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# Trip-based MFD Model With Tradable Credit Scheme: Investigating Modal Equilibrium

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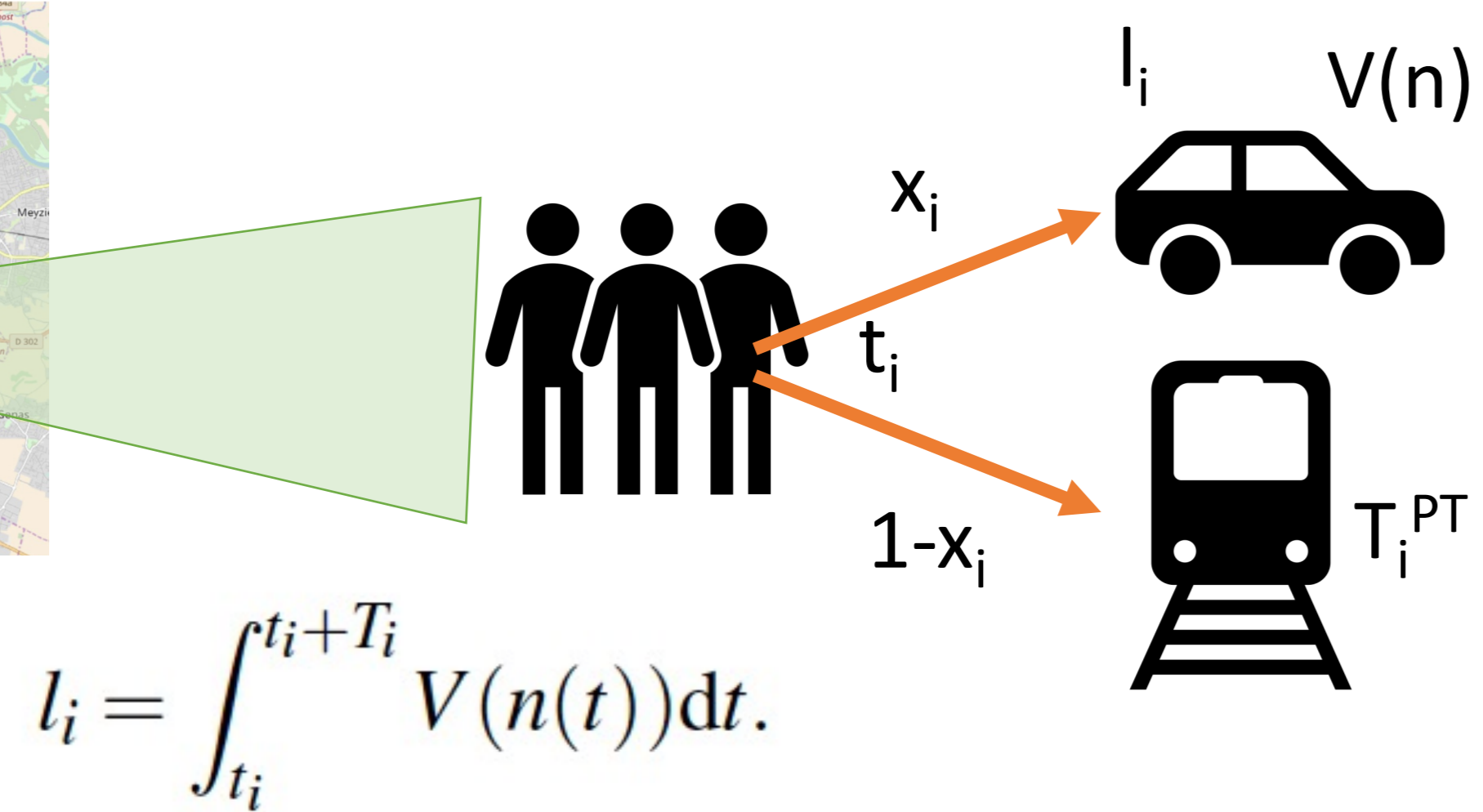
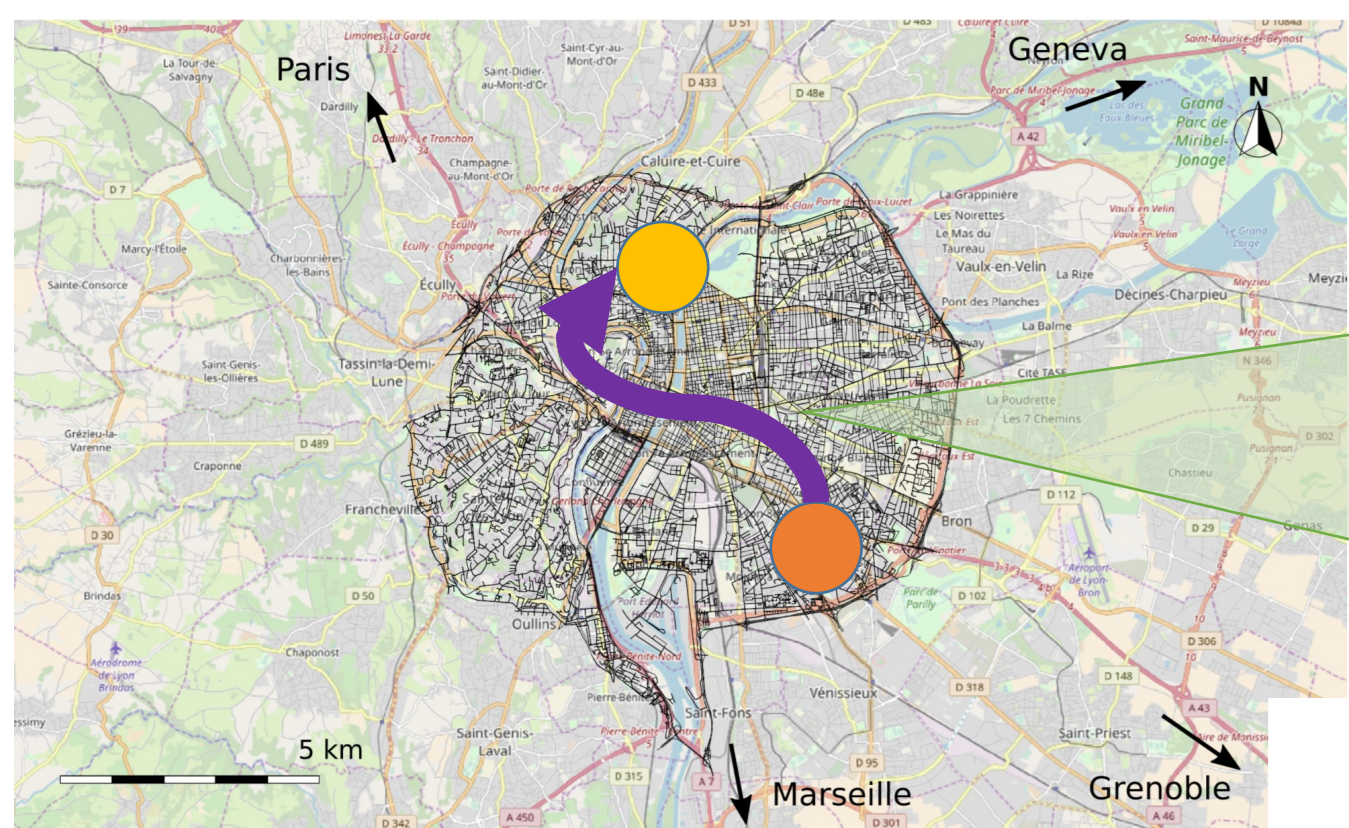
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## Motivation

