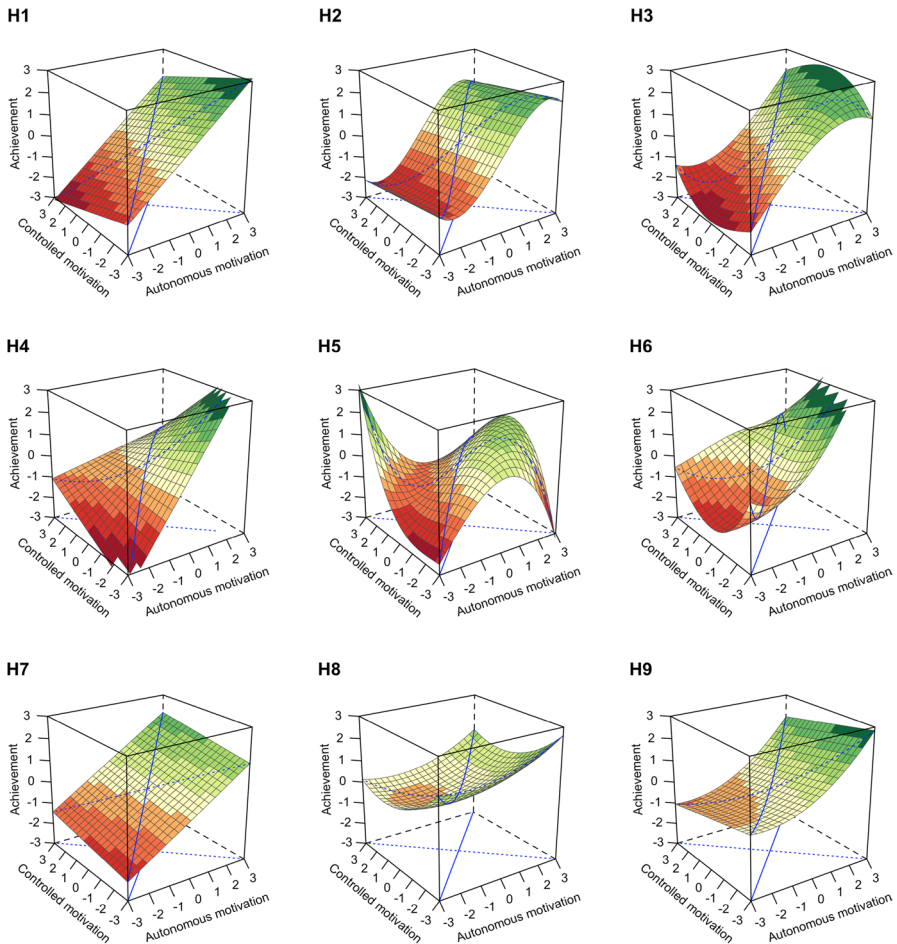


Fig. 2 Response surfaces corresponding to alternative hypotheses on the relations of autonomous and controlled motivation to achievement. Note. Hypotheses are organized to guide the analyses of linear and nonlinear processes compatible with SDT's motivational spectrum (H1 to H3), interactive and nonlinear-interactive processes partly contradicting the motivational spectrum (H4 to H6), and other previously reported processes (H7 to H9)

controlled regulatory styles ($\beta_{\text{external}} = -.03$, $\beta_{\text{introjected}} = -.01$; J. L. Howard et al., 2021).

As a complement to variable-centered studies, person-centered studies assume that the student population is heterogeneous and comprises subpopulations with distinct motivational profiles, which relate differently to a target outcome (e.g., achievement; M. C. Howard & Hoffman, 2018). This approach can provide indirect evidence on interactive processes between autonomous and controlled motivation, by comparing mean achievement levels between subgroups of students defined by categories of motivation (e.g., high autonomous and low controlled and high autonomous and high controlled). The evidence is “indirect” in the sense that categorical profiles are

used to shed light on a continuous interactive process, and no direct test for dependencies of one effect (of autonomous motivation) upon another (of controlled motivation) are obtained. Nonetheless, the evidence is suggestive. For example, among student profiles reporting low amounts of autonomous motivation, those reporting higher levels of controlled motivation appeared to reach higher achievement levels in some studies (Gillet et al., 2017), and equivalent (Vansteenkiste et al., 2009) or poorer achievement in others (Hayenga & Corpus, 2010). Conversely, among profiles reporting high amounts of autonomous motivation, higher levels of controlled motivation were related to lower (Hayenga & Corpus, 2010; Vansteenkiste et al., 2009) or equivalent levels of achievement (Gillet et al., 2017; Ratelle et al., 2007). Translated in terms of an interaction, the evidence from person-centered studies suggests that experiencing controlled motivation may have a beneficial, neutral, or deleterious effect on achievement when autonomous motivation is low, and a deleterious or neutral effect when autonomous motivation is high.

Both variable- and person-centered studies converge in inconsistent findings about the relation of controlled motivation to achievement. These inconsistencies may reflect differences in the clustering solution (for person-centered studies) and, more generally, differences in measures, in national contexts considered (i.e., Belgium, Canada, China, France, Germany, Spain, Sweden), or both. There is a research need to test whether more convergent findings occur using international data, or whether cross-national differences remain that would qualify the universality of SDT's motivational spectrum (Fig. 1B; Norenzayan & Heine, 2005). From a more analytic perspective, inconsistencies could also reflect unobserved complexities, such as nonlinear effects of controlled motivation on achievement that appear in the guise of positive, null, or negative linear effects depending on the sample; but also interactive processes with autonomous motivation not accounted for in the modeling strategy, whereby controlled motivation would compensate for lack of autonomous motivation (Gillet et al., 2017). Response surface analysis based on cubic polynomial regression appears optimal to provide direct evidence on nonlinear and interactive effects (see Figs. 1C and 2).

Previous Response Surface Analyses of Interactive and Nonlinear Processes

Two studies were the first to use RSA to investigate the interactive and nonlinear relations of autonomous and controlled motivation to achievement indicators (i.e., grade point average and grades on Turkish language or mathematics) among Canadian university students (Brunet et al., 2015) and Turkish secondary school students (Mouratidis et al., 2021). The study by Brunet and her colleagues found evidence for main effects of autonomous (positive effect) and controlled motivation (negative effect), but did not find evidence for nonlinear or interactive effects (Brunet et al., 2015). The corresponding response surface is depicted in Fig. 2 (H1), which aligns well with expectations from SDT's motivational spectrum. By contrast, Mouratidis et al. (2021) found evidence for both main effects and nonlinear effects in the form of quadratic effects of X and Y (sample 1) and a quadratic effect of X (sample 2; no significant interaction effects were found). The finding on main effects aligned with

SDT's expectations (see Fig. 2I), but the finding on quadratic effects of autonomous and controlled motivation did not because it indicated that non-regulated states associated with amotivation (i.e., low levels on both controlled and autonomous motivation) were as adaptive for achievement as identified regulation (high levels on both controlled and autonomous motivation; see Fig. 2H). Due to a lack of convergent findings, the authors concluded that autonomous motivation was the only clear predictor of achievement among secondary students (Mouratidis et al., 2021).

These pioneering studies were limited by the fact that the nonlinear or interactive processes explored were limited to second-order effects (i.e., parameters b_3 to b_5), but did not explore third-order effects (e.g., b_6 to b_9 ; Fig. 1C). This is problematic because third-order effects account for a wider range of nonlinear or interactive response surfaces, which could better approximate the motivational processes underlying the data (see Fig. 2). To their defense, cubic RSA was not available at the time of these studies. Nonetheless, it is possible that the omission of third-order effects limited the identification of nonlinear or interactive processes.

A Set of Nine Hypotheses for Response Surface Analysis

Based on the literature and on a cubic RSA approach, several hypotheses about the relations of autonomous motivation and controlled motivation to achievement can be made, which are organized in three sets. Technical details on hypothesis generation are reported in the Appendix. Here, we focus on the substantive meaning of these hypotheses. Following previous RSA (Brunet et al., 2015; Mouratidis et al., 2021), we focus hypotheses on the lines of congruence (LOC) and incongruence (LOIC), that is, where the amounts of autonomous motivation and controlled motivation coincide or oppose each other, respectively. In speculating interactive and nonlinear processes along the LOC and LOIC, these hypotheses offer explanations for inconsistencies in the effects of controlled motivation or in motivational profiles reported in previous findings. They also illustrate how motivational processes might differ from one national context to another (thereby limiting SDT's universality), even though the evidence on this matter is insufficient to anticipate in which way exactly (J. L. Howard et al., 2021; Taylor et al., 2014).

Linear and Nonlinear Processes

The first set of hypotheses corresponds to linear and nonlinear variants of SDT's bidimensional taxonomy (Fig. 1B). Aligning with meta-analytic findings (Taylor et al., 2014) and some RSA applications (Brunet et al., 2015), hypothesis 1 (H1) assumes that the positive effect of autonomous motivation is larger than the negative effect of controlled motivation, thereby creating a response plane with positive trends along the LOC and the LOIC, with achievement levels culminating in situations reflecting internal regulation (i.e., high AM and low CM), followed by those reflecting integration (i.e., high AM and average CM) and identification (i.e., high AM and high CM; see Fig. 2, H1). In this hypothesis, higher-order effects are expected to be null ($b_3 : b_9 = 0$), meaning that the surface is strictly linear. In

contrast, hypothesis 2 (H2) modifies H1 by introducing a negative cubic effect of autonomous motivation. This modifies the response surface by delimiting threshold points beyond which the beneficial (deleterious) effects of autonomous motivation (controlled motivation) reverse in valence, also called “reversal effects” (see “Method” section). This would occur, for example, if extreme degrees of external regulation (e.g., very low AM and very high CM) prove more adaptive for mathematics performance than less extreme degrees (e.g., low AM and high CM). Such nonlinearities could account not only for the negative, albeit non-significant effects on achievement of controlled-regulatory styles reported in some meta-analyses (J. L. Howard et al., 2021) but also for inconsistent findings concerning the educational benefits of experiencing controlled motivation in the absence of autonomous motivation, or conversely (Gillet et al., 2017; Hayenga & Corpus, 2010; Ratelle et al., 2007; Vansteenkiste et al., 2009). Hypothesis 3 is a refined version of H2 that confines reversal points to the LOIC, thus specifying that no reversal effects occur on the LOC axis where autonomous and controlled motivation levels coincide.

