



**HAL**  
open science

# Optimisation of the insulation domain for heat transfer in building wall

Zhanat Karashbayeva, Suelen Gasparin, Julien Berger

► **To cite this version:**

Zhanat Karashbayeva, Suelen Gasparin, Julien Berger. Optimisation of the insulation domain for heat transfer in building wall. 11th INTERNATIONAL CONFERENCE ON INVERSE PROBLEMS IN ENGINEERING: THEORY AND PRACTICE (ICIPE 2024), Jun 2024, Buzios, Brazil. hal-04660099

**HAL Id: hal-04660099**

**<https://hal.science/hal-04660099v1>**

Submitted on 23 Jul 2024

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Optimisation of the insulation domain for heat transfer in building wall

Zhanat Karashbayeva<sup>a b \*</sup>, Suelen Gasparin<sup>c</sup>, Julien Berger<sup>a</sup>

<sup>a</sup> Laboratoire des Sciences de l'Ingénieur pour l'Environnement (LaSIE), UMR 7356 CNRS, La Rochelle Université, CNRS, 17000, La Rochelle, France

<sup>b</sup> Astana IT University, 010000, Astana, Kazakhstan

<sup>c</sup> Cerema, BPE Research team, 44200 Nantes, France

\*corresponding author, e-mail address : zhanat.karashbayeva@univ-lr.fr,

## Abstract

The objective of this paper is to minimize the energy consumption of a building wall by finding the optimal locations of the insulation elementary blocks. The direct problem considers a 2D steady state heat transfer equation. A density (in the mathematical sense) function is introduced to represent the thermal properties of the wall, *i.e.* insulation blocks and load bearing materials. On the outside boundary surface, the convection and incident short-wave radiation flux vary with space. The objective function is defined as the total heat flux on the inside part of the wall. It is optimized respecting the constraint of proportion insulation/load bearing material given by standard wall configurations. The optimal locations of the insulation material blocks are retrieved using an exchange algorithm. It ensure to find a local minimum solution with a reduced computational effort. Finally, a case study located in Paris is investigated. Results show the efficiency of the numerical strategy. On the top of that, the aim of the study is reached by obtaining an improved wall energy efficiency with optimized insulation locations.

**Key words:** topology optimization problem, parameter estimation problem, design problem, two-dimensional heat transfer, building energy efficiency.

## 1 INTRODUCTION

Within the environmental context, the building energy efficiency represents a major issue. Building enclosures have been elaborated to protect the occupants from weather daily variations and to minimize the heat transfer loss. For the past 50 years, several models have been developed to represent the physical phenomena occurring in building enclosures to assess its energy efficiency. A first review is provided in [1] with a recent update in [2].

Despite all the simulation programs developed, the building enclosures are often designed under the following assumption: the incident radiation and convective heat flux are spread uniformly on the enclosure, *i.e.* it only varies according to time and not to space. As a consequence, the enclosures are conceived by association of multiple plane layers composed of different materials. However, in [3], it has been demonstrated that the variation of the incident flux with space cannot be omitted to accurately design efficient wall. Thus, the scientific issue is the following: given a flux varying with space, can the insulation position in the building enclosure can be optimized?

Several works have been proposed in the literature to answer such question. In [4], the thermal and structure design of lightweight concrete block is optimized. In [5], the same problem is considered with a more general definition of the holes geometry in the block. Recent development has been proposed in [6] combining both thermal and mechanical efficiency of building structure through topology optimization. These results also highlights the possibility of designing new structure with increased energy efficiency and of elaborating them using 3D printing [7].

However, in the above-mentioned works, the heat transfer model is steady-state, assuming only convective transfer at the boundary conditions. In addition, the incident short-wave radiation flux, that depends on both space, is neglected. Those assumptions are in accordance with the actual building simulation programs. The latter generally requires the transmittance of the enclosure or layer as inputs parameters. According to standards, the transmittance is defined in steady state conditions with convective transfer as boundary conditions. This article proposes to go further by carrying thermal design optimization of

enclosures considering the steady heat transfer phenomena, including the space depending incident radiation and convection flux. To achieve this objective, an efficient strategy is proposed. The optimization problem is solved using a density-based method. To cut the computational cost, the direct problem is solved using an advanced numerical method. Furthermore, the optimization problem is solved using an exchange algorithm, which ensure to find a local minimum, with a reduced computational effort.

The structure of the article is the following. Section 2 presents the physical model of steady heat transfer in two-dimensional enclosure, considering space varying incident short wave radiation. Then, Section 3 describes the design problem with the optimization strategy. A case study of an enclosure located in Paris, France is investigated in Section 4. Last, some conclusions and outlooks are discussed in Section 5.

## 2 DIRECT PROBLEM

### 2.1 Description of Physical Model

The physical problem considers two dimensional stationary heat transfer through a multi-layered wall as illustrated in Figure 1. The spatial domain is defined by  $\Omega = [0, L] \times [0, H]$ , where  $L$  [m] and  $H$  [m] are the thickness and height of the wall, respectively. The physical problem to compute the temperature of the wall  $T$  [K] can be formulated as:

$$\frac{\partial}{\partial x} \left( \lambda(x, y) \cdot \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( \lambda(x, y) \cdot \frac{\partial T}{\partial y} \right) = 0, \quad \forall x \in \Omega_x, \forall y \in \Omega_y, \quad (1)$$

where  $\lambda$  [ $\text{W} \cdot \text{m}^{-1} \cdot \text{K}^{-1}$ ] is the heat conductivity, which varies with space according to the different layers.