- ✘ We implement a Tradable Credit Scheme (TCS) to foster modal shift from personal cars to public transportation (PT) to reduce congestion and pollution during the 7:00-10:00 morning commute in Lyon.
- ✘ We use the trip-based MFD framework to represent the congestion. It accounts for the departure time and trip length of each traveler. The congestion dynamic is represented by the uniform mean speed on the network.

## Model description and assumptions



- ✘ For each group  $i$ , departure time  $t_i$  and travel distance  $l_i$  are fixed.
- ✘ Degree of freedom is car share  $x_i$ . The rest rides PT. PT travel time  $T_i^{PT}$  depends on the PT operations (timetables).
- ✘ City model: one reservoir with trip-based MFD.
- ✘ Each user gets  $\kappa$  credits, needs  $\tau$  credits to take the car, 0 for the PT, and can trade them on a market.
- ✘ Offer and demand determine credit price  $p$ .

## An iterative quadratic optimization based on travel times linearization

- ✘ Travel costs perception

$$\begin{cases} C_{i,car} = \alpha T_i(\mathbf{x}) + \tau p; \\ C_{i,PT} = \alpha T_{i,PT}, \end{cases}$$

- ✘ Logit-based decision model

$$\psi_i(\mathbf{x}, p) = \frac{e^{-\theta C_{i,car}}}{e^{-\theta C_{i,car}} + e^{-\theta C_{i,PT}}}$$

- ✘ Modal equilibrium

$$\begin{aligned} \mathbf{x} &= \psi(\mathbf{x}, p); \\ \tau \sum_{i=1}^N x_i &\leq N\kappa; \\ x_i &\geq 0 \quad \forall i; \\ x_i &\leq 1 \quad \forall i; \\ p &\geq 0, \end{aligned}$$

- ✘ QP formulation

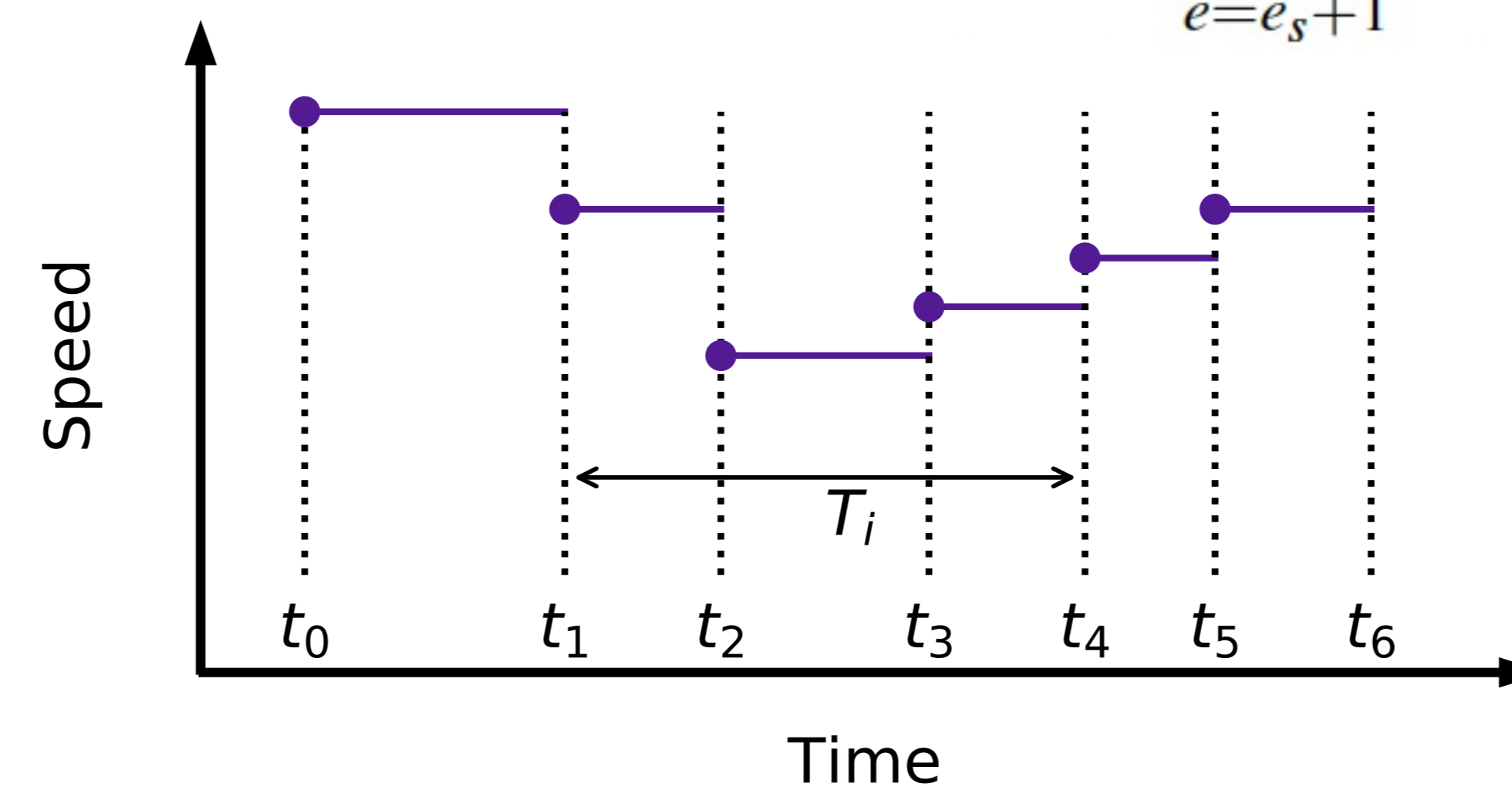
$$J = \frac{1}{2} \left\| (\tilde{\nabla} \psi - \mathbf{I}_x) \cdot \Delta \tilde{\mathbf{x}} + \psi_0 - \mathbf{I}_x \cdot \tilde{\mathbf{x}}_0 \right\|_2^2 + \frac{1}{2} \eta p^2,$$

- ✘ Travel time linearization

$$T_i = T_{0,i} + \nabla \mathbf{T}_i \cdot \Delta \mathbf{x} + o(\Delta \mathbf{x}).$$

- ✘ Split between events

$$\nabla \mathbf{T}_i = \sum_{e=e_s+1}^{e_e} \nabla \mathbf{T}_e.$$



- ✘ Differentiate event types

- ✘ Entry-entry

$$\nabla \mathbf{T}_e = 0.$$

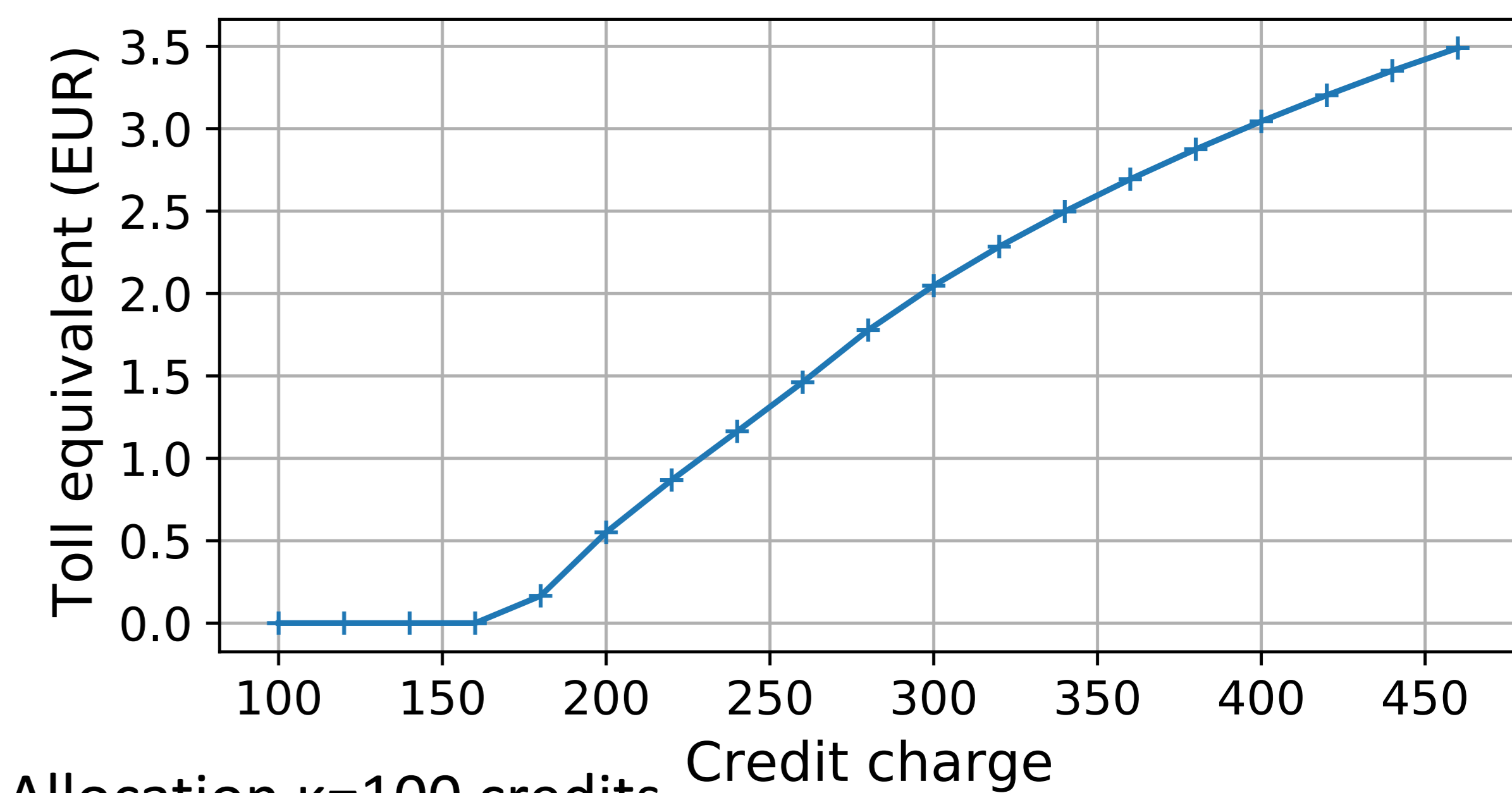
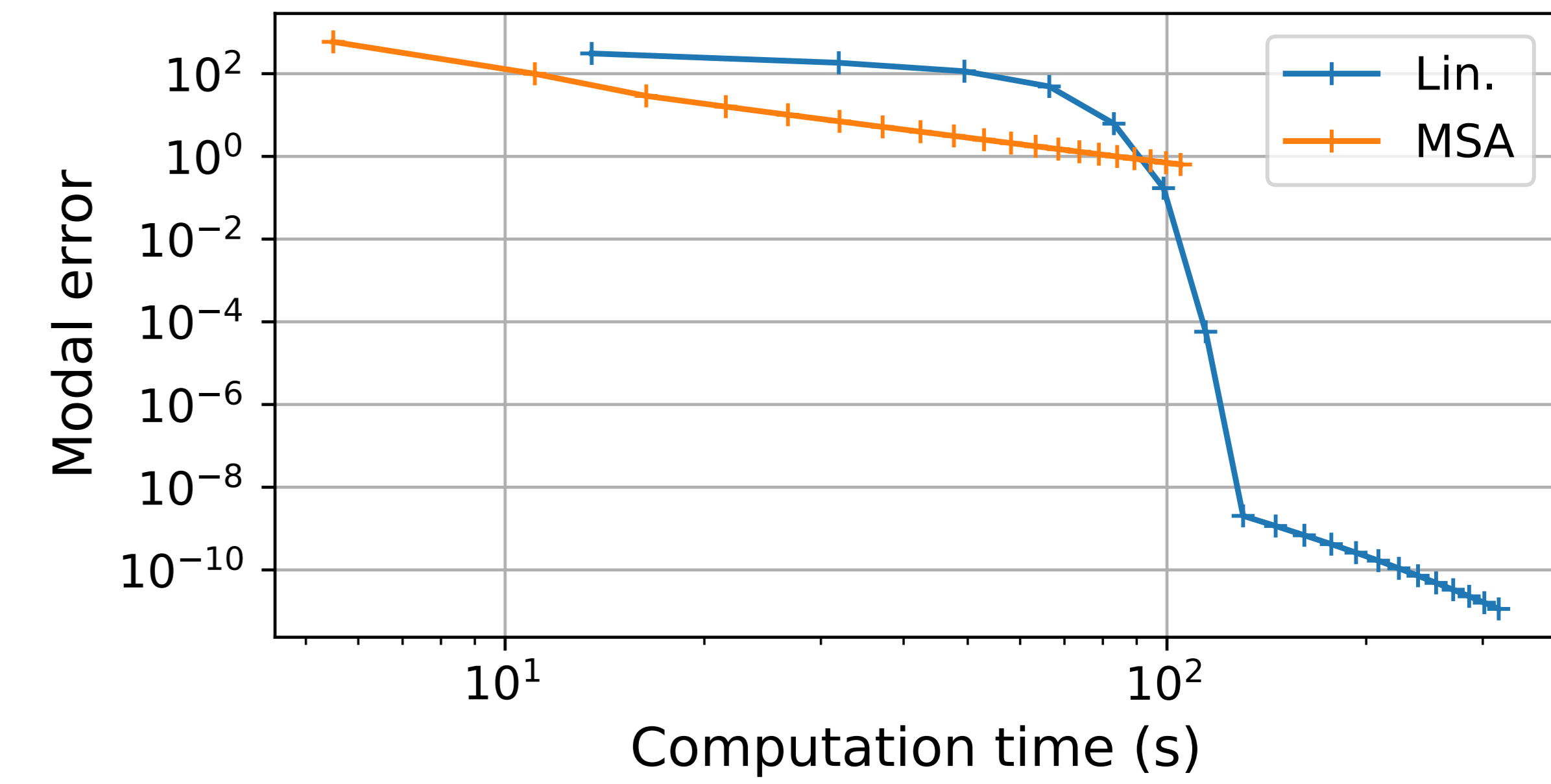
- ✘ Exit-entry

$$\nabla \mathbf{T}_e = -\nabla \mathbf{t}_{e-1} = -\sum_{g=1}^{e-1} \nabla \mathbf{T}_g.$$

