



HAL
open science

Generalized performance criteria for identified models

Bernard Vau, Henri Bourlès

► **To cite this version:**

Bernard Vau, Henri Bourlès. Generalized performance criteria for identified models. 20th SysID 2024 - IFAC, Jul 2024, Boston (MA), United States. hal-04654378

HAL Id: hal-04654378

<https://hal.science/hal-04654378>

Submitted on 19 Jul 2024

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Generalized performance criteria for identified models

Bernard Vau* Henri Bourlès**

* *Control Department, Exail, 94385 Bonneuil-sur-Marne, France
(bernard.vau@exail.com).*

** *SATIE, Ecole Normale Supérieure Paris-Saclay, 4 avenue des sciences, 91190, Gif-sur-Yvette, France
(henri.bourles@satie.ens-cachan.fr)*

Abstract: It is shown that some usual criteria evaluating the performances of an identified model with respect to experimental data, like the FIT criterion, can be not well-suited to fast sampled systems. This leads us to propose some generalized criteria where the signals are filtered by transfer functions belonging to an orthonormal basis. An interpretation of this filtering in the frequency domain is proposed. The basis poles selection is equivalent to making a specification about the criteria in function of the expected use of the identified model.

Keywords: Identification, Model validation, Model performance assessment, Fit

1. INTRODUCTION

In any system identification process, the step of model validation and model performance assessment is crucial, since it allows (or not) to retain this model for simulation and/or prediction purposes, and for control law design. Numerous validation methods have been associated for a long time to classical identification approaches. The most well known are the whiteness test of residuals and the cross correlation test between the prediction error and the system input (Söderström and Stoica (1989), Ljung (1999)), which provide, by using statistical concepts, a binary response about the model validity. Independently, the performance of an identified model is most of the time evaluated by computing the FIT which is an indicator depending on the ratio between the sum of squares of residuals and the sum of squares of the centred measured output. The FIT (also called NRSME) yields a more qualitative indication about the model matching to experimental data, and it is widely employed for comparing continuous-time identification methods (Pascu et al. (2019)) or discrete-time ones (Pillonetto et al. (2022)). Moreover some well-known software packages (such as Matlab[®] Identification Toolbox) provide also routines that compute it.

In (Muroi and Adachi (2015)) it is pointed out that the FIT criterion is not without any shortcoming as it can provide negative values in some cases, and that led the authors to propose two other criteria in particular the triangle criterion (based on the triangle inequality) denoted as TRI in the sequel.

All these criteria make use of the sums of signals squares (the system output, the model output, or the residual) in time domain. These sums are asymptotically related to the signals 2-norm, and by shifting in the frequency domain that implies an integration of these signals power spectrum density (PSD) implicitly carried out over a linear frequency scale. This scale is not consistent with respect to the logarithmic frequency scale which is employed es-

pecially for representing fast sampled models.

In (Vau and Bourlès (2022)) a similar remark has been made about the validation test assessing the whiteness of residuals which is well suited in practice only if one expects to represent the model in a linear frequency scale. In the same reference it is proposed a generalized whiteness test using Generalized Orthonormal Bases of transfer Functions (GOBF) so as to address the case of fast sampled systems, or to validate reduced order models.

The purpose of the present article is to propose generalized FIT and TRI criteria using some signal filtering by GOBF. The basis poles selection of GOBF allows for adapting the test in function of the expected use of the model. It is shown that this basis poles selection entails implicitly a distortion of the frequency scale on which the model performance is assessed. Thus the basis poles of GOBF play the role of tuning parameters allowing to specify on which frequency band (either large or narrow) the identified model performance is evaluated. The paper is structured as follows:

In Section 2, the expressions of FIT and TRI criteria are recalled, and an interpretation in the frequency domain is provided. Then two examples show the limits of these classical criteria when applied to fast sampled systems.

In Section 3, after a short presentation of GOBF and their associated frequency (the Hambo frequency), two new criteria called H-FIT and H-TRI are proposed. These criteria use a filtering by orthonormal functions, and an impact of this filtering in the frequency domain is analysed. Finally, these new criteria are applied to the two examples of the previous section.