The boundary condition at the interface between the wall  $\Gamma_1$  and the outside defines the balance between diffusive, radiative and convective fluxes. It is expressed as:

$$\lambda(x, y) \cdot \frac{\partial T}{\partial x} = h^L(y) \cdot (T - T_\infty^L) - q_\infty^L(y), \quad x = 0, \quad (2)$$

where  $h^L$  [ $\text{W} \cdot \text{m}^{-2} \cdot \text{K}^{-1}$ ] is the surface heat transfer coefficient and  $T_\infty^L$  [K] is the temperature of the air defined according to climatic data. The incident radiation flux  $q_\infty^L$  [ $\text{W} \cdot \text{m}^{-2}$ ] is also defined with climatic data of the sun beams. It varies according to the space, due to the variation of the sunlit  $h$  according to the urban scene. It includes the direct, diffuse and reflected components as described in [3]. It is computed using pixel counting technique as described in [?]. The surface heat transfer coefficient depends on height  $y$  and on the wind velocity  $v_\infty$  [ $\text{m} \cdot \text{s}^{-1}$ ] [8]:

$$h^L(y) = h_{10} + h_{11} \cdot \frac{v_\infty}{v_0} \cdot \left( \frac{y}{y_0} \right)^\beta, \quad (3)$$

where  $(h_{10}, h_{11})$  [ $\text{W} \cdot \text{m}^{-2} \cdot \text{K}^{-1}$ ] are surface coefficients,  $v_0 = 1 \text{ m} \cdot \text{s}^{-1}$  and  $y_0 = 65.33 \text{ m}$  are reference quantities, and  $\beta = 0.32$  is the velocity variation coefficient. On the inside interface  $\Gamma_3$ , the diffusive heat transfer in the wall equals the convective ones. Thus, the following boundary condition is set:

$$\lambda(x, y) \cdot \frac{\partial T}{\partial x} = -h^R(y) \cdot (T - T_\infty^R), \quad x = L, \quad (4)$$

where the inside heat transfer coefficient  $h^R$  [ $\text{W} \cdot \text{m}^{-2} \cdot \text{K}^{-1}$ ] is defined as [?]:

$$h^R(y) = h_{20} + (h_{21}^R - h_{20}^R) \cdot \sin\left(\frac{\pi y}{H}\right), \quad (5)$$

$(h_{20}, h_{21}^R)$  [ $\text{W} \cdot \text{m}^{-2} \cdot \text{K}^{-1}$ ] are surface coefficients, and  $T_\infty^R$  [K] is the inside building temperature. The latter is set-up for the occupant's thermal comfort. At top and bottom of the facade, an adiabatic conditions is set. Thus, the boundary condition at  $y = 0$  and  $y = H$  are expressed as:

$$-\lambda(x, y) \cdot \frac{\partial T}{\partial y} = 0, \quad y = 0, \quad -\lambda(x, y) \cdot \frac{\partial T}{\partial y} = 0, \quad y = H. \quad (6)$$

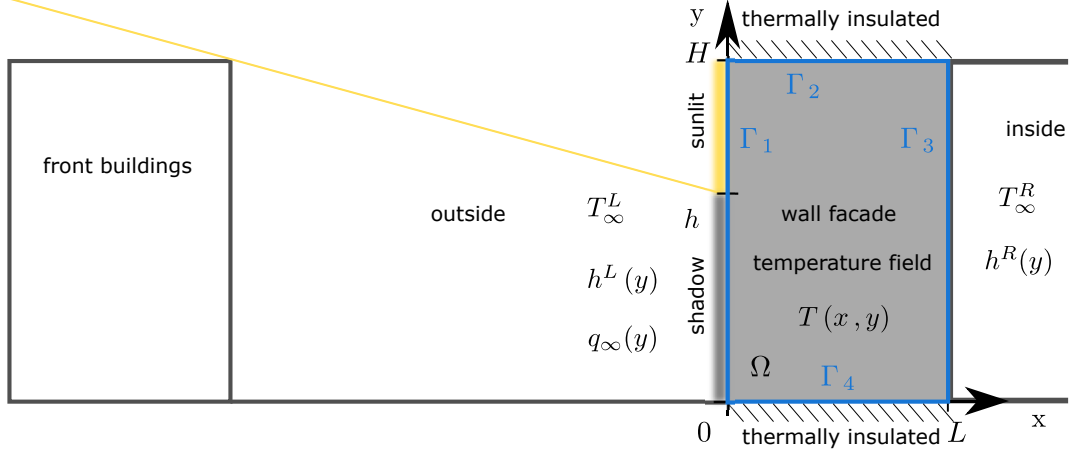


Figure 1. *Illustration of the domain under investigation.*

## 2.2 Thermophysical properties

The objective is to improve the energy efficiency of the wall by optimizing the insulation domain. By assumptions, the wall is composed of two materials. The so-called reference case corresponds to the usual plane layers configuration. It is denoted by sub-script  $\circ$ . In such case, the material properties are defined as piece-wise functions:

$$\lambda(x, y) = \lambda_{\circ} = \begin{cases} \lambda_1, & x \leq x_0 \\ \lambda_2, & x > x_0 \end{cases}.$$

The heat conductivity coefficients  $\lambda_1$  and  $\lambda_2$  corresponds to two different materials. Namely, material  $\lambda_1$  operates as a load materials (for structural purpose) and material  $\lambda_2$  acts as an insulators (for energy efficiency purpose). Depending on the value of  $x_0$ , one can represents external or internal thermal insulation configurations, abbreviated by ETI and ITI, respectively.

The second possibility to define the insulation domain is to divide the insulation domain into  $N$  elementary volumes and to optimize the position of each insulation element. For this, the material properties is defined by:

$$\lambda_{\boxplus}(x, y) = \lambda_1 + (\lambda_2 - \lambda_1) \cdot f(x, y), \quad (7)$$

where  $f(x, y)$  is a function that enables the transition between the material properties of the load material with subscript 1 and the insulator, with subscript 2. This function can be seen as a density (scaled with no unit) function that balance between the two material behavior. Here the function  $f$  is defined using a denoted coarse grid approach, hereinafter abbreviated by CRS. For this, the whole physical domain  $\Omega$  is divided into  $N$  elementary volumes  $\Omega_j$  such as:

$$\Omega = \bigcup_{j \in \mathcal{J}} \Omega_j,$$

where

$$\mathcal{J} = \{1, \dots, N\},$$

is the set of possible positions of an elementary volume  $\Omega_j$ . For each square elementary volume,  $(x_j, y_j)$  are the coordinates of its center. Then, we define as  $\xi$  the set of locations of insulation volumes. Note that  $\xi$  is a subset of  $\mathcal{J}$  and that the number of elements in the set  $\xi$  gives the number of elementary volumes of insulation  $N_i \leq N$ :

$$\text{card } \xi = N_i.$$

Then, the function  $f$  from Eq. (7) is defined by:

$$f(x, y) = \begin{cases} 1, & (x, y) \in \bigcup_{j \in \xi} \Omega_j, \\ 0, & (x, y) \notin \bigcup_{j \in \xi} \Omega_j. \end{cases}$$