Interactive and Interactive-Nonlinear Processes

The second set of hypotheses corresponds to different kinds of interactive processes, which aim to account for the finding that the effect of controlled motivation on achievement may be positive at low values of autonomous motivation (compensatory effect; Gillet et al., 2017) and negative at high values of autonomous motivation (disabling effect; Hayenga & Corpus, 2010; Vansteenkiste et al., 2009). According to hypothesis 4 (H4), this interactive process takes the form of a negative interaction effect ($b_4 < 0$), which introduces an inverted-U shaped (concave) curvature along the LOC and a U shaped (convex) curvature along the LOIC. Substantively, these curvatures modify SDT’s bidimensional taxonomy (Fig. 1B) by assuming that situations reflecting non-regulation (i.e., low AM and low CM) relate more negatively to achievement than those reflecting external regulation (i.e., low AM and high CM). Hypothesis 5 (H5) assumes instead that the interactive process is more complex due to the presence of nonlinearity along the LOIC. Here, the relative advantage of external regulation over non-regulation is observed only upon reaching reversal points situated at extreme values of external regulation (i.e., very low AM and very high CM). Conversely, hypothesis 6 (H6) specifies an interactive process where the nonlinearity occurs along the LOC, but not along the LOIC, resulting in the observation that the relative advantage of external regulation over non-regulation may not occur at extreme values of non-regulation (i.e., very low AM and very low CM), due to the presence of a reversal effect (see Fig. 2, H6). Both nonlinear-interactive processes (H5 and H6) could account for inconsistencies in person-centered studies reporting significant or non-significant differences in achievement levels between student profiles characterized by congruent or incongruent levels of autonomous and controlled motivation (Gillet et al., 2017; Hayenga & Corpus, 2010; Ratelle et al., 2007; Vansteenkiste et al., 2009). Indeed, in the presence of nonlinearity on the LOC and LOIC, and depending on the clustering solution obtained, qualitatively similar profiles (e.g., low AM and low CM) might reflect motivational states situated above

or below the reversal points, thereby yielding quantitatively different results on the relation of such profiles to achievement.

Other Processes

Hypothesis 7 (H7) accounts for the observation that controlled motivation may have a positive main effect (Taylor et al., 2014), which portrays identification (high AM and high CM) as the state where students reach the highest levels of achievement. Hypotheses 8 and 9 correspond to the significant effects reported by Mouratidis et al. (2021). More formally, H8 specifies a “bowl” surface driven by positive quadratic effects of both autonomous and controlled motivations, in which the lowest levels of achievement are obtained at lower-than-average autonomy and higher-than-average control levels. H9 describes instead a single quadratic effect of autonomous motivation that introduces convexity in the surface, but in a way that does not modify H1 meaningfully (no reversal effects occur within the range of realistic observations).

Study Aims and Hypotheses

The present series of studies (studies 1 and 2) aimed to provide new evidence on the relations of autonomous and controlled motivation to achievement in mathematics, using a flexible analytical framework and more robust identification techniques. First, international data using a common research design is needed to verify that the inconsistent findings in the literature are substantive (e.g., cross-national differences) rather methodological in nature (e.g., measurement or modeling differences). Second, stronger RSA techniques, notably based on cubic polynomial regression modeling, are required to obtain more valid evidence on nonlinear or interactive processes. The present study offers to address these research needs, while answering the following research questions:

RQ1: Are the effects of autonomous and controlled motivation on achievement in mathematics concordant with the motivational spectrum assumed for SDT (Fig. 1B)?

RQ2: Is there evidence for nonlinear or interactive processes that modify the effects autonomous and controlled motivation in a nontrivial way?

RQ3: Are there systematic cross-national differences in these effects on achievement?

Concerning RQ1, two hypotheses are conjectured based on variable-centered studies, which both assume a positive effect of autonomy on achievement, but differ with regard to the negative (H1) or positive (H7) effect of controlled motivation. The expectations for RQ2 are formalized through seven interactive and nonlinear processes, five of which are derived analytically based on person-centered studies (H2 to H6), and two of which are derived empirically from previous RSA findings (H8 and H9; Fig. 2). Concerning RQ3, the literature is too limited to conjecture on cross-national differences, so the present study will be exploratory in this regard. It should

also be noted that previous research has been characterized by different measurement approaches (i.e., motivational types vs. gradients of control and autonomy), uneven designs (i.e., different outcome measures, different national contexts), and few investigations of nonlinear effects. The full scope of findings therefore provides limited information on the possible effects that may exist. Accordingly, non-conjectured hypotheses should also be accounted for, using a mixed paradigm for confirmatory and exploratory findings in response surface analyses (Núñez-Regueiro & Jehel, 2022, 2024a). In this perspective, study 1 aimed to provide answers to the research questions using the most recent TIMSS data currently available (TIMSS 2019; Mullis et al. 2020). Study 2 aimed to probe the robustness of findings from study 2, by cross-validating results with an earlier TIMSS database (TIMSS 2015; Mullis et al., 2016) and with a measurement approach to motivational constructs more closely matching SDT studies (i.e., using the Academic Motivation Scale; Valierand et al., 1992).

Study 1: TIMSS 2019

Method

Participants

Data was obtained on eighth grade students from 37 countries around the world, using the TIMSS 2019 International Database (see Table 1; Mullis et al., 2020).¹ Since 1995, TIMSS has assessed mathematics and science among eighth graders every four years, using a quality assurance program to ensure the good implementation of the framework in all participating countries. Participants of TIMSS 2019 were sampled using stratified weights at the school and classroom levels, thus ensuring the national representativeness of the data. After removal of students whose scores in mathematics were too low to be reliably estimated (9.8% of students; TIMSS flag), the total sample consisted of 181,763 students, each country accounting for 2% to 3% of the total sample, except for Australia (4.7%), France (3.9%), Malaysia (3.5%), the USA (4.4%), and the United Arab Emirates (10.6%). To balance the relative weight of each country in multigroup solutions, the latter samples were reduced to represent the average sample (i.e., $n=4,130$ random draws). The final sample was therefore composed of 152,825 8th grade students ($M_{age} = 14.3 \pm 0.6$, 49.7% female adolescents).

Measures

Measures for autonomous and controlled motivation in mathematics were self-reported by students as part of TIMSS's context questionnaire, whereas achievement scores in mathematics were obtained using TIMSS's assessment framework

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Table 1 Descriptive statistics

	N (%)	X = autonomous motivation		Y = controlled motivation		Z = achievement		Correlations		
		ω	M (SD)	ω	M (SD)	k	M (SD)	X:Y	X:Z	Y:Z
All	152,825 (100)	0.935	2.7 (0.8)	0.895	2.9 (0.6)	0.98	506.3 (96.5)	0.544	0.144	0.030
Australia	4,130 (2.7)	0.943	2.6 (0.8)	0.909	2.9 (0.5)	0.98	535.3 (83.3)	0.505	0.406	0.215
Bahrain	5,179 (3.4)	0.939	2.7 (0.9)	0.897	2.9 (0.6)	0.90	495.2 (86.7)	0.578	0.210	0.138
Chile	3,480 (2.3)	0.936	2.6 (0.8)	0.890	2.9 (0.5)	0.96	463.6 (71.2)	0.427	0.228	0.089
Cyprus	3,327 (2.2)	0.941	2.6 (0.9)	0.884	2.9 (0.6)	0.98	508.9 (75.3)	0.545	0.364	0.246
Egypt	5,798 (3.8)	0.888	3.2 (0.7)	0.860	3.2 (0.5)	0.99	434.8 (84.6)	0.483	0.137	0.122
England	3,143 (2.1)	0.935	2.5 (0.8)	0.880	3.0 (0.5)	0.97	525.1 (78.8)	0.436	0.263	0.077
Finland	4,583 (3.0)	0.940	2.4 (0.8)	0.908	2.7 (0.6)	0.99	516.4 (67.2)	0.511	0.465	0.312
France	4,130 (2.7)	0.925	2.6 (0.7)	0.892	2.8 (0.5)	0.97	494.0 (64.1)	0.484	0.366	0.152
Georgia	2,886 (1.9)	0.922	2.9 (0.8)	0.891	3.0 (0.5)	0.96	478.2 (80.4)	0.492	0.278	0.117
Hong Kong	3,167 (2.1)	0.936	2.5 (0.8)	0.916	2.6 (0.6)	1.00	582.7 (81.6)	0.513	0.295	0.237
Hungary	4,331 (2.8)	0.935	2.4 (0.8)	0.882	2.7 (0.6)	0.97	536.5 (82.5)	0.543	0.425	0.255
Iran	5,155 (3.4)	0.936	3.0 (0.8)	0.893	3.0 (0.6)	0.99	465.1 (83.8)	0.615	0.261	0.138
Ireland	3,993 (2.6)	0.937	2.5 (0.8)	0.884	2.9 (0.6)	0.99	529.7 (66.8)	0.496	0.347	0.176
Israel	3,476 (2.3)	0.937	2.6 (0.9)	0.877	3.1 (0.5)	0.98	527.0 (92.1)	0.464	0.184	0.083
Italy	3,398 (2.2)	0.951	2.5 (0.9)	0.880	2.7 (0.6)	0.98	505.6 (66.1)	0.557	0.377	0.145
Japan	4,411 (2.9)	0.945	2.4 (0.7)	0.867	2.6 (0.5)	0.99	594.7 (82.2)	0.450	0.419	0.236
Jordan	5,820 (3.8)	0.916	3.1 (0.8)	0.886	3.2 (0.5)	0.99	435.2 (75.7)	0.491	0.166	0.163
Kazakhstan	4,123 (2.7)	0.928	3.0 (0.6)	0.904	2.8 (0.5)	0.93	496.6 (77.4)	0.573	0.155	0.044
Kuwait	3,571 (2.3)	0.926	2.6 (0.9)	0.886	2.9 (0.6)	1.00	423.0 (79.9)	0.539	0.196	0.125
Lebanon	4,018 (2.6)	0.893	2.9 (0.8)	0.882	3.0 (0.6)	1.00	438.0 (64.4)	0.554	0.211	0.184
Lithuania	3,611 (2.4)	0.932	2.6 (0.7)	0.882	2.8 (0.5)	0.97	523.8 (77.3)	0.531	0.264	0.131
Malaysia	4,130 (2.7)	0.910	3.0 (0.6)	0.873	3.0 (0.5)	0.99	501.7 (87.1)	0.466	0.225	0.189
New Zealand	5,595 (3.7)	0.938	2.6 (0.8)	0.904	2.9 (0.6)	0.98	510.8 (86.8)	0.495	0.364	0.160
Norway	4,204 (2.8)	0.945	2.5 (0.8)	0.899	2.9 (0.5)	0.97	516.0 (72.7)	0.529	0.440	0.242

Table 1 (continued)

	N (%)	X = autonomous motivation		Y = controlled motivation		Z = achievement		Correlations		
		ω	M (SD)	ω	M (SD)	k	M (SD)	X:Y	X:Z	Y:Z
Oman	5,374 (3.5)	0.895	3.1 (0.7)	0.873	3.1 (0.5)	0.99	438.5 (90.5)	0.511	0.239	0.211
Portugal	3,206 (2.1)	0.949	2.6 (0.9)	0.898	2.9 (0.6)	0.97	510.3 (69.5)	0.549	0.375	0.296
Qatar	3,236 (2.1)	0.928	2.6 (0.9)	0.910	2.9 (0.6)	0.97	466.0 (88.3)	0.569	0.263	0.198
Romania	4,052 (2.7)	0.922	2.6 (0.8)	0.905	2.8 (0.6)	0.99	504.9 (88.5)	0.639	0.325	0.178
Russian Fed	3,790 (2.5)	0.924	2.8 (0.7)	0.905	2.8 (0.6)	0.98	549.0 (78.3)	0.591	0.302	0.142
Saudi Arabia	4,496 (2.9)	0.939	2.8 (0.9)	0.870	3.0 (0.6)	0.98	422.7 (79.0)	0.555	0.159	0.123
Singapore	4,786 (3.1)	0.937	2.8 (0.8)	0.864	2.9 (0.5)	0.99	611.7 (85.2)	0.455	0.332	0.138
South Korea	3,778 (2.5)	0.944	2.3 (0.7)	0.896	2.6 (0.6)	0.99	609.3 (90.2)	0.551	0.418	0.407
Sweden	3,736 (2.4)	0.946	2.5 (0.8)	0.876	2.8 (0.5)	0.97	513.0 (71.4)	0.494	0.385	0.128
Taipei	4,810 (3.1)	0.945	2.4 (0.8)	0.905	2.4 (0.6)	0.97	613.2 (93.2)	0.572	0.430	0.395
Turkey	3,642 (2.4)	0.926	2.9 (0.8)	0.878	3.0 (0.6)	0.99	507.3 (99.2)	0.565	0.255	0.218
United Arab Emir	4,130 (2.7)	0.929	2.9 (0.8)	0.892	3.0 (0.6)	0.97	484.7 (91.1)	0.549	0.209	0.167
USA	4,130 (2.7)	0.941	2.6 (0.9)	0.895	3.0 (0.5)	0.98	526.2 (90.8)	0.461	0.291	0.149

Note. ω , omega total; κ , score point agreement between raters of mathematics achievement items. All correlations are significant at 0.1%

(Fishbein et al., 2021). All measures were scaled using item response theory and validated for commensurability across national contexts (Martin et al., 2020).

