



HAL
open science

Continuous control of the angular spread in Ambisonics

Thibaut Carpentier

► **To cite this version:**

Thibaut Carpentier. Continuous control of the angular spread in Ambisonics. 156th Convention of the Audio Engineering Society (AES), Jun 2024, Madrid, Spain. hal-04663367

HAL Id: hal-04663367

<https://hal.science/hal-04663367v1>

Submitted on 24 Sep 2024

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Audio Engineering Society

Convention Paper

Presented at the AES 156th Convention
2024 June 15–17, Madrid, Spain

This paper was peer-reviewed as a complete manuscript for presentation at this convention.

Continuous control of the angular spread in Ambisonics

Thibaut Carpentier¹

¹*STMS Lab, IRCAM – CNRS – Sorbonne Université – Ministère de la Culture*

Correspondence should be addressed to Thibaut Carpentier (thibaut.carpentier@ircam.fr)

ABSTRACT

When mixing or post-producing audio scenes, it is often desirable to modify the spatial characteristics of the sound field. In this article we present a method for controlling the angular spread of a sound source or a sound scene encoded in Ambisonics. To do this, we develop a mathematical approach allowing the norm of the energy vector to be continuously varied, by simulating fractional Ambisonics orders. We derive this approach for the 3D and 2D cases, then present the objective results in particular in the form of equivalent directivity patterns. The technique proposed here corrects and completes two previous publications on related topics [1, 2].

1 Introduction

Sound designers, composers and mixers need tools to creatively manipulate the spatial image during production, reproduction or post-production of 3D sound scenes (synthesized or recorded). When working with Ambisonics scenes, spatial transformations of the sound field can be obtained by applying a (frequency-independent) transformation matrix in the spherical harmonics domain [3, Chapter 5.2]. Several transformation operators have been proposed previously in the literature, such as 3D rotation effects, mirroring, directional emphasis, warping, etc.

The objective of this study is to propose a tool allowing to continuously control the angular spread – or spatial extent – of a virtual source or an Ambisonics scene. Indeed, the angular spread (denoted σ_E) of sources is an attribute known to be particularly important for composers and sound designers [4, 5]. The creation of a spreading effect, sometimes called “divergence” or “focus”, can be achieved by distributing (coherently or incoherently) the energy from a

phantom source to a certain number of speakers so as to increase or reduce the perceived extent of the virtual source [6, 7, 8, 9]. In a previous work [1], a technique had been proposed allowing to blur an Ambisonics scene by gradually reducing the encoding order N of the scene (while preserving the overall energy, under diffuse field hypothesis). A limitation of this previous study is that it does not allow intuitive control of the angular spread, as the latter varies non-linearly with the chosen “blur factor” (see [1, Eq. 14 and Fig. 6]). In this paper, we propose a new approach that provides simple control of σ_E .

2 Background

2.1 Vector models

Vector models [10], derived from Makita’s localization theory [11], are frequently used to objectively evaluate amplitude-panned sources reproduced with a loudspeaker setup. While vector models characterize the acoustic propagation of sounds, they are used as “psychoacoustic” cues and claimed to predict the perceived

direction of a virtual source [12]. The velocity vector \mathbf{V} assumes an amplitude combination of the loudspeaker contributions, and is used to predict the apparent direction of a virtual source in the low frequency domain (up to approximately 700 Hz) [13]. In the high frequency domain, it is more relevant to consider an energetic superposition of the signals, and the energy vector \mathbf{E} is used. It writes [13, Eq. 14] :

$$\mathbf{E} = \frac{\sum_{k=1}^K g_k^2 \mathbf{u}_k}{\sum_{k=1}^K g_k^2} = r_E \mathbf{u}_E, \quad (1)$$

where K is the number of loudspeakers, g_k denotes the gain of the k^{th} speaker, and \mathbf{u}_k its (normalized) direction. The norm of the energy vector, denoted r_E (with $0 \leq r_E \leq 1$), represents the concentration of energy in the direction \mathbf{u}_E . The direction \mathbf{u}_E of the energy vector is used as an indicator of the perceived direction of the virtual source, while its norm r_E correlates with the perceived width of the source [14, 15] : the shorter the energy vector, the more broadly the energy is distributed over the loudspeakers and the more the source is perceived as “wide”. Several authors have proposed analytical formulas to quantify this angular spread :

$$\sigma_E = \begin{cases} \arccos(r_E) & \text{according to [12, 16],} \\ 2 \arccos(2r_E - 1) & \text{see [17, Eq. 11],} \\ 186.4^\circ (1 - r_E) + 10.7^\circ & \text{see [14, Eq. 22].} \end{cases} \quad (2)$$

The first two formulas can be interpreted geometrically, the angular spread corresponding to the angular aperture of the area of space in which the energy is distributed. The third formula is derived from perceptual studies [18].