For  $j \in \xi$ , the volume  $\Omega_j$  is defined using the R-functions method [9, 10]:

$$\Omega_j = \Omega_{Rx} \wedge_{\tau} \Omega_{Ry}, \quad \forall j \in \xi,$$

with

$$\Omega_{Rx} = \left\{ x \in [0, 1] \mid \left| \frac{h^2 - 2(x - x_j)}{2h} \right| \geq 0 \right\},$$

$$\Omega_{Ry} = \left\{ y \in [0, 1] \mid \left| \frac{h^2 - 2(y - y_j)}{2h} \right| \geq 0 \right\},$$

where  $h$  is the half length of each square volume and  $\wedge_{\tau}$  is the “and” logical operator:

$$x_1 \wedge_{\tau} x_2 \equiv \frac{1}{1 + \tau} \left( x_1 + x_2 - \sqrt{x_1^2 + x_2^2 - 2\tau x_1 x_2} \right).$$

Then, to construct the complete domain of insulation  $\Omega_i$ ;

$$\Omega_i = \bigcup_{j \in \xi} \Omega_j,$$

the “union”  $\vee_{\tau}$  logical operator is employed:

$$x_1 \vee_{\tau} x_2 \equiv \frac{1}{1 + \tau} \left( x_1 + x_2 + \sqrt{x_1^2 + x_2^2 - 2\tau x_1 x_2} \right).$$

In our computations, the parameter  $\tau$  is set to 0. A schematic illustration of the coarse grid approach is presented in Figure 2. For this example, the domain is divided into  $N = 9$  elementary volumes. The set of possible positions of an elementary volume is  $\mathcal{J} = \{1, \dots, 9\}$ . Two volumes of insulation are represented  $N_i = 2$ , placed at the locations  $\xi = \{5, 7\}$ .

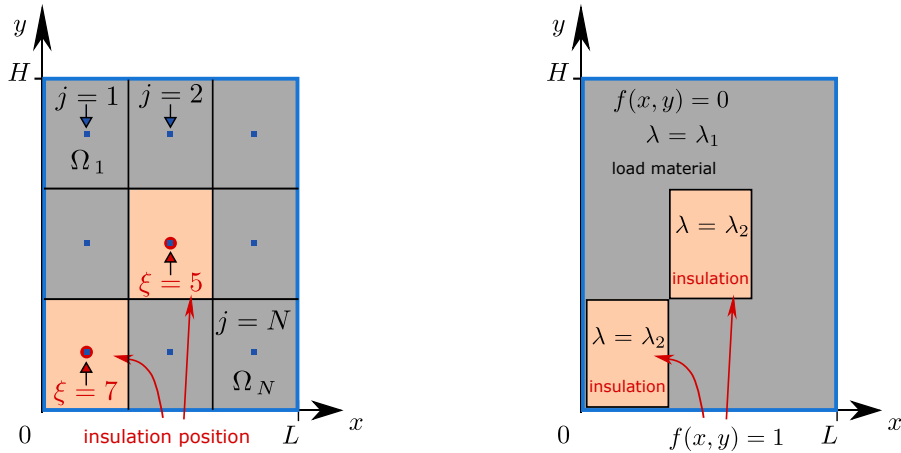


Figure 2. Illustration of definition of the material properties using a coarse grid approach.

### 3 DESIGN PROBLEM

#### 3.1 Definition

The objective is to optimize the energy efficiency of the wall by finding the optimal locations of the insulation elementary volumes. Given the definition of the material properties in Section 2.2, the space

of parameters is the set of locations of insulation volumes  $\xi$ . The energy efficiency of the wall is defined regarding the heat flux on the inside surface. The latter is positive since we have a steady state problem with a positive inward flux on the outside surface. Thus, to improve the energy efficiency, the objective is to maximize the inside surface heat flux and the optimization problem can be formulated by:

$$\xi^* = \arg \min_{\xi \in \mathcal{J}} \mathcal{C}(\xi), \quad (8)$$

where the cost function  $\mathcal{C}$  [ $\text{W} \cdot \text{m}^{-2}$ ] is defined as minus the average flux on the inside surface:

$$\mathcal{C} = \frac{-1}{H} \int_0^H q(x = L, y) dy, \quad (9)$$

where the flux in the  $x$  direction is defined by:

$$q = -(\lambda_{\boxplus}(x, y)) \nabla T \cdot \mathbf{e}_x, \quad (10)$$

where  $\mathbf{e}_x$  is the unit vector of the  $x$  axis.

The cost function Eq. (9) needs to be minimized under the following constraint:

$$S_{\boxplus} \leq S_{\circ}, \quad (11)$$

where

$$S_{\boxplus} = \int_0^H \int_0^L \lambda_{\boxplus}(x, y) dx dy, \quad S_{\circ} = \int_0^H \int_0^L \lambda_{\circ}(x, y) dx dy.$$

The constraint (11) means that the proportion of insulation and load materials should not be higher than one of the reference case, for a given  $x_0$ .

### 3.2 Optimization strategy

The exhaustive search of the optimal solution requires  $\binom{N_i}{N}$  computations of the direct problem. It corresponds to all possible combination of choosing  $N_i$  insulation locations out of  $N$  possible position. Even, with a number of total elementary volumes scaling with a few ten, the computational burden of the exhaustive strategy is too high. In addition, the optimization problems involves integer elements (the locations of insulation volumes  $\xi$  in the set of all possible positions  $\mathcal{J}$ ) under the volume constraint from Eq. (11). Algorithm such as genetic ones can be used to solve such optimization problem with integer parameters. However, the computational cost is also very expensive. Therefore, the strategy adopted in this work is based on an exchange algorithm [11]. It runs as follows over the iterations  $m$ .