Autonomous Motivation Autonomy was measured by the *Students Like Learning Mathematics* scale, which consists of nine items assessing the degree of enjoyment or pleasure students experience in mathematics (sample items=*I enjoy learning mathematics, I like to solve mathematics problems, I look forward to mathematics lessons, I learn many interesting things in mathematics*). Reflecting internal PLOC in the form of pleasure and genuine interest in the learning activity, this scale aligned with items used in previous SDT studies to measure autonomous motivation [sample items from SDT studies=*why do you go to school/study?*] *Because I experience pleasure and satisfaction while learning new things, For the pleasure that I experience in broadening my knowledge about subjects which appeal to me* (Li et al., 2018; Ratelle et al., 2007; Taylor et al., 2014), *Because of the fun and enjoyment that this goal provides me* (Brunet et al., 2015), *Because I am highly interested in doing this* (Gillet et al., 2017; Vansteenkiste et al., 2009), *Because it is fun* (Mouratidis et al., 2021)]. The items were rated on 4-point Likert scale (1 = agree a lot, 4 = disagree a lot), and were encoded reversely so that higher scores reflected higher levels of autonomous motivation. The scale showed excellent reliability in all countries ($\omega_{\text{autonomy}} = [.888, .951]$; Table 1).

Controlled Motivation The *Students Value Mathematics* scale was used as a measure of controlled motivation. This scale is composed of nine items rated on a 4-point Likert scale (1 = agree a lot, 4 = disagree a lot, reverse encoded) that assess the social and economic rewards and incentives that students associate with learning mathematics (sample items=*I need to do well in mathematics to get the job I want, It is important to learn about mathematics to get ahead in the world, I need to do well in mathematics to get into the university, My parents think that it is important that I do well in mathematics*). Reflecting external PLOC in the form of social compliance and approval from others, this scale aligned well with items used to measure controlled motivation in previous SDT studies [sample items from SDT studies=*why do you go to school/study?*] *Because I want to have “the good life” later on, In order to obtain a prestigious job later on, Because I need at least a high school diploma in order to find a high-paying job later on* (Li et al., 2018; Ratelle et al., 2007; Taylor et al., 2014), *I feel obligated to have this goal* (Brunet et al., 2015), *Because that’s what others (parents, friends, etc.) expect me to do* (Gillet et al., 2017; Vansteenkiste et al., 2009), *Because that’s the rule* (Mouratidis et al., 2021)], and showed excellent reliability ($\omega_{\text{control}} = [.864, .916]$; Table 1).

Achievement Mathematics achievement was assessed using standardized tests, comprising 211 items covering numbers (30%), algebra (29%), geometry (21%), and data and probability (20%). The items were pretested and selected to be robust to differential item functioning across countries, thus ensuring that test scores reflected the same abilities in mathematics for all students in the sample. According to TIMSS

2019, interrater agreement for scoring mathematics items ranged from 90% to 100% (Table 1; Martin et al., 2020).

Analytic Strategy

Response Surface Analysis A general comparative framework was used to identify best-fitting polynomial solutions for response surface analysis (Núñez-Regueiro & Juhel, 2022, 2024a, 2024b). This framework builds on the comparison of 37 families of polynomials (Núñez-Regueiro & Juhel, 2023), which are specified by imposing parametric constraints on the cubic model $z = b_0 + b_1x + b_2y + b_3x^2 + b_4xy + b_5y^2 + b_6x^3 + b_7x^2y + b_8xy^2 + b_9y^3$. More precisely, each family is defined by zero, non-zero, and/or proportional constraints on a single parametric order (i.e., first order = b_1 to b_2 , second order = b_3 to b_5 , and third order = b_6 to b_9), and by zero constraints on parameters above its highest parametric order. Thus, unique families exist for first-order (3 families, linear effects), second-order (12 families, quadratic effects), and third-order polynomials (22 families, cubic effects). Using this strategy, families were compared on several criteria, including statistical significance of parameters defining the family, tests of absolute fit against the saturated cubic model (i.e., full cubic model with no constraints), evidence of good fit to the data (e.g., $CFI \geq 0.95$, $TLI \geq 0.95$, $RMSEA \leq 0.06$, $SRMR \leq 0.08$), and evidence of parsimony (e.g., Akaike weights). This identification strategy enabled selecting best-fitting response surfaces in a reliable and quasi-exhaustive manner, allowing for either confirmatory findings (based on conjectured hypotheses H1–H9) or exploratory findings based on unexpected families providing a better fit to the data.

To ensure that the parametric constraints that define each family are structurally valid, effect sizes of predictors must be commensurate (Núñez-Regueiro & Juhel, 2024c). This was obtained by standardizing autonomous and controlled motivation scores to unit variance and zero means, while accounting for TIMSS sampling weights (i.e., student weights accounting for missing data and stratified sampling). To facilitate inferences, the outcome of achievement was standardized in the same way. Moreover, sampling weights were used in the estimation of polynomial families to ensure that final results had strong validity in the national population.

Once a family was retained as providing the best fit for a country or a group of countries (see “[Structural invariance testing](#)” section), its response surface was interpreted by probing the relevance of the curvatures in the surface. Curvatures were considered when they appeared within the confines of the bagplot of realistic predictor values (i.e., values contained within the quartiles of both autonomous and controlled motivation) and were further probed by identifying reversal points where the behavior changes sign ($\frac{\partial z}{\partial x} = 0.0$), and acceleration points where the response reaches noteworthy change in behavior (i.e., $|\frac{\partial z}{\partial x}| = 0.30$; Núñez-Regueiro & Juhel, 2024b; Humberg et al., 2020). Reversals (accelerations) were identified for students situated above extrema (acceleration) lines on the LOC or LOIC and with achievement in mathematics significantly different from what would be expected from the main trend. Significant differences were established by observing that the 95%

one-tailed confidence interval of the expected outcome value on the LOC or LOIC of a given student situated above reversal or acceleration lines did not contain the outcome value observed for this student. This probing was implemented to prevent unwarranted extrapolations from the graphical projection of the surface and to offer more precise estimates of the degree of validity of SDT's predictions in the TIMSS data. Effect sizes for explained variance were considered small ($R^2 = .02$), medium ($R^2 = .13$), or large ($R^2 = .26$) following guidelines in behavioral sciences (Cohen, 1988). Also, to shed light on the added value of nonlinear-interactive polynomial solutions over linear ones, response surfaces for both solutions were considered.

Structural Invariance Testing To identify groups of national samples that shared common structural relations, a bottom-up strategy was privileged. First, the comparative framework for response surface analysis (presented above) was conducted on each country separately, thus obtaining the ranking of families providing best-fitting models. Second, country samples with a common best-fitting model in the top rankings (i.e., among the five best-fitting families) were united as plausible candidates for a group. Four groups of countries were explored (see “Results” section). Third, the validity of the common model in each group was tested by verifying that the structural relations did not vary across units within the group. Building on simulation studies on power to detect unwarranted constraints in single and multi-group structural equation models (Chen, 2007; Hu & Bentler, 1998; Jorgensen et al., 2018), invariance was assumed when the differences in model fit between the single group solution (constraining model parameters to be equal across units) and the multigroup solution (allowing model parameters to differ across units) were trivial (i.e., $\Delta CFI \geq -0.01$, $\Delta Mc \geq -0.015$, $\Delta RMSEA \leq 0.015$, $\Delta SRMR \leq 0.030$), and when the single group solution provided adequate fit to the data according to fixed cutoff values (i.e., $CFI \geq 0.95$, $TLI \geq 0.95$, $RMSEA \leq 0.06$, $SRMR \leq 0.08$). Analyses were estimated on *lavaan* (Núñez-Regueiro & Juhel, 2024c; Rosseel, 2012), using the comparative framework for polynomial regression and response surface analysis of the *RSAtools* package (Núñez-Regueiro & Juhel, 2024c).

Results

Descriptive Statistics

On average, students reported levels of autonomous motivation situated at the midpoint of the scale (i.e., between 2 = “disagree” and 3 = “agree”; $M_{\text{autonomous}} = 2.6 \pm 0.8$) and slightly higher levels of controlled motivation ($M_{\text{controlled}} = 2.8 \pm 0.6$; see Table 1). A large, positive association between the two kinds of motivation was apparent across all countries ($r = .544$, $p < .001$, $\text{range}(r) = [.427, .639]$), indicating that students who experienced pleasure and enjoyment in mathematics also tended to study mathematics for its social or instrumental rewards. This correlation was higher than ($r = [.29, .32]$ Mouratidis et al., 2021; Vansteenkiste et al., 2005) or equivalent to ($r = [.48, .50]$; Li et al., 2018; Taylor et al., 2014) those reported in previous SDT

studies. In addition, achievement levels were higher among students experiencing autonomous motivation ($r = .248, p < .001$) and, to a lesser extent, among students experiencing controlled motivation ($r = .080, p < .001$). Due to their high correlation, regression analyses accounting for the joint effects of autonomous and controlled motivation on achievement are needed to untangle these observations.

Identification of Polynomial Regression Models

Thirty-seven families of polynomial regression models were estimated and compared for each country of the TIMSS 2019 data (see SM1 and SM2). The aim of these preliminary analyses was to identify best-fitting families in each country and, among these, groups of countries sharing best-fitting solutions according to structural invariance tests. Altogether, polynomials including nonlinear or interactive effects of autonomous and controlled motivation on achievement (i.e., interactive, quadratic, cubic, or interactive-quadratic effects) always provided a better fit than solutions including linear effects only (i.e., additive effects). Moreover, common best-fitting complex polynomials were found, which enabled the identification of four groups of countries (see Table 2).