2.2 “max- r_E ” optimization

When decoding an N -order Ambisonics stream, Daniel et al. have shown that it is possible to optimize the reproduction by maximizing the norm r_E of the energy vector. This is achieved by applying weighting coefficients $d_{N,n}$ (for $0 \leq n \leq N$) to the Ambisonics components. In the 3D case, this leads to (see [12, Eq. A.64] or [3, Eq. A.45]) :

$$r_E(N) = \frac{2 \sum_{n=1}^N n d_{N,n} d_{N,n-1}}{\sum_{n=0}^N (2n+1) d_{N,n}^2}. \quad (3)$$

Daniel et al. [12, 13] showed how to choose the weights $d_{N,n}$ so as to maximize r_E . The solution is expressed as [12, Eq. 3.89]

$$\forall n \leq N, d_{N,n} = \mathcal{P}_n(\eta), \quad (4)$$

where \mathcal{P}_n denotes the Legendre polynomial of order n , and $\eta \equiv r_E^{\max}(N)$ is the largest root of $\mathcal{P}_{N+1}(\cdot)$, that is $\mathcal{P}_{N+1}(\eta) = d_{N,N+1} = 0$. The roots of the polynomials \mathcal{P}_n are not known analytically, but an approximate solution was proposed by Zotter et al. [15, Eq. 38]

$$r_E^{\max}(N) \approx \cos\left(\frac{137.9^\circ}{N+1.52}\right), \quad (5)$$

which is presented in Figure 1. As a consequence, Eq. 4 can be rewritten as [15, Eq. 10] :

$$\forall n \leq N, d_{N,n} = \mathcal{P}_n\left(\cos\left(\frac{137.9^\circ}{N+1.52}\right)\right). \quad (6)$$

3 Proposed method

3.1 Continuous variation of r_E

Offering continuous control over the angular spread σ_E (or equivalently over r_E) amounts to simulating fractional Ambisonics orders $\nu \in \mathbb{R}^+$, such that the energy vector varies as (see Figure 1) :

$$r_E^{\max}(\nu) \approx \cos\left(\frac{137.9^\circ}{\nu+1.52}\right). \quad (7)$$

In [2, paragraph 7.2], a simple solution to this problem was proposed, by generalizing Eq. 6 to fractional orders, such that

$$\forall n \leq N, d_{\nu,n} = \mathcal{P}_n\left(\cos\left(\frac{137.9^\circ}{\nu+1.52}\right)\right). \quad (8)$$

It turns out that this solution, which seemed “obvious”, is unfortunately incorrect. It is displayed in red dashed lines in Figure 1. We observe that the energy vector resulting from these weightings $d_{\nu,n}$ does not match the expected $r_E(\nu)$ curve (Eq. 7). The error is more pronounced for lower orders ($\nu \leq 4$).

In the following paragraphs, we will develop a new approach that ensures an exact solution. To do this, we return to the notion of fractional Ambisonics orders ν .

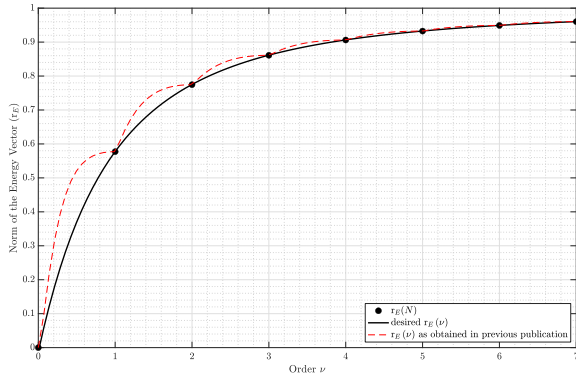


Fig. 1: Evolution of r_E as a function of the Ambisonics fractional order ν . The discrete markers \bullet represent $r_E^{\max}(N)$ (Eq. 5). In solid black line, the desired shape of $r_E^{\max}(\nu)$ (Eq. 7). In red dashed line, the results of the (erroneous) approach proposed in [2, paragraph 7.2].

3.2 Synthesizing fractional orders

A method for designing directional patterns of fractional orders was originally proposed by Huang et al. [19, 20]. The method basically consists in creating a linear combination of harmonic representations of order $N-1$ and N . For a fractional order ν such that $N-1 \leq \nu \leq N$, we write

$$d_{\nu,n} = \begin{cases} \alpha d_{N,n} + (1-\alpha) d_{N-1,n} & \text{if } 0 \leq n < N \\ \alpha d_{N,n} & \text{if } n = N. \end{cases} \quad (9)$$

The “interpolation” factor α (such that $\alpha \in \mathbb{R}$ and $0 \leq \alpha \leq 1$) obviously depends on the fractional order ν ; this weighting between orders $N-1$ and N must be determined according to an imposed design criterion. In [19, 20], this framework was applied to circular and spherical microphone arrays, and the tuning parameter α was determined so as to optimize the directivity factor or the white noise gain of the arrays. In [2], the author has applied a similar methodology to the design of other beampatterns relevant for Ambisonics processing. Various design criteria have been examined such as: maximizing the directivity index (hypercardioid pattern), maximizing the front-to-back ratio (supercardioid pattern), canceling the rear lobes (cardioid pattern or *in-phase*), etc. In the present study, the criterion of interest is the maximization of the norm $r_E(\nu)$, a problem commonly referred to as “max- r_E ” optimization [12, 13].