**Step 1.** At  $m = 0$ , an initial configuration  $\xi^0$  is defined where  $N_i = \text{card } \xi^0$ . Note that constraint Eq. (11) is verified by setting:

$$N_i \leq N_{\circ},$$

where  $N_{\circ}$  is the number of insulation volumes in the reference case (external or internal thermal insulation configurations). For such configuration, the cost function  $\mathcal{C}(\xi^{(0)})$  is computed according to Eq. (9) and by solving the governing equations (16)–(21).

**Step 2.** The second step consists in exchanging the position labels of the current configuration with ones that correspond to vacant sites so as to maximally decrease the cost function. It is performed by determining the labels  $(i^*, j^*)$  such that:

$$(i^*, j^*) = \arg \max_{(i, j) \in \mathcal{S}^{(m)}} \Delta(i, j), \quad (12)$$

where

$$\mathcal{S}^{(m)} = \xi^{(m)} \times (\mathcal{J} \setminus \xi^{(m)}),$$

so that  $\mathcal{S}^{(m)}$  contains all possible exchanges of points, at which insulation volumes currently resides by labels which are filled by load material. The quantity  $\Delta(i, j)$  evaluates the relative changes in the cost function:

$$\Delta(i, j) = \left( \mathcal{C}(\xi_{i \leftrightarrow j}) - \mathcal{C}(\xi^{(m)}) \right) \cdot \left( \mathcal{C}(\xi^{(m)}) \right)^{-1}, \quad (13)$$

where  $\xi_{i \leftrightarrow j}$  means the configuration in which label position  $i$  has been replaced by label  $j$ .

**Step 3.** If the relative decrease in the cost function is lower than a set tolerance  $\eta$

$$\Delta(i^*, j^*) \leq \eta, \quad (14)$$

then the algorithm stops since  $\xi^{(m)}$  is a locally optimal configuration. Otherwise, the iterations continues by setting  $\xi^{(m+1)} \leftarrow \xi^{(m)}$  and  $m \leftarrow m + 1$  and coming back to Step 2. The Algorithm 1 synthesizes the procedure. Regarding the computational resources of the optimization strategy, the theoretical CPU time is given by the following formula:

$$t_{\text{cpu}} = N_m \cdot t_{\text{cpu}}^{\text{dp}} \cdot N_{\text{dp}}, \quad (15)$$

where  $t_{\text{cpu}}^{\text{dp}}$  is the CPU time spend for one calculation of direct problem,  $N_{\text{dp}}$  is the number of the calculations of direct problem. The latter is given by:

$$N_{\text{dp}} = 1 + (N - N_i) \cdot N_i.$$

It is explained by the fact at each iteration, one computation of the direct problem for the current configuration is needed at Step 5 of Algorithm 1. In addition, the direct problem is computed for the  $N_i$  possibilities of changing one volume of insulation with one of the volume of load material. It corresponds to step 7 of Algorithm 1. An important remark is that those computations are independent and can be performed in parallel.

---

**Algorithm 1** Exchange algorithm to determine the local optimal configuration of insulation positions.

---

- 1: Sample candidate design  $\xi^0$  with card  $\xi^0 \leq N_o$  ▷ Step 1
  - 2: Compute cost function  $\mathcal{C}(\xi^{(0)})$
  - 3:  $m = 0$
  - 4: **while**  $\Delta(i^*, j^*) \geq \eta$  **do**
  - 5:     State  $\mathcal{S}^m$  ▷ Step 2
  - 6:     Determine labels  $(i^*, j^*)$  according to Eq. (12)
  - 7:     Compute  $\Delta(i^*, j^*)$  with Eq. (13)
  - 8:      $m = m + 1$
  - 9: **end while**
  - 10: Set optimal configuration  $\xi^* = \xi^{m-1}$
- 

## 4 CASE STUDY

### 4.1 Description

This case study considers a house located in Paris, France. The height and width of the wall are  $H = 3$  m and  $L = 35$  cm. The building wall is composed from concrete and wood fiber insulation. The material properties of the wall are taken from French standards [12] and shown in Table 1. The reference cases are defined for external thermal insulation, denoted as ETI, and as internal insulation, denoted as ITI. In both cases, the length of the insulation and concrete materials are 15 and 20 cm, respectively. It corresponds to a reference area  $S_o = 0.45$  m<sup>2</sup>.

Table 1. *Thermal properties of the materials.*

<i>Layer</i>	<i>Heat conductivity</i> [W · m <sup>-1</sup> · K <sup>-1</sup> ]	<i>Volumetric heat capacity</i> [MJ · m <sup>-3</sup> · K <sup>-1</sup> ]	<i>Length in reference case</i> [m]
<i>Concrete</i>	1.4	2	0.2
<i>Insulation</i>	0.05	0.85	0.15

Regarding the outside and inside surfaces, the outside heat transfer coefficient  $h^L$  and the inside heat transfer coefficient varies according to height as defined in Eq.(3) and Eq.(5), respectively. The following parameters are used  $h_{10} = 5.82$  W · m<sup>-2</sup> · K<sup>-1</sup>,  $h_{11} = 3.96$  W · m<sup>-2</sup> · K<sup>-1</sup>,  $h_{20} = 0.1$  W · m<sup>-2</sup> · K<sup>-1</sup> and  $h_{21} = 3$  W · m<sup>-2</sup> · K<sup>-1</sup>. The outside wind velocity  $v_\infty = 6.93$  m · s<sup>-1</sup>. The temperature of the

air  $T_{\infty}^L = 10.8^{\circ}\text{C}$  and the inside building temperature  $T_{\infty}^R = 19^{\circ}\text{C}$ . As defined in the physical model, adiabatic conditions are set for the top and bottom boundaries of the facade.