2. ANALYSIS OF THE CLASSICAL MODEL PERFORMANCE CRITERIA

2.1 FIT and TRI criteria

According to (Ljung (1999), p.500) the FIT is a normalized criterion, defined as

$$FIT = \left(1 - \frac{\sqrt{\sum_{t=1}^N \varepsilon^2(t)}}{\sqrt{\sum_{t=1}^N (y(t) - \bar{y})^2}} \right) \cdot 100 \quad (1)$$

where $\{y(t)\}$ is the measured output sequence, $\{\varepsilon(t)\}$ is the residual sequence with $\varepsilon(t) = y(t) - \hat{y}(t)$, $\{\hat{y}(t)\}$ being the predicted output sequence of the model, and $\bar{y} = \frac{1}{N} \sum_{t=1}^N y(t)$ the mean of $\{y(t)\}$, and N is the sample number.

The FIT is expressed in percent, and its maximum value is 100% if the residual is identically null. Note that the FIT takes negative values if $\sum_{t=1}^N \varepsilon^2(t) > \sum_{t=1}^N (y(t) - \bar{y})^2$.

In (Muroi and Adachi (2015)), a normalized criterion called TRI is proposed where

$$TRI = \left(1 - \frac{\sqrt{\sum_{t=1}^N \varepsilon^2(t)}}{\sqrt{\sum_{t=1}^N y^2(t) + \sum_{t=1}^N \hat{y}^2(t)}} \right) \cdot 100 \quad (2)$$

Its value is guaranteed to be superior to 0% and inferior to 100% owing to the triangle inequality ($\sqrt{\sum_{t=1}^N \varepsilon^2(t)} < \sqrt{\sum_{t=1}^N y^2(t) + \sum_{t=1}^N \hat{y}^2(t)}$).

2.2 Interpretation in the frequency domain

Let us consider the asymptotic expressions of these criteria as N tends towards infinity, one obtains immediately

$$\lim_{N \rightarrow \infty} FIT = \left(1 - \frac{\sqrt{\int_{-\pi}^{\pi} \Phi_{\varepsilon}(\omega) d\omega}}{\sqrt{\int_{-\pi}^{\pi} \Phi_{y-\bar{y}}(\omega) d\omega}} \right) \cdot 100 \quad (3)$$

$$\lim_{N \rightarrow \infty} TRI = \left(1 - \frac{\sqrt{\int_{-\pi}^{\pi} \Phi_{\varepsilon}(\omega) d\omega}}{\sqrt{\int_{-\pi}^{\pi} \Phi_y(\omega) d\omega + \int_{-\pi}^{\pi} \Phi_{\hat{y}}(\omega) d\omega}} \right) \cdot 100 \quad (4)$$

where Φ_{ε} , $\Phi_{y-\bar{y}}$, are the Power spectrum Density (PSD) of $\{\varepsilon(t)\}$, $\{y - \bar{y}(t)\}$, respectively. Equations (3) and (4) imply a sum over the linear frequency scale ω .

If the system to be assessed is fast sampled, it will be necessarily represented in a Bode diagram with a logarithmic scale $\bar{\omega} = \log(\omega)$. In this case, the asymptotic expressions of FIT and TRI become

$$\lim_{N \rightarrow \infty} FIT = \left(1 - \frac{\sqrt{\int_{-\pi}^{\pi} \Phi_{\varepsilon}(e^{\bar{\omega}}) e^{\bar{\omega}} d\bar{\omega}}}{\sqrt{\int_{-\pi}^{\pi} \Phi_{y-\bar{y}}(e^{\bar{\omega}}) e^{\bar{\omega}} d\bar{\omega}}} \right) \cdot 100 \quad (5)$$

$$\lim_{N \rightarrow \infty} TRI = \left(1 - \frac{\sqrt{\int_{-\pi}^{\pi} \Phi_{\varepsilon}(e^{\bar{\omega}}) e^{\bar{\omega}} d\bar{\omega}}}{\sqrt{\int_{-\pi}^{\pi} \Phi_y(e^{\bar{\omega}}) e^{\bar{\omega}} d\bar{\omega} + \int_{-\pi}^{\pi} \Phi_{\hat{y}}(e^{\bar{\omega}}) e^{\bar{\omega}} d\bar{\omega}}} \right) \cdot 100 \quad (6)$$

and a weighting term $e^{\bar{\omega}}$ appears in the integral, showing that the low frequencies are severely underweighted in the criteria expressions. For this reason, the traditional expressions of FIT and TRI are not consistent with respect to the logarithmic frequency scale and are not always relevant to assess fast sampled systems.