Group 1 included 14 countries (Australia, Chile, Cyprus, England, Finland, France, Hong Kong, Hungary, Italy, Lithuania, New Zealand, Russian Federation, Singapore, and Sweden) who shared a best-fitting solution in the family “cubic effect of x ” (family 16), that is, a polynomial model including a cubic effect of autonomy ($b_6 \neq 0$) and fixing other third-order polynomial parameters to zero ($b_7 = b_8 = b_9 = 0$). This polynomial ranked among the top 5 best-fitting solutions in most of these countries, contributed to increase the explained variance by 6% to 30% relative to the linear effects model, and showed excellent fit to the data (e.g., CFI ≥ 0.974 , RMSEA ≤ 0.033). Constraining parameters to be equal in this group resulted in a model with excellent fit to the data ($\chi^2(3) = 9.0, p = .029$, CFI = 0.997, TLI = 0.991, RMSEA = 0.011, SRMR = 0.001), which showed negligible differences compared to the multigroup solution allowing parameters to differ by country (i.e., $\Delta\text{CFI} = -0.002$, $\Delta\text{Mc} < 0.001$, $\Delta\text{RMSEA} = 0.001$, $\Delta\text{SRMR} = 0.001$). This model explained 10.7% of the variance in achievement or an 8.1% increase relative to the additive model (i.e., $R^2 = .099$).

The second group comprised 12 countries (Ireland, Israel, Japan, Kazakhstan, Kuwait, Norway, Qatar, Romania, South Korea, Taipei, United Arab Emirates, and USA) coalescing around the polynomial family “level-dependent incongruence effect rotated by Y ” (family 36), which is characterized by equality constraints on interactive-quadratic effects and the cubic effect of autonomy (i.e., $b_6 = -b_7 = b_8$), and by the free estimation of the cubic effect of control ($b_9 \neq 0$). This model fitted satisfactorily all countries in this group (e.g., CFI ≥ 0.903 , RMSEA ≤ 0.061 , SRMR ≤ 0.004) and contributed to increase the explained variance by 5% to 48% relative to the linear effects model. Inspection of model parameters (SM2) of the single group solution ($\chi^2(2) = 4.8, p = .190$, CFI = 0.996, TLI = 0.987, RMSEA = 0.00, 0 SRMR = 0.003) and of invariance tests ($\Delta\text{CFI} = -0.007$, $\Delta\text{Mc} = -0.001$, $\Delta\text{RMSEA} = 0.018$, $\Delta\text{SRMR} = -0.001$) all contributed to show that the countries

Table 2 Fit indices of best-fitting polynomial solutions (vs. additive models)

Country	Solution	wAIC	chi2	CFI	RMSEA	R2	Rank
Australia	CUBICX	0.256 (0.000)	2.0 (57.3)	1.000 (0.888)	0.000 (0.054)	0.166 (0.148)	2 (32)
Chile	CUBICX	0.237 (0.000)	1.1 (41.1)	1.000 (0.787)	0.000 (0.042)	0.057 (0.044)	1 (35)
Cyprus	CUBICX	0.229 (0.000)	1.3 (51.5)	1.000 (0.899)	0.000 (0.049)	0.154 (0.139)	1 (31)
England	CUBICX	0.134 (0.000)	3.0 (36.9)	1.000 (0.868)	0.000 (0.040)	0.080 (0.070)	3 (32)
Finland	CUBICX	0.236 (0.000)	4.4 (104.8)	0.999 (0.919)	0.009 (0.054)	0.221 (0.205)	2 (32)
France	CUBICX	0.013 (0.000)	8.1 (49.9)	0.989 (0.915)	0.023 (0.042)	0.134 (0.124)	6 (31)
Hong Kong	CUBICX	0.011 (0.000)	10.5 (63.2)	0.974 (0.806)	0.033 (0.058)	0.113 (0.094)	4 (33)
Hungary	CUBICX	0.275 (0.000)	1.8 (37.3)	1.000 (0.940)	0.000 (0.039)	0.166 (0.156)	1 (32)
Italy	CUBICX	0.146 (0.000)	3.2 (35.0)	0.999 (0.926)	0.006 (0.039)	0.133 (0.123)	3 (28)
Lithuania	CUBICX	0.029 (0.000)	9.2 (41.1)	0.980 (0.868)	0.024 (0.040)	0.081 (0.072)	4 (28)
New Zealand	CUBICX	0.092 (0.000)	4.7 (41.5)	0.997 (0.924)	0.011 (0.034)	0.102 (0.095)	4 (30)
Russian Fed	CUBICX	0.164 (0.000)	4.9 (40.6)	0.995 (0.901)	0.013 (0.036)	0.087 (0.079)	3 (33)
Singapore	CUBICX	0.078 (0.000)	7.5 (81.5)	0.993 (0.878)	0.017 (0.046)	0.114 (0.101)	4 (34)
Sweden	CUBICX	0.013 (0.000)	13.1 (37.9)	0.982 (0.925)	0.029 (0.039)	0.130 (0.123)	5 (26)
Ireland	LEVDEPINCONGY	0.092 (0.000)	7.5 (72.2)	0.988 (0.849)	0.027 (0.051)	0.114 (0.099)	2 (31)
Israel	LEVDEPINCONGY	0.296 (0.000)	1.3 (46.7)	1.000 (0.744)	0.000 (0.040)	0.042 (0.031)	2 (33)
Japan	LEVDEPINCONGY	0.052 (0.000)	7.6 (58.7)	0.994 (0.938)	0.024 (0.042)	0.177 (0.168)	6 (27)
Kazakhstan	LEVDEPINCONGY	0.000 (0.000)	22.2 (53.2)	0.903 (0.749)	0.046 (0.039)	0.038 (0.032)	7 (34)
Kuwait	LEVDEPINCONGY	0.356 (0.000)	0.3 (27.5)	1.000 (0.860)	0.000 (0.030)	0.045 (0.038)	1 (33)
Norway	LEVDEPINCONGY	0.439 (0.000)	1.8 (50.7)	1.000 (0.927)	0.000 (0.047)	0.193 (0.180)	1 (34)
Qatar	LEVDEPINCONGY	0.140 (0.000)	5.0 (103.0)	0.992 (0.665)	0.020 (0.071)	0.099 (0.067)	2 (36)
Romania	LEVDEPINCONGY	0.311 (0.000)	0.1 (24.5)	1.000 (0.947)	0.000 (0.029)	0.104 (0.098)	1 (31)
South Korea	LEVDEPINCONGY	0.000 (0.000)	28.0 (136.2)	0.971 (0.842)	0.061 (0.076)	0.219 (0.193)	3 (31)
Taipei	LEVDEPINCONGY	0.000 (0.000)	26.8 (204.1)	0.982 (0.835)	0.048 (0.078)	0.223 (0.193)	2 (32)
United Arab Em	LEVDEPINCONGY	0.000 (0.000)	32.2 (115.2)	0.936 (0.652)	0.050 (0.062)	0.071 (0.050)	11 (34)
USA	LEVDEPINCONGY	0.126 (0.000)	2.2 (91.6)	0.999 (0.678)	0.007 (0.071)	0.104 (0.072)	2 (32)
Bahrain	NONPARALASYM	0.067 (0.000)	3.0 (40.5)	1.000 (0.841)	0.000 (0.033)	0.047 (0.040)	7 (31)

Table 2 (continued)

Country	Solution	wAIC	chi2	CFI	RMSEA	R2	Rank
Egypt	NONPARALASYM	0.163 (0.000)	4.8 (148.8)	0.994 (0.512)	0.010 (0.059)	0.049 (0.025)	3 (34)
Iran	NONPARALASYM	0.000 (0.000)	29.0 (49.7)	0.940 (0.889)	0.041 (0.037)	0.078 (0.074)	15 (28)
Jordan	NONPARALASYM	0.001 (0.000)	7.2 (60.2)	0.967 (0.593)	0.021 (0.050)	0.041 (0.025)	8 (32)
Lebanon	NONPARALASYM	0.150 (0.000)	1.0 (31.8)	1.000 (0.865)	0.000 (0.032)	0.054 (0.046)	1 (31)
Oman	NONPARALASYM	0.000 (0.000)	35.4 (154.8)	0.965 (0.708)	0.035 (0.066)	0.097 (0.072)	2 (32)
Saudi Arabia	NONPARALASYM	0.002 (0.000)	17.3 (77.5)	0.929 (0.592)	0.030 (0.048)	0.036 (0.023)	11 (32)
Turkey	NONPARALASYM	0.002 (0.000)	19.2 (67.3)	0.964 (0.834)	0.034 (0.048)	0.090 (0.078)	12 (34)
Georgia	QUADXQUADY	0.054 (0.017)	4.9 (9.0)	1.000 (0.985)	0.000 (0.012)	0.070 (0.069)	5 (21)
Malaysia	QUADXQUADY	0.179 (0.000)	3.4 (21.4)	1.000 (0.953)	0.000 (0.022)	0.070 (0.066)	1 (29)
Portugal	QUADXQUADY	0.050 (0.004)	5.6 (13.2)	0.999 (0.986)	0.007 (0.019)	0.158 (0.156)	6 (30)
Group							
Group 1	CUBICX	—	9.0 (147.7)	0.997 (0.928)	0.011 (0.034)	0.107 (0.099)	—
Group 2	LEVDEPINCONGY	—	4.8 (118.9)	0.996 (0.757)	0.010 (0.051)	0.072 (0.055)	—
Group 4	NONPARALASYM	—	20.8 (239.9)	0.976 (0.680)	0.018 (0.044)	0.041 (0.029)	—
Group 3	QUADXQUADY	—	4.3 (64.9)	1.000 (0.905)	0.000 (0.032)	0.072 (0.065)	—

Note. CUBICX, cubic effect of X; LEVDEPINCONGY, level-dependent incongruence effect rotated by Y; NONPARALASYM, non-parallel asymmetric congruence and incongruence effects; QUADXQUADY, quadratic effects of X and Y. Rank is relative to 37 families of polynomials (SMI). Values in parentheses correspond to a model with additive, linear effects (ADDITIVE)

in this group shared similar family parameters. The resulting model contributed to explain 7.8% of the variance ($R^2 = .072$), which represented a 30.9% increase relative to the additive model ($R^2 = .055$).

The countries from group 3 (Bahrain, Egypt, Iran, Jordan, Lebanon, Oman, Saudi Arabia, and Turkey) showed more diversity in their patterns of structural relations, but could be united around the polynomial family of “non-parallel asymmetric congruence and incongruence effect” (family 25; $CFI \geq 0.929$, $RMSEA \leq 0.041$; Table 2). This family assumes equality constraints between the interactive-quadratic and cubic effect of controlled motivation (i.e., $b_7 = b_9$), and zero constraints for the remaining third-order parameters ($b_6 = b_8 = 0$). It showed satisfactory fit to the data for the group ($\chi^2(3) = 20.8$, $p < .001$, $CFI = 0.976$, $TLI = 0.929$, $RMSEA < 0.018$, $SRMR = 0.001$) and was robust to cross-national differences within the group ($\Delta CFI = -0.013$, $\Delta Mc < 0.001$, $\Delta RMSEA = 0.012$, $\Delta SRMR = 0.002$). Although the variance in mathematics explained by this model was modest ($R^2 = .041\%$), it represented a 41% increase from the additive model ($R^2 = .029\%$), underlining important higher-order effects.

The fourth and final group included the three countries remaining (Georgia, Malaysia, and Portugal). These countries shared a common complex solution in the polynomial “quadratic effects of x and y ” (family 13), which estimates the quadratic effects of autonomy and control on achievement ($b_3 \neq 0$, $b_5 \neq 0$) but constrains other second-order ($b_4 = 0$) and third-order effects ($b_6 = b_7 = b_8 = b_9 = 0$) to be null. This complex model showed excellent fit to the data ($\chi^2(5) = 4.3$, $p = .532$, $CFI = 1.000$, $TLI = 1.004$, $RMSEA < 0.001$, $SRMR = 0.003$) and structural invariance in the group ($\Delta CFI < 0.001$, $\Delta Mc = 0.001$, $\Delta RMSEA < 0.001$, $\Delta SRMR < 0.001$), explaining 7.2% of achievement levels ($R^2 = .072$), representing an increase in explained variance of 11% relative to the linear model ($R^2 = .065$).