The investigations are carried out for the day corresponding to the highest value of the outside incident radiation flux of the January. The facade is located in the urban area with a building of 3 m height located at 5 m in front of the studied facade. The outside incident radiation heat flux  $q_{\infty}^L$  and the surface heat transfer coefficient  $h^L$  varies according to height of the facade as shown in Figure 3(a).

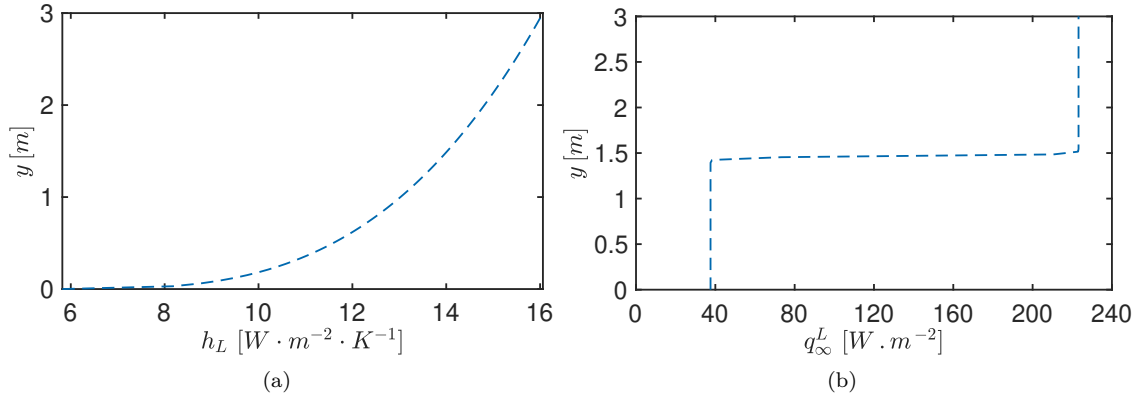


Figure 3. (a) *Space variation of the surface heat transfer coefficient and (b) of the incident radiation heat flux.*

## 4.2 Results

The design problem is solved using the optimization procedure described in Section 3 with its algorithm presented in Section 3.2. As initial configurations in the algorithm ( $m = 0$ ), the internal, external and random insulation distribution cases are considered. With this choice, the improvement of the energy efficiency of the design can be compared along the iterations with the standard configuration. Note also that parallel computing is used for the optimization. The dimensionless time and space discretizations of the DU FORT-FRANKEL scheme are  $\Delta t = 10^{-3}$  and  $\Delta x = \Delta y = 10^{-2}$ . As mentioned earlier, for the two reference cases, 42, 8% of the wall area is covered by insulation material. Thus, given the constraint (11), the number of elementary volumes of insulation in the design problem is  $N_i = 0.428 N$ . Here,  $N_i$  is approximated to the nearest natural number. In this case study  $N = 49$  with the corresponding  $N_i = 21$  is investigated.

The optimal position of insulation volumes for three different initial configurations are given in Figure 4. It shows that the insulation position tends to the similar locations, for all three different designs of initial configurations. However, number of iterations needed for the optimization process of three different initial configurations, are different. The ITI initial configuration case requires  $m = 18$  iterations, while ETI and random initial configuration cases need  $m = 8$  and  $m = 10$  iterations, respectively. Moreover, in order to check the consistency of the algorithm, the optimization problem is solved with  $N = 25$  elementary volumes, with corresponding  $N_i = 11$  elementary volumes of insulation. As shown in Figure 4(h), despite the different number of  $N$  elementary volumes, the shape of the optimized configuration is similar with the  $N = 49$  cases. The number of iterations needed for computations is 6 iterations and is lower than  $N = 49$  cases.

More details on changes of the insulation locations iteration by iteration is given in Figure 5. Iterations start at  $m = 0$  from the ETI configuration, after the algorithm changes one insulation volume place at each iteration. It ensure to obtain a local minimum of the optimization problem.

The Figure 6(a) shows information about convergence of the optimization algorithm of both  $N = 49$  and  $N = 25$  elementary volume cases. In general, it can be seen that despite of the different number of elementary values, the value of the cost function is similar, around  $-1.8 \text{ W} \cdot \text{m}^{-2}$ . Thus, the energy efficiency of the wall has been improved by 52.4%. Regarding the  $N = 49$  case, the cost function value obtained using the ETI and random initial configurations are same. It is consistent with the fact that same optimized configuration designs are obtained as show in Figures 4(d) and 4(f). ITI initial configuration design case has a lower optimized cost function compared to others. Consequently, for further analysis, the optimized design obtained from ITI initial configuration design case is used.



The optimal distribution illustrated in Figure 4(b) corresponds to an horizontal band of diffusive material (concrete) on the top of the facade which corresponds to the sunlit area as shown in Figure 3(b). The heat flux calculated with the initial and optimized designs at  $x = 0$  and  $x = L$  are illustrated in Figure 6(b) and 6(c), respectively. It is confirmed that the heat flux on the inside surface is positive due to a positive inward flux on the outside facade. In addition, as remarked in Figure 3(b), at the bottom of the facade (below the 1.5 m) the value of the incident radiation heat flux is lower. Consequently, Figure 6(b) shows lower value of the heat flux at the same height. However, it can be seen that the value of the heat flux calculated with optimized design at  $x = 0$  is increased. Moreover, Figure 6(c) shows the significantly increase of the heat flux value with optimized design at the inner sight of the wall.

Table 2 presents the CPU time needs to solve the optimization problem, computed theoretically as defined by Eq. (15) (denoted  $t_{\text{cpu}}$ ), and using parallel computing (denoted  $t_{\text{cpu}}^{\parallel}$ ). As it can be seen, the theoretical CPU time without parallel computing is higher. By using parallel computing it is decreased at least by 1.5 times.

Table 2. Computational cost of the algorithm to determine an optimal design.