2.3 Two basic examples

Example 1: Consider the continuous-time "true" system G_0 given by the following transfer function

$$G_0(s) = \frac{s^2}{s^2 + 2 \cdot 10^{-3}s + 10^{-6}} \frac{\frac{s^2}{\omega_1^2} + \frac{2s}{\omega_1} + 1}{\frac{s^2}{\omega_2^2} + \frac{0.01s}{\omega_2} + 1} \quad (7)$$

where $s \equiv \frac{d}{dt}$, and $\omega_1 = 0.0003 \text{ rad/s}$, $\omega_2 = 0.001 \text{ rad/s}$. Let us assume that the identification process yields a second order model G such that

$$G(s) = \frac{s^2}{s^2 + 2 \cdot 10^{-3}s + 10^{-6}} \quad (8)$$

One considers the discretized (with ZOH) counterparts of G_0 and G denoted as G_{0d} and G_d , with a sampling period $T_s = 1$. The Bode diagram of G_{0d} and G_d are displayed in Fig. 1

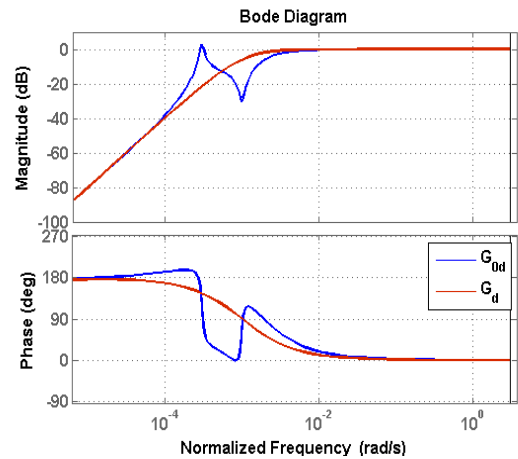


Fig. 1. Bode diagrams of G_{0d} and G_d (first example)

The mismatch between the two systems in low frequency is obvious if one observes the Bode diagram, however neither the FIT criterion nor the TRI criterion¹, which yield values very close to 100%, inform the user of a major matching default between these two systems (see Table 1).

¹ In this paper all the FIT and TRI results have been obtained for a centred and white system input

Table 1. Example 1: criteria values

Criterion	Result (%)
FIT	98.1
TRI	99.1

The reason is that the misfit occurs in low frequency where the weighting function e^{ω} is close to 0 and where the systems magnitude is not very large. To understand these misleading results one can also represent these two systems on the linear scale (the one on which is performed integration): the two systems are hardly distinguishable (fig. 2).

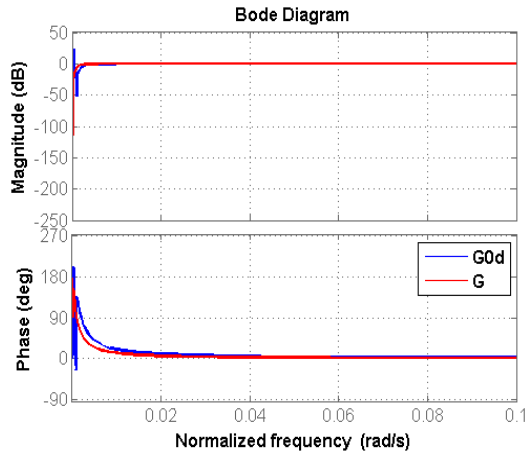


Fig. 2. Bode diagrams of G_{0d} and G_d (first example) on a linear scale

Example 2: The "true" system G_0 is now a model proposed as a benchmark in the model reduction community (Chahlaoui and Dooren (2002)), the system is sampled yielding G_{0d} , and the model G_d that we assume to be identified differs from G_{0d} only in high frequency, such that $G_d(q) = G_{0d}(q) \frac{1-0.4q^{-1}}{1} \frac{1-0.4q^{-1}}{1} \frac{1}{0.6^2}$, (q being the forward shift operator), see in Fig. 3