Interpretation of Response Surfaces Across Groups

Group 1: Nonlinearities Driven by Autonomous Motivation

The response surfaces for the models in group 1 showed evidence for motivational processes mostly compatible with SDT’s bidimensional structure (Fig. 1A). Aligning with H1, the additive model (Fig. 3A) indicated a large positive effect of autonomous motivation on achievement ($b_1 = 0.344$, $p < .001$) and a weaker negative effect of controlled motivation ($b_2 = -0.060$, $p < .001$), resulting in a response surface culminating for states reflecting internal regulation (i.e., high AM and low CM). Aligning with H2, the cubic polynomial model (Fig. 4A) showed that the effect of autonomous motivation was nonlinear, due to a negative cubic effect ($b_1 = -0.033$, $p < .001$) that introduced asymmetric curvatures in the LOC and LOIC. More precisely, beyond extreme points on the LOC (see green lines, Fig. 4A), sizable reversal effects occurred that contradicted the motivational spectrum at high and low values of autonomous motivation. Thus, students reporting extreme states reflecting identification ($AM = CM > 1.960 SD$; 6%) or non-regulation ($AM = CM < 2.100 SD$; 7%) had lower- and higher-than-expected achievement levels in mathematics

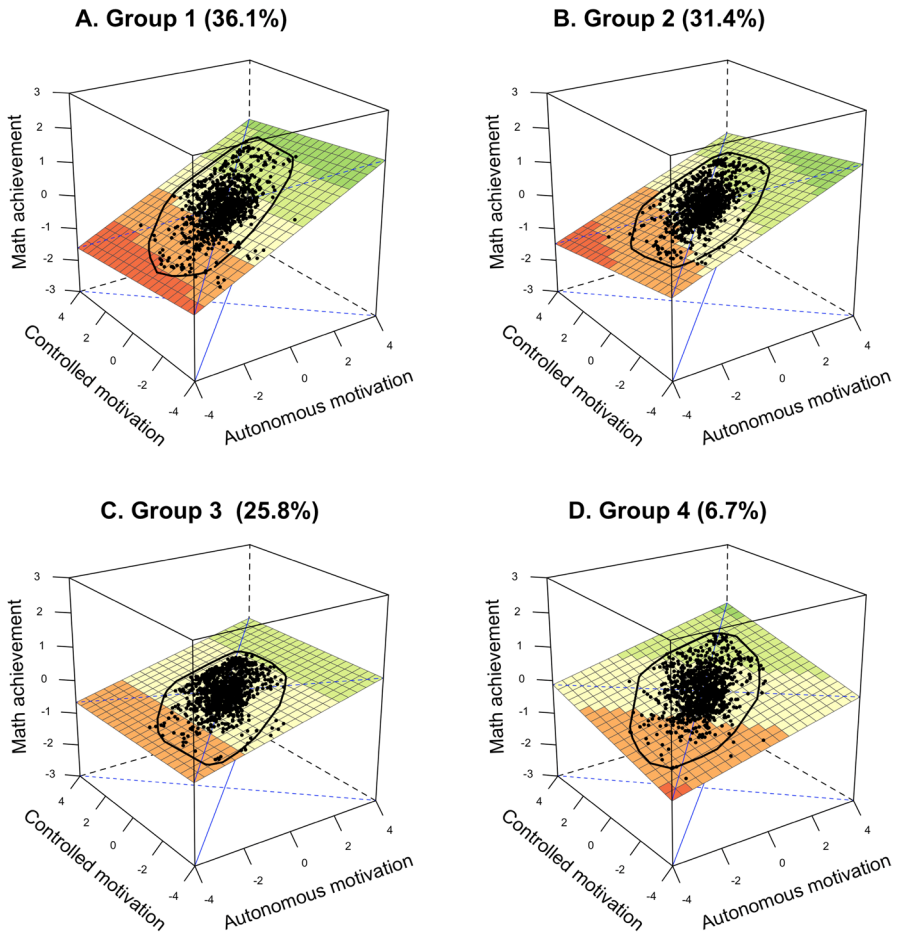


Fig. 3 Response surfaces of additive models for TIMSS 2019. Note. N , 152,825 students from 37 countries. Group 1, Australia, Chile, Cyprus, England, Finland, France, Hong Kong, Hungary, Italy, Lithuania, New Zealand, Russian Federation, Singapore, and Sweden; group 2, Ireland, Israel, Japan, Kazakhstan, Kuwait, Norway, Qatar, Romania, South Korea, Taipei, United Arab Emirates, and USA; group 3, Bahrain, Egypt, Iran, Jordan, Lebanon, Oman, Saudi Arabia, and Turkey; group 4, Georgia, Malaysia, and Portugal. For each group, 1000 randomly drawn data points are projected. Parameters are reported in Table 3

(Fig. 4.2). Part of these students even reached lower- and higher-than-average scores (respectively, 41% and 57% of them; Fig. 4.2). A similar pattern of reversal effects was apparent over the LOIC (where $AM = -CM$), but these reversals concerned very few students (<1%; Fig. 4.3). Overall, the nonlinear processes from group 1 concerned 39.2% of TIMSS students, with 100% (additive model) or 85% (cubic model) of them corroborating SDT's motivational spectrum.

Table 3 Model parameters of group solutions TIMSS 2019

Model	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	b_9
<i>Group 1 (14 countries; 55,207 students; 36.1% of students)</i>									
ADDITIVE	0.344	-0.060	0	0	0	0	0	0	0
<i>p</i> values	0.000	0.000	Fixed	Fixed	Fixed	Fixed	Fixed	Fixed	Fixed
CUBICX	0.462	-0.058	-0.002	0.012	-0.017	-0.033	0	0	0
<i>p</i> values	0.000	0.000	0.778	0.246	0.011	0.000	Fixed	Fixed	Fixed
<i>Group 2 (12 countries; 47,914 students; 31.4% of students)</i>									
ADDITIVE	0.272	-0.102	0	0	0	0	0	0	0
<i>p</i> values	0.000	0.000	Fixed	Fixed	Fixed	Fixed	Fixed	Fixed	Fixed
LEVDEPIN- CONGY	0.359	-0.119	-0.007	0.050	-0.096	-0.028	0.028	-0.028	0.011
<i>p</i> values	0.000	0.000	0.211	0.031	0.000	0.000	0.000	0.000	0.007
<i>Group 3 (8 countries, 39,482 students; 25.8% of students)</i>									
ADDITIVE	0.169	0.000	0	0	0	0	0	0	0
<i>p</i> values	0.000	0.968	Fixed	Fixed	Fixed	Fixed	Fixed	Fixed	Fixed
NONPARAL- ASYM	0.187	0.043	0.037	0.038	-0.112	0	-0.016	0	-0.016
<i>p</i> values	0.000	0.003	0.000	0.001	0.000	Fixed	0.000	Fixed	0.000
<i>Group 4 (3 countries; 10,222 students; 6.7% of students)</i>									
ADDITIVE	0.162	0.129	0	0	0	0	0	0	0
<i>p</i> values	0.000	0.000	Fixed	Fixed	Fixed	Fixed	Fixed	Fixed	Fixed
QUADXQUADY	0.165	0.130	0.054	0	-0.027	0	0	0	0
<i>p</i> values	0.000	0.000	0.000	Fixed	0.000	Fixed	Fixed	Fixed	Fixed

Note. ADDITIVE, additive (linear) effects of X and Y ; CUBICX, cubic effect of X ; LEVDEPIN-CONGY, level-dependent incongruence effect rotated by Y ; NONPARALASYM, non-parallel asymmetric congruence and incongruence effects; QUADXQUADY, quadratic effects of X and Y . Parameters b_1 to b_9 describe the cubic polynomial model (see Fig. 1C). Fit indices are reported in Table 2, whereas response surfaces are interpreted in Figs. 3 and 4

Group 2: Nonlinear-Interactions Between Autonomous and Controlled Motivations

The response surfaces in group 2 were more complex. For this group, the additive model (Fig. 3B) aligned with H1 by showing positive and negative effects of autonomous ($b_1 = 0.272$, $p < .001$) and controlled motivation ($b_2 = -0.102$, $p < .001$) on achievement, with the former dominating in terms of effect size. However, the cubic polynomial (Fig. 4B) told a more qualified story due to the presence of a negative level-dependent incongruence effect of autonomous motivation ($b_6 = -b_7 = b_8 = -0.028$, $p < .001$), mildly rotated by controlled motivation ($b_9 = 0.011$, $p = .007$). More precisely, the nonlinear effect of autonomous motivation made extreme motivational states to contradict the main trends of H1 (as in group 1), but the interactive process made this nonlinearity dependent upon levels of controlled motivation. This resulted in a nonlinear-interactive process in between H5 and H6, in which reversal effects were marked at high congruent and incongruent

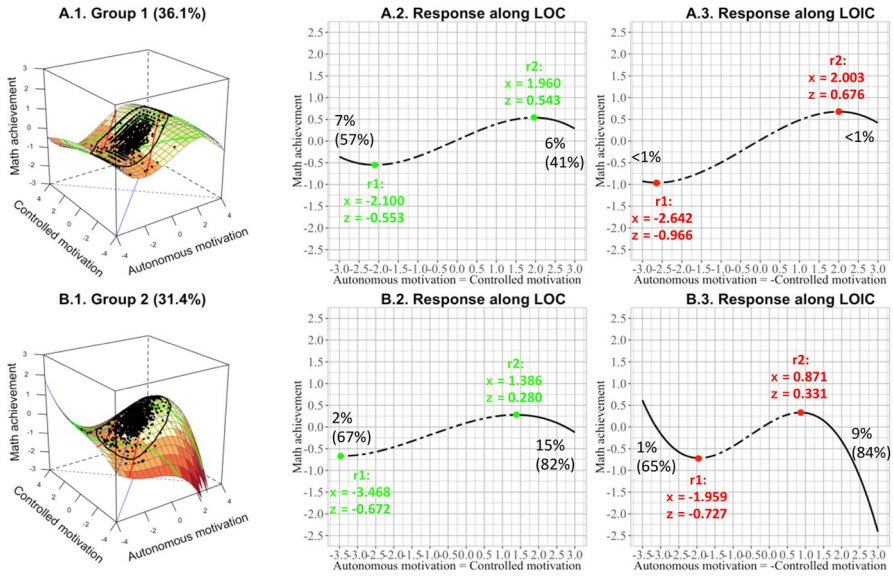


Fig. 4 Response surfaces of polynomial models for TIMSS 2019. Note. Lines in green and red intersect reversal (“r”) or acceleration points (“a”) on the LOC and LOIC, respectively. Coordinates correspond to reversal or acceleration points on the LOC and LOIC. Percentages correspond to students concerned by significant response reversals or accelerations beyond these lines, while percentages in parentheses identify those among them who also reach above- ($z > 0$) or below-average ($z < 0$) achievement in mathematics. Other features are defined as in Fig. 3. Model parameters are reported in Table 3