3.3 Determination of α according to the max- r_E criterion

According to Eq. 3, the energy vector r_E for fractional orders ν can be written in terms of the weights $d_{\nu,n}$ (Eq. 9) :

$$r_E(\nu) = \frac{2 \sum_{n=1}^N n d_{\nu,n} d_{\nu,n-1}}{\sum_{n=0}^N (2n+1) d_{\nu,n}^2}. \quad (10)$$

Here we will expand the numerator $\text{num}(\nu)$ and the denominator $\text{den}(\nu)$ of this expression; to do this, we separate the terms $n < N$ and $n = N$:

$$\begin{aligned} \text{den}(\nu) &= \sum_{n=0}^N (2n+1) d_{\nu,n}^2 \\ &= \sum_{n=0}^{N-1} (2n+1) d_{\nu,n}^2 + (2N+1) d_{\nu,N}^2. \end{aligned} \quad (11)$$

Inserting the expressions for the weights $d_{\nu,n}$ (Eq. 9) for $n < N$ and $n = N$ respectively, we obtain

$$\begin{aligned} \text{den}(\nu) &= \sum_{n=0}^{N-1} (2n+1) (\alpha d_{N,n} + (1-\alpha) d_{N-1,n})^2 \\ &\quad + (\alpha d_{N,N})^2, \end{aligned} \quad (12)$$

which, after some developments and simplifications, leads to a simple formulation of $\text{den}(\nu)$ as a function of α :

$$\text{den}(\nu) = \alpha^2 A + 2\alpha(1-\alpha)B + (1-\alpha)^2 C, \quad (13)$$

where we introduced the following notations

$$\begin{cases} A = \sum_{n=0}^N (2n+1) d_{N,n}^2, \\ B = \sum_{n=0}^{N-1} (2n+1) d_{N,n} d_{N-1,n}, \\ C = \sum_{n=0}^{N-1} (2n+1) d_{N-1,n}^2. \end{cases} \quad (14)$$

It should be noted here that A , B and C are known and constant (i.e. independent of ν or α , and dependent only on the chosen maximum Ambisonics order N). They can be determined analytically since all terms $d_{N,n}$ are known (cf. Eq. 6).

We proceed in the same way with the numerator :

$$\begin{aligned} \text{num}(\nu) &= 2 \sum_{n=1}^N n d_{\nu,n} d_{\nu,n-1} \\ &= 2 \sum_{n=1}^{N-1} n d_{\nu,n} d_{\nu,n-1} + 2N d_{\nu,N} d_{\nu,N-1} . \end{aligned} \quad (15)$$

Replacing the weights $d_{\nu,n}$ (Eq. 9), we obtain after some elementary developments :

$$\begin{aligned} \text{num}(\nu) &= \alpha^2 D + \alpha(1-\alpha)E \\ &\quad + \alpha(1-\alpha)F + (1-\alpha)^2 G , \end{aligned} \quad (16)$$

with the constants D, E, F, G defined as follows

$$\begin{cases} D = 2 \sum_{n=1}^N n d_{N,n} d_{N,n-1} , \\ E = 2 \sum_{n=1}^N n d_{N,n} d_{N-1,n-1} , \\ F = 2 \sum_{n=1}^{N-1} n d_{N-1,n} d_{N,n-1} , \\ G = 2 \sum_{n=1}^{N-1} n d_{N-1,n} d_{N-1,n-1} . \end{cases} \quad (17)$$

As noted previously, these terms D, E, F, G are entirely determined, and can be easily evaluated.