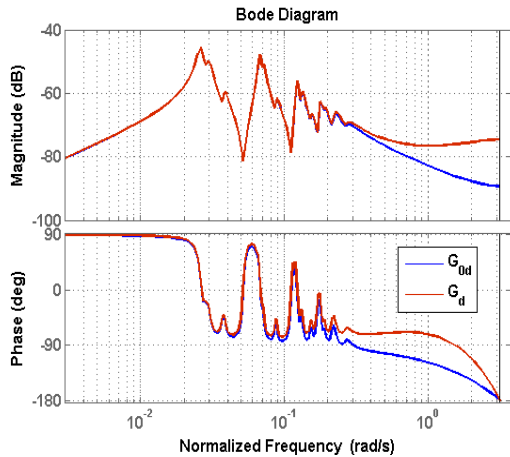


Fig. 3. Bode diagrams of G_{0d} and G_d (second example)

Even if the matching between the two systems is almost perfect at the frequencies of almost all poles and zeros of the system (between $2 \cdot 10^{-2}$ rad/s and $3 \cdot 10^{-1}$ rad/s) and

if the step responses of the two systems are hardly distinguishable as shown in Fig 4), the FIT criterion provides a value generally considered as poor (in a deterministic context): 59.8% (see Table 2)

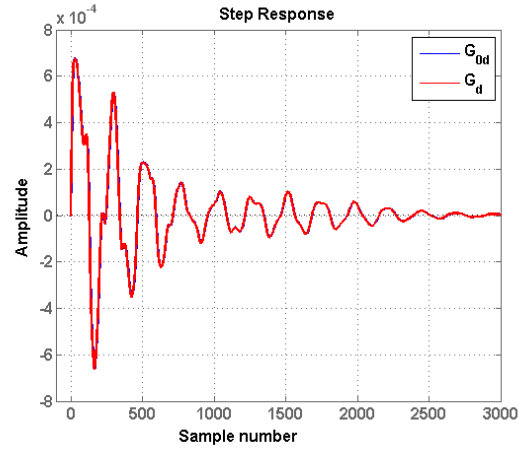


Fig. 4. Step responses G_{0d} and G_d (second example)

Table 2. Example 2: criteria values

Criterion	Result (%)
FIT	59.8
TRI	80.8

These two examples highlight the lack of reliability of the classical model performance criteria when applied to fast sampled systems. Similarly, it has been shown in (Vau and Bourlès (2022)) that the classical whiteness test of residuals may be not reliable for validating fast sampled models. In order to overcome this issue, an extension of the classical whiteness test established on GOBF has been proposed. In the sequel of the present paper we also propose to apply GOBF to the FIT and TRI criteria so as to make them more relevant in function of the expected use of the model.

3. GENERALIZED PERFORMANCE CRITERIA

3.1 Review on GOBF and the Hambo frequency

Before describing the novel criteria proposed here, it is necessary to recall some basic notions about GOBF and the Hambo transform. The generalized bases of orthogonal transfer function are generated from an all-pass transfer function (Blaschke product) $G_b(z) = \prod_{j=0}^{n_p-1} \frac{-z \cdot \bar{p}_j + 1}{z - p_j}$ where p_j ($|p_j| < 1$) are the basis poles and n_p the basis poles number. There exists an orthonormal state space realization such that $G_b(z) = D_b + C_b (zI - A_b)^{-1} B_b$, with (see chap. 2 of (Heuberger et al. (2005)))

$$\begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix} \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix}^T = I \quad (9)$$

The orthonormal transfer function vectors $V_k(z)$ with $k = 1, 2, \dots$ and of size $(n_p, 1)$ are given by the relation $V_k(z) = (zI - A_b)^{-1} B_b G_b^{k-1}(z)$. Orthonormality between these functions holds, because of the orthonormal state space realisation of $G_b(z)$. Particular configurations of n_p

and p_k correspond to well known cases: $n_p = 1, p_0 = 0$ is the classical z^{-1}, z^{-2}, \dots basis, and $n_p = 1, |p_0| < 1$ correspond to the Laguerre basis. The Hambo operator λ is given by $\lambda^{-1} = G_b(z)$. The mapping $\lambda \mapsto z$ is n_p valued in a domain including the unit circle, and the z_j $j = 1, \dots, n_p$ so that $\lambda^{-1} = G_b(z_j)$ are the eigenvalues of $\mathcal{N}(1/\lambda)$ with $\mathcal{N}(\lambda) = A_b + (\lambda - D_b)^{-1}C_b$.