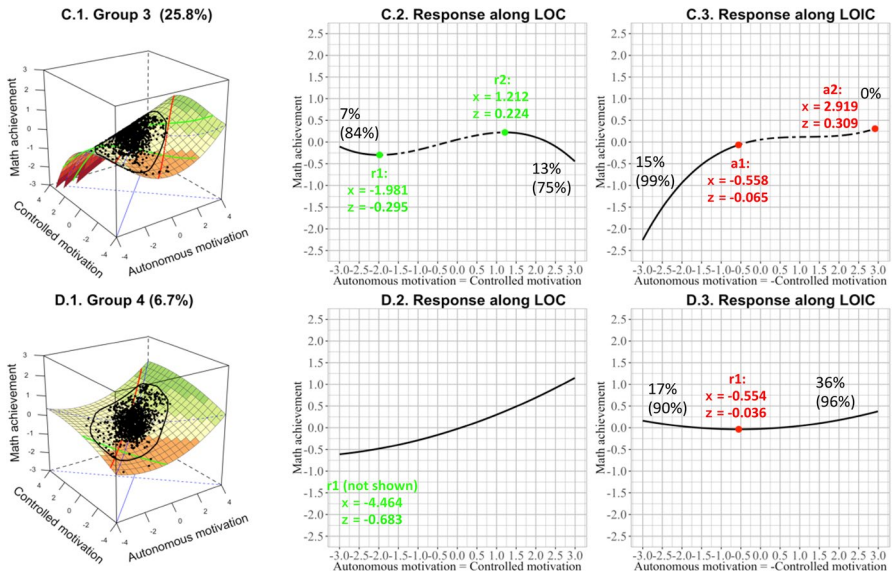


Fig. 4 (continued)

values of autonomous and controlled motivations (versus low and high congruent values in group 1). Thus, students experiencing motivational levels reflecting high-intensity identification ($AM = CM > 1.386SD$; 15%; Fig. 4.B2) or high-intensity internal regulation ($AM = -CM > 0.871SD$, or a $0.871 * 2 = 1.742SD$ relative excess in AM; 9%, Fig. 4B.3) had lower-than-expected achievement levels in mathematics. Most of these students also reached lower-than-average achievement (i.e., 82% and 84%, respectively; Fig. 4B). On the contrary, reversal effects were negligible for states reflecting high-intensity non-regulation (i.e., $AM = CM < -3.468SD$, 2%; Fig. 4B.2) or high-intensity external regulation LOIC (i.e., $AM = -CM < -1.959SD$, 1%; Fig. 4B.3). Taken together, the processes in group 2 (31.4% of students across countries) were compatible with SDT in 100% (additive model) and 73% of cases (polynomial model).

Group 3: Non-parallel Nonlinearities Driven by Autonomous and Controlled Motivations

Contrary to previous groups, the additive solution of group 3 partly contradicted SDT assumptions by showing that controlled motivation had no significant effect on achievement ($b_2 < 0.001$, $p = .968$), which created a response surface in which the LOC and LOIC showed identical positive trends equal to the effect of autonomous motivation ($b_1 = 0.169$, $p < .001$; Fig. 3C). As a consequence, achievement differences between distinct regulatory styles located on the LOC and LOIC (e.g., identification vs. internal regulation, non-regulation vs. external regulation, see Fig. 1B) were not apparent. The polynomial solution (Fig. 4C) nevertheless found that this contradictory pattern was partly due to the omission of the negative and positive cubic nonlinearities running along the LOC and LOIC, respectively (i.e., non-parallel asymmetric congruence and incongruence effect, $b_7 = b_9 = -0.016$, $p < .001$). When included, nonlinearities revealed that motivational states related to achievement in a way more compatible with SDT. For example, an acceleration effect situated on the LOIC showed that upon reaching motivational states reflecting a high degree of external regulation (i.e., $AM = -CM < -0.558SD$, or a $0.558 * 2 = 1.116SD$ relative excess in CM; 15% of students, Fig. 4C.3), achievement levels dropped abruptly. Similarly, the reversal effect occurring at levels reflecting high-intensity identification (i.e., $AM = CM > 1.212SD$; 13%, Fig. 4C.2) indicated that this state was less adaptive than levels reflecting internal regulation (i.e., $AM = -CM > 0$; Fig. 4C.3), although it was still more adaptive than external regulation in terms of achievement levels. These nonlinearities overall aligned with SDT assumptions about the achievement effects of motivational processes. In fine, only the reversal effect occurring at levels reflecting high-intensity non-regulation (i.e., $AM = CM < -1.981SD$; 7%, Fig. 4C.2) unambiguously contradicted SDT assumptions. Thus, the processes observed in group 3 (12.5% of students across countries) corroborated SDT's motivational spectrum for 50% of students for the additive model (positive effect of autonomous motivation) and in 93% of cases for the polynomial model.

Group 4: Non-informative Nonlinearities by Autonomous and Controlled Motivations

The response surfaces for the final group of countries were quite different from previous ones and partly contradicted SDT. The additive model for this group showed indeed not only a positive effect of autonomous motivation on achievement (as in other groups; $b_1 = 0.162$, $p < .001$) but also a positive effect of controlled motivation ($b_2 = 0.129$, $p < .001$). Aligning with H7, this resulted in a response surface in which the highest levels of achievement were found for motivational states reflecting identified regulation instead of internal regulation, and the lowest levels for those reflecting non-regulation instead of external motivation (Fig. 3D). The polynomial model for this group modified these observations only mildly (Fig. 4D1). More precisely, the reversal effects contradicting this trend suggested that at high intensity, external regulation or internal regulation patterns (i.e., $AM = -CM < -0.554SD$, or $AM = -CM > -0.554SD$; 17%, Fig. 4D.3) resulted in higher-than-expected achievement. Yet, these reversal effects did not change the relative standing of motivational regulations from the additive model. In other words, the nonlinear solution was not more informative than the linear solution. Overall, processes in group 4 (6.7% of TIMSS students) contradicted SDT assumptions for 50% of students.

Discussion of Study 1

Using the TIMSS 2019 database, study 1 found that the combinatory effects of autonomous and controlled motivation on achievement in mathematics were largely reducible to three fundamental polynomial models accounting for 25% to 36% of data each (i.e., in groups 1, 2, and 3) and 93% of all data together. These models largely corroborated SDT assumptions, while also showing specific nonlinear and interactive processes indicating optimal and suboptimal thresholds in motivational regulations. A fourth model accounting for a small portion of the data (7% of students, group 4) contradicted SDT's motivational spectrum and did not present very informative nonlinearities. Before interpreting these findings to provide answers to our research questions (see "General Discussion" section), some methodological issues need to be addressed. First, it is unknown to what extent each of the four patterns identified generalize to other datasets, or whether they are attributable to noise in the data (e.g., sampling variance across countries). Second, although the measures used in TIMSS showed similarities with standard measures of autonomous and controlled motivations used in the literature, this difference in operationalization could cast doubt as to the relevance of present findings to SDT research. For these reasons, a replication study using a different TIMSS dataset, as well as more standard SDT measures, is warranted.

Study 2: TIMSS 2015 and the Academic Motivation Scale

Method

Data

Two datasets were used to test the external validity of results from study 1. The first was the international database of TIMSS 2015, identical in design to TIMSS 2019 (Mullis et al., 2020), except for some countries missing out or appearing in TIMSS 2015 (see SM-4). The final sample was comparable to the one used in study 1 ($N=169,269$ 8th grade students from 36 countries), and showed similar correlations among constructs as well. The second dataset was drawn from a study on college students in a Southern California university ($N=270$; $M_{\text{age}} = 18.9$, 59% female adolescents), which was made publicly available by the authors (Pfund et al., 2018). Although focused on stimulant use, the study contained data on achievement (grade point average) and the Academic Motivation Scale (AMS; Vallerand et al., 1992), a standard measure often used in SDT studies. The measures for autonomous and controlled motivations were highly reliable (8 items, $\omega = .878$, for both measures) and correlated strongly ($r = .651$). In reference to the SDT scale, we shall call this dataset the “AMS dataset.”

Analytic Strategy

The same methodology was used as in study 1, only differing for the AMS dataset by the absence of sampling weights (not available) and of cross-national invariance testing in the AMS dataset (not applicable). Other analytical features were as before. Due to space limitations, detailed results (e.g., fit indices and model parameters) are reported in SM4.

Results

Analyses of TIMSS 2015 strongly replicated the three major structural relations from study 1 (i.e., groups 1 to 3), both in terms of the best-fitting polynomial families identified (including structural invariance within groups) and of the resulting response surfaces (see Fig. 5A, B, C). Variations occurred in their prevalence and in countries included (group 1=Canada, Chile, England, Hong Kong, Hungary, Ireland, Iran, Italy, Lebanon, Malaysia, Norway, New Zealand, Singapore, Slovenia, Sweden, Taipei, and USA; group 2=Australia, Bahrain, Israel, Japan, Kazakhstan, Lithuania, Oman, Qatar, Russia, and South Korea; group 3=Egypt, Georgia, Jordan, Morocco, Malta, and United Arab Emirates), although they still accounted for 93% of TIMSS students (as in study 1). Yet, the fourth group did not replicate well, showing a reverse nonlinearity from the one found for group 4 of study 1 (see Fig. 5D). This, as well as the observations that this group was small (6.6% of students), comprised different countries from study 1 (i.e., Kuwait, Saudi Arabia, and

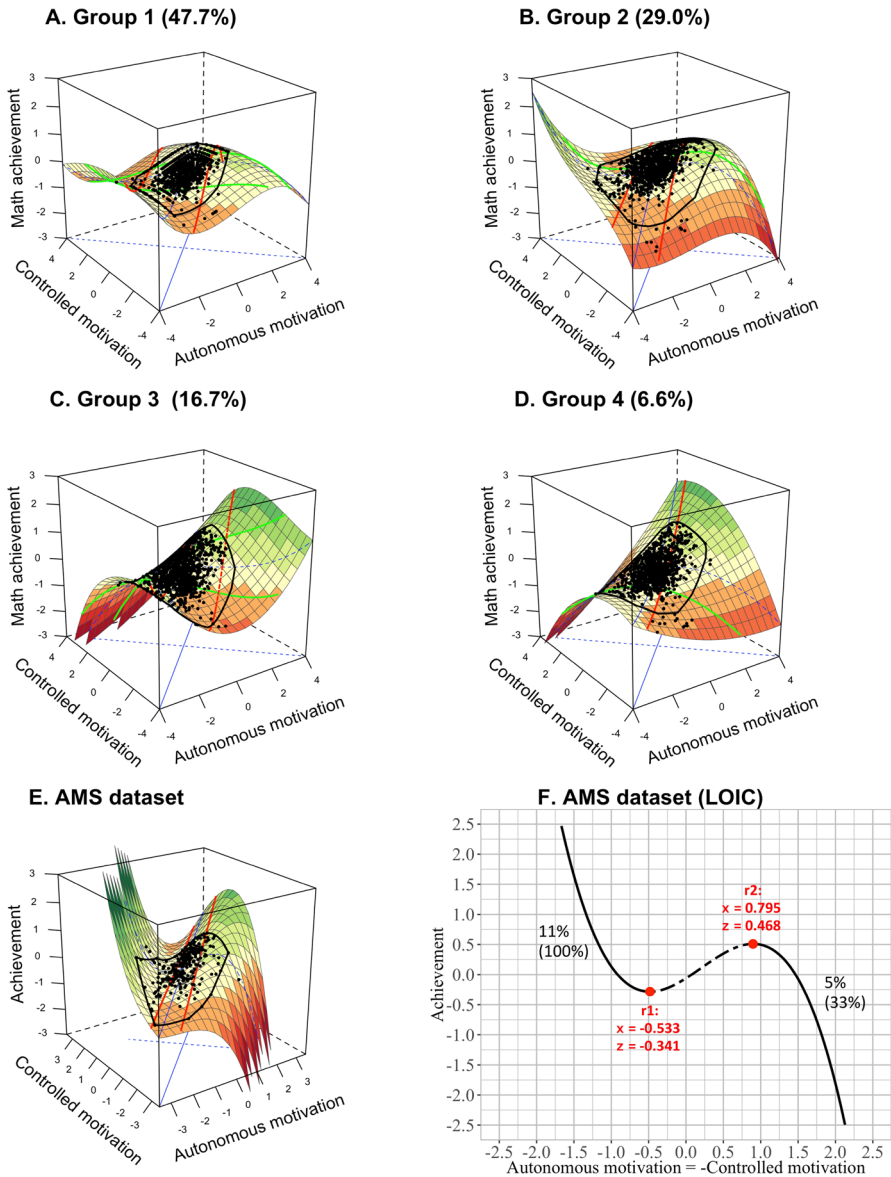


Fig. 5 Response surfaces for TIMSS 2015 and AMS datasets. Note. $N=169,269$ 8th grade students from 36 countries (TIMSS 2015 dataset) and 270 college students (AMS dataset). Model parameters and fit indices are reported in SM4