<i>Initial configuration</i>	<i>Elem. volumes</i>	$t_{\text{cpu}}^{\text{dp}}$ [s]	$N_{\text{dp}}$ [-]	$N_k$ [-]	$t_{\text{cpu}}$ [h]	$t_{\text{cpu}}^{\parallel}$ [h]	$t_{\text{cpu}}^{\parallel} / t_{\text{cpu}}$ [%]
<i>ITI</i>	$N = 49, N_i = 21$	1.16	588	18	3.41	2.26	66.28
<i>ETI</i>		1.21		8	1.58	0.96	60.76
<i>Random</i>		1.20		10	1.96	1.19	60.71
<i>ETI</i>	$N = 25, N_i = 11$	1.28	154	6	0.33	0.19	57.58

## 5 CONCLUSION

In this paper, the problem of optimizing the insulation position in building wall was considered. The direct 2D steady heat transfer problem was solved numerically using DU FORT-FRANKEL method. The standard building wall composed of two blocks of materials (insulation and load bearing) were replaced by a new design, described mathematically with a density function. The latter is defined by a number of elementary volume which position needs to be optimized regarding the wall energy efficiency. The cost function was minimized taking into account the constraint of proportion of the insulation and load bearing blocks given by the standard wall case. To solve the optimization problem, an exchange algorithm is used to reduce the computational cost and ensure to find a local minimum. The optimal location of the insulation material was obtained. The results of this paper revealed that the applied algorithm converges relatively fast, spending less than 20 iterations. Moreover, the energy efficiency of the wall was improved with an optimized design different from the standard building walls. The consistency of the optimized design was verified with different initial guess configuration and number of elementary volume. The study can be extended for transient heat transfer problem, researching the behaviour of the optimal positions of the insulation material during the different seasons.

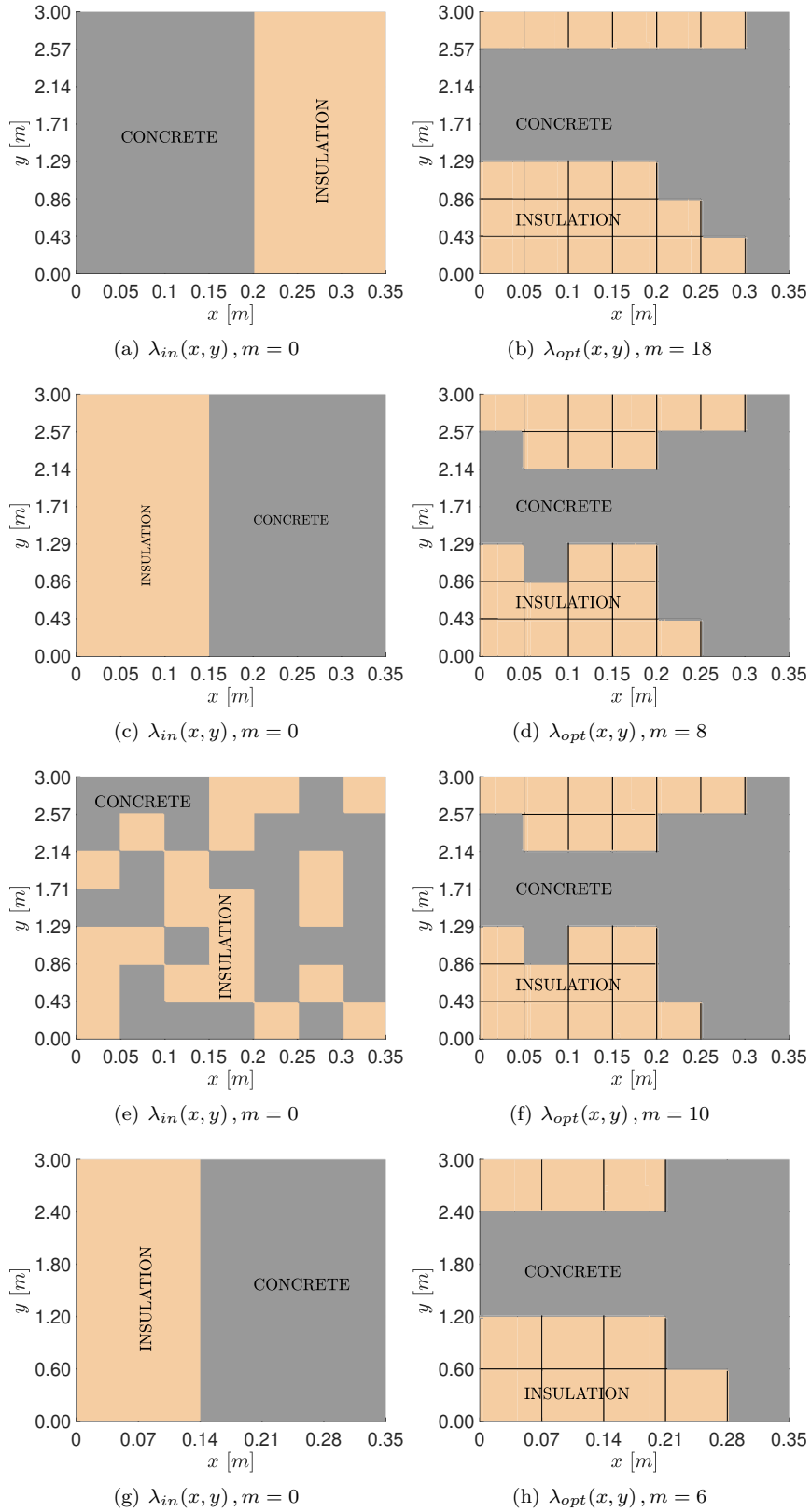


Figure 4. Design of the initial (a,c,e,g) and optimized configurations (b,d,f,h) starting with ITI distribution (a,b), ETI distribution (c,d,g,h), random distribution (e,f). Number of elementary volume is  $N = 49$  (a-f), and  $N = 25$  (g,h).