A transform is associated to GOBF: the Hambo transform. Since G_b is all-pass, a function expressing the phase of $G_b(e^{i\omega})$, called the function β was introduced in Shipp et al. (1996). It can be associated to the operator λ , such that $\beta : [0, 2\pi[\rightarrow [0, 2\pi n_p]$ with $G_b(e^{i\omega}) = e^{-i\beta(\omega)}$ for all $\omega \in [0, 2\pi]$, and $\lambda^{-1} = e^{-i\beta(\omega)}$. The Hambo frequency ω_λ is by definition $\omega_\lambda = \beta(\omega)$, and it is shown in Shipp et al. (1996) (see also Heuberger et al. (2005) chap. 8) that

$$\omega_\lambda = \beta(\omega) = \sum_{k=0}^{n_p-1} \eta_k + 2 \arctan \left(\left(\frac{1 + \rho_k}{1 - \rho_k} \right) \tan \left(\frac{\omega - \eta_k}{2} \right) \right) \quad (10)$$

where $p_k = \rho_k e^{i\eta_k}$. The derivative of function β plays a great role: this is the distortion rate from ω to ω_λ , and it is shown in (Heuberger et al. (2005) (Chap.3)) that

$$\beta'(\omega) = \frac{d\omega}{d\omega_\lambda} = V_1^T(e^{i\omega})V_1(e^{-i\omega}) \quad (11)$$

moreover, $V_1^T(e^{i\omega})V_1(e^{-i\omega})$ is a particular expression of the reproducing kernel of the Hilbert space associated with the GOBF.

3.2 Expressions of the generalized indicators

At first, one selects n_p basis poles of GOBF (the effect of this selection will be discussed below), and one computes A_b and B_b . Secondly, all the signals entering in the expression of FIT and TRI are filtered by the first function of the basis namely V_1 with $V_1(z) = (zI - A_b)^{-1}B_b$ according to

$$x_f(t) = V_1(q)x'(t) \quad (12)$$

with $x'(t) = x(t)$ if $0 \leq t \leq N$ and $x'(t) = 0$ if $t > N$, where $x = y, \hat{y}, \varepsilon$. Note that $y_f(t), \hat{y}_f(t)$ and $\varepsilon_f(t)$ are column vectors of size n_p . The generalized indicators called H-FIT and H-TRI are defined as follows

H - FIT =

$$\lim_{N_h \rightarrow \infty} \left(1 - \frac{\sqrt{\sum_{t=1}^{N_h} \varepsilon_f^T(t)\varepsilon_f(t)}}{\sqrt{\sum_{t=1}^N (y_f(t) - \bar{y}_f)^T (y_f(t) - \bar{y}_f)}} \right) \cdot 100 \quad (13)$$

H - TRI =

$$\lim_{N_h \rightarrow \infty} \left(1 - \frac{\sqrt{\sum_{t=1}^{N_h} \varepsilon_f^T(t)\varepsilon_f(t)}}{\sqrt{\sum_{t=1}^N y_f^T(t)y_f(t) + \sum_{t=1}^N \hat{y}_f^T(t)\hat{y}_f(t)}} \right) \cdot 100$$

In practice H-FIT and H-TRI are computed by truncating the sums of (13) and (14) to a finite number of terms $N_h \gg N$. This approximation is valid since V_1 is necessarily stable in (12).

3.3 Interpretation of the filtering effect

Now, let us define the sequence $\{x_f(t)\}$ such that $x_f(t) = V_1(q)x(t)$. One has immediately

$$\lim_{N \rightarrow \infty} \sum_{t=1}^N x_f^T(t)x_f(t) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \Phi_x(\omega)V_1^T(e^{i\omega})V_1(e^{-i\omega})d\omega \quad (15)$$

and according to the function β in (10) and (11) one can write

$$\lim_{N \rightarrow \infty} \sum_{t=1}^N x_f^T(t)x_f(t) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \Phi_x(\omega)\beta'(\omega)d\omega = \frac{1}{2\pi} \int_{-\pi}^{\pi} \tilde{\Phi}_x(\omega_\lambda)d\omega_\lambda \quad (16)$$

where $\tilde{\Phi}_x(\omega_\lambda) = \Phi_x(\omega)|_{\omega=\beta^{-1}(\omega_\lambda)}$

Thus one verifies that the signals filtering by V_1 in the H-FIT and H-TRI expressions (13), (14), is equivalent to transpose the original criteria (3), (4) in the Hambo frequency scale. The function β for Laguerre bases ($n_p = 1$) has been provided in Oliveira (1995) and one displays it for various basis poles in Fig 5.