Having developed the numerator and denominator, we can now express the norm of the energy vector as a function of α

$$\begin{aligned} r_E(\nu) &= \frac{\text{num}(\nu)}{\text{den}(\nu)} \\ &= \frac{\alpha^2 D + \alpha(1-\alpha)(E+F) + (1-\alpha)^2 G}{\alpha^2 A + 2\alpha(1-\alpha)B + (1-\alpha)^2 C} . \end{aligned} \quad (18)$$

We recognize here a second degree equation in α

$$a\alpha^2 + b\alpha + c = 0 , \quad (19)$$

with the following (constant) coefficients

$$\begin{cases} a = D - (E+F) + G - r_E(\nu)(A - 2B + C) , \\ b = (E+F) - 2G - 2r_E(\nu)(B - C) , \\ c = G - Cr_E(\nu) . \end{cases} \quad (20)$$

The determinant of the quadratic equation is $\Delta = b^2 - 4ac$, and we admit here that this determinant is always positive ($\forall \nu \geq 0, \Delta \geq 0$). If such result is not trivial to prove analytically, its verification can be done numerically. We display in Figure 2 the numerical evaluation of Δ for different values of the fractional order ν , and for $N = 7$.

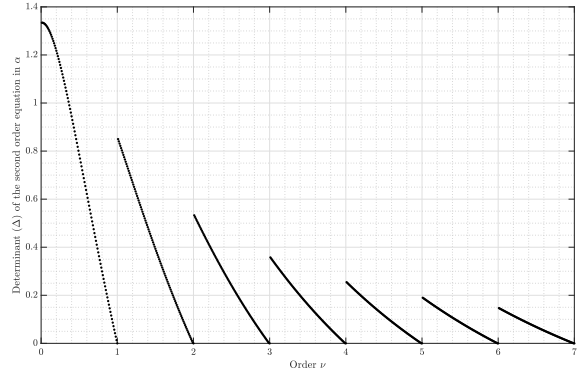


Fig. 2: Determinant Δ of the quadratic equation in α (Eq. 19) for various values of ν in $[0, 7]$.

Solving the quadratic equation is then straightforward. Out of the two real solutions, we retain the one satisfying $0 \leq \alpha \leq 1$. To conclude, we have shown how to determine α according to the desired fractional order ν . Consequently, the weights $d_{\nu,n}$ can be evaluated for any fractional order ν .

The method proposed in this section considered 3D Ambisonics sound field. The approach can be straightforwardly adapted to the 2D case; the corresponding results are presented in the Appendix (Section 5).

3.4 Continuous control of the spread

We note here ζ a control parameter, expressed in %, which will allow linear variation of the angular spread σ_E . Such a linear control law ζ is written in the form

$$\sigma_E = \frac{(\sigma_E^{\max} - \sigma_E^{\min})}{(100\% - 0\%)} \zeta + \sigma_E^{\min} , \quad (21)$$

where σ_E^{\min} and σ_E^{\max} denote the minimum and maximum achievable spread values. For a given order N , the minimum spread is given by

$$\sigma_E^{\min} = 2 \arccos \left(2 \cos \left(\frac{137.9^\circ}{N + 1.52} \right) - 1 \right) , \quad (22)$$

using here the formula from Epain et al. [17] (see Eq. 2). The maximum spread obviously corresponds to $\sigma_E^{\max} = 360^\circ$.

For a continuous control of the spread, the procedure is as follows :

1. the user specifies the desired control parameter ζ (with $0 \leq \zeta \leq 100$),
2. the corresponding angular spread σ_E is calculated (Eq. 21),
3. the norm of the energy vector r_E is derived (Eq. 2),
4. the corresponding fractional order ν is determined (Eq. 7),
5. the interpolation factor α is obtained by solving Eq. 19,
6. finally, the weights $d_{v,n}$ are evaluated (Eq. 9).

The evolution of the corresponding quantities (σ_E , r_E , ν , α) is represented in Figure 3 (a, b, c, and d respectively), as a function of the control parameter ζ .

Finally, the results of our approach are presented in Figure 4, showing the equivalent directivity patterns, for different values of ζ between 0 and 100%. As expected, we observe a continuous (and linear) variation of the angular spread, as manifested by the widening of the main lobe. For $\zeta = 0\%$, we obtain an optimal max- r_E pattern for the given order $N = 7$, and corresponding to an angular spread $\sigma_E^{\min} \approx 60^\circ$. As can be observed in Figure 3(c), we synthesize almost integer order patterns for $\nu \approx 2\%$ ($N = 6$), $\nu \approx 4\%$ ($N = 5$), $\nu \approx 8\%$ ($N = 4$), $\nu \approx 13\%$ ($N = 3$), $\nu \approx 21\%$ ($N = 2$), and $\nu \approx 37\%$ ($N = 1$). For these values of ν , we observe in Figure 4 the known max- r_E patterns, as presented for example in [12, Fig. 3.14].

3.5 Implementation

The proposed method was developed in C++ language and integrated into the Spat5 library for Max [21, 22], namely in the external object `spat5.hoa.directivity` which implements weighting laws $d_{N,n}$ according to various design criteria. The equations introduced in the previous paragraphs are numerically simple, therefore computationally efficient, and well suited to real-time uses (dynamic variations of the spread σ_E).