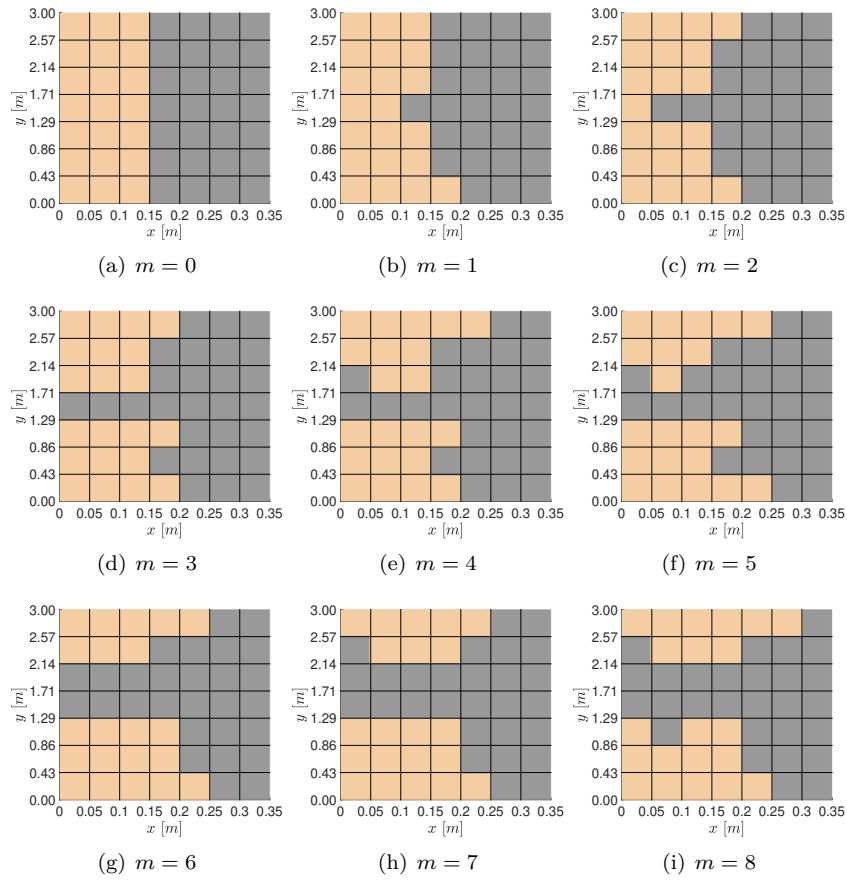


Figure 5. *Insulation configurations according to the iterations of the exchange algorithm.*

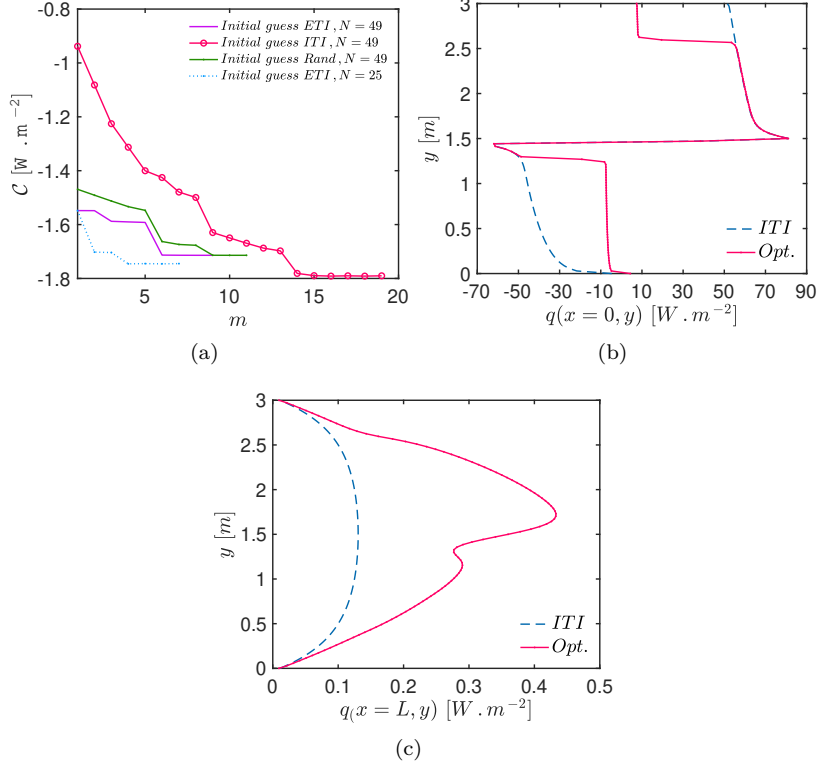


Figure 6. (a) Convergence of the optimization method. Heat flux on the outside surface ( $x = 0$ ) (b) and on the inside surface ( $x = L$ ) (c) for the optimized and initial ITI configurations.

## 6 NOMENCLATURE

<i>Latin letters</i>		
$c$	volumetric heat capacity	[J/(m <sup>3</sup> .K)]
$h^L, h_{10}, h_{11}$	surface heat transfer coefficient	[W/(m <sup>2</sup> .K)]
$v_\infty$	velocity	[m/s]
$v_0$	velocity variation coefficient	[m/s]
$T$	temperature	[K]
$t$	time	[s]
$x$	horizontal space coordinate	[m]
$y$	vertical space coordinate	[m]
$H$	building facade height	[m]
$L$	thickness of the wall	[m]
$t_f$	final time	[s]
$q_\infty^L$	incident radiation flux	[W/m <sup>2</sup> ]
$h^R, h_{20}, h_{21}$	inside heat transfer coefficient	[W/m <sup>2</sup> .K]
$f(x, y)$	function that enables the transition between the material properties	[-]
$\mathcal{J}$	set of possible positions of an elementary volume	[-]
$N_i$	number of elementary volumes	[-]
$N$	number of elementary volumes of insulation	[-]
$C$	cost function	[W/m <sup>2</sup> ]
$q$	total heat flux on the inside and outside part of the wall	[W/m <sup>2</sup> ]
$S$	area	[m <sup>2</sup> ]