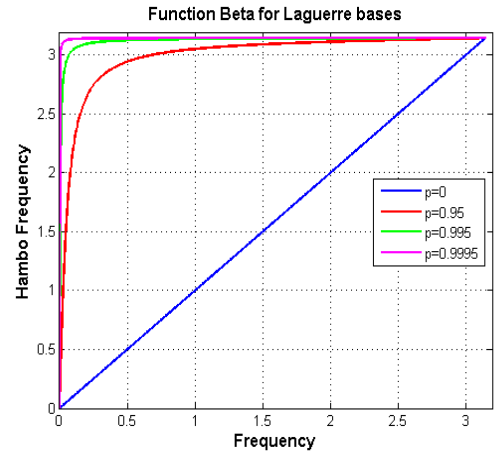


Fig. 5. Function β for various Laguerre bases

Let us go back to Example 1 of Subsection 2.3. One represents also in Fig. 6 $\Phi_y(\omega_\lambda)$ (over the Hambo frequency).

According to (3) and (4) the integration in FIT and TRI is performed over the linear frequency scale. In this scale, corresponding to $p = 0$, the system modes are almost invisible (see Fig. 6): that explains why FIT yields a so misleading value. As p tends towards 1, these modes appears more clearly in the diagram expressed over the Hambo (Laguerre) frequency scale, and accordingly

(14) $\tilde{\Phi}_x(\omega_\lambda)$ is not a power spectral density in the Hambo domain as defined in chap.3 of (Heuberger et al. (2005)) but its trace

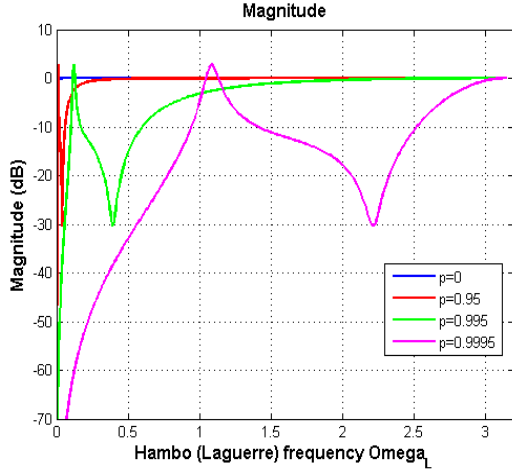


Fig. 6. $\tilde{\Phi}_y(\omega_\lambda)$ in the Hambo (Laguerre) frequency scale the values provided by H-FIT and H-TRI become more relevant.

Table 3. System of Example 1: criteria values of H-FIT and H-TRI for various Laguerre bases

Basis pole	p=0	p=0.95	p=0.995	p=0.9995
H-FIT (%)	98.1	88.2	62.0	10.6
H-TRI (%)	99.1	94.1	81.8	55.9

One can also employ a basis with more poles, for example $p = [0.5 \ 0.95 \ 0.995 \ 0.9995]$, and again one represents $\tilde{\Phi}(\omega_\lambda)$ in function of ω_λ in Fig. 7. The corresponding criteria results are displayed in Table 4, giving a more global appreciation of the model matching over a large band.

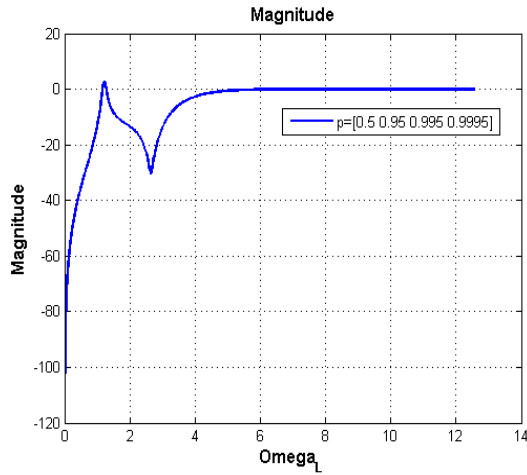


Fig. 7. $\tilde{\Phi}_y(\omega_\lambda)$ in the Hambo frequency scale for $p = [0.5 \ 0.95 \ 0.995 \ 0.9995]$

Table 4. System of Example 1: criteria values of H-FIT and H-TRI for a four basis poles (0.5 0.95 0.995 0.9995)