Figure 5 illustrates the implementation in Max, with a graphical representation of the weights $d_{v,n}$, a visualization of the equivalent directivity pattern, the calculation of various metrics, and the application coefficients $d_{v,n}$ to an Ambisonics audio stream.

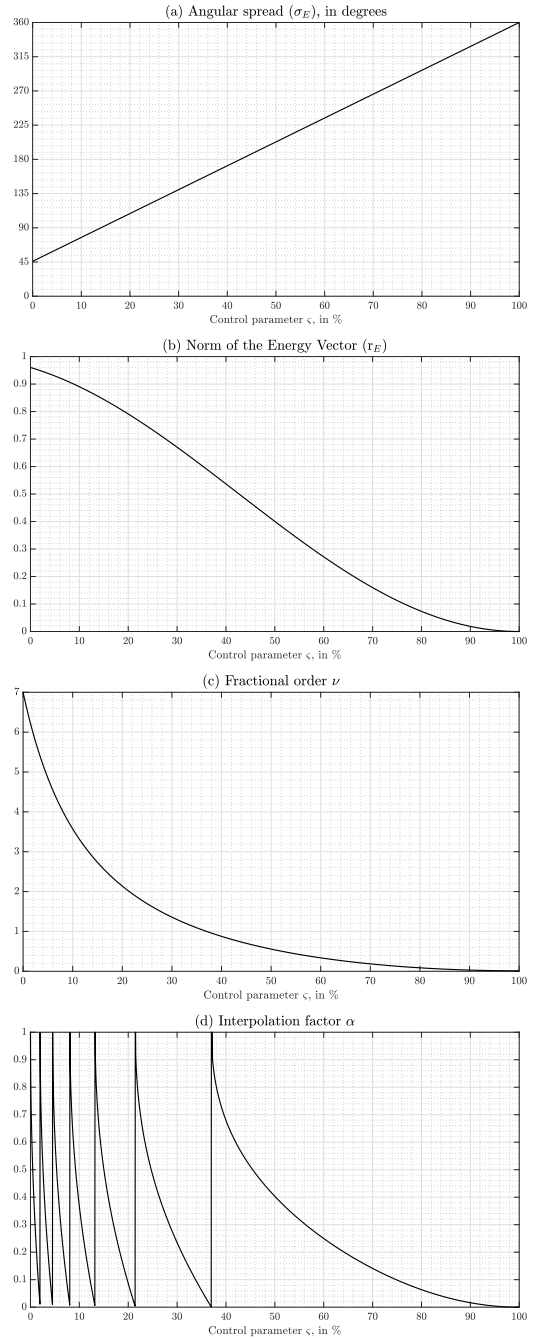


Fig. 3: (a) Angular spread σ_E , (b) Energy vector r_E , (c) Fractional order ν , and (d) Interpolation factor α , as function of the control parameter ζ . The figures are for $N = 7$.

4 Conclusion

In this article we presented a method for controlling the angular spread of a virtual source (or a sound field) encoded in Ambisonics. The approach is based on a continuous variation of the energy vector norm, obtained by simulating fractional orders. The technique, mathematically simple, is integrated into the Max environment for interactive audio applications. We have shown that the expected objective results are achieved, and informal listenings confirmed the effectiveness of the method.

However, the proposed solution also has limitations : the framework is indeed based on a “coherent” spreading approach, that is to say that the signal from a virtual source is sent fully correlated to all active loudspeakers; it has been shown that this can give rise to ambiguous auditory events or timbral colorations [23, 8], and this is why several authors have suggested alternative incoherent spreading techniques, typically based on decorrelation or frequency-dependent filtering [24, 25, 26]. It would therefore remain to conduct perceptual studies comparing the proposed approach to other state of the art strategies.