Turkey), and that the countries from group 4 of study 1 were now found in other fundamental groups (i.e., Georgia in group 2, Malaysia in group 1), all suggested that the pattern from group 4 did not reflect stable structural relations between motivational constructs and achievement. Instead, this group could be viewed as unifying

data from countries that, perhaps for methodological reasons (e.g., poor quality of protocol or sampling on the year of testing), did not align well with more fundamental processes found in other groups.

Concerning the AMS dataset, the best-fitting polynomial solution coincided with H3 (i.e., asymmetric congruence effect, family 20; Fig. 5E), although with much stronger reversal effects. Here, the response surface was as expected from the motivational spectrum for most data points (i.e., 84% of students). However, students experiencing states reflecting high-intensity internal regulation ($AM = -CM > 0.795SD$; 5% of students) or external regulation ($AM = -CM < -0.533SD$; 11% of students) had lower- and higher-than-expected achievement levels in mathematics (see Fig. 5F). Similar to groups 1 and 2 of the TIMSS datasets, this pattern reflected nonlinearity along the LOIC. The difference with groups 1 and 2 was that the nonlinearity did not affect motivational states situated on the LOC of the motivational spectrum (i.e., non-regulation and identification).

General Discussion

The present research explored novel ways to understand how motivational processes relate to achievement among school students. Based on SDT, motivational states were defined as a function of the interplay between autonomous (i.e., being driven by one's joy and pleasure in conducting tasks in mathematics) and controlled motivations (i.e., being driven by social rewards or pressures to conduct the tasks), thus creating a variety of states either supporting or undermining learning processes at school. The validity of this motivational spectrum was then tested by examining the joint effects of autonomous and controlled motivation on achievement in mathematics, using international data on 8th grade students from 37 countries (TIMSS 2019 dataset), cross-validation data (TIMSS 2015 dataset, AMS dataset), and novel techniques for cubic RSA. This design enabled testing the validity of SDT's predictions across the globe (RQ1), exploring the existence of nonlinear or interactive effects on achievement (RQ2), while also accounting for cross-national differences in the motivational processes under investigation (RQ3). Overall, the findings showed that the motivational spectrum had strong validity to predict achievement levels in diverse educational settings, thus supporting SDT as a robust, universal theory. These novel findings have implications for future research analyzing the impact of motivation on youth development and wellbeing, and for interventions aimed at supporting motivation at school.

Validity of SDT's Motivational Spectrum to Predict Achievement at School

In its basic form, the motivational spectrum expected from SDT assumes that autonomous motivation has a positive effect on achievement levels, and that controlled motivation has a weaker, negative effect. For the great majority of students in the world (93% of students from 34 countries), these predictions were fully supported by the data and contributed to explain 4% to 11% of achievement in mathematics, which

can be considered nontrivial, moderate effect sizes (Cohen, 1988). These results are particularly strong considering the fact that the measures used in the research design were coherent with previous research and showed excellent psychometric properties, being highly reliable and invariant across countries. Moreover, these observations concerned countries from various regions of the world, with quite distinct cultures not directly compared hitherto (i.e., from Europe, North America, South America, the Middle East, East Asia, Southeast Asia). To date, these findings represent one of the strongest evidence in support of SDT's universality across educational contexts (Ryan & Deci, 2017). The fact that effect sizes differed substantially between groups also suggests that the universality in question is modest ("functional") rather than strong ("accessible"), in the sense that the motivational processes undergirding achievement worked in fundamentally the same way (except for nonlinear-interactive processes, see next section), but at varying intensities (Norenzayan & Heine, 2005).

Evidence for Nonlinear and Interactive Effects of Autonomous and Controlled Motivation on Achievement

The present study also contributed to the field by exploring nonlinearities and interactions in motivational processes, using an innovative framework for RSA that allows for the comparison of linear, quadratic, and cubic polynomials (Núñez-Regueiro & Juhel, 2022, 2024a). This framework was useful not only in formulating precise hypotheses about interactions or nonlinearities but also in identifying best-fitting solutions among non-formulated hypotheses. Aligning with H2, a subset of countries (group 1, 36% of TIMSS 2019 students from 14 countries) showed evidence for a cubic effect of autonomous motivation on achievement, which was characterized by linear effects compatible with SDT for the great majority of students (85%) and by reversal effects contradicting SDT for students situated at the higher or lower end of predictor values (15% of students). More precisely, the reversal effects indicated that predictor values corresponding to extreme states of non-regulation (i.e., $-2SD$ in autonomous and controlled motivation) were positively related to achievement, instead of negatively for less extreme states. This nonlinearity provides an explanation for why person-centered studies have reported that student profiles characterized by low autonomous and high controlled motivation performed better (Gillet et al., 2017), worse (Hayenga & Corpus, 2010), or similarly compared to students with low levels on both motivations (Ratelle et al., 2007; Vansteenkiste et al., 2009). Indeed, these three inconsistent findings could simply reflect the identification of different profiles situated, respectively, beyond, on-point, or before the reversal effect, thus bearing contradictory relations to achievement. Conversely, states reflecting high-intensity identification ($+2SD$ in both autonomous and controlled motivation) were negatively related to achievement, instead of positively for low-intensity identified states. Again, this nonlinearity might explain why profiles characterized by high autonomous and controlled motivation (vs. high autonomous and low controlled) were found to perform worse (Hayenga & Corpus,

2010; Vansteenkiste et al., 2009) or equivalently (Gillet et al., 2017; Ratelle et al., 2007) depending on the clustering solution.

Other more sizable nonlinearities were found that additionally involved variations in levels of controlled motivation. For group 2 (31% of TIMSS 2019 students from 12 countries), a nonlinear-interactive process was observed that partly aligned with H5 and H6 and concerned 1 in 4 students in the group (24%). According to this process, students experiencing motivational states reflecting strong identification or strong internal regulation (i.e., 1SD above average levels of autonomous motivation, and opposite or equivalent levels of controlled motivation) performed worse in mathematics than what was expected from SDT and, for most of them (80%–90%), worse than the average student as well. These processes thus suggested that, although experiencing autonomous motivation (i.e., interest, pleasure) is generally beneficial to achievement, high-intensity forms of autonomous motivation might have a deleterious impact on achievement. To our knowledge, this finding is novel and may require theoretical complements, perhaps based on school burnout processes (as discussed hereafter).

For group 3 (26% of TIMSS 2019 students from 8 countries) the nonlinearities affected both autonomous and controlled motivational processes, resulting in a strongly complex response surface. Here, acceleration and reversal effects negatively affected achievement levels for 28% of students, notably those experiencing motivational states associated with external regulation (i.e., low autonomous and high controlled motivation) or identified regulation (high autonomous and controlled motivations). These nonlinearities could be interpreted as corroborating the deleterious impact of high levels of controlled motivation, be it at high (identification) or low levels (external regulation) of autonomous motivation. Stressing external incentives to learn mathematics (e.g., to get a good-paying job, to comply with parental authority, or because one “needs to”) could severely undermine the learning process at school, even among students genuinely interested in the discipline. This conclusion is fully concordant with SDT, but was only apparent in this group upon accounting for nonlinear effects, which underlines the heuristic value of exploring cubic polynomial models. As for group 1, the nonlinear process in group 3 also resulted in reversal effects for extreme states reflecting non-regulation, which related to unexpected higher-than-average achievement.

Besides providing explanations for inconsistent findings in the literature, these novel findings on interactive and nonlinear processes extend the current state of knowledge. They open a debate on the generality of SDT for students experiencing extreme situations of autonomous and controlled motivation, for whom complementary theories might be needed. For instance, reversal effects observed for motivational states reflecting identification (groups 1 and 2) or internal regulation (group 2) are reminiscent of school burnout processes affecting students highly interested and highly engaged in school activities (e.g., in selective or difficult academic contexts), but whose work resources are put under severe strain by their heightened awareness of school demands and of the social import of school success (Núñez-Regueiro et al., 2022; Tuominen-Soini & Salmela-Aro, 2014; Walburg, 2014). Could it be that students with a genuine interest in learning are at increased risk of experiencing deleterious burnout, notably when they focus

excessively on the learning activities and the desire to succeed at school? Some cultural contexts—yet unidentified but represented by countries in these groups—might expose students to burnout processes to a greater extent, perhaps as a result of more competitive labor markets or school-to-work transitions (e.g., job polarization or deregulation, and high graduation rates). In such contexts, being more critical about the pleasure and interest taken in school activities could result in diminished stress levels (see “cynical” profile in Tuominen-Soini & Salmela-Aro, 2014), possibly benefiting the learning process. In other words, “letting go” on autonomous motivation and focusing only on the external rewards to study could provide a means to succeed for some of these students. This hypothesis is coherent with previous data (see “controlled” profile in Gillet et al., 2017) and with reversal effects associated with external regulation (groups 1 and 2), although this only concerned few students (less than 2%).

Complementarily, reversal effects associated with non-regulated motivational states (groups 1 and 3) might be related to processes of learned helplessness among students experiencing chronic school failure, which are known to disrupt perceived contingencies between engagement of school tasks and achievement, and to instill lethargic or depressed states (Núñez-Regueiro, 2017; Maier & Seligman, 1976; Quiroga et al., 2013). It remains to be understood why these negative motivational states can create higher-than-expected (and sometimes higher-than-average) achievement. One explanation could be that students in these situations reach such intense negative motivational states where they have “nothing to lose,” that they outgrow their lethargic or depressed states and reengage with their learning process anew (e.g., for academically failing students admitted into alternative schooling structures; Nicholson & Putwain, 2015).

Finally, although other nonlinearities were observed in group 4 (7% of TIMSS 2019 students from 3 countries), these were hardly informative in terms of achievement processes and seemed not to generalize well across datasets. Intriguingly, similar response surfaces were reported in previous RSA applications with high school or college students (Brunet et al., 2015; Mouratidis et al., 2021). Whether the latter findings revealed unstable processes as in group 4, overlooked cubic nonlinearities not explored in the modeling strategy, or were not comparable due to their use of different measures of motivation, remain to be tested. In this respect, study 2 showed clear cubic nonlinearities using similar measures as in these studies (i.e., the AMS dataset), thereby indicating that the measurement problem is probably not as important as the modeling strategy or the sample considered in explaining inconsistencies with previous RSA applications.

Cross-National Differences in Motivational Processes Relating to Achievement

A final contribution of this study was revealing to what extent motivational processes varied across countries, using a common research design and formal tests for structural invariance. Although the introduction of nonlinearities and interactions in the models risked confounding substantial differences with parametric noise (i.e., sampling variance specific to each country), the comparative framework for RSA

and multigroup invariance testing enabled identifying a limited number of best-fitting solutions with strong validity in each group of countries, which replicated almost perfectly in a new dataset (i.e., solutions for groups 1 to 3 in TIMSS 2015). This means that the observed nonlinear-interactive processes reflected robust cross-national commonalities (within group) and differences (between groups) that could be considered normative hypotheses for future tests of cubic relations of autonomous and controlled motivation to achievement. For simpler linear relations, one might expect the motivational spectrum to generalize in most educational contexts (as in groups 1 and 2), but the possibility remains for contradictory findings among non-normative samples with unclear solutions (as in group 4) or with clear but overlooked cubic nonlinearities (as in group 3). Based on the present findings, cubic nonlinearities seem to provide a more complete and accurate understanding of motivational processes at school by informing on nontrivial reversal and acceleration effects, both in large assessments of international dataset (TIMSS) and in more specific research contexts based on SDT (AMS dataset).

Limitations

Three major limitations must be considered in the light of the research design. The first limitation regards the samples and the measures used to investigate SDT's motivational processes. Because the measures were specific to mathematics and 8th grade students, the relations reported above should be tested and replicated using more diversified samples (e.g., among younger or older students) and distinct learning processes (e.g., in language arts, science, or other academic disciplines). The 2019 TIMSS data contained achievement measures for 4th graders, but no measures were available for autonomous and controlled motivation for this grade. Complementary research using other international data (e.g., PISA and PIRLS) could be conducted to address this issue, while also accounting for students in complementary countries.

A second limitation concerns the nature of measures used. Although we argued for the similarities between TIMSS measures and SDT's measures of autonomous and controlled motivation, the former did not reflect all the nuances of the latter. Since structural relations revealed in this study drew mainly from the TIMSS data, complementary research is needed to probe to robustness of these findings using more traditional measures. For instance, our application of the AMS measurement framework revealed reversal effects closely related to those found for groups 1 and 2 of the TIMSS datasets. Similarly, new applications of cubic RSA using other renowned SDT measures (e.g., variants of the Academic Self-Regulation Questionnaire; Ryan & Connell, 1989; Vansteenkiste et al., 2005) might reveal specific nonlinear or interactive processes, which could inform SDT theory and educational applications.

A third limitation to consider is the causality undergirding motivational processes. The present study used cross-sectional data that informed on structural relations on a given time, but did inform on longitudinal relations. Complementary research using a longitudinal design is therefore needed to shed light on the directionality of

relations over time (e.g., whether autonomy causes achievement or conversely). One challenge to this aim consists in using appropriate modeling strategies to disaggregate processes of change relative to oneself (within-person change) versus relative to others (between-person change), lest spurious findings are obtained on longitudinal relations between constructs (Marsh et al., 2024; Núñez-Regueiro, Juhel, Bressoux, & Nurra, 2021; Hamaker et al., 2015).

Methodological and Practical Implications

Besides corroborating SDT, the present study has heuristic value for quantitative research and interventions on student motivation. In terms of research, the methodology illustrated the added value of using a modeling framework based on cubic polynomials. For instance, whereas previous applications had concluded to the absence of consistent nonlinearities in the relations of autonomous and controlled motivation to achievement (Brunet et al., 2015; Mouratidis et al., 2021), the present findings showed that the quadratic polynomial models explored in these studies—which were the only available at the time—provided poor fit to the data in most cases (except in the anomalous group 4), and that cubic models needed to be accounted to identify fundamental nonlinear-interactive processes across datasets TIMSS 2015 and TIMSS 2019. Using a comparative framework of polynomials as well as dedicated techniques to probe response curvatures (i.e., reversal and accelerations effects) is therefore highly recommended for future applications exploring SDT's motivational spectrum. Accounting for cubic nonlinearities or interactive processes might be particularly valuable to overcome inconsistent results on the relation of controlled motivation to achievement, as these can partly be explained by the omission of higher-order effects in the modeling strategy (as exemplified by group 3) or by the indirect approximation of nonlinear processes via person-centered analyses (as exemplified by group 1). Similarly, cubic RSA can be useful to complementary SDT research also dwelling on bidimensional approaches, including the circumplex model of teacher autonomy-controlling behavior (Aelterman et al., 2019), or the person-environment fit model between student basic psychological needs and environmental supplies (Núñez-Regueiro, Juhel, & Wang, 2024; Núñez-Regueiro, Santana-Monagas, & Juhel, 2024).

From a more educational standpoint, the validation of the bidimensional structure of motivations in explaining achievement levels has heuristic value, because it reveals alternative pathways to attain optimal student motivation (i.e., internal regulation) from non-regulated states (see Fig. 1B), with general applicability across cultures (i.e., functional universality; Norenzayan & Heine, 2005). The external pathway attempts to gradually move students from non-regulation to controlled forms of regulation (i.e., external regulation and introjection) to more autonomous ones (i.e., identification and integration), eventually reaching internal regulation (i.e., low controlled and high autonomous motivation). This is the approach privileged by SDT-based interventions, notably in the form of social and personal incentives such as enjoying positive student–teacher relationships (Assor et al., 2018; Hornstra et al., 2020). The risk, here, is for students to remain in controlled regulatory states,

without evolving toward more autonomous states. By contrast, the internal pathway aims at facilitating the emergence of internal regulation without resorting to incentives from extrinsic rewards, as proposed by SDT interventions based on mindfulness theory (Brown & Ryan, 2004). This can be done by introducing game, mystery, or personal relevance in instructional designs (Schutte & Malouff, 2022) or, more directly, by supporting the emergence of absorbing experiences using exercises aimed at sustaining self-consciousness of thought processes (Creswell, 2017). The risk, here, is for learning goals to become too opaque (vs. explicit) for low-achieving students with learning difficulties, thus increasing disparities in acquisitions between high and low achievers (alike minimally guided instruction; Kirschner et al., 2006). All in all, both pathways might be considered for different student profiles.

One key approach of significance to both pathways is the framing of messages to engage students in class. Preliminary findings showed indeed that the kind of motivation (autonomous, controlled) that teachers put forward to underscore the benefits of engaging tasks in class contributes to determine the corresponding motivational state among students (León et al., 2024; Santana-Monagas et al., 2022). In this perspective, informing and training teachers on strategic engaging messages could be a strong lever to determine student motivational processes in a way that supports learning processes at school and, more generally, self-determined behavior in youth development.

More generally, by providing an explanatory framework for these two motivational pathways, the bidimensional spectrum enables a dynamic and corrective relation to pedagogy, in which educators can calibrate their dosage of controlling and autonomous appeals depending on the location of students' motivation on the pathway that is chosen. This dynamic, spectral approach to pedagogy appears more intuitive and comprehensive than what is afforded by a categorical framework (i.e., SDT's taxonomy of motivational regulations), in which the exact number and educational implications of categories of regulations are difficult to establish. In considering this approach, it is important to expect for reversals in motivational processes, which concerned a substantial proportion of students in this study (from 15% of students in group 1 to 27% in group 2). These reversals provide valuable information for targeting more specific learning situations, notably among students bestowing high-intensity autonomous or controlled motivational orientations (i.e., where reversal effects occur), for whom motivational appeals may work differently from other students in supporting learning processes.

Appendix. Details on Cubic Polynomial Modeling

The cubic polynomial model is of the form $z = b_0 + b_1x + b_2y + b_3x^2 + b_4xy + b_5y^2 + b_6x^3 + b_7x^2y + b_8xy^2 + b_9y^3$ and consists of three qualitatively distinct parameters responsible for trends (b_1 to b_2), curvatures (b_3 to b_5), and asymmetric curvatures (b_6 to b_9) in the response surface z . Predefined parametric constraints on b_1 to b_9 can then be applied to create families of polynomial models (there are currently 37 families or FMs), the combination of which can

generate over 2000 congruence hypotheses (for a list, see Núñez-Regueiro & Juhel, 2022, 2024a). In this method, the lines of congruence (LOC, $x = y$) and incongruence (LOIC, $x = -y$) have heuristic value for hypothesis generation, because substituting y by x values rewrites the cubic model twofold:

$$z(\text{LOC}) = b_0 + (b_1 + b_2)x + (b_3 + b_4 + b_5)x^2 + (b_6 + b_7 + b_8 + b_9)x^3$$

$$z(\text{LOIC}) = b_0 + (b_1 - b_2)x + (b_3 - b_4 + b_5)x^2 + (b_6 - b_7 + b_8 - b_9)x^3$$

By imposing constraints on these two models, precise hypotheses about the behavior of the outcome along LOC and LOIC can be formulated. In this study, we conceived nine hypotheses about the combined effects of autonomous motivation (x) and controlled motivation (y) on achievement (z). For example, H1 assumed that AM ($b_1 > 0$) and CM ($b_2 < 0$) had positive and negative effects on achievement, respectively, and that the effect size of AM was larger than that of CM ($|b_1| > |b_2|$). First-order metaparameters for the slopes along the LOC (i.e., $u_1 = b_1 + b_2$) and the LOIC (i.e., $v_1 = b_1 - b_2$) were therefore positive. H2 was a nonlinear variant of H1 in which a negative cubic effect of AM was allowed for (i.e., $b_6 < 0$; Fig. 2B). Hypothesis 3 was a variant of H2 itself, in the form of a negative asymmetric congruence effect (i.e., $b_6 = \frac{b_7}{-3} = \frac{b_8}{3} = -b_9 > 0$, FM20) that confined the asymmetric curve to the LOIC. This was due to the fact that the third-order metaparameter of the LOC is canceled out by the constraints of this family (i.e., $u_3 = 0$), creating a ridge in the surface, tilted positively by the slope along the LOC ($u_1 > 0$). Similar rationales were used to determine the parametric constraints corresponding to the remaining hypotheses (H4 to H9). For details on polynomial families, rationales for their combination, dedicated software, and illustrative examples, see references (Núñez-Regueiro & Juhel, 2022, 2024a, 2024b, 2024c, 2024c; Núñez-Regueiro et al., 2024; Núñez-Regueiro et al., 2024).

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Data Availability The TIMSS datasets used in this research are freely available on the TIMSS website (<https://timssandpirls.bc.edu/timss-landing.html>). The AMS dataset is provided as open data by Pfund and her colleagues on the Open Science Framework (<https://osf.io/j2g7t/>). The software used for conducting the analyses is freely available on the Comprehensive R Archive Network (<https://CRAN.R-project.org/package=RSAtools>).

Declarations

Conflict of Interest The authors declare no competing interests.

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