Basis poles	$p = [0.5 \ 0.954 \ 0.995 \ 0.9995]$
H-FIT (%)	79.7
H-TRI (%)	89.8

Since the fast sampled linear systems are usually represented over the logarithmic frequency scale necessarily

different from the Hambo scale on which is implicitly performed the integration in H-FIT and H-TRI, an indicator is necessary to characterize the weighting effect induced by the use of GBOF. This indicator is the distortion rate from the logarithmic frequency scale $\bar{\omega}$ to the Hambo frequency scale ω_λ . This distortion rate is known as the function χ and has been introduced in (Vau and Bourlès (2021)) with

$$\chi(\omega) = \frac{1}{\pi} \omega \beta'(\omega) = \frac{1}{\pi} \omega V_1^T(e^{i\omega}) V_1(e^{-i\omega}) \quad (17)$$

Note that the term $\frac{1}{\pi}$ has been introduced for a normalization purpose. One has also

$$\chi(e^{\bar{\omega}}) = \frac{1}{\pi} e^{\bar{\omega}} \sum_{j=0}^{n_p-1} \frac{1 - |p_j|^2}{|1 - \bar{p}_j e^{i e^{\bar{\omega}}}|^2} \quad (18)$$

Nice properties have been presented in (Vau and Bourlès (2021)):

- a conservation principle holds (which is a consequence of the orthonormality of the basis functions)

$$\int_{-\infty}^{\log(\pi)} \chi(e^{\bar{\omega}}) d\bar{\omega} = 1 \quad (19)$$

- If p_j is real and sufficiently close to the unit circle, the function $\bar{\omega} \mapsto e^{\bar{\omega}} \frac{1 - |p_j|^2}{|1 - \bar{p}_j e^{i e^{\bar{\omega}}}|^2}$ has a maximum at a frequency close to the pole frequency.

the proofs can be found in (Vau and Bourlès (2021)).

$\Phi(\omega)$ is represented in the top graph of Fig. 8, whereas the functions χ corresponding to the basis poles employed so far are displayed in the bottom graph. The conservation principle can be observed since the area below the curves in the bottom plot are the same.

Let us use now H-FIT and H-TRI to assess the system corresponding to the second example of Subsection 2.3. Since most of the poles and zeros of the system are located within a decade around $7.10^{-2} rad/s$, this suggests to propose a two poles basis ($n_p = 2$) with poles having a frequency equal to 7.10^{-2} , and a damping equal to 0.5, corresponding $p = 0.9638 \pm 0.0585i$. The corresponding function χ is displayed in the bottom plot of Fig. 9. The associated criteria values are provided in Table 5. These values are coherent with the model matching at this frequencies.

Table 5. System of Example 2: criteria values of H-FIT and H-TRI for a two poles basis

Basis pole	$p = 0.9638 \pm 0.0585i$
H-FIT (%)	91.5
H-TRI (%)	95.7

4. CONCLUDING REMARKS

The classical performance criteria for assessing identified models, such as FIT and TRI are well suited for discrete-time systems, if one expects to represent them on the usual linear frequency scale. These criteria may lead to a deceptive model evaluation for fast sampled systems. Some generalized criteria H-FIT and H-TRI established

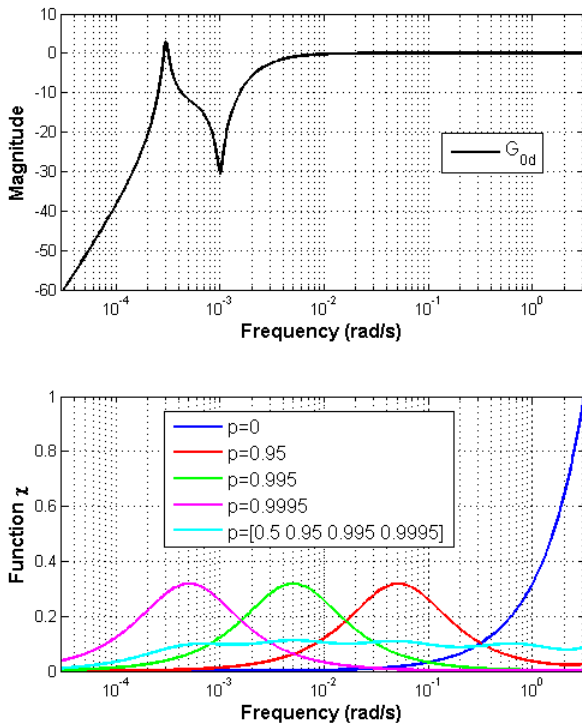


Fig. 8. System magnitude (example 1) (top), and functions χ for various basis poles