References

- [1] Carpentier, T., “Ambisonic spatial blur,” in *Proc. of the 142nd Convention of the Audio Engineering Society (AES)*, Berlin, Germany, 2017.
- [2] Carpentier, T., “Spherical beampatterns with fractional orders,” in *Proc. of the Forum Acusticum, 10th Convention of the European Acoustics Association (EAA)*, pp. 607 – 614, Torino, Italy, 2023, doi:10.61782/fa.2023.0531.
- [3] Zotter, F. and Frank, M., *Ambisonics: A Practical 3D Audio Theory for Recording, Studio Production, Sound Reinforcement, and Virtual Reality*, Springer, Cham, Switzerland, 1st edition, 2019, doi:10.1007/978-3-030-17207-7.
- [4] Peters, N., Marentakis, G., and McAdams, S., “Current Technologies and Compositional Practices for Spatialization: A Qualitative and Quantitative Analysis,” *Computer Music Journal*, 35(1), pp. 10 – 27, 2011, doi:10.1162/COMJ_a_00037.
- [5] Baalman, M. A., “Spatial Composition Techniques and Sound Spatialisation Technologies,” *Organised Sound*, 15(3), pp. 209 – 218, 2010, doi:10.1017/S1355771810000245.
- [6] Pulkki, V., “Uniform spreading of amplitude panned virtual sources,” in *Proc. of the IEEE Workshop on Applications of Signal Processing to Audio and Acoustics (WASPAA)*, pp. 187 – 190, New Paltz, NY, USA, 1999, doi:10.1109/aspaa.1999.810881.
- [7] Rumsey, F., *Spatial Audio*, Routledge, New York, NY, USA, 1st edition, 2001, doi:10.4324/9780080498195.
- [8] Roginska, A. and Geluso, P., *Immersive Sound: The Art and Science of Binaural and Multi-Channel Audio*, Routledge, New York, NY, USA, 1st edition, 2018, doi:10.4324/9781315707525.
- [9] Ziemer, T., *Psychoacoustic Music Sound Field Synthesis: Creating Spaciousness for Composition, Performance, Acoustics and Perception*, Springer, Cham, Switzerland, 2019, doi:10.1007/978-3-030-23033-3.
- [10] Gerzon, M. A., “General metatheory of auditory localization,” in *Proc. of the 92nd Convention of the Audio Engineering Society (AES)*, Vienna, Austria, 1992.
- [11] Makita, Y., “On the directional localization of sound in the stereophonic sound field,” in *EBU Review*, volume 73, pp. 102 – 108, 1962.
- [12] Daniel, J., *Représentation de champs acoustiques, application à la transmission et à la reproduction de scènes sonores complexes dans un contexte multimédia*, Ph.D. thesis, Université de Paris VI, 2000.
- [13] Daniel, J., Rault, J.-B., and Polack, J.-D., “Ambisonics Encoding of Other Audio Formats for Multiple Listening Conditions,” in *Proc. of the 105th Convention of the Audio Engineering Society (AES)*, San Francisco, CA, USA, 1998.
- [14] Frank, M., *Phantom Sources using Multiple Loudspeakers in the Horizontal Plane*, Ph.D. thesis, Institute of Electronic Music and Acoustics, Graz, Austria, 2013.
- [15] Zotter, F. and Frank, M., “All-Round Ambisonic Panning and Decoding,” *Journal of the Audio Engineering Society*, 60(10), pp. 807 – 820, 2012.

- [16] Bertet, S., Daniel, J., Parizet, E., and Warusfel, O., “Investigation on Localisation Accuracy for First and Higher Order Ambisonics Reproduced Sound Sources,” *Acta Acustica united with Acustica*, 99, pp. 642 — 657, 2013, doi:10.3813/aaa.918643.
- [17] Epain, N., Jin, C., and Zotter, F., “Ambisonic Decoding With Constant Angular Spread,” *Acta Acustica united with Acustica*, 100(5), pp. 928 — 936, 2014, doi:10.3813/AAA.918772.
- [18] Frank, M., Marentakis, G., and Sontacchi, A., “A simple technical measure for the perceived source width,” in *Proc. of the Fortschritte der Akustik (DAGA)*, pp. 691 – 692, Düsseldorf, Germany, 2011.
- [19] Huang, G., Chen, J., and Benesty, J., “A flexible high directivity beamformer with spherical microphone arrays,” *Journal of the Acoustical Society of America*, 143(5), pp. 3024 – 3035, 2018, doi:10.1121/1.5038275.
- [20] Huang, G., Chen, J., and Benesty, J., “Design of Planar Differential Microphone Arrays With Fractional Orders,” *IEEE/ACM Transactions on Audio, Speech, and Language Processing*, 28, pp. 116 – 130, 2019, doi:10.1109/TASLP.2019.2949219.
- [21] Carpentier, T., Noisternig, M., and Warusfel, O., “Twenty Years of Ircam Spat: Looking Back, Looking Forward,” in *Proc. of the 41st International Computer Music Conference (ICMC)*, pp. 270 – 277, Denton, TX, USA, 2015.
- [22] Carpentier, T., “Spat: a comprehensive toolbox for sound spatialization in Max,” *Ideas Sónicas*, 13(24), pp. 12 – 23, 2021.
- [23] Blauert, J., *Spatial Hearing: The Psychophysics of Human Sound Localization*, The MIT Press, Cambridge, MA, USA, 1st edition, 1983, doi:10.7551/mitpress/6391.001.0001.
- [24] Zotter, F., Frank, M., Kronlachner, M., and Choi, J.-W., “Efficient Phantom Source Widening and Diffuseness in Ambisonics,” in *EAA Symposium on Auralization and Ambisonics*, pp. 69 – 74, Berlin, Germany, 2014, doi:10.2478/aoa-2013-0004.
- [25] Zotter, F. and Frank, M., “Phantom Source Widening by Filtered Sound Objects,” in *Proc. of the 142nd Convention of the Audio Engineering Society (AES)*, Berlin, Germany, 2017.
- [26] McCormack, L., Politis, A., and Pulkki, V., “Rendering of source spread for arbitrary playback setups based on spatial covariance matching,” in *Proc. of the IEEE Workshop on Applications of Signal Processing to Audio and Acoustics (WASPAA)*, New Paltz, NY, USA, 2021, doi:10.1109/waspaa52581.2021.9632724.

5 Annex: derivation of the proposed method in the 2D case

Here we use the same methodology as in the Section 3, with similar notations, and we adapt the equations to the 2D case. The results are provided without demonstration.

Daniel [12, Eq. A.56] established that

$$r_E(N) = \frac{2 \sum_{n=1}^N d_{N,n} d_{N,n-1}}{d_{N,0}^2 + 2 \sum_{n=1}^N d_{N,n}^2}. \quad (23)$$

The weights $d_{N,n}$ that maximize r_E are [12, Eq. 3.88] :

$$\forall n \leq N, d_{N,n} = \mathcal{T}_n(\eta), \quad (24)$$

where $\mathcal{T}_n(\cdot)$ denotes the Chebyshev polynomial (of the first kind) of order n such that $\mathcal{T}_n(\cos \theta) = \cos(n\theta)$, and $\eta \equiv r_E^{\max}(N)$ is the greatest root of $\mathcal{T}_{N+1}(\cdot)$ given by [13, Eq. 19] :

$$r_E^{\max}(N) = \cos\left(\frac{\pi}{2N+2}\right). \quad (25)$$

Thus the weights $d_{N,n}$ are written [12, Eq. 3.88] :

$$\forall n \leq N, d_{N,n} = \cos\left(\frac{n\pi}{2N+2}\right). \quad (26)$$

Considering fractional orders ν :

$$r_E(\nu) = \frac{2 \sum_{n=1}^N d_{\nu,n} d_{\nu,n-1}}{d_{\nu,0}^2 + 2 \sum_{n=1}^N d_{\nu,n}^2} = \frac{\text{num}(\nu)}{\text{den}(\nu)}, \quad (27)$$

with :

$$\text{num}(\nu) = 2 \sum_{n=1}^N d_{\nu,n} d_{\nu,n-1}, \quad (28)$$

and

$$\text{den}(\nu) = d_{\nu,0}^2 + 2 \sum_{n=1}^N d_{\nu,n}^2, \quad (29)$$

we obtain after some developments and simplifications :

$$\begin{aligned} \text{num}(\nu) &= \alpha^2 D + \alpha(1-\alpha)E \\ &+ \alpha(1-\alpha)F + (1-\alpha)^2 G, \end{aligned} \quad (30)$$

where :

$$\begin{cases} D = 2 \sum_{n=1}^N d_{N,n} d_{N,n-1}, \\ E = 2 \sum_{n=1}^N d_{N,n} d_{N-1,n-1}, \\ F = 2 \sum_{n=1}^{N-1} d_{N-1,n} d_{N,n-1}, \\ G = 2 \sum_{n=1}^{N-1} d_{N-1,n} d_{N-1,n-1}. \end{cases} \quad (31)$$

$$\text{den}(\nu) = \alpha^2 A + 2\alpha(1-\alpha)B + (1-\alpha)^2 C, \quad (32)$$

$$\begin{cases} A = d_{N,0}^2 + 2 \sum_{n=1}^N d_{N,n}^2 = (N+1), \\ B = d_{N,0} d_{N-1,0} + 2 \sum_{n=1}^{N-1} d_{N,n} d_{N-1,n}, \\ C = d_{N-1,0}^2 + 2 \sum_{n=1}^{N-1} d_{N-1,n}^2 = N. \end{cases} \quad (33)$$

As in the 3D case, we find the relevant value of α by solving the quadratic equation :

$$a\alpha^2 + b\alpha + c = 0, \quad (34)$$

with the following (constant) coefficients

$$\begin{cases} a = D - (E + F) + G - r_E(\nu)(A - 2B + C), \\ b = (E + F) - 2G - 2r_E(\nu)(B - C), \\ c = G - Cr_E(\nu). \end{cases} \quad (35)$$

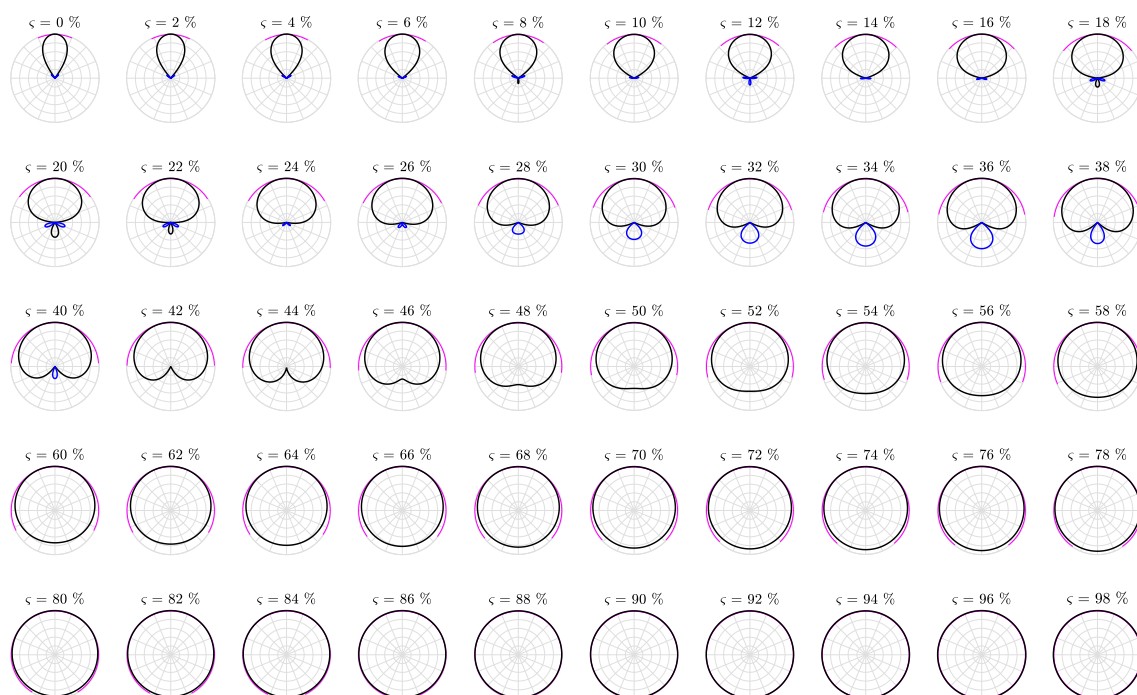


Fig. 4: Equivalent directivity pattern for various values of ζ . The angular spread σ_E , depicted in purple arc, varies linearly between $\sigma_E^{\min} \approx 60^\circ$ ($N = 7$) and $\sigma_E^{\max} = 360^\circ$. The radial scale is logarithmic with 6 dB/division.

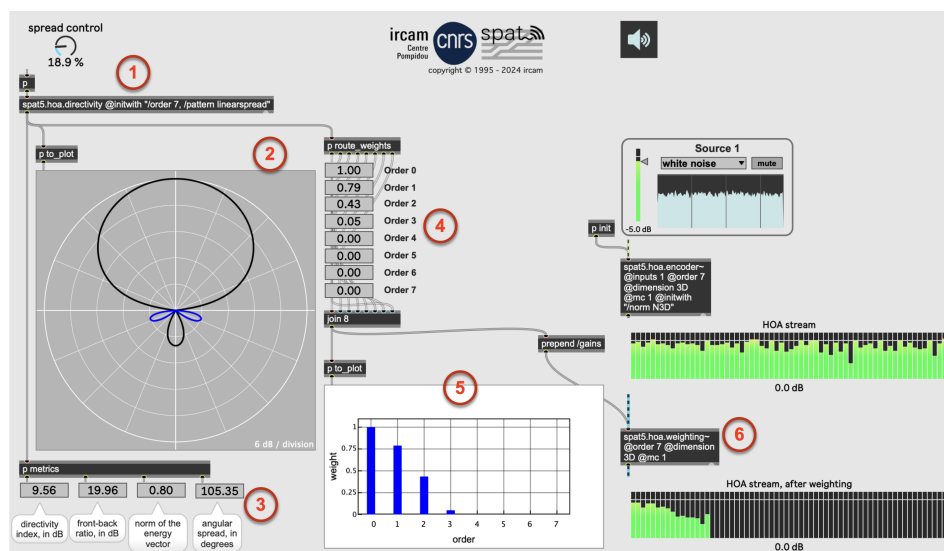


Fig. 5: Example of implementation in the Max environment. ① Calculation of the coefficients $d_{v,n}$ as function of the spread parameter ζ (here, $\zeta = 18.9\%$). ② Equivalent directivity pattern. ③ Calculation of various metrics, based on the coefficients $d_{v,n}$ (we note for example that $\sigma_E \approx 105^\circ$ and $r_E \approx 0.80$). ④ Numerical display of coefficients $d_{v,n}$. ⑤ Graphical display of the coefficients $d_{v,n}$, in the form of bar graph. ⑥ Application of coefficients $d_{v,n}$ to an Ambisonics audio stream (here with $N = 7$).