<i>Greek letters</i>		
$\lambda$	heat conductivity	[W/(m.K)]
$\xi$	set of locations of insulation volumes	[-]
$\Delta x, \Delta y$	dimensionless space step	[-]

<i>Subscripts and superscripts</i>	
<i>in</i>	initial
$L_{\infty}$	outside boundary
$R_{\infty}$	inside boundary
o	reference case
$\boxplus$	coarse grid

<i>Abbreviations</i>	
ITI	internal insulation
ETI	external insulation
CRS	coarse grid
Rand	random
Opt	optimized
CPU	central processing unit
DP	direct problem

## 7 ACKNOWLEDGEMENTS

The authors acknowledge DSG 2021 and the French National Research Agency (ANR) as part of the “JCJC CE-22 AAPG 2023” programme (project TOPS) for the financial support. This research is funded by the Science Committee of the Ministry of Science and Higher Education of the Republic of Kazakhstan (Grant No. AP19175447).

## 8 REFERENCES

### References

- [1] M. Woloszyn and C. Rode. Tools for performance simulation of heat, air and moisture conditions of whole buildings. *Building Simulation*, 1:5–24, 01 2008. [1](#)
- [2] N. Mendes, M. Chhay, J. Berger, and D. Dutykh. *Numerical methods for diffusion phenomena in building physics*. Springer, 2019. [1](#)
- [3] Julien Berger, Suelen Gasparin, Walter Mazuroski, and Nathan Mendes. An efficient two-dimensional heat transfer model for building envelopes. *Numerical Heat Transfer, Part A: Applications*, 79(3):163–194, 2021. [1](#), [2](#), [13](#)
- [4] L. C. Sousa, C. F. Castro, C. C. António, and H. Sousa. Topology optimisation of masonry units from the thermal point of view using a genetic algorithm. *Construction and Building Materials*, 25(5):2254–2262, 2011. [1](#)
- [5] M. Bruggi and A. Taliercio. Design of masonry blocks with enhanced thermomechanical performances by topology optimization. *Construction and Building Materials*, 48:424–433, 2013. [1](#)
- [6] G. Vantighem, V. Boel, M. Steeman, and W. De Corte. Multi-material topology optimization involving simultaneous structural and thermal analyses. *Structural and Multidisciplinary Optimization*, 59(3):731–743, 2019. [1](#)

- [7] G. Vantighem, W. De Corte, M. Steeman, and V. Boel. Density-based topology optimization for 3D-printable building structures. *Structural and Multidisciplinary Optimization*, 60(6):2391–2403, 2019. 1
- [8] Williams H McAdams. *Heat transmission*. Krieger Pub Co, 1985. 2
- [9] V. L. Rvachev, T. I. Sheiko, V. Shapiro, and I. Tsukanov. On completeness of RFM solution structures. *Computational Mechanics*, 25(2):305–317, 2000. 4
- [10] V. Shapiro and I. Tsukanov. Meshfree simulation of deforming domains. *Computer-Aided Design*, 31(7):459–471, 1999. 4
- [11] A. Atkinson, A. Donev, and R. Tobias. *Optimum Experimental Designs, with SAS*. Oxford Statistical Science Series. Oxford University Press, Oxford, New York, 2007. 5
- [12] Journal Officiel de la République Française. Arrêté du 26 octobre 2010 relatif aux caractéristiques thermiques et aux exigences de performance énergétique des bâtiments nouveaux et des parties nouvelles de bâtiments, 2010. 6

## 9 RESPONSIBILITY NOTICE

The authors are solely responsible for the printed material included in this paper.

## A MOVING FROM THE TRANSIENT HEAT TRANSFER TO THE STEADY STATE

Furthermore, the direct problem is solved by transforming the problem into a transient one which is solved until steady state being reached. For this, we define  $\tilde{T}$  verifying the problem of:

$$c(x, y) \cdot \frac{\partial \tilde{T}}{\partial t} = \frac{\partial}{\partial x} \left( \lambda(x, y) \cdot \frac{\partial \tilde{T}}{\partial x} \right) + \frac{\partial}{\partial y} \left( \lambda(x, y) \cdot \frac{\partial \tilde{T}}{\partial y} \right), \quad \forall x \in \Omega_x, \forall y \in \Omega_y, \forall t \in \Omega_t, \quad (16)$$

where  $c$  [ $\text{J} \cdot \text{m}^{-3} \cdot \text{K}^{-1}$ ] is the volumetric heat capacity. The boundary conditions are expressed as:

$$\lambda(x, y) \cdot \frac{\partial \tilde{T}}{\partial x} = h^L(y) \cdot (\tilde{T} - T_\infty^L) - q_\infty^L(y), \quad x = 0, \quad (17)$$

$$\lambda(x, y) \cdot \frac{\partial \tilde{T}}{\partial x} = -h^R(y) \cdot (\tilde{T} - T_\infty^R), \quad x = L, \quad (18)$$

$$-\lambda(x, y) \cdot \frac{\partial \tilde{T}}{\partial y} = 0, \quad y = 0, \quad (19)$$

$$-\lambda(x, y) \cdot \frac{\partial \tilde{T}}{\partial y} = 0, \quad y = H. \quad (20)$$

Initial condition is expressed as:

$$\tilde{T} = T_{in}(x, y), \quad \forall (x, y) \in \Omega, t = 0. \quad (21)$$

where  $T_{in}$  is a given function of space. The transient problem (16) - (21) is solved using the DU FORT-FRANKEL [3]. It is investigated over the time domain  $\Omega_t = [0, t_f]$ , where the final time  $t_f$  [s] is defined such as:

$$\left. \frac{\partial \tilde{T}}{\partial t} \right|_{t=t_f} \leq \epsilon, \quad \forall (x, y) \in \Omega,$$

where  $\epsilon$  is a constant set by user. In our computations it is set to  $10^{-2}$ . Thus, in the end, the solution of Eq. (1) is given by:

$$T(x, y) = \tilde{T}(x, y, t_f).$$