on Generalized Orthonormal Basis functions have been introduced so as to provide more relevant indicators. These novel criteria offer, *via* the basis poles selection, a flexibility regarding the frequency band on which the model is assessed. This band can be large, over several decades, in accordance with Bode Diagrams represented on a logarithmic frequency scale. The band can also be narrow, which is useful for the performance evaluation of reduced order models, or even very narrow to assess "almost punctual" models. Finally, we stress out that there cannot exist an universal performance indicator, and the criterion must necessarily be chosen in function of the expected use of the model. This is the strength of H-FIT and H-TRI to provide to the engineer or the researcher some tuning parameters to specify beforehand what they expect from this model.

REFERENCES

- Chahlaoui, Y. and Dooren, P.V. (2002). A collection of benchmark examples for model reduction of linear time invariant dynamical systems. *Slicot working note, on the internet*.
- Heuberger, P., den Hof, P.V., and Wahlberg(eds), B. (2005). *Modelling and Identification with rational orthonormal basis functions*. Springer, London.
- Ljung, L. (1999). *System Identification Theory for the user, second edition*. Prentice Hall, Upper Saddle River.
- Muroi, H. and Adachi, S. (2015). Model validation criteria for system identification in time domain. *17th IFAC Symposium on System Identification*.

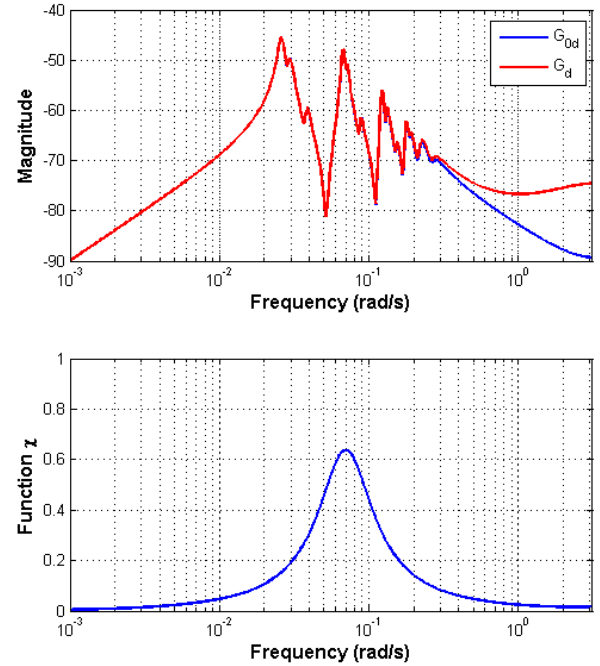


Fig. 9. System (example 2) magnitude (top), and function χ for $p = 0.9638 \pm 0.0585i$

- Oliveira, T. (1995). On the determination of optimal pole position of laguerre filters. *IEEE Trans. on signal processing*, 43, 2079–2087.
- Pascu, V., Garnier, H., Ljung, L., and Janot, A. (2019). Benchmark problems for continuous-time identification: design, aspects, results and perspectives. *Automatica*, 107, 511–517. doi:10.1016/2019.06.011.
- Pillonetto, G., Chen, T., Chiuso, A., Nicolao, G.D., and Ljung, L. (2022). *Regularized System Identification: Learning Dynamic Models from Data*. Springer. doi: 10.1007/978-3-030-95860-2.
- Söderström, T. and Stoica, P. (1989). *System Identification*. Prentice Hall, Upper Saddle River.
- Shipp, F., Gianone, L., Bokor, J., and Szabó, Z. (1996). Identification in generalized orthogonal basis- a frequency domain approach. *Proc. 13th IFAC World congress*.
- Vau, B. and Bourlès, H. (2021). Closed-loop output error identification algorithms with predictors based on generalized orthonormal transfer functions: Convergence conditions and bias distribution. *Automatica*, 125. doi: 10.1016/2020.109377.
- Vau, B. and Bourlès, H. (2022). A robust whiteness test for the identification of discrete-time linear models: use of orthonormal transfer functions. *Automatica*, 139. doi: 10.1016/2022.110174.