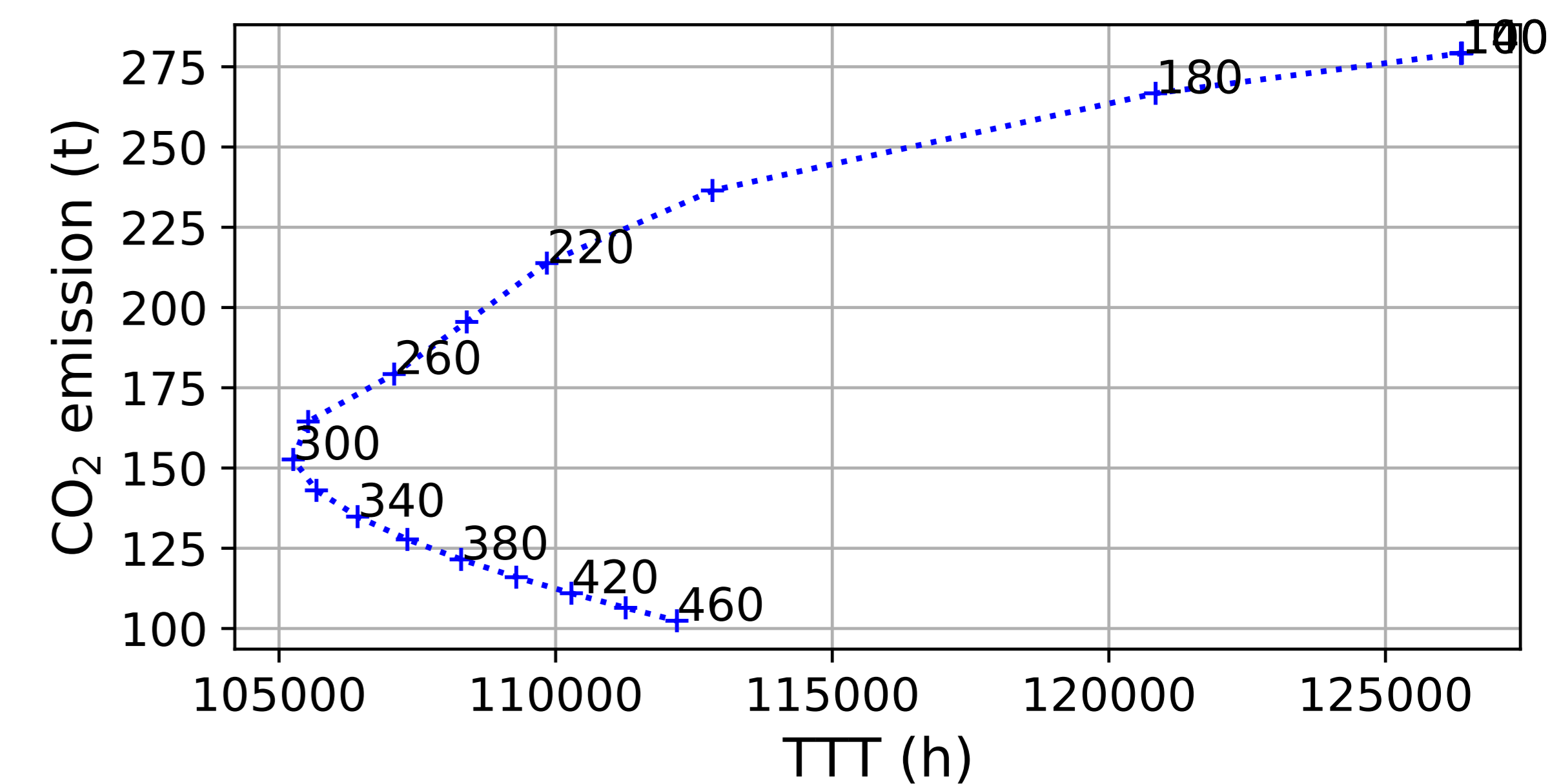
- ✘ \*-exit

$$\nabla \mathbf{T}_e = -\frac{1}{V_e} \left( T_e \nabla V_e + \sum_{g=e_{i,s}+1}^{e-1} \nabla \mathbf{T}_g V_g + T_g \nabla V_g \right).$$

## Results



Allocation  $\kappa=100$  credits



## Conclusion

- ✘ TCS improves total travel time by about 15% and carbon emission by around 45%.
- ✘ From a methodological point of view, iterative quadratic optimization based on travel times linearization reaches better precision than